Vertical cross-spectral phases in neutral atmospheric flow

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Version of record first published: 20 Aug 2012


To link to this article: http://dx.doi.org/10.1080/14685248.2012.711524

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Vertical cross-spectral phases in neutral atmospheric flow

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(Received 11 April 2012; final version received 07 July 2012)

The cross-spectral phases between velocity components at two heights are analyzed from observations at the Høvsøre test site and from the field experiments under the Cooperative Atmosphere-Surface Exchange Study in 1999. These phases represent the degree to which turbulence sensed at one height leads (or lags) in time the turbulence sensed at the other height. The phase angle of the cross-wind component is observed to be significantly greater than the phase for the along-wind component, which in turn is greater than the phase for the vertical component. The cross-wind and along-wind phases increase with stream-wise wavenumber and vertical separation distance, but there is no significant change in the phase angle of vertical velocity, which remains close to zero. The phases are also calculated using a rapid distortion theory model and large-eddy simulation. The results from the models show similar order in phasing, but the slopes of the phase curves are slightly different from the observations, especially for low wavenumbers.

Keywords: cross-spectra; phases; atmospheric turbulence; wavenumber; vertical distance

1. Introduction

The structure of atmospheric turbulence can be analyzed in terms of two-point statistics such as normalized cross spectra (also known as coherences), which are typically studied both experimentally and theoretically as a function of horizontal separation distance for homogeneous turbulence in the atmospheric surface layer [1, 2]. The coherences of the along-wind, cross-wind, and vertical velocity components \((u, v, w)\) decrease with increasing separation distance, as seen from both observations and theory [2].

In this paper, we attempt to answer the research question, “how and why are the cross-spectral phases with a vertical separation different, for different velocity components in the neutral atmospheric boundary layer (ABL)?”

We investigate cross spectra with particular emphasis on the associated phases \(\phi\) for vertical separations \(\Delta z\), using observations at Høvsøre [3, 4] and from Cooperative Atmosphere-Surface Exchange Study in 1999 (CASES-99) [5, 6]. No investigation of the vertical phase angles for all three velocity components (i.e., \(\phi_u, \phi_v, \text{and} \phi_w\)), including their behavior in the ABL, has been noted in the literature. Mann [2] studied \(\phi_{vw}\) (the phase angle between \(v\) and \(w\)) for horizontal separations, and \(\phi_{uw}(\equiv \phi_u)\) and \(\phi_{uw}\) for vertical separations, where the \(w\)-component was measured further from the surface. Few experimental
investigations have been done on the phases. Heidrick et al. [7] experimentally studied the phases of the axial velocity component in fully developed pipe flow using measurements taken at two different points, where the separation vector was oriented at different angles to the mean flow. Komori et al. [8] studied the phase angle between the vertical velocity component and the temperature in stably stratified open-channel flow. Both Heidrick et al. [7] and Komori et al. [8] assumed turbulent motions approach as wavelike motions. The Sandia (Veers) method [9], which is used in wind engineering for load calculations on wind turbines, assumes an average of zero phase between any two points because of an exponential form of the coherence function as given in Ref. [10]. The Mann method [11], based on the Mann spectral tensor model [2] and widely used in wind engineering, does give non-zero phases.

In addition to the observations, we also evaluate the phase angles from the Mann spectral tensor model [2], which incorporates rapid distortion theory (RDT) [12,13], and from data generated by large-eddy simulation (LES) [14]. The phases are determined by calculating the two-point cross spectra of velocity components and corresponding spectra as defined in Section 1.2. The observations and the models used for the analysis are described in Section 2 and Section 3, respectively. The results from the observations and the models are given in Section 4. In Section 5, we discuss more details and the mechanism explaining the systematic behavior of the phases, followed by conclusions in Section 6.

1.1. Motivation

Mann [2] modeled the evolution of turbulence induced by uniform shear using RDT [12,13] in a neutral surface layer. Mann [11] used the model of [2] to develop a method to simulate the three-dimensional wind in the time domain. The model in [2] and the method in [11] are the industry standards for aeroelastic calculation of wind turbine loads [15]. Turbulence simulations from [11] show systematic behavior in \( u, v, \) and \( w \) fluctuations in the rotor plane of a horizontal axis wind turbine, and when used to predict the respective phase angles between two heights, we see that \( \phi_v > \phi_u > \phi_w \) for \( k_1 \Delta z \leq 1 \), where \( k_1 \) is a stream-wise wavenumber. We expect that this behavior in phasing is due to the vertical shear. In order to confirm that, we analyze observed and LES data in more detail.

