Atmospheric Kinetic Energy Spectra from Global High-Resolution Nonhydrostatic Simulations

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ABSTRACT

Kinetic energy (KE) spectra derived from global high-resolution atmospheric simulations from the Model for Prediction Across Scales (MPAS) are presented. The simulations are produced using quasi-uniform global Voronoi horizontal meshes with 3-, 7.5-, and 15-km mean cell spacings. KE spectra from the MPAS simulations compare well with observations and other simulations in the literature and possess the canonical KE spectra structure including a very-well-resolved shallow-sloped mesoscale region in the 3-km simulation. There is a peak in the vertical velocity variance at the model filter scale for all simulations, indicating the underresolved nature of updrafts even with the 3-km mesh. The KE spectra reveal that the MPAS configuration produces an effective model resolution (filter scale) of approximately $6\Delta x$. Comparison with other published model KE spectra highlight model filtering issues, specifically insufficient filtering that can lead to spectral blocking and the production of erroneous shallow-sloped mesoscale tails in the KE spectra. The mesoscale regions in the MPAS KE spectra are produced without use of kinetic energy backscatter, in contrast to other results reported in the literature. No substantive difference is found in KE spectra computed on constant height or constant pressure surfaces. Stratified turbulence is not resolved with the vertical resolution used in this study; hence, the results do not support recent conjecture that stratified turbulence explains the mesoscale portion of the KE spectrum.

1. Introduction

The atmospheric kinetic energy (KE) spectrum has a canonical structure depicted schematically in Fig. 1. With KE plotted as a function of horizontal wavenumber, the spectrum is characterized by a shallow-sloped region at low wavenumbers and global scales, a steeper-sloped power-law region at intermediate wavenumbers corresponding to wavelengths of baroclinic cyclones, followed by the mesoscale region with a shallower slope, close to $-5/3$, that leads to fully three-dimensional turbulence at the smallest scales. These characteristics appear in many observational analyses, most notably in the analyses of the Global Atmospheric Sampling Program (GASP) aircraft observations by Nastrom and Gage (1985) and of the Measurement of Ozone by Airbus In-Service aircraft (MOZAIC) observations analyzed by Lindborg (1999). KE spectra can also be calculated from high-resolution global and regional atmospheric model simulations (e.g., Hamilton et al. 2008; Terasaki et al. 2009; Skamarock 2004), and these spectra reproduce many of the characteristics of the canonical spectrum. Many questions have been raised about the spectrum, the dynamics underlying it, and the ability of atmospheric models to reproduce it. In this paper we examine KE spectra produced by the nonhydrostatic global atmospheric solver within the Model for Prediction Across Scales (MPAS; Skamarock et al. 2012) and discuss the implications of these spectra on questions concerning atmospheric spectra and model capabilities.

There is no clear consensus on the dynamics producing many aspects of the observed spectrum, and there are even questions concerning some of the specific characteristics of the canonical structure itself. For example, while the $k^{-3}$ region is commonly thought to be the result of an enstrophy cascade, O’Gorman and Schneider (2007) have demonstrated that an enstrophy cascade is not necessary to produce the $k^{-3}$ region of the spectrum.

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Lovejoy et al. (2009) have recently argued that the $k^{-3}$ region is not, in fact, present in the atmosphere and is entirely the result of analysis errors. In addition, the dynamics producing the $k^{-5/3}$ mesoscale spectrum behavior continue to be debated. The dynamics underlying the spectrum are of more than just academic interest. Atmospheric prediction systems are designed and configured in part based on an understanding of these dynamics that, for example, have implications for forecast error growth and spinup time scales and filtering and subfilter-scale physics (e.g., Skamarock 2004; Shutts 2005).

An atmospheric model's ability to reproduce the canonical spectrum is often presented as evidence of the correctness of a model's formulation, implementation, and configuration. The KE spectrum also provides a direct measure of model filter effects and a model's effective resolution—that is, the filter scale of the model. A number of questions have arisen regarding the relationship of model configuration to the correctness of the simulated spectrum, and perhaps the most significant of these are concerned with vertical resolution (Brune and Becker 2013) and the forms and magnitude of filtering (e.g., Laursen and Eliasen 1989; Yuan and Hamilton 1994; Skamarock 2004).

We have performed global simulations with the non-hydrostatic atmospheric model in MPAS (MPAS-A) at nominally uniform mesh spacings of 3–15 km. We report on the KE spectra from those simulations in this paper and discuss their implication for model formulation and configuration and for dynamics. In section 2 we describe the model configuration and test period. We outline the computation of the KE spectra from the simulation results in section 3 and present the global KE spectra from the 3-km MPAS simulation. In section 4 we compare the MPAS KE spectra with those from other models published in the literature. We discuss model configuration issues related to MPAS and other model spectra in section 5, and we briefly discuss the implications of the MPAS results on the dynamics underlying the KE spectra in section 6. The paper concludes with a summary section 7.

### 2. Simulation period and MPAS configuration

The results presented herein are computed from 20-day simulations initialized at 0000 UTC 15 January 2009 and extending to 0000 UTC 4 February 2009. Initial fields for MPAS are taken from the Climate Forecast System Reanalysis (CFSR; Saha et al. 2010). This time period included a short but energetic MJO event that is the focus of another study.

