Linear theory of the sea breeze in a thermal wind

Philippe Drobinski,^{a*} Richard Rotunno^{b†} and Thomas Dubos^a

^a*Institut Pierre Simon Laplace/Laboratoire de Météorologie Dynamique, Palaiseau, France*

^b*National Center for Atmospheric Research, Boulder, Colorado, USA*

*Correspondence to: P. Drobinski, IPSL/LMD, Ecole Polytechnique, Palaiseau 91128, France.

E-mail: philippe.drobinski@lmd.polytechnique.fr

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This article investigates the linear dynamics of the sea breeze in an along-shore thermal wind shear. The present analysis shows that the sea-breeze circulation is tilted towards the slanted isentropes associated with the thermal wind. At a critical value of the thermal wind shear, the tilt of the sea-breeze circulation becomes equal to the slope of the background isentropes. The present analysis also shows a spatial shift between the heating pattern and the sea-breeze circulation. The present linear theory is then applied to interpret measurements made in the vicinity of New York City where there is a warm-season synoptic southwesterly jet. It is compared with observations and past numerical simulations. Agreement is found with respect to the enhanced along-coast wind that follows the tilted isentropes, the order of magnitude of the isentropes tilt and the clockwise rotating wind hodograph showing the jet maximum peaking at 1800 solar time. There is a disagreement between theory and observations on the phase lag between the jet maximum and the cross-shore pressure gradient maximum. However, this disagreement can reasonably be attributed to either the angle made by the synoptic jet to the coastline and/or the presence of friction. The inland spatial shift of the breeze indicated by the theory might also be indirectly confirmed by the coastal inland wind observations of a larger diurnal amplitude for a stronger synoptic jet. Copyright © 2011 Royal Meteorological Society

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1. Introduction

The mesoscale structure of the sea-breeze circulation has in the past been idealized as the response of a rotating, stratified fluid to a diurnally varying differential surface heating (e.g. Walsh, 1974; Rotunno, 1983; Niino, 1987; Dalu and Pielke, 1989; Baldi *et al.*, 2008; Qian *et al.*, 2009; Drobinski and Dubos, 2009). These studies have been useful for the extraction of some basic qualitative features of the theoretical sea-breeze circulation, such as its aspect ratio, phase relative

to the diurnal heating cycle, and modification by friction. An aspect that has received relatively less attention is the effect of the summertime land–sea temperature contrast (thermal wind) on the diurnally varying sea-breeze circulation. In this article we report on a simple extension to the theory of Rotunno (1983; R83 hereafter) to include a base-state thermal wind.

The effect of an onshore/offshore background wind on the dynamics of the sea breeze has been heavily investigated through numerical simulation (e.g. Estoque, 1962; Walsh,

1974; Pielke 1974; Bastin and Drobinski, 2006; Bastin *et al.*, 2006) and observations (e.g. Bastin *et al.*, 2005, 2006; Drobinski *et al.*, 2006, 2007) and only recently through analytical techniques (Qian *et al.*, 2009). The effect of an along-shore thermal wind has received comparatively little attention. Burk and Thompson (1996) modelled the summertime thermally balanced coastal jet on the west coast of the USA. They found that the diurnal sea-breeze circulation is superimposed on the summertime northerly coastal jet and is strongly influenced by elevated and variable coastal terrain as well as relatively cold coastal sea-surface temperatures. Recently a similar phenomenon has been identified on the east coast of the USA (although without the strong topographic effects). Colle and Novak (2010; CN10 hereafter) find that there is a warm-season southerly jet in the coastal regions of New York and New Jersey that often displays a strong diurnal modulation. These jet winds are vertically confined to the boundary layer (< 300 m) and display a seasonal maximum occurrence in June–July suggesting that summertime land–sea temperature contrast is important to the jet occurrence. The jet events exhibit a diurnal cycle, rotating in a clockwise sense on a wind hodograph. However, compared to CN10 observations, the effect is much more obvious along the US west coast where the northerly jet and depressed marine layer is a quasi-permanent feature of the summertime flow. The coastline is straighter as well.