Shear-induced turbulence may have an effect on wind turbine loading. Sathe et al. [4] showed that in the ABL under stably stratified conditions, where turbulence is suppressed, large wind gradients lead to increased fatigue loading on the turbine rotor. Due to the presence of the ground in situations where there is no flow reversal or flow separation, one expects that the wind approaches a given point \((x, y, z)\) faster at heights farther from the ground than at smaller \(z\) (closer to the surface), and so the turbulence sensed at the higher point “leads” in time that sensed at the lower point for the same \((x, y)\).

The sketch given in Figure 1 illustrates the deformation of an “eddy” by uniform shear. The eddy hits the turbine plane first at point \(a\), then at \(b\). With increasing wind turbine diameter, the bending moments due to the vertical shear become more prominent. The combination of fatigue loads on wind turbines, due both to wind shear and shear-induced turbulence – with its ability to induce significant coherent phase differences across the vertical extent of a turbine rotor – motivates investigation of the phases in more detail. However, the actual consequences for loads will not be investigated here.

1.2. Definitions

The phases are calculated from complex cross spectra. The cross spectrum between velocity components \(u_i(t)\) \((i = 1, 2, 3)\) and \(u_j(t)\) \((j = 1, 2, 3)\) at heights \(z_1\) and \(z_2\), respectively, is
Figure 1. Sketch of the eddy stretching due to the shear. The turbulence sensed at point \( a \) leads in phase with respect to the turbulence sensed at point \( b \) in the rotor plane of a horizontal axis turbine.

defined as

\[
\chi_{ij}(f, \Delta z) = \langle \hat{u}_i(f, z_1)\hat{u}_j^*(f, z_2) \rangle, \tag{1}
\]

where \( f \) is frequency, \( \Delta z = z_2 - z_1 \), \( \langle \rangle \) denotes ensemble averaging, \( * \) denotes complex conjugate, and \( \hat{u}_i(f, z_1) \) is the complex-valued Fourier transform of the \( i \)th velocity component \( u_i(t) \) at height \( z_1 \). The phase between the two velocity components is then

\[
\varphi_{ij}(f, \Delta z) = \arg(\chi_{ij}(f, \Delta z)). \tag{2}
\]

The coherences known as “squared coherences” \([16]\) are frequently used in wind engineering \([17]\) and calculated from the cross spectra and the single-point power spectra \([18]\) via

\[
\text{coh}_{ij}(f, \Delta z) = \frac{|\chi_{ij}(f, \Delta z)|^2}{F_i(f, z_1)F_j(f, z_2)}, \tag{3}
\]

where \( F_i(f, z) = \langle \hat{u}_i(f)\hat{u}_i^*(f) \rangle \) is the single-point power spectrum of the \( i \)th velocity component \( u_i(t) \) at height \( z \).

If we assume that Taylor’s hypothesis of “frozen turbulence” is valid, then the measured time series can be related to spatial fluctuations. So for the stream-wise direction,
single-point measurements can be related through $k_1 = 2\pi f / U$, where $U$ is the stream-wise mean wind speed.

2. Observations

Two different datasets are used to investigate the vertical cross-spectral phases: the Høvsøre test site in Denmark and the CASES-99 field experiment. Both provide a unique opportunity to investigate the phase angles between wind components as a function of the vertical separation distance and the distance from the ground.

2.1. Høvsøre

The measurements are taken from the 116.5 m tall mast at the Høvsøre test site on the west coast of Denmark. Sonic anemometers, sampling at 20 Hz and measuring in three dimensions, are installed on the mast at heights of 10, 20, 40, 60, 80, and 100 m. The land to the east of the mast can be considered as flat, homogeneous terrain. On the west side of the mast, land extends 1500 m to the North Sea coast, including a dune which can affect the flow. Five wind turbines are situated to the north of the mast. More details about the location and instrumentation can be found in Refs. [3, 4].

To avoid the effects from the wind turbine wakes and focus on flow over essentially uniform terrain, winds are selected from directions between 60° and 120°, and the data limited to when the 80 m mean wind speeds fall between 8 and 9 m s$^{-1}$. The calculations are done for data corresponding to neutral stability conditions, i.e., when the Obukhov length $L_o$ measured at $z = 10$ m is $|L_o| \geq 500$ m. The height interval chosen in the phase analysis spans 40 – 100 m. Analysis is done using seven years of data from 2004 to 2010.