The MPAS model is described in Skamarock et al. (2012). It uses a spherical centroidal Voronoi mesh (nominally hexagons) for its horizontal tiling of the sphere along with a C-grid staggering of the prognostic variables. The quasi-uniform meshes we use in this study are based on icosahedral meshes (hexagonal) with average cell-centered spacings of 15, 7.5, and 3 km. This corresponds to 2 621 442, 10 485 762, and 65 536 002 cells to tile the sphere, respectively. The first two of these meshes correspond to 9 and 10 successive bisections of the base icosahedron, and the 3-km mesh represents a division of the 15-km mesh by a factor of 5 (see Heikes and Randall 1995). The centroidal Voronoi meshes are produced by applying the iterative Lloyd's method to these bisected meshes (Rindler et al. 2008). The MPAS mesh utilizes 41 vertical layers and we configure the hybrid terrain-following vertical height coordinate in MPAS, described in Klemp (2011), such that the horizontal coordinate surfaces are nearly constant height surfaces above approximately 15 km above sea level.

MPAS uses a split-explicit Runge–Kutta time-integration technique and we use time steps of 60, 30, and 12 s on the 15-, 7.5-, and 3-km meshes, respectively. A fourth-order horizontal filter is employed in these simulations using hyperviscosities of $1.6875 \times 10^{11}$, $2.109375 \times 10^{10}$, and $1.35 \times 10^{9}$ m$^4$ s$^{-1}$ on the 15-, 7.5-, and 3-km meshes, respectively; this represents a scaling of the hyperviscosity by a factor of $\Delta t^4/\Delta x$. The hyperviscosity multiplying the divergence term in the horizontal momentum mixing [Skamarock et al. 2012, their Eq. (16)] is increased by a factor of 10 for these meshes to remove grid-scale noise in the solution. The hyperviscosities used here are about a factor of 10 less than that used in

![Schematic of canonical atmospheric kinetic energy spectra.](image-url)
many global models (e.g., see Jablonowski and Williamson 2006), except for the divergent portion for horizontal momentum. A second-order horizontal mixing using eddy viscosities computed with the Smagorinsky subgrid mixing model is also active in these simulations. The mixing length used in the Smagorinsky formulation is equal to the mean cell spacing (i.e., 15, 7.5, and 3 km) and the Smagorinsky coefficient $c_s = 0.125$. There is no lower bound applied to this eddy viscosity term. The spatially third-order transport scheme described in Skamarock and Gassmann (2011) is used in these simulations, and the shape-preserving (monotonic) option is used for all moist species but not for potential temperature. Fourth- and second-order horizontal filtering is not applied to the moist species because adequate filtering is implicit in the shape-preserving constraint applied in the transport scheme. A gravity wave–absorbing layer is active above 22 km as described in Klemp et al. (2008).

A suite of atmospheric physics taken from the Advanced Research Weather Research and Forecasting (WRF) Model (ARW) are used for these tests. We employ the Monin–Obukhov surface-layer scheme from WRF v3.5, the YSU planetary boundary layer scheme from WRF v3.4.1, the Noah land surface model using four layers from WRF v3.3.1, the Tiedtke convective parameterization from WRF v3.3.1, the WRF single-moment 6-class microphysics scheme (WSM6) microphysics from WRF v3.5, and the GCM version of the Rapid Radiative Transfer Model (RRTMG) radiation scheme from WRF v3.4.1; the schemes are described in the ARW description provided in Skamarock et al. (2008). Vertical diffusion in the free atmosphere is provided by the planetary boundary layer scheme; vertical diffusion is not included in the dynamics.

3. KE spectra and comparison with observations
   
a. KE spectra computation

   In this paper we present spectra for KE from the horizontal wind components, the vertical wind component, and the potential temperature. The centroidal Voronoi mesh does not directly lend itself to the computation of KE spectra given its unstructured nature and lack of a global coordinate system. To compute spectra, we interpolate the MPAS fields to a uniform latitude–longitude mesh having a mesh spacing one-half that of the MPAS mesh. The interpolation uses cell-centered horizontal velocities produced by MPAS from a radial basis function reconstruction using the prognosed velocities on the edges of a given cell (see Skamarock et al. 2012); the prognosed vertical velocity and the potential temperature have the same horizontal locations as the cell centers on the MPAS mesh. We make use of the Delauney triangular mesh, the dual of the Voronoi mesh, in the interpolation to the latitude–longitude mesh. For each latitude–longitude point, we determine the Delauney triangle that contains it, and we use a barycentric interpolation within this triangle, making use of the fact that the cell-centered values are located at the vertices of the triangles. This interpolation is $C_0$ continuous across the MPAS mesh. A spherical harmonics transform is applied to these latitude–longitude fields and the resulting 2D wavenumber decomposition is then summed over spherical harmonics with the same total spherical wavenumber to produce the 1D spectrum. To further understand the nature of the KE spectrum, the spherical harmonics representation of the horizontal velocity field is decomposed into a horizontally divergent component and a separate rotational (vortical) component with 1D spectra produced from each. All the spectra are truncated at the minimum wavelength resolvable on the MPAS mesh (i.e., the 2$\Delta$ wavelength).