In the present article we present a simple theoretical description of the above-described phenomena through a modification of the theory for the two-dimensional sea-breeze circulation proposed in R83. Using a simple co-ordinate transformation to geostrophic coordinates (Hoskins and Bretherton 1972), the solutions found in R83 can, for the simple case of spatially constant along-coast wind shear, be used directly. Upon transformation of that solution back to fixed Cartesian coordinates, the effects of constant along-coast shear on the sea breeze can be easily deduced.

In section 2 we introduce the linear model, coordinate transform and solution with its salient features described. Section 3 illustrates the solution for prescribed heating. In section 4 we compare the latter features with the modelled and observed features of the sea-breeze circulations embedded in the coastal jets of the west coast of the USA. Section 5 concludes the study.

2. The linear model

2.1. Equations of motion

Consider the Cartesian coordinate system shown in Figure 1 with the coastline at the origin and with the dark and light shadings representing sea and land, respectively. For the purposes of the present simple exposition, we consider the horizontally uniform, constant shear, along-coast jet

$$V = V_0 - \Lambda z, \quad (1)$$

where V_0 is a constant and Λ is the vertical wind shear. Assuming thermal-wind balance we have

$$f \frac{\partial V}{\partial z} = \frac{\partial B}{\partial x}, \quad (2)$$

where the base-state buoyancy $B(x, z) \equiv g\Theta/\Theta_0$, g is the gravitational acceleration, $\Theta(x, z)$ is the base-state

Figure 1. The dark and light shadings represent the cooler sea and the warmer land, respectively. The thermal wind is illustrated by the arrows indicating decreasing southerly wind with height.

potential temperature and Θ_0 is a constant reference value. Substituting Eq. (1) into Eq. (2) we have

$$\frac{\partial B}{\partial x} = -f\Lambda, \quad (3)$$

so that

$$B = N^2 z - f\Lambda x + B(0, 0), \quad (4)$$

where N is the Brunt frequency (assumed constant) and $B(0, 0)$ is a constant.

The equations of motion linearized about the base-state wind $V(z)$ are

$$\left. \begin{aligned} \frac{\partial u}{\partial t} - fv &= -\frac{\partial \phi}{\partial x} + F_u, \\ \frac{\partial v}{\partial t} - \Lambda w + fu &= F_v, \\ \frac{\partial w}{\partial t} - b &= -\frac{\partial \phi}{\partial z} + F_w, \\ \frac{\partial b}{\partial t} + N^2 w - f\Lambda u &= Q + F_b, \\ \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} &= 0, \end{aligned} \right\} \quad (5)$$

where (u, v, w) are the components of the perturbation wind vector in the directions (x, y, z) , respectively, $\phi = p/\rho_0$ is the perturbation pressure divided by a reference air density, b is the perturbation buoyancy and (F_u, F_v, F_w, F_b) are friction terms which will first be taken equal to zero. The boundary conditions are $w = 0$ at $z = 0$ and the requirement for bounded solutions as $x^2 + z^2 \rightarrow \infty$. In the following, we consider N to be constant and prescribe $Q = Q(x, z, t)$.

From Eq. (5), one can derive a single equation for the streamfunction ψ ($u = \partial\psi/\partial z$ and $w = -\partial\psi/\partial x$):

$$\left(\frac{\partial^2}{\partial t^2} + N^2 \right) \frac{\partial^2 \psi}{\partial x^2} + \left(\frac{\partial^2}{\partial t^2} + f^2 \right) \frac{\partial^2 \psi}{\partial z^2} + 2f\Lambda \frac{\partial^2 \psi}{\partial x \partial z} = -\frac{\partial Q}{\partial x}. \quad (6)$$

Herein we take $Q = H(x, z) \sin \omega t$ (where $\omega = 2\pi/24$ h is the diurnal frequency and $t = 0$ corresponds to sunrise), so that $\psi \sim \sin \omega t$. Since $N \simeq 10^{-2} \text{ s}^{-1}$ and $\omega \simeq 0.73 \times 10^{-4} \text{ s}^{-1}$, $N^2 \gg \omega^2$; under the latter conditions, Eq. (6) becomes

$$N^2 \frac{\partial^2 \psi}{\partial x^2} + (f^2 - \omega^2) \frac{\partial^2 \psi}{\partial z^2} + 2f\Lambda \frac{\partial^2 \psi}{\partial x \partial z} = -\frac{\partial Q}{\partial x}. \quad (7)$$

Following R83, we set

$$H = \frac{Q_{\max}}{\pi} \left\{ \frac{\pi}{2} - \tan^{-1} \left(\frac{x}{x_0} \right) \right\} \exp \left(-\frac{z}{z_0} \right), \quad (8)$$

where $Q_{\max}$, x_0 , z_0 are the amplitude, horizontal and vertical scales of the heating, respectively. When $x_0 \rightarrow 0$, $Q(x, z, t)$ is a step function in x that decays exponentially away from the lower boundary, similar to the heating function used in Drobinski and Dubos (2009).