2.2. CASES-99

The CASES-99 was conducted over relatively flat grassland near Leon, Kansas, US during October 1999. Mean and fluctuating wind components in three dimensions were sampled at 20 Hz from sonic anemometers at six levels on a 60 m scaffolding tower. Although data were collected throughout the diurnal cycle, CASES-99 was primarily focused on the stable, nocturnal boundary layer, including transition periods. Poulos et al. and Sun et al. [5, 6, 19] described the experiment and discussed some of the results. Here, we use the observations from 40 m and 55 m on the night of 17 October, which had the maximum nighttime wind; for this case, the mean wind averaged over these two heights was $\sim 12$ m s$^{-1}$ from the north and $|L_o| \geq 200$ m.

3. Modeling

Two different models are used to predict and analyze the phases in comparison with the observations: the Mann spectral velocity tensor model and National Center for Atmospheric Research’s LES model.

3.1. Spectral tensor model

The Mann spectral velocity tensor model incorporates RDT [12, 13] with an assumption of a mean uniform shear, plus a wavenumber-dependent eddy lifetime, to estimate the structure of turbulence over uniform flat terrain, which has been extended to cover gently varying
orography [20]. The model calculates the evolution of turbulence in Fourier modes from an initial isotropic state, the energy spectrum of which is given by the von Kármán form [21].

The Mann model contains three adjustable parameters:

- A length scale $L$ describing the size of energy-containing eddies.
- A non-dimensional anisotropy parameter $\Gamma$ used in the parameterization of the eddy lifetime.
- A measure of the energy dissipation $\alpha \epsilon^{2/3}$, where the Kolmogorov constant $\alpha = 1.7$ and $\epsilon$ is the rate of viscous dissipation of specific turbulent kinetic energy.

The analytical form of the spectral velocity tensor in [2] is a function of these three parameters and can be expressed as $\Phi_{ij}(k, L, \Gamma, \alpha \epsilon^{2/3})$, where $k = (k_1, k_2, k_3)$ is the three-dimensional wave vector. The modeled cross spectra, which also become functions of the three parameters, are given as

$$\chi_{ij}(k_1, L, \Gamma, \alpha \epsilon^{2/3}, \Delta y, \Delta z) = \int \Phi_{ij}(k, L, \Gamma, \alpha \epsilon^{2/3}) \exp(i(k_2 \Delta y + k_3 \Delta z)) dk_\perp, \quad (4)$$

where $\int dk_\perp \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_2 dk_3$ and $\Delta y$ is the transverse separation distance. The three parameters are determined by fitting model single-point power spectra $F_i(k_1, L, \Gamma, \alpha \epsilon^{2/3}) = \chi_{ii}(k_1, L, \Gamma, \alpha \epsilon^{2/3}, 0, 0)$ (no summation) to the measured single-point power spectra through chi-squared fitting as given in Ref. [2].

Figure 2 gives an example of a model fit of power spectra to the Høvsøre data at 100 m height illustrating extraction of $L$, $\Gamma$, and $\alpha \epsilon^{2/3}$. One hundred seventy-six 30 min time series are used to calculate spectra and co-spectra from the measurements. The three parameters are subsequently used as an input to calculate numerically the cross spectrum between any two velocity components through Equation (4). Thus, for vertical separations ($\Delta y = 0$), the model cross spectra and phases are expressed as $\chi_{ij}(k_1, L, \Gamma, \alpha \epsilon^{2/3}, \Delta z)$ and $\varphi_{ij}(k_1, L, \Gamma, \Delta z)$, respectively. The model phases are unaffected by $\epsilon$.

The distortion of the wave vector due to shear $dU/dz$ is given by $k(t) = (k_1, k_2, k_30 - k_1(dU/dz)t)$, with the initial wave vector $k_0 = (k_1, k_2, k_30)$. The model assumes a uniform shear, so $dU/dz$ is constant with height, which is an approximation, but we do not expect that a non-zero $d^2U/dz^2$ would significantly alter the results. In addition to the uniform shear, the vertically inhomogeneous effect of blocking due to the surface (e.g., ground) was included in [2]; however, it does not produce significantly different results. Nevertheless, as discussed above, $\chi_{ij}$, $F_i$, and $\varphi_{ij}$ are functions of $L$ that itself depends on the distance $z$ from the ground. In this way, the model treats vertical inhomogeneity in application.