   MPAS uses a hybrid terrain-following geometric height vertical coordinate (Klemp 2011). The horizontal surfaces do not correspond to constant height surfaces, although they are nearly so in the upper atmosphere. We present constant height spectra at two different characteristic levels. Tropospheric spectra are produced by interpolating values to constant height surfaces [above mean sea level (MSL)] between 8.5 and 10.5 km, at 1-km intervals, computing spectra on these surfaces, and then vertically averaging the resulting spectra. Stratospheric spectra are computed in the same manner using levels between 16 and 18 km MSL. The mean vertical mesh spacing is approximately 750 m where the tropospheric spectra are computed and approximately 900 m where the stratospheric spectra are computed. We also present spectra computed on a 200-hPa constant pressure surface for comparison to the other spectra. In all these cases, the variables are vertically interpolated to the surfaces before being interpolated to the latitude–longitude mesh. All spectra herein are computed by averaging spectra computed over the 10-day period at 6 hourly intervals from 0000 UTC 20 January to 0000 UTC 30 January 2009. Initially, the higher-wavenumber components in the KE spectrum have little energy given that the CFSR data from which MPAS is initialized has a mesh spacing of 40 km. The KE spectra in these simulations spin up within 12–18 h (e.g., Skamarock 2004). By simulating 5 days before sampling the spectrum we avoid any spinup issues.

b. Simulation results and observations

   The horizontal KE spectra for the 3-km global MPAS simulation are shown in Fig. 2. Both the tropospheric (left panel) and stratospheric (right panel) spectra show
the general characteristics depicted in the canonical spectrum in Fig. 1; the total KE has a shallow slope at the largest wavelengths (above a few thousand kilometers; i.e., the global scale), a steeper-slope region at wavelengths between a few thousand kilometers and several hundred kilometers (the synoptic scale), followed by a transition to a shallower-sloped region for wavelengths less than several hundred kilometers (mesoscale and finer scales). The slopes steepen again in the tails, indicating the effect of model filters on the kinetic energy (Skamarock 2004). Figure 3 plots the slope of the model-derived spectra given in Fig. 2. The slope of the tropospheric spectrum is slightly greater than $-\frac{5}{3}$ in the synoptic scale and slightly less than $-\frac{5}{3}$ in the mesoscale. The mesoscale transition region occurs at somewhat longer wavelengths in the stratosphere compared to the troposphere. The shallower slope in the global scale is evident, but it is difficult to assign or compute any approximate value to the slope given the few wavenumbers present and the spread in their KE. The stratospheric spectrum is similar to the tropospheric spectrum, with a transition to a slightly shallower-sloped mesoscale beginning at slightly longer wavelengths.

The MPAS spectra are similar to observationally analyzed spectra presented by Nastrom and Gage (1985, their Fig. 3) using GASP observations and by Lindborg (1999, their Fig. 7) using the MOZAIC observations. Specifically, the simulated and observed spectra show transitions between a steeper synoptic-scale slope to a shallower mesoscale slope at wavelengths of several hundred kilometers. The transitions between synoptic and mesoscale are gradual and a distinct transition wavelength or wavenumber is not identifiable, as is the case with the observational analyses. The mesoscale transition in the MPAS KE spectra occurs at somewhat smaller wavelengths than that deduced from observations. There is evidence for the shallower-sloped spectra in the global scales in the observational analyses—for example, in Boer and Shepherd (1983, their Fig. 1) and Nastrom and Gage (1985, their Fig. 3). In the modeled KE spectra, there is more energy in the synoptic scales in the troposphere than in the stratosphere while there is a similar amount of energy in the mesoscale in the two regions. Nastrom and Gage (1985, their Fig. 6) show small differences in the KE levels in the troposphere compared to the stratosphere in the mesoscale, with some indication that only small differences persist into the synoptic scale, although their analyses are truncated at a wavelength of 800 km. Also note that we have computed our stratospheric KE at levels well above the tropopause (16–18 km MSL), while the GASP flight levels contributing to the analysis for stratospheric spectra were just above the tropopause.

Figure 4 depicts the variance (spectra) for potential temperature from the 3-km global MPAS simulation for the troposphere and stratosphere. It shows a similar wavenumber dependence as found in the KE spectra, consistent with the observational analysis of Nastrom and Gage (1985, their Fig. 3). The stratospheric potential temperature spectrum shows more variance than the troposphere at all wavelengths except those between
500 and 1000 km. The stratospheric potential temperature variance is a factor of 10 larger than that in the troposphere throughout the mesoscale in the MPAS results. The analysis in Nastrom and Gage (1985, their Fig. 6) indicates that the stratospheric variance is factor of 4 larger than the tropospheric variance in the mesoscale. We would expect the variance to grow with increasing altitude in the stratosphere owing to the increased stability (Gage and Nastrom 1986); hence, the difference is likely explained by our use of a higher altitude band to represent the stratosphere. The transition to the shallower-sloped mesoscale occurs at slightly longer wavelengths in the potential temperature spectra than in the MPAS KE spectra.