2.2. Non-dimensional equations

Non-dimensionalizing the governing equation (7) with

$$\left. \begin{aligned} x &= L\bar{x}, & z &= z_0\bar{z}, & t &= \omega^{-1}\bar{t}, \\ Q &= Q_{\max}\bar{Q}, & \psi &= WL\bar{\psi}, & \Lambda &= \Lambda_c\bar{\Lambda}, \end{aligned} \right\} \quad (9)$$

where

$$\Lambda_c^2 \equiv \frac{N^2(f^2 - \omega^2)}{f^2}, \quad L \equiv \frac{z_0 N}{\sqrt{f^2 - \omega^2}} \quad \text{and} \quad W \equiv \frac{Q_{\max}}{N^2},$$

we have

$$\frac{\partial^2 \bar{\psi}}{\partial \bar{x}^2} + 2\bar{\Lambda} \frac{\partial^2 \bar{\psi}}{\partial \bar{x} \partial \bar{z}} + \frac{\partial^2 \bar{\psi}}{\partial \bar{z}^2} = -\frac{\partial \bar{Q}}{\partial \bar{x}}. \quad (10)$$

Without the heating term, Eq. (10) is the familiar equation for symmetric instability (e.g. Bluestein, 1993, p. 317) with instability occurring if $\bar{\Lambda}^2 > 1$. In the present application, we consider the stable case $\bar{\Lambda}^2 < 1$ and situations in which $f > \omega$ (latitude $> 30^\circ$); with the latter restrictions, Eq. (10) is elliptic and may be transformed into

$$\frac{\partial^2 \bar{\psi}}{\partial \xi^2} + \frac{\partial^2 \bar{\psi}}{\partial \zeta^2} = -\frac{1}{\sqrt{1 - \bar{\Lambda}^2}} \frac{\partial \bar{Q}}{\partial \xi}, \quad (11)$$

where

$$\xi = \frac{\bar{x} - \bar{\Lambda}\bar{z}}{\sqrt{1 - \bar{\Lambda}^2}}, \quad \zeta = \bar{z}. \quad (12)$$

The coordinate transformation (12) is well known in the theory of frontal circulations, as are the solution techniques for the Poisson equation (11) (e.g. Hakim and Keyser, 2001). Here we note that Eq. (11) is identical to (14) of R83 with $\beta = 1/\sqrt{1 - \bar{\Lambda}^2}$ and thus the solutions given in R83 may be used directly.

2.3. Solutions

The solution of Eq. (11) for the point source $-4\pi\delta(\xi - \xi')\delta(\zeta - \zeta') \sin \bar{t}$ far away from boundaries is the Green's function (omitting the time dependence)

$$G_\infty(\xi - \xi', \zeta - \zeta') = -\ln\{(\xi - \xi')^2 + (\zeta - \zeta')^2\} \quad (13)$$

(Morse and Feshbach, 1953, p. 798). With the source located at $\xi' = \zeta' = 0$, the dimensional form of the argument of the logarithm in (13) is proportional to

$$\alpha x^2 - 2\gamma xz + z^2, \quad (14)$$

where $\alpha \equiv (f^2 - \omega^2)/N^2$ and $\gamma \equiv \Lambda f/N^2 (= \partial z/\partial x)_B$ in the light of Eq. (4)). With a rotation of coordinates, (14) can be expressed as

$$\begin{aligned} &(\alpha \cos^2 \epsilon - 0.5\gamma \sin \epsilon + \sin^2 \epsilon) \hat{x}^2 \\ &+ (\alpha \sin^2 \epsilon + 0.5\gamma \cos \epsilon + \cos^2 \epsilon) \hat{z}^2, \end{aligned} \quad (15)$$