### 3.2. LES model

The pseudospectral LES code of Sullivan and Patton [14] simulates the ABL over flat, homogeneous terrain, with high temporal and spatial resolution. A database containing LES results from this code has been previously established, for different ABL states and surface conditions; here we use LES results for a neutral ABL, consisting of 1000 instantaneous volumes (snapshots) saved every two time steps. The domain size for the analyzed neutral case is $2400 \times 2400 \times 1000 \, m^3$, with horizontal resolution $\delta x = \delta y = 4 \, m$ and vertical resolution $\delta z = 2.5 \, m$. A geostrophic wind of $u_g = 5 \, m \, s^{-1}$ is imposed, and the time step, which is determined based on a CFL (due to Courant–Friedrichs–Lewy condition) number
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Figure 2. Example of the model fit of single-point spectra to the Høvsøre data at 100 m height to determine the three parameters in the Mann model [2]. One hundred seventy-six 30 min time series are used to calculate spectra from the measurements at Høvsøre. Measurements: $u$-spectrum; --*, $v$-spectrum; --□, $w$-spectrum; -- ○, co-spectrum of $u$ and $w$; --○-. Model spectra and co-spectrum: $u$; ---*, $v$; -----, $w$; -----, $uw$; · · ·.

of 0.75, is nearly constant at $\sim 0.7$ s. The simulation meets the “high-accuracy zone” LES criteria established by Brasseur and Wei [22], with essentially resolution-independent results.

The surface is characterized by a roughness of $z_0 = 0.3$ cm and a virtual potential temperature of 300 K, with zero mean surface heat flux prescribed and the surface momentum fluxes dictated by $z_0$. A uniform virtual potential temperature is initially imposed up to the top of the boundary layer, which is capped by a virtual potential temperature inversion having $dT/dz = 0.003$ K m$^{-1}$. The boundary layer “top” can thus act dynamically and its structure varies, with a diagnosed mean boundary layer depth $z_i$ of 616 m. The “snapshots” used for the analysis correspond to simulation times long after the initial “spin-up” of the code ($\sim 1$ h, much larger than the rough ABL timescale $\sim z_i/u_g$ [23] and larger than the large-eddy turnover timescale $z_i/u_*$, where $u_*$ is the friction velocity). The (cross) spectra are calculated for each instantaneous snapshot in horizontal planes at two given heights and then averaged over all snapshots. Due to horizontal homogeneity, statistics are constant over the horizontal plane.

In the next section, the results from the observations and the models are provided, followed by discussion in Section 5.

4. Results

The phases from the Høvsøre observations are shown in Figure 3(a) and the coherences in Figure 3(b), along with the predictions from the Mann model. As described in Section 3.1, the three adjustable parameters in the model are determined by fitting the one-dimensional power spectra of the model to that from the data at heights 40 and 100 m (see Figure 2).
The phases (a) and the coherences (b) between 40 m and 100 m at Høvsøre for a neutral ABL with the predictions from the Mann model \[2\]. Phase angles from the measurements: \(\varphi_u\); – □ –, \(\varphi_v\); – □ –, \(\varphi_w\); – ○ –. Model phases: \(\varphi_u\); --- , \(\varphi_v\); – – – , \(\varphi_w\); — — . Similar notations are followed for the coherences.

The average of the parameters at the two heights is used to calculate the model cross spectra. The slopes of the phase curves predicted by the model are different than those calculated from the measurements. However, the model is able to predict the order in phasing, \(\varphi_v > \varphi_u > \varphi_w\), for \(k_1 \Delta z \leq 1\).

The model overestimates the \(u\)-, \(v\)-, and \(w\)-coherence for \(k_1 \Delta z \leq 1\). So at a given length scale, the fluctuations at two corresponding heights in the modeled coherent eddies are more correlated than those from the observation. It is also observed that the modeled phases are smaller than the phase angles from the measurements.

The phase angles from the CASES-99 are shown in Figure 4(a) between heights 40 and 55 m, along with the predictions from the Mann model. The Mann parameters are determined in the same way as described for the Høvsøre case. The same order in phasing

Figure 3. The phases (a) and the coherences (b) between 40 m and 100 m at Høvsøre for a neutral ABL with the predictions from the Mann model \[2\]. Phase angles from the measurements: \(\varphi_u\); – □ –, \(\varphi_v\); – □ –, \(\varphi_w\); – ○ –. Model phases: \(\varphi_u\); --- , \(\varphi_v\); – – – , \(\varphi_w\); — — . Similar notations are followed for the coherences.