4. Model-derived spectra

The MPAS global tropospheric and stratospheric spectra depicted in Fig. 2 reveal some subtle structure for which we do not have sufficient observations to verify. The transition from the synoptic and mesoscale regimes in the full kinetic energy begins at approximately 200 km wavelengths where the rotational KE is still behaving as \( k^{-3} \). The transition occurs because the divergent portion of the KE has a shallower slope and it is becoming comparable in magnitude to the rotational portion. At wavelengths between 200 and 50 km, the slope of the divergent KE component is approximately constant (\( \sim -5/3 \)) while the slope of the rotational component of the KE is becoming shallower. At wavelengths smaller than approximately 50 km the slope of the divergent component of the KE has transitioned to a slope slightly shallower than \( -5/3 \), as does the full KE spectrum. A similar behavior of the spectrum components is evident in the stratosphere with the differences being that the synoptic–mesoscale transition occurs at longer horizontal wavelengths, the divergent portion of the kinetic energy is significantly larger than the rotational component in the stratospheric mesoscale regime, and the KE spectrum exhibits a slope significantly shallower than \( -5/3 \). The rotational component of the KE in the stratosphere is consistently smaller by almost an order of magnitude than that in the troposphere at the synoptic and smaller scales. The divergent components have similar magnitudes in the troposphere and stratosphere in the mesoscale region, with the stratospheric divergent energy having something less than twice the energy of the tropospheric energy at a given wavelength. This behavior is consistent with the results from idealized simulations of baroclinic waves in a channel (Waite and Snyder 2013, 2009), where it is observed that the rotational KE decays with height while the divergent component does not. Waite and Snyder argue that this behavior is consistent with the model of waves propagating upward through a balanced flow that decays with height above the tropopause.

Many other global model–derived spectra have been published for the purpose of demonstrating the correct
behavior of a model, to understand the sensitivity of spectra and model energetics to model configuration, and to further understand the atmospheric physics responsible for the spectra.

At this time, the highest-resolution global model–derived spectra have been analyzed using simulations from the Nonhydrostatic Icosahedral Atmospheric Model (NICAM) global model employing a mesh with an average cell-center spacing of approximately 3.5 km (Terasaki et al. 2009). NICAM uses an icosahedral (hexagonal) horizontal mesh and, as with MPAS, the velocities are interpolated to a latitude–longitude mesh for analysis. The NICAM spectra, however, are presented as a function of zonal wavenumber averaged between latitude bands, making it difficult to attribute a physical wavelength to a wavenumber. The NICAM spectra from the 3.5-km simulation (Terasaki et al. 2009, their Fig. 1), representing an average between 40° and 50° N on the 200-hPa surface, are similar to the MPAS spectra shown in Fig. 2; the global-, synoptic-, and meso- and cloud-scale characteristics of the spectrum appear in both. The NICAM results also exhibit a slope slightly steeper than $-\frac{3}{2}$ in the synoptic scales and slightly less than $-\frac{5}{3}$ in the mesoscale, consistent with the MPAS results. Figure 3 in Terasaki et al. (2009) shows KE as a function of pressure level. Comparing the 500- and 200-hPa spectra, the transition to the mesoscale occurs at longer wavelengths, consistent with the MPAS spectra.

The NICAM results presented in Fig. 1 from Terasaki et al. (2009) depict the spectrum for the vertical velocity from the 3.5-km simulation, in addition to the spectra for the other wind components, as a function of zonal wavenumber averaged between 40° and 50° N. The vertical velocity possesses much less energy than the horizontal velocity spectra and there is little variation as a function of horizontal wavenumber. Vertical velocity spectra for the 3-km MPAS simulation are shown in Fig. 5. These spectra, averaged over the globe, are also relatively flat (note the KE scale) but show evidence of two peaks: one at the synoptic scale at a few thousand kilometers and a second at the filter scale of the model between 4 and 6 $D_h$ (i.e., between 12 and 18 km). The synoptic-scale peak presumably is associated with vertical motions associated with large-scale waves. We believe that the filter-scale peak is likely associated with grid-scale convection, waves generated by convection, and other marginally or under-resolved small-scale processes.

A detailed examination of relatively high-resolution global model spectra was performed by Hamilton et al. (2008) using results from the Atmospheric GCM for the Earth Simulator (AFES) global spectral model. The KE spectra presented in Hamilton et al. (2008, their Fig. 3), analyzed on a 200-hPa surface, compare well with the MPAS spectra also analyzed on a 200-hPa surface and presented in Fig. 6. The AFES spectrum has a synoptic-scale slope only marginally greater than $-\frac{3}{2}$ but a significant mesoscale region with a slope somewhat shallower than $-\frac{5}{3}$ (see Hamilton et al. 2008, their Fig. 3), though the AFES T639 configuration ($\Delta h \sim 30$ km) resolves only a small portion of the mesoscale regime. The divergent component of the spectrum is also given in both the AFES and MPAS results and both show that the divergent component becomes comparable to the rotational component in the mesoscale regime. The MPAS spectra (Figs. 2 and 6) show the rotational component transitioning to a shallower slope in the mesoscale, similar to the behavior of the full spectrum and the divergent component. The spectrum for the rotational component is not given for the AFES analysis, but given that the slopes of the full spectrum and divergent spectrum are almost the same in the mesoscale, the difference between the full and divergent energies (i.e., the rotational energy) likely exhibit the same characteristic of transitioning to a shallower slope in the mesoscale region. The dominance of the divergent KE in the mesoscale regime in the stratosphere cannot be directly assessed in the AFES results because no divergent spectra above 200 hPa are presented. However, the full KE spectra at 70 hPa (Hamilton et al. 2008, their Fig. 8) shows a mesoscale transition beginning at spherical wavenumbers less than 100, consistent with the
MPAS results for the stratosphere (Fig. 2, right column), suggesting that the behavior of the AFES divergent energy is also similar to that from MPAS.