where

$$\tan(2\epsilon) = \frac{2\gamma}{1 - \alpha}. \quad (16)$$

For typical values of the meteorological parameters, $(\alpha, \gamma) \ll 1$; consequently Eq. (16) indicates that $\epsilon \simeq \gamma$ and Eq. (15) reduces to $\alpha \hat{x}^2 + \hat{z}^2$. Thus we deduce that surfaces of constant ψ are ellipses, with the ratio of major to minor axes being $N(f^2 - \omega^2)^{-1/2}$ (as in (19) of R83), but tilt so that the major axes align with the tilt of the isentropes $\Lambda f/N^2$. The horizontal scale of the land/sea-breeze cell $z_0 \alpha^{1/2}$ is thus comparable to the Rossby radius $z_0 N/f$ (Drobinski and Dubos, 2009).

The solution of Eq. (11) for the two point sources $-4\pi\delta(\xi - \xi')\delta(\zeta \pm \zeta') \sin \bar{t}$, which satisfies the lower boundary condition $\psi(\xi, \zeta = 0, \bar{t}) = 0$, is the modified Green's function (again omitting the time dependence)

$$G_0(\xi - \xi', \zeta - \zeta') = -\ln \left[\frac{(\xi - \xi')^2 + (\zeta - \zeta')^2}{(\xi - \xi')^2 + (\zeta + \zeta')^2} \right]. \quad (17)$$

To help investigate the spatial dependence of the solution, we express Eq. (17) in terms of the coordinates $(\bar{x}, \bar{z})$ and consider a point source located at the coast $\bar{x} = 0$ and at some level $\bar{z} = \bar{z}_s$; thus the image source is located at $(\xi', \zeta') = (-(1 - \bar{\Lambda}^2)^{-1/2} \bar{\Lambda} \bar{z}_s, \bar{z}_s)$; with the latter specifications, the right-hand side of Eq. (17) becomes

$$-\ln \left[\frac{\{\bar{x} - \bar{\Lambda}(\bar{z} - \bar{z}_s)\}^2 + (1 - \bar{\Lambda}^2)(\bar{z} - \bar{z}_s)^2}{\{\bar{x} - \bar{\Lambda}(\bar{z} + \bar{z}_s)\}^2 + (1 - \bar{\Lambda}^2)(\bar{z} + \bar{z}_s)^2} \right]. \quad (18)$$

Figure 2 displays the Green's function G_0 for a source located at $\bar{z}_s = 1$ for $\bar{\Lambda} = 0$ and $\bar{\Lambda} = 0.9$. Several features may be deduced directly from the formulae. First, we note that G_0 is symmetric with $\bar{\Lambda} = 0$ and tilted for $\bar{\Lambda} = 0.9$, as expected from the solution for the point source discussed above. Second, we see that the horizontal decay away from the (tilted) maximum is more rapid in the case with $\bar{\Lambda} = 0.9$. To deduce the latter analytically, we fix the height $\bar{z} = \bar{z}_s$ so that (18) becomes

$$\ln \left\{ 1 + 4 \left(1 - \bar{\Lambda}^2 \right) \left(\frac{\bar{z}_s}{\bar{x}} \right)^2 \right\} \simeq 4 \left(1 - \bar{\Lambda}^2 \right) \left(\frac{\bar{z}_s}{\bar{x}} \right)^2. \quad (19)$$

The horizontal scale of G_0 is therefore proportional to $\sqrt{1 - \bar{\Lambda}^2}$ in $(\bar{x}, \bar{z})$ coordinates. One can similarly examine the vertical decay for fixed $\bar{x}$; setting $\bar{x} = 0$, one can show that $G_0 \sim \bar{\Lambda} \bar{z}_s / \bar{z}$. However, when the Green's function is applied to a given heating function (see below), the vertical scale of the sea-breeze cell cannot be smaller than the vertical scale of the heating itself, and therefore the vertical scale of the sea-breeze cell will be ~ 1 , independent of $\bar{\Lambda}$.

Figure 2. Green's function (Eq. (18)) with point source located at (0, 1) for (a) $\bar{\Lambda} = 0$ and (b) $\bar{\Lambda} = 0.9$.