Figure 4. A comparison of phases from (a) CASES-99 and (b) LES for a neutral ABL with the phases from the Mann uniform shear model. Mann model: \(\varphi_u\); --- , \(\varphi_v\); – – – , \(\varphi_w\); — — . CASES-99 and LES: \(\varphi_u\); – □ –, \(\varphi_v\); – □ –, \(\varphi_w\); – ○ –.
Table 1. The three parameters in the Mann model obtained from single-point power spectra from Høvsøre, CASES-99, and LES data, via chi-squared fits [2]. The averages of the parameters obtained at given heights $z_1$ and $z_2$ are provided. Refer to Figure 2 for an example.

<table>
<thead>
<tr>
<th>Data</th>
<th>$z_1$ [m]</th>
<th>$z_2$ [m]</th>
<th>$\Gamma$</th>
<th>$L$ [m]</th>
<th>$\alpha \epsilon^{2/3}$ [m${}^{4/3}$s${}^{-2}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Høvsøre</td>
<td>40</td>
<td>100</td>
<td>3.3</td>
<td>40</td>
<td>0.044</td>
</tr>
<tr>
<td>CASES-99</td>
<td>40</td>
<td>55</td>
<td>3.5</td>
<td>85</td>
<td>0.008</td>
</tr>
<tr>
<td>LES</td>
<td>50</td>
<td>100</td>
<td>2.9</td>
<td>52</td>
<td>0.028</td>
</tr>
</tbody>
</table>

is seen in the CASES-99. The phase $\varphi_w$ is close to zero, which can also be seen at Høvsøre. The Mann model underestimates $\varphi_u$ and $\varphi_v$, as compared to the observations at both sites.

Figure 4(b) shows the phases between heights 50 and 100 m from the LES of a neutral ABL. The LES spectra are fitted with the Mann model and the model parameters are obtained to predict the phases. We also observe $\varphi_u > \varphi_v > \varphi_w$.

The three adjustable parameters in the Mann model obtained from the Høvsøre, CASES-99 and LES are provided in Table 1.

5. Discussion
In this section, we describe some more details about the behavior of the phases. As we have seen in Section 4, RDT is capable of explaining the fact that $\varphi_v > \varphi_u > \varphi_w$. However, it is not entirely clear why this ordering is inevitable. In this section, the inequality is supported with simple physical and geometrical arguments.

Thirty-minute time series are used for the analysis of the Høvsøre data. For the neutral ABL, we obtain an ensemble of $n = 176$ realizations. As per the definition of the ensemble average, we require an infinite number of realizations in order to obtain (cross) spectra as defined in Section 1.2. So, due to the limited $n$, there is uncertainty in the estimated (cross) spectra and hence in the corresponding coherences and phases. Kristensen and Kirkegaard [24] showed that the estimated coherence is systematically overestimated. Letting the true coherence to be denoted as coh, the estimated coherence $\langle \text{coh} \rangle$ is given due to [24] by

$$\langle \text{coh} \rangle = \alpha_1$$

with

$$\alpha_1 = 1 - \frac{n - 1}{n} (1 - \text{coh})^n \text{$_2F_1$(n, n; n + 1; coh)},$$

where $_2F_1$ is the hypergeometric function. We see that $n$ is large enough to give almost insignificant overestimation of the coherences. For example, for $n = 176$ we get $\langle \text{coh}_{176} \rangle = 0.5014$ for coh = 0.5 and $\langle \text{coh}_{176} \rangle = 0.0155$ for coh = 0.01 from Equations (5) and (6).

For winds from the west (i.e., from the North Sea), the flow is essentially inhomogeneous, so we might expect the phase angles to be different. But, quite surprisingly, when we examine the phase angles for winds selected between 240° and 300°, we find the same difference and ordering of the phases with slightly greater values. Also when we analyze the data for combinations of heights other than 40 and 100 m, we observe three things: first, with increasing $\Delta z$, both $\varphi_u$ and $\varphi_v$ increase for $k_1 \leq 0.01$, with no significant change in $\varphi_w$;
second, for a given $\Delta z$, moving further from the surface we see a slight decrease in $\varphi_u$ and $\varphi_v$ but no systematic effect upon $\varphi_w$; and third, we still get the same order in phasing, i.e., $\varphi_v > \varphi_u > \varphi_w$ (irrespective of the values of $z_1$ and $z_2$, as long as the coherence is non-zero). When we analyze the phases for mean wind speed intervals greater than 8–9 m s$^{-1}$ (corresponding to 80 m height), the phases are insignificantly affected, with the same difference and ordering. In addition to the phases being almost unaffected by the mean wind speed, they show the same trend as in the two cases described above. In this regard, if we observe Figures 3(a) and 4(a), the phases from CASES-99 are smaller than those from Høvsøre (as is $\Delta z$).