The MPAS spectral characteristics are also similar to those presented in Burgess et al. (2013), computed from European Centre for Medium-Range Weather Forecasts (ECMWF) Integrated Forecast System (IFS) analyses from the T1279 operational configuration. The mesoscale transition in the spectra occurs at lower wavenumbers for higher levels in the stratosphere (see Burgess et al. 2013, their Fig. 1), similar to that found in the MPAS results in Fig. 2. In the stratosphere, the divergent component of the KE spectrum in the mesoscale is greater than the rotational component (see Burgess et al. 2013, their Fig. 6c), and the divergent spectrum is found to have a slope of $-1.4$ (i.e., somewhat shallower than $-5/3$), again consistent with the MPAS results.

The global model–derived horizontally divergent components of the KE spectra reported in the literature all show a shallow slope throughout the entire wavenumber space after approximately spherical wavenumber 10 (e.g., Figs. 2 and 5). There are a number of potential sources of divergent kinetic energy in the free atmosphere—for example, moist convection, flow interaction with fronts, and flow interaction with topographic features. While we are not attempting to determine the energy input from these sources in this paper, we do wish to point out that the energy input from topographic sources should scale to the topographic spectrum. Figure 7 depicts the variance in the topography field as a function of spherical wavenumber and horizontal wavelength for the MPAS terrain, on the 3-, 7.5-, and 15-km meshes, in addition to the spectrum from the U.S. Geological Survey (USGS) Global 30 arc s elevation dataset (GTOPO30; available at https://lta.cr.usgs.gov/GTOPO30) from which the MPAS terrain is taken. The variances have been plotted as compensated variances (variance $\times k^{5/3}$) to more easily identify features. The terrain variance has a slope of approximately $-2$ from wavenumber 10 to approximately wavenumber 300 (as reported in Balmino 1993), followed by a quick transition to a slope somewhat shallower than $-5/3$. The terrain on the MPAS meshes is a filtered version of that interpolated from the USGS dataset, and the effects of the filtering are evident in the tailing off of the terrain variances at wavelengths of approximately 4–6Dx; the variances are nearly identical at longer wavelengths. The divergent portion of the tropospheric KE spectra from MPAS also show a small transition to a slope shallower than $-5/3$ in the vicinity of spherical wavenumber 300 (Fig. 2, left panel, and Fig. 6), but it is not as abrupt as that observed in the terrain variance. The divergent stratospheric spectrum (Fig. 2, right panel) has a slope shallower than $-5/3$ over most of its range. The relationship between the KE spectra and the terrain has not been quantified, but terrain forced vertically propagating gravity waves are an often-observed phenomena and we expect it should play some role in the dynamics underlying
the KE spectra. It should be noted, however, that atmospheric simulations that have no terrain forcing have also produced a mesoscale KE regime—for example, in the regional forecast of a cyclone over the Pacific Ocean (Skamarock 2004, his Fig. 8) and in the baroclinic wave simulations in a periodic channel (Waite and Snyder 2013, 2009).

Terasaki et al. (2011) compute a KE spectrum from the Japan Meteorological Agency (JMA) TL959L60 analyses using a normal-mode decomposition to separate Rossby waves from gravity waves. The KE is integrated over the entire depth of the atmosphere, and their result (Terasaki et al. 2011, their Fig. 1) shows the mesoscale transition in the spectrum when the gravity wave (divergent) component of the KE spectrum becomes larger than the Rossby wave component at about zonal wavenumber 80. The Rossby wave component maintains a $k^{-3}$ behavior in this transition in contrast to the rotational spectra in other analyses, but it begins to become more shallow for zonal wavenumbers greater than 200. Given the nature of the analysis (a zonal- and normal-mode decomposition), it is difficult to compare the results more directly with KE decompositions using horizontal divergence and rotation.

Hamilton et al. (2008) also present the time evolution of the total KE for spherical wavenumbers 360, 480, and 600 (see their Fig. 7). The spinup time is found to be somewhat less than a day, consistent with MPAS results presented in Fig. 8 and those given in Skamarock (2004) for simulations from a regional version of the Advanced Research WRF Model. The spinup time should be comparable to an eddy turnover time. A characteristic time can be estimated from the spectral energy density $E(k)$, where $E(k)$ has units of $L^3/T^2$ and wavenumber $k$ has units of $1/L$, as

$$\tau = \left(\frac{k^{-3}}{E(k)}\right)^{1/2}.$$  

Using Lindborg’s [1999, his Eq. (71)] functional fit to the atmospheric spectrum

$$E(k) = d_1 k^{-5.3} + d_2 k^{-3},$$

where $d_1 = 9.1 \times 10^{-4}$ and $d_2 = 3 \times 10^{-10}$, we find that for the synoptic-scale regime [small $k$, where the second term on the left-hand side of Eq. (2) dominates], the turnover time scale [Eq. (1)] is approximately 16 h and is independent of $k$. As one enters the mesoscale regime and higher wavenumbers, the time scale decreases quickly with increasing wavenumber. The MPAS results in Fig. 8 also show that the spinup time in the stratosphere is similar to that in the troposphere. Overall, the KE spinup times observed in models, and estimated from theory, are consistent.