One further feature that will be useful for later interpretation is the calculation of the cross-coast velocity $\partial\bar{\psi}/\partial\bar{z}$ at $\bar{z} = 0$. From Eq. (18) we deduce

$$\left. \frac{\partial G_0}{\partial \bar{z}} \right|_{\bar{z}=0} = \frac{-4\bar{z}_s}{\bar{x}^2 + 2\bar{\Lambda}\bar{x}\bar{z}_s + \bar{z}_s^2}, \quad (20)$$

which shows that, for $\bar{\Lambda} > 0$, the maximum cross-coast wind at the ground moves inland by the distance $-\bar{\Lambda}\bar{z}_s$.

3. Solutions for prescribed heating

The solution to Eq. (11) for the heating function (Eq. (8)) is given by (24) of R83, which in the present notation is

$$\begin{aligned} \bar{\psi}(\xi, \zeta, \bar{t}) = & -\frac{\beta \sin \bar{t}}{4\pi} \iint \ln \left[\frac{(\xi - \xi')^2 + (\zeta - \zeta')^2}{(\xi - \xi')^2 + (\zeta + \zeta')^2} \right] \\ & \times \frac{\partial \bar{H}(\xi', \zeta')}{\partial \xi'} d\xi' d\zeta', \end{aligned} \quad (21)$$

which in $(\bar{x}, \bar{z})$ coordinates becomes, using Eq. (8),

$$\begin{aligned} \bar{\psi}(\bar{x}, \bar{z}, \bar{t}) = & \frac{\beta \sin \bar{t}}{4\pi} \iint \ln \left[\frac{\{\bar{x} - \bar{x}' - \bar{\Lambda}(\bar{z} - \bar{z}')\}^2 + (1 - \bar{\Lambda}^2)(\bar{z} - \bar{z}')^2}{\{\bar{x} - \bar{x}' - \bar{\Lambda}(\bar{z} - \bar{z}')\}^2 + (1 - \bar{\Lambda}^2)(\bar{z} + \bar{z}')^2} \right] \\ & \times \frac{1}{\pi} \frac{\bar{x}_0 \exp(-\bar{z}')}{\bar{x}_0^2 + \bar{x}'^2} d\bar{x} d\bar{z}'. \end{aligned} \quad (22)$$

The solutions for $\bar{\psi}$ for $\bar{\Lambda} = 0, 0.5$ and 0.9 are shown in Figures 3 and 4; the zero-shear case reproduces the solution shown in figure 2a of R83. In the presence of shear, the linear response displays an asymmetric shape of the breeze cell with a seaward tilt ($x > 0$), which increases with the shear ($\sim \Lambda f/N^2$). Onshore ($x < 0$), the sea-breeze cell seems to be compressed to a shallower depth and to extend farther inland than in the absence of shear. However the sea-breeze aspect ratio is maintained independent of the shear, as shown previously by the scaling analysis. The intensity of the breeze also increases with shear.

The sea-breeze strength and direction as a function of time, visualized by hodographs, is a key aspect of the sea-breeze dynamics. Figure 5 shows the wind hodographs (polar diagram where wind direction is indicated by the angle from the centre axis and its strength by the distance from the centre), rotating clockwise from the south at $\bar{t} = 0$ (sunrise),

as a function of distance to the shore and for various values of the shear $\bar{\Lambda}$. At a fixed height ($\bar{z} = 0.1$ in Figure 5), the present solutions indicate that the sea breeze is weaker offshore for stronger shear and the magnitude of the diurnal wind variation decreases with increasing shear, whereas the opposite behaviour is found onshore. Indeed, onshore ($x < 0$), Figures 5(a) and (b) show that the magnitude of the wind fluctuations is larger in the presence of shear. Offshore, the sea breeze tilt implies that near-surface wind decreases rapidly for increasing distance from the shore. Since the tilt increases with shear, despite the intensification of the sea breeze with shear, for a given distance from the shore and a given height, the near-surface wind decreases with increasing shear. Onshore, the absence of tilt and the increase of the breeze intensity with shear implies that, for a given distance from the shore and a given height, the near-surface wind increases with increasing shear.