Sathe et al. [4] showed the variation of the three parameters specifying the spectra as a function of mean wind speed at different atmospheric stabilities. For the neutral case, there is no significant variation in $L$ or $\Gamma$ with the mean wind speed between 3 and 16 m s$^{-1}$, but the $\alpha \epsilon^{2/3}$ parameter varies significantly as expected. Interestingly, as we discussed in Section 3.1, the model phases are functions of $L$ and $\Gamma$ but not $\alpha \epsilon^{2/3}$. With increasing $\Gamma$, which represents the degree of turbulence anisotropy, phase angles of all three velocity components increase and the phase curves shift upward. From Ref. [4], for the variations of $L$ and $\Gamma$ with mean wind speed, the standard deviations are $\sim 5.2$ m and $\sim 0.11$, respectively, which has no appreciable effect upon the model phases. This aspect of the model prediction is also consistent with the fact that the phases at Høvsøre are not dependent on the mean wind speed over the intervals described in the previous paragraph. In this regard, if we observe Figures 3(a) and 4(b), we see that the LES phases compare well with those from Høvsøre, although the average of the mean wind speeds at 50 and 100 m in the LES is $\sim 4$ m s$^{-1}$. The vertical variation of the three parameters is described in Ref. [25].

As observed from the results in Section 4, modeled $\varphi_u$ and $\varphi_v$ are smaller than those from the measurements; we do not know the exact reason for this, but it could be due to inhomogeneity and as discussed in Section 3.1, the $L$ parameter roughly follows $z$ but does not strictly represent $z$.

The LES results shown here are based on spatial calculations. When the phases are calculated using simulated time series (i.e., in time-frequency domain) and using Taylor’s hypothesis, we obtain a different result, as shown in Figure 5. We do not know the precise reason behind this, but what is noticeable from Figure 5 is that the temporal-$u$-phase is systematically smaller than the spatial-$u$-phase and the temporal-$v$-phase is greater than the spatial-$v$-phase.

![Figure 5](image-url)  
**Figure 5.** The LES phases based on time- and space-domain calculations. The temporal phases: $\varphi_u$; $\varphi_v$; $\varphi_w$. The spatial phases: $\varphi_u$; $\varphi_v$; $\varphi_w$.  

spatial-$v$-phase. The differences between the time- and space-domain calculations likely result from the limited domain size, applicability of Taylor’s hypothesis, and the effect of the LES code zeroing the mean vertical velocity over the horizontal plane at each height and time step. In this connection, we suggest that two pulsed lidars mounted at two different heights on a meteorological mast, staring upwind and measuring at many range gates simultaneously, could shed light on the differences between the temporal and spatial spectral phases. Several such lidars have been deployed in the field [26] but not in this configuration.

5.1. Mechanism

The three-dimensional velocity field $\mathbf{u}(x)$ is decomposed into Fourier modes, so the entire field can be written as a sum (or more precisely an integral) of terms of the form $\mathbf{u}(k) \exp(-i k \cdot x)$, where $\mathbf{u}(k)$ is a complex vector, the Fourier amplitude. According to Equation (4), all three-dimensional Fourier modes $\mathbf{u}(k)$ with a particular value of $k_1$ contribute to the cross spectrum at that one-dimensional wavenumber. In the following, we consider qualitatively all these contributions to the cross spectrum and how they change under the action of a shear $dU/dz$.

Because of incompressibility, $\nabla \cdot \mathbf{u}(x) = 0$, the velocity amplitudes are perpendicular to the wave vectors: $k \cdot \mathbf{u}(k) = 0$. Furthermore, since the Fourier amplitude of the vorticity $\omega(k) = -i k \times \mathbf{u}(k)$, then $k$, $\mathbf{u}(k)$, and $\omega(k)$ are mutually perpendicular. In RDT, the vorticity equation is

$$\frac{D}{Dt} \omega(k) = \omega(k) \cdot \nabla \mathbf{u}(k), \tag{7}$$

where $k(t) = (k_1, k_2, k_3) - k_1 (dU/dz) t$ as mentioned in Section 3.1.