5. Model configuration considerations

There are a number of aspects of model design and configuration that affect the KE spectrum. In the following subsection we address three aspects of model configurations that have been raised in the literature: deep convection parameterization, vertical resolution, and model filters.

a. Convective parameterization and model physics

Convective cells and their statistics (e.g., precipitation, vertical flux divergence, etc.) may not be explicitly resolved until the mesh spacing is of order 100 m or less (Bryan et al. 2003) [i.e., in the large-eddy simulation (LES) regime]. Realistic convective system structure, however, can be captured using mesh spacings of a few kilometers or less without the use of a deep convection parameterization (Weisman et al. 1997). Over the last decade, several modeling groups have performed simulations using mesh spacing greater than 10 km without the use of a deep convection parameterization—for example, NICAM (Satoh et al. 2008; Terasaki et al. 2009) and the Geophysical Fluid Dynamics Laboratory (GFDL) High Resolution Atmospheric Model (HiRAM) (Chen and Lin 2011). Outside the LES regime, deep convective updrafts appear in simulations as unphysically large laminar plumes, and the explicit simulation of convection in this manner is itself a form of parameterization. The decision to not use a parameterization on very coarse meshes can have significant negative impacts such as a delay in the onset of convection, a slowing of convective system evolution, and excessive transport of moisture into the upper atmosphere (Weisman et al. 1997). However, beneficial aspects have been found in these simulations, including the occurrence of propagating convective systems (often lacking in simulations where deep convection is parameterized) and a beneficial shift in maximum surface precipitation over continents from local noon (the time of maximum heating) to later in the afternoon, as supported by observation.

We have performed simulations using 7.5- and 15-km meshes for our test period with and without the Tiedtke deep convection parameterization; compensated KE spectra for these tests are presented in Fig. 9 along with the 3-km MPAS KE spectra for comparison. We present the spectra in terms of a compensated spectra (KE $\times k^{5.3}$) so as to better observe the small but noticeable differences among the spectra. In the troposphere the KE is about a factor of 2 greater in both the 15- and 7.5-km simulations without convective parameterization compared to the 3-km simulation.
between spherical wavenumbers 130 and 400. The difference is somewhat less than 2 in the simulations using convective parameterization for the 15-km mesh simulation. The differences are somewhat larger in the stratosphere. Results on 7.5- and 15-km meshes from the NICAM model are presented in Fig. 2 of Terasaki et al. (2009) (glevel-9 and glevel-10 meshes), along with their 3.5-km simulation. The NICAM results also show a pronounced upward shift in the spectra when convective parameterization is not used by a maximum factor of approximately 3 for the 15-km mesh between zonal wavenumbers 10 and 100 and for the 7.5-km mesh between zonal wavenumbers 10 and 60 (compared to the NICAM explicit 3.5-km simulation). Simulations produced using coarser meshes and deep convective parameterizations have KE spectra that compare better with the explicit MPAS 3-km simulation (Fig. 6) and explicit NICAM 3.5-km simulation Terasaki et al. (2009, their Fig. 2). Thus, the omission of a deep convection parameterization for mesh spacings greater than 3–4 km

FIG. 8. Tropospheric and stratospheric kinetic energy spectra for the 3-km MPAS simulations at simulation times 0, 6, 12, and 18 h. The day 5–15 averages are plotted for reference.

Fig. 9. Tropospheric and stratospheric compensated kinetic energy spectra (KE × k^{−5/3}) from MPAS global simulations with and without the parameterization of deep convection (cp).
appears to degrade the KE spectra in what are considered well-resolved portions of the spectrum—that is, wavelengths above 6\(\Delta h\) for MPAS and well into the resolved regime for NICAM.

Figure 10 depicts the vertical velocity spectra for these MPAS simulations. All the spectra show very similar peak values at the synoptic scales (spherical wavenumber 10) and a second peak at the filter scale of the model at 4–6 times the mesh spacing. The spectra for the 15- and 7.5-km simulations without convective parameterization follow the 3-km spectrum for spherical wavenumbers less than about 40 and 70, respectively, where their energy levels begin their rise toward the filter-scale peak. The 15- and 7.5-km parameterized convection simulations have more energy in the synoptic scales, although less at the filter-scale peak compared with their nonparameterized counterparts. We would expect that a physical peak in the vertical velocity spectra would occur at the scale of typical convective updraft diameter of a few kilometers (e.g., Bryan et al. 2003, their Fig. 10). This length scale is well below the resolution limit of the global MPAS and NICAM simulations, and as expected the vertical velocity spectra in Fig. 10 show no clear indications of convergence.

While the differences between parameterized and unparameterized deep convection simulations in the total KE spectra could be considered small relative to the variability of the spectra between models and atmospheric flow regimes (e.g., Skamarock 2004, their Fig. 5), the scales and magnitudes of the vertical transport are likely very different as implied by the vertical velocity spectra in Fig. 10 and as illustrated in the modeling results of Weisman et al. (1997), Bryan et al. (2003) and others.