3.1. Spatial lag between heating and breeze cell

Equation (20) indicates a spatial shift between the heating pattern and the sea-breeze response. Figure 6(a) displays the horizontal wind component $\bar{u}$ close to the surface. It shows that the $\bar{u}$ disturbance is shifted onshore as $\bar{\Lambda}$ increases (see also Figure 3) and that the magnitude of the near-surface wind increases with increasing shear onshore while it decreases with increasing shear offshore.

Figure 6(b) shows the position $\bar{x}_{\max}$ of the near-surface wind maximum as a function of $\bar{\Lambda}$. Consistent with Eq. (20) implying that $\bar{x}_{\max} \sim -\bar{\Lambda}\bar{z}_s$, Figure 6(b) shows evidence of a linear relationship between $\bar{x}_{\max}$ and $\bar{\Lambda}$.

4. Similarities to and differences from observations

In CN10's climatological study, the southwesterly near-surface jet that develops primarily during the warm season east of the northern New Jersey coast and south of Long Island ranges typically between 11 and 17 m s⁻¹ (Figures 2 and 3 of CN10). The wind directions for the jet (Figure 6 of CN10) trace out a nearly elliptical orbit for the 24 h period, similar to the inertial rotation of the sea breeze (e.g. Neumann, 1984). CN10 discusses a geostrophic adjustment during the day for the near-surface jet, so by the end of the day the flow is quasi-balanced on a larger scale. The scale of the adjustment is relatively large (Rossby radius), so the enhanced winds extend well offshore (Figures 13d

of the isentrope tilt ranges between about 2 and 5×10^{-3} (Figures 15 and 16 of CN10). The vertical shear of the horizontal wind $\Lambda \sim 4\text{--}5 \times 10^{-3} \text{ s}^{-1}$ (Figures 13 and 14 of CN10), and the thermal stratification $N \sim 10^{-2} \text{ s}^{-1}$ (tephigrams in Figures 11, 15 and 16 of CN10). For the region surrounding New York City, $f \sim 9.5 \times 10^{-5} \text{ s}^{-1}$, so the scaling variables are

$$\Lambda_c \equiv \frac{N}{f} \sqrt{f^2 - \omega^2} = 6.4 \times 10^{-3} \text{ s}^{-1}$$

$$\text{and } L \equiv \frac{z_0 N}{\sqrt{f^2 - \omega^2}} \sim 50 \text{ km}$$

with $z_0 \sim 0.3 \text{ km}$, therefore $\bar{\Lambda} \sim 0.6\text{--}0.8$. In our model, the tilt of the isentropes is $\Lambda f/N^2 \sim 6.0 \times 10^{-3} \text{ s}^{-1}$, which is larger than the tilt simulated in CN10 (between 2 and 5×10^{-3}), but still within an acceptable range considering the simplicity of our linear approach.

There is also a good qualitative comparison with the wind hodographs (Figures 6 and 12 in CN10) (Figure 5). Indeed, the wind hodographs in CN10 display a clockwise rotation in agreement with the present theory. The jet maximum is reached at $\sim 1800 \text{ LST}$, which in non-dimensional time would be $\bar{t} = \pi$. In our model, $\bar{u}$ is in advance and $\pi/2$ (i.e. 6 h) out of phase with $\bar{\phi}$. Conversely $\bar{v}$ and $\bar{\phi}$ are in phase, so that at sunrise and sunset ($\bar{t} = 0$ and $\bar{t} = \pi$), the breeze blows parallel to the coast with high pressure to the right. The jet maximum is $\sqrt{u^2 + (V + v)^2}$. If $V \gg (u, v)$, then the jet maximum is obtained when the synoptic wind jet blows in the same direction as the $\bar{v}$ -component of the breeze, i.e. from the southwest. This corresponds to high pressure over the sea at $\bar{t} = \pi$, consistent with the CN10 observations. The effect of the shear, $\bar{\Lambda}$, on the hodograph cannot be assessed quantitatively. Qualitatively, the wind measurements in CN10 are collected inland near the shore. The climatological hodograph (Figure 6 in CN10) corresponds to a jet maximum of about 10 m s^{-1} , whereas the hodograph of the case-study (Figure 12 in CN10) corresponds to a jet maximum of about 15 m s^{-1} . We cannot directly infer the value of the wind shear $\bar{\Lambda}$ (except for the CN10 numerical simulation where $\bar{\Lambda} \sim 0.6\text{--}0.8$), but we can postulate that the stronger the jet maximum, the stronger the shear. The climatological hodograph displays typical diurnal wind variation of about 7 m s^{-1} (between 5 and 12 m s^{-1} ; Figure 6 in CN10), whereas the magnitude of the variations reach 10 m s^{-1} for the strong jet case-study (between 2 and 12 m s^{-1} ; Figure 12 in CN10). This difference, also predicted theoretically in Figure 5(b), might be attributed to the spatial shift of the sea breeze inland and the sea-breeze intensification with shear onshore, as discussed in the previous section. Even though this interpretation must be taken with care (because of measurement uncertainties, typically of the order of 1 m s^{-1}), the behaviour of the observed hodographs might be an indirect validation of the inland shift of the breeze in the presence of a superimposed synoptic along-shore jet.