In Figure 6, Fourier modes are illustrated, with wavefronts in the vertical $x$–$z$ plane for some lagging, leading, and normal modes before and after shearing (Figures 6(a) and 6(d), respectively). The corresponding wave vectors are shown in Figures 6(b) and 6(e). All modes shown have a fixed horizontal wavenumber $k_1$, but for some the fluctuations lead at height $z_2$ relative to $z_1$, some have approximately zero phase difference, while for others they lag. The corresponding vorticity $\omega(k)$ perpendicular to $k$ is shown in Figures 6(c) and 6(f). In isotropic turbulence, leading and lagging modes are equally energetic, so the resulting phase is zero. Introducing a linear shear as in [2] changes this situation. Here, RDT (Equation (7)) predicts that modes with vorticity roughly aligned with the principal axis of strain become more energetic, while modes with vorticity aligned with the principal axis of compression are suppressed. The consequences for the phase are illustrated by looking at lagging, neutral, and leading modes corresponding, respectively, to red, green, and blue in Figure 6.

For the $u$-component, the zero phase mode would typically have the most energy $\langle |\mathbf{u}(k)|^2 \rangle$ due to the shape of the energy spectrum, because $k = |k|$ is smallest for all Fourier modes contributing to the cross spectrum with horizontal wavenumber $k_1$; but the energy in the $u$-component is zero because $\mathbf{u}(k)$ is perpendicular to $k$. After some time, the shear will tilt $\mathbf{u}(k)$ (if it has a vertical component) and some energy will be transferred to the $u$-component, introducing a leading phase. The phase-lagging mode provides the most energy to the $u$-component, when $\mathbf{u}(k)$ is in the $x$–$z$ plane (in the plane of the figure). That implies that the vorticity $\omega(k)$ is mainly perpendicular to the paper plane, which implies that the mode is neither suppressed nor enhanced, since there is no vortex stretching. The
same could be said about the leading mode, and we conclude that only because the most energetic modes are tilted forward, we expect a slightly leading phase of the $u$-component.

Modes with a lot of $v$ energy have $\mathbf{u}(k)$ pointing mainly perpendicular to the paper plane. That implies that $\omega(k)$ will be roughly aligned with the plane. For the mode with lagging phase (red, in Figure 6), the vorticity is then compressed leading to reduced energy, conversely for the leading modes. Here, the vorticity is stretched implying amplified fluctuations and the zero phase is also tilting as in the $u$-component case discussed above. In conclusion, distortion by the mean shear and tilting both enhance leading modes.

The zero phase mode has a lot of $w$-energy, but the vorticity is mainly perpendicular to the paper plane, so no amplification occurs. As $\mathbf{u}(k)$ tilts forward, energy is transferred away from the $w$-component and into the $u$-component, so phase shifting for $w$ is suppressed. For the leading mode, which is blue in Figure 6, vorticity is still mainly perpendicular to the paper plane, so no amplification, but $w$ will decrease because of the tilting of $\mathbf{u}(k)$. For the lagging phase, the tilting increases $w$ at the expense of $u$. Summing up the various contributions, we must conclude that the $w$-phase must be very small or even negative.

With these qualitative arguments, one can intuitively deduce the inequality $\varphi_v > \varphi_u > \varphi_w$.

6. Conclusions

The main goal of this study is to understand how and why the vertical cross-spectral phases in the neutral ABL behave as observed. Phases of the cross spectra of all three velocity components show systematic behavior: $\varphi_v > \varphi_u > \varphi_w$ for $k_1 \Delta z \leq 1$. $\varphi_u$ and $\varphi_v$ tend to increase with $k_1 \Delta z$, but $\varphi_w$ remains close to zero. We expect that this behavior is due to
the vertical shear, and we show that this is consistent with simple physical and geometrical arguments. RDT and LES both predict the observed phase ordering.

Acknowledgments
This study is a part of the Ph.D. project funded by Siemens Wind Power A/S and WindScanner.dk, which is funded by the Danish Agency for Science. We are also obliged to the COMWIND project funded by Danish Council of Strategic Research (DSF-contract: 09-067216). The National Center for Atmospheric Research is sponsored by the National Science Foundation.

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