With the increasing use of ensembles, a question arises concerning the need for explicit parameterization of KE backscatter. Shutts (2005) argues for the introduction of a backscatter parameterization into the ECMWF IFS model based on physical arguments and on results demonstrating that its addition in the T799 IFS produces a KE spectrum that has a reasonable representation of the mesoscale transition that was lacking in the nonparameterized IFS (see Shutts 2005, his Figs. 6 and 7). T799 represents a minimum wavelength of approximately 50 km, and examination of the parameterized IFS suggests that the filter scale of the model is at approximately 300 km (see Shutts 2005, his Fig. 6, and Burgess et al. 2013, their Fig. 1); thus, one should not expect to observe the mesoscale transition given that the model is strongly damping the mesoscale. IFS KE analyses at T1299 by Augier and Lindborg (2013, their Fig. 2) also do not produce a mesoscale transition. Although Augier and Lindborg suggest that model filters are partly responsible for this, they also note significant dissipation in the synoptic scales, suggesting that traditional explicit spatial and temporal damping mechanisms cannot explain all the observed dissipation in the IFS. Models that are tuned so that no filter scale is evident in the spectral tail (e.g., with an unphysical steepening of the spectrum at the wavelengths approaching 2\(\Delta h\)) can produce a mesoscale spectrum at relatively coarse resolutions (e.g., Hamilton et al. 2008, at T639). Models that more heavily filter the shortest wavelengths, such as MPAS, show a transition to the mesoscale when that transition is at wavelengths longer than the filter scale (approximately 6\(\Delta h\) for MPAS; see Fig. 2). Thus, we can find no evidence that KE
backscatter is required to recover the appropriate mesoscale energetics; rather, we find that most models are able to reproduce the main characteristics of the observed KE spectra without backscatter.

b. Vertical resolution and model filters

In a recent paper, Brune and Becker (2013) report results showing a marked degradation in the KE spectrum as the number of cells in a vertical column is reduced in a perpetual-January simulation using a spectral GCM with simplistic parameterizations of radiative and latent heating with T330 truncation. They compare spectra produced using 100 vertical levels (approximately 200-m vertical mesh spacing in the midtroposphere through the lower stratosphere) with those produced using 30 vertical levels (approximately 1500-m vertical mesh spacing in the midtroposphere through the lower stratosphere). Figure 1 in Brune and Becker (2013) depicts the spectra for these simulations at approximately the 520- and 225-hPa pressure levels. The 30-level simulation shows significantly higher levels of energy in the mesoscale compared to the 100-level simulation, both in the rotational and divergent components of the total KE, along with a transition to the shallower mesoscale spectral slope beginning around total wavenumber 60 compared to a transition beginning at approximately wavenumber 100. The rotational and divergent KE components intersect at approximately wavenumber 130 in the 100-level run but do not intersect in the 30-level run. Based on these results, Brune and Becker conclude that the coarse vertical resolution (e.g., that used in many climate models) is insufficient for producing the correct KE spectrum and simulating the underlying dynamics. They also note that even their high vertical resolution is marginal for resolving stratified turbulence postulated in some studies as being responsible for the mesoscale spectrum (e.g., Lindborg 2006).

The MPAS simulations were performed on a mesh containing 41 vertical levels (41 mesh cells in every vertical column), with a vertical mesh spacing of 115 m for the lowest model level stretching to 1091 m at the model top located at 30 km MSL. We find significant variation of the spectra as a function of height in our results (e.g., Fig. 2), similar to that found by Brune and Becker (2013; their Figs. 1a and 1c). For example, the increasing ratio of the divergent to the rotational KE in the mesoscale with increasing (decreasing) height (pressure) is reproduced in our spectra, along with an earlier transition to the mesoscale slope with increasing height. To a large extent these same features are found in the NICAM results of Terasaki et al. (2009), the AFES results analyzed in Hamilton et al. (2008), and the high-resolution JMA analyses in Terasaki et al. (2011), all produced at relatively coarse vertical resolution in the free troposphere and stratosphere.

The conclusions of Brune and Becker (2013) call into question the validity and the physical relevance of the spectra reported in other studies, but models using similarly coarse vertical resolution (this study included) have not found the anomalous behavior of the KE spectra reported by Brune and Becker. After examining the Brune and Becker spectra, we believe that there is insufficient energy and enstrophy sinks in their 30-level configuration. This insufficient filtering leads to spectral blocking and the contamination of the mesoscale spectrum beyond wavenumber 30 that is evident in the results. Wavenumber 30 represents wavelengths greater than 20Δx in these simulations, so the contamination of the spectra (the erroneous appearance of the mesoscale) extends far into horizontal wavelengths that are usually considered very well resolved. We conclude from these results that model filters need to be tuned for both the horizontal and the vertical resolution of a model configuration. Importantly, we believe that model filters should be tuned to produce an obvious filter scale (i.e., tuned to produce a steepening tail to the spectra). Model configurations should not be tuned to reproduce the spectrum to the resolution limits (2Δx) of the model because model numerics are highly inaccurate at the shortest wavelengths and unphysical spectral blocking and erroneous energetics (spectra) can result from insufficient energy and enstrophy sinks. There is no obvious filter scale in the Brune and Becker results (no steepening tail to the spectra), and it is not possible to determine if the mesoscale region in the spectra are the result of unphysical spectral blocking even in the 100-level results (Brune and Becker 2013, their Fig. 1a).