Although the similarities between CN10 observations and our linear model reveal a broad agreement with respect to some of the basic flow features, our simple model assumptions limit a more quantitative one-to-one correspondence. Indeed, there are significant differences between CN10 observations and our model:

- the observed/modelled evolution of the near-surface jet is tightly correlated with the evolution of a meridional pressure gradient, which cannot be included in the 2D linear model;
- the observed jet maximum occurs about 2 to 5 h later than the pressure gradient maximum, while they are in phase in the linear model.

Regarding this last aspect, in our model, $\bar{v}$ and $\partial\phi/\partial x$ are in phase, so that the pressure gradient maximum should be found when $\bar{v}$ is maximum (i.e. at sunset, $\bar{t} = \pi$). In CN10, the jet maximum occurs about 2 to 5 h later than the pressure gradient maximum. So, the phase lag between the jet maximum and the cross-shore pressure or temperature gradient is apparently not consistent with our theory. The explanation for this difference could be twofold: (i) the absence of a cross-shore wind which can interact with the sea-breeze, and (ii) the absence of friction in our model which is known to play a major role on the phasing between the sea-breeze cell and surface heating (R83). The effect of a cross-shore background wind is geometrical, and results from the fact that the wind speed is a maximum when $U_0 u + V_0 v$ is a maximum if $U_0 \neq 0$. Taking for u and v the values given by the current theory with $U_0 = 0$, and since they are then in quadrature, one finds a phase lag χ given by $\chi = \tan^{-1} U_0 |u|/V_0 |v|$. Considering a 10 m s^{-1} offshore synoptic jet making an angle of 45° with the shore to the left, one finds a jet maximum occurring about 2 h 45 min after the maximum cross-shore pressure gradient, which is not inconsistent with CN10, considering the shape of the concave coast line where the cross-shore pressure gradient as well as the wind components are measured. Accounting for friction, by adding a linear friction term (R83; Drobinski and Dubos, 2009), i.e. $(F_u, F_v, F_w, F_b) = -\nu(u, v, w, b)$ (where ν is a friction coefficient), is equivalent to the transformation $\partial/\partial t \rightarrow \mathcal{L} = \partial/\partial t + \nu$. To investigate the effect of friction, we compute the circulation budget of the breeze flow:

$$C = \int_0^{+\infty} \{u(x, 0, t) - u(x, +\infty, t)\} dx. \quad (23)$$

The vertical branch may be neglected in the hydrostatic approximation. The evolution of C is obtained from Eqs (5) and (23), so

$$\begin{aligned} \mathcal{L}C &= \underbrace{f \int_0^{+\infty} \{v(x, 0, t) - v(x, +\infty, t)\} dx}_{FV} \\ &\quad - \underbrace{\int_0^{+\infty} \{b(-\infty, z, t) - b(+\infty, z, t)\} dz}_{PG} \end{aligned}$$

where FV , PG are the Coriolis and pressure gradient contributions. The pressure gradient term PG is directly related to the buoyancy through the hydrostatic approximation ($PG = B$ in R83 and Drobinski and Dubos, 2009). Following the methodology of R83, one can easily show that the circulation budget is not affected by the presence of the thermal wind-induced shear Λ , and is the same as in R83 (which is not surprising since Eq. (11) is identical to Eq. (14) of R83). We can thus show that the breeze circulation and heating display a phase lag

$$\chi_1 = \tan^{-1} \left\{ \frac{2\nu\omega}{f^2 + \nu^2 - \omega^2} \right\}$$

(for $f^2 + \nu^2 > \omega^2$ since $f > \omega$) (R83). Similarly, the phase lag between buoyancy or pressure gradient and heating is $\chi_2 = \tan^{-1}(\omega/\nu)$ (R83). The Coriolis term FV thus displays a phase lag χ_1 with respect to the pressure gradient PG , FV being a maximum after the pressure gradient maximum is reached. Figure 7 displays phase lag χ_1 of the Coriolis force relative to pressure gradient as a function of the linear friction parameter ν for $f = 10^{-4} \text{ s}^{-1}$. It effectively shows that friction induces a lag between FV and PG , and thus between ν and the pressure gradient $\partial\phi/\partial x$. Figure 7 also shows that χ_1 quickly tends to a value of about $\pi/4$, i.e. 3 h, which is close to the value observed by CN10. At this stage, it is difficult to conclude which is the dominant process that can explain the phase lag between the pressure gradient maximum and the jet maximum. A full nonlinear modelling system in an idealized framework would be more suited to address this issue.

5. Conclusion

This article investigates the linear dynamics of the sea breeze in a thermal wind. It shows that the breeze cell modified

Figure 5. Near-surface wind perturbation hodograph ($\bar{z} = 0.1$) starting at $\bar{t} = 0$, with data plotted every $2\pi/24$ time units ($\circ$ marker), for $\bar{\Lambda} = 0$ (black), 0.5 (red) and 0.9 (green) and for (a) $\bar{x} = -1.5$, (b) $\bar{x} = -0.5$, (c) $\bar{x} = 0.5$ and (d) $\bar{x} = 1.5$. In (a) and (d), the radius of the circle represents $\bar{u} = 0.2$, whereas in (b) and (c), it represents $\bar{u} = 0.4$.

Figure 6. (a): Wind component $\bar{u}$ as a function of $\bar{x}$ for $\bar{\Lambda} = 0$ (black), 0.5 (red) and 0.9 (green) at $\bar{t} = \pi/2$. (b): Onshore distance $\bar{x}_{\max}$ of the maximum wind speed $|\bar{u}|$ as a function of $\bar{\Lambda}$ at $\bar{t} = \pi/2$.

Figure 7. Phase lag χ_1 of the Coriolis force relative to pressure gradient as a function of the linear friction parameter ν for $f = 10^{-4} \text{ s}^{-1}$.

by the along-shore jet has a tilt $O(\bar{\Lambda})$. When $\bar{\Lambda} = 1$, the tilt of the breeze cell becomes equal to the slope of the background isentropes (which are in thermal wind balance with the wind shear). The critical slope of the sea-breeze tilt thus corresponds to the slope of the isentropes tilt. The present theory also predicts a spatial displacement between the heating pattern and the sea-breeze response, as the u disturbance moves onshore with increasing the thermal wind shear $\bar{\Lambda}$.

The present linear theory is also used to interpret measurements made in the vicinity of New York City in the presence of a sustained synoptic southwesterly jet which occurs predominantly during the spring and summer (more than two events per month; CN10). There are very consistent results between the observations and the theory: the enhanced along-coast wind basically follows the tilted isentropes with a similar isentropes tilt, the clockwise-rotating wind hodograph showing the jet maximum peaking at 1800 LST (i.e. sunset at $\bar{t} \sim \pi$) is also predicted by the theory. The phase lag between the jet maximum at 1800 LST and the cross-shore pressure gradient maximum about 2–5 h earlier is less straightforward, but can be attributed to the angle made by the synoptic jet with respect to the coastline and/or the presence of friction. However, there is a need to quantify the contribution of the two processes, which is outside the scope of this study and is left for future work. The inland spatial shift of the breeze might also be indirectly confirmed by the coastal inland wind observations of larger diurnal amplitude for a stronger synoptic jet (thermal wind shear).

Finally, all the comparisons that can reasonably be made between such a simple model and a very complex reality show a broad agreement of some of the basic flow features. However a deeper modelling study, where the restrictions can be relaxed one at a time, is needed to include some key aspects of the cases observed in the 'real atmosphere' as, for instance, an explicit expression of the meridian pressure gradient. This is left for future work.

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