6. Implications for KE spectrum dynamics

Lovejoy et al. (2009) argue that the synoptic-scale-to-mesoscale transition in the observed KE spectrum, from behaving approximately as $k^{-3}$ to $k^{-5/3}$, is the result of assuming that the aircraft from which the observations are obtained follow constant height surfaces. The aircraft, however, more closely follows constant pressure surfaces, and Lovejoy et al. argue that this leads to an erroneous break in the KE spectrum where one does not exist. They also argue that spectra on constant height surfaces should behave approximately as $k^{-5/3}$ throughout the meso- and synoptic scales. Model-based studies, including this one, all find a steeply sloped KE spectrum in the synoptic scales, with a spectral slope of $-3$ or slightly greater. Most modeling studies, including this one, also find a synoptic-scale-to-mesoscale transition at wavelengths somewhere
around several hundred kilometers depending on the altitude or pressure level; those that do not are lacking resolution relative to their configuration and filtering. In this study, moreover, we have presented KE spectra on constant height surfaces and on constant pressure surfaces, and the different spectra show similar characteristics, including an obvious spectral break. Our study further confirms the results and conclusions presented by Frelich and Sharman (2010); that is, we find no evidence in our results to support the hypothesis that the synoptic–mesoscale spectral break is an artifact of an analysis assumption.

The results from this study show that high vertical resolution is not necessary for a model to capture the general structure of the atmospheric KE spectra. This does not, in and of itself, demonstrate or prove that theories invoking stratified turbulence to explain the mesoscale spectrum and transition (e.g., Lindborg 2006) are erroneous; the details of the turbulence may be unimportant, or the models may be producing the correct spectrum for the wrong reasons. If models are producing qualitatively correct spectra for the wrong reasons, however, then the responsible model formulation and configuration errors are shared among numerous models.

Finally, we point out that the canonical spectrum is precisely that—canonical. Both observations and numerous model analyses show that the KE spectra vary in time and by region, with variations in slope in the global scale, synoptic scale, and mesoscale, and in the position and extent of their transition regions. The rich dynamics of the atmosphere and forcing mechanisms need to be taken into account in any theory for the atmospheric KE spectrum.

7. Summary

From the results presented in this paper and our review of previous modeling-based and observational studies, we summarize and conclude as follows:

1) MPAS KE and potential temperature spectra show a well-resolved mesoscale region with a shallow mesoscale slope. The spectra are qualitatively similar to other observation-derived and model-derived spectra.

2) MPAS spectra show the expected variation with height; that is, the transition between synoptic-scale and mesoscale regimes occurs at longer wavelengths for increasing altitude and the synoptic-scale energy decreases with increasing height.

3) The mesoscale region of the KE spectra is located in the region where the divergent portion of the spectrum is of similar or greater magnitude compared with rotational component of the spectra.

4) The vertical velocity spectra are not converged at the higher wavenumbers; there is a peak in the vicinity of the model filter scale at all model resolutions.

5) In these simulations, removing the deep convection parameterization on coarse meshes (Δh = 7.5 and 15 km) increases the KE and the vertical velocity variance in the mesoscale relative to that in parameterized simulations. The energy is also increased in the resolved portion of the spectrum relative to the convection-permitting simulation (Δh ~ 3 km).

6) All MPAS spectra show clear evidence of a filter scale. Lack of a filter scale in the spectrum (a downturned tail at the highest wavenumbers) may indicate an unphysical buildup of energy (spectral blocking) and contamination of the spectrum into what are considered to be well-resolved wavelengths. Model filters and physics need to be tuned to both the horizontal and vertical mesh spacing.

7) A stochastic backscatter parameterization is not needed to produce a realistic mesoscale regime in the spectrum, consistent with most high-resolution model KE spectra analyses.

8) KE spectra computed on constant height and constant pressure surfaces possess the same qualitative (canonical) character, and we find no evidence supporting the conjecture (Lovejoy et al. 2009) that the steeper synoptic-scale spectral slope is an artifact of analysis assumptions.

9) The vertical resolution employed in this study is relatively coarse. Given the similarity of the simulated spectra with observations and other models, the results do not support theories (e.g., Lindborg 2006) stressing the importance of stratified turbulence to explain the $k^{-5/3}$ mesoscale spectra.

The dynamics underlying the atmospheric KE spectrum are still poorly understood three decades after the observational study of Nastrom and Gage (1985) revealed the general structure of the spectrum. In particular, there is no emerging consensus on the dynamics underlying the mesoscale portion of the spectrum where horizontally divergent motions are important. Most models capture the general structure of the spectrum, but there is significant sensitivity to the parameterization of model physics and to implicit and explicit model filtering that can mask the underlying dynamics or, in more serious cases, produce erroneous spectra. With the advent of faster computers and nonhydrostatic global models, the mesoscale portion of the spectrum can now be well resolved including explicit representation of much of the underlying physics including that associated with terrain and deep convection. Much work remains.
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REFERENCES


