

# Effect of surface thermal heterogeneity on the structure and mixing intensity in strongly stable boundary-layer flows: a DNS study

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## 1. Introduction

Representation of stably stratified boundary-layer turbulence is a serious problem in modelling engineering and environmental flows. For example, numerical weather prediction and climate models suffer from large errors associated with the stably stratified flows that are characterized by weak and often intermittent turbulence and low mixing intensity. It is still largely unclear how the problem can be cracked. One outstanding issue is the strongly stable boundary-layer turbulence over heterogeneous surfaces. In the present study, we use direct numerical simulation (DNS) to gain some insight into the effect of surface thermal heterogeneity on the mean and turbulence structure and mixing intensity in strongly stable boundary-layer flows.

We focus on physical processes at work in strongly stratified boundary-layer flows over thermally heterogeneous surfaces. To this end, we use an idealized plane Couette flow set-up as a proxy for real-world flows (e.g., turbulent flows in the atmosphere and the ocean). The plane Couette flow configuration has often been used to study various aspects of neutral, convective and stably stratified turbulence (see, e.g., [1, 2, 3, 4, 5, 6] and references therein). To the best of the authors' knowledge, DNS of Couette flows over thermally heterogeneous surfaces has not been performed so far.

## 2. Flow Configuration and Simulations Performed

A plane Couette flow is considered. The fluid depth is  $H$ , the lower boundary is at rest, and the upper boundary moves with a constant velocity  $U_u$ . Stable buoyancy stratification is maintained by a potential temperature difference  $\Delta T = T_u - T_l$  between the upper and lower boundaries. The Boussinesq approximation and the linear equation of state are used. The problem is considered in dimensionless form, using the length scale  $H$ , velocity scale  $U_u$ , time scale  $H/U_u$ , temperature scale  $\Delta T$ , and dimensionless potential temperature  $\theta = (T - T_l) / (T_u - T_l)$  (potential temperature will be referred to as simply “temperature” in what follows). The governing equations include three dimensionless parameters, viz., the Reynolds number,  $Re = U_u H / \nu$ , the Prandtl number,  $Pr = \nu / \kappa$ , and the Richardson number,  $Ri = gH\alpha_T\Delta T / U_u^2$ , where  $\nu$  is the kinematic molecular viscosity,  $\kappa$  is the molecular temperature conductivity,  $g$  is the acceleration due to gravity, and  $\alpha_T$  is the thermal expansion coefficient.

Periodic boundary conditions for both velocity and temperature are applied in both stream-

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<sup>1</sup> Printed book of Proceedings will contain the short papers (4 pages) and the full-length keynote papers (12 pages). The CD-Proceedings will contain all full-length papers (see Section 14 for recommended length).

wise  $x$  and spanwise  $y$  horizontal directions. At the lower surface  $z = 0$  ( $z$  being the vertical coordinate), the streamwise  $u$ , spanwise  $v$  and vertical  $w$  velocity components are all zero. At the upper surface  $z = 1$ ,  $v = w = 0$  and  $u = 1$ . The following Dirichlet boundary conditions are used for temperature:

$$\begin{aligned} \theta &= \delta\theta \sin[2\pi nx/L_x] & \text{at } z = 0, \\ \theta &= 1 + \delta\theta \sin[2\pi n(x - U_{ut})/L_x] & \text{at } z = 1, \end{aligned} \quad (1)$$

where  $t$  is time,  $L_x$  is the domain size in the  $x$  direction,  $\delta\theta$  is the (dimensionless) amplitude of the temperature variations at the upper and lower surfaces, and  $n$  is the number of cold and warm stripes (the number of surface temperature waves). A conventional homogeneous plane Couette flow configuration is recovered with  $\delta\theta = 0$ . In the heterogeneous cases,  $\delta\theta > 0$  but, importantly, the horizontal-mean surface temperature is the same as in the homogeneous case.

In all simulated cases, the fixed values of  $\text{Pr} = 1$ ,  $\text{Re} = 10^4$ ,  $\text{Ri} = 0.25$ , and  $n = 4$  are used. The number of grid points is 512, 512, and 256 in the  $x$ ,  $y$ , and  $z$  directions, respectively. The domain size in the vertical direction is 1, and the domain size in both horizontal directions is 8. Four simulations are performed: one with homogeneous lower and upper surfaces (Hom), and three with the heterogeneous surfaces, using  $\delta\theta = 0.25$  (Het25),  $\delta\theta = 0.50$  (Het50), and  $\delta\theta = 0.75$  (Het75). The DNS data are averaged over horizontal planes and the resulting profiles are then averaged over several thousand time steps (covering more than 150 time units  $H/U_u$  in heterogeneous cases). In what follows, an overbar denotes a horizontal-mean quantity, a prime denotes a fluctuation about a horizontal mean, and the angle brackets denote the quantities averaged over time. The DNS code used in the present study is described in detail in [2, 3].

### 3. Results

The stratification is strong enough to fully quench turbulence over homogeneous surfaces, resulting in velocity and temperature profiles that vary linearly with height. However, turbulence survives over heterogeneous surfaces. As seen from Fig. 1, the mean temperature  $\Theta = \langle \bar{\theta} \rangle$  is

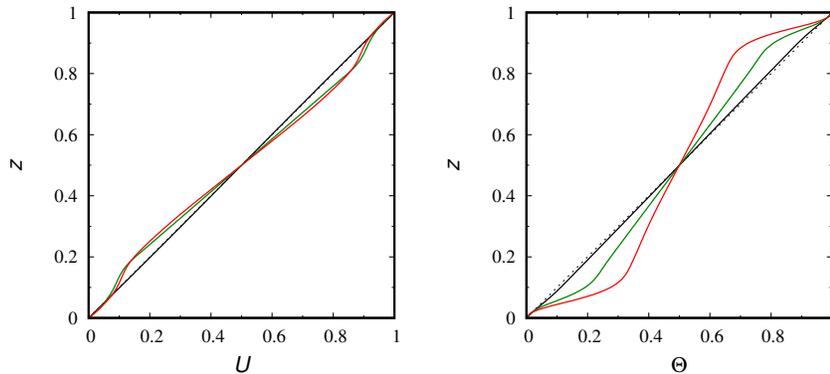


Figure 1: Streamwise component of mean velocity (left panel), and mean temperature (right panel) from simulations Het25 (black), Het50 (green), and Het75 (red). Thin black dotted line shows the laminar solution.

affected much stronger by the surface thermal heterogeneity than the mean velocity  $U = \langle \bar{u} \rangle$ . As  $\delta\theta$  increases, the flow becomes increasingly mixed with respect to  $\Theta$ . The effect is substantial

at large values of  $\delta\theta$  in the simulations Het50 and Het75. In the simulation Het25, the effect of surface heterogeneity is weak, and the temperature profile remains almost linear.

Both the molecular diffusion and the turbulence contribute to the downward, i.e., the down-gradient, transfer of horizontal momentum (not shown). The total (diffusive plus turbulent) heat flux  $w\theta = \langle w'\theta' \rangle$  is directed downward, Fig. 2. However, the turbulent contribution to the heat flux appears to be positive, i.e., up the gradient of the mean temperature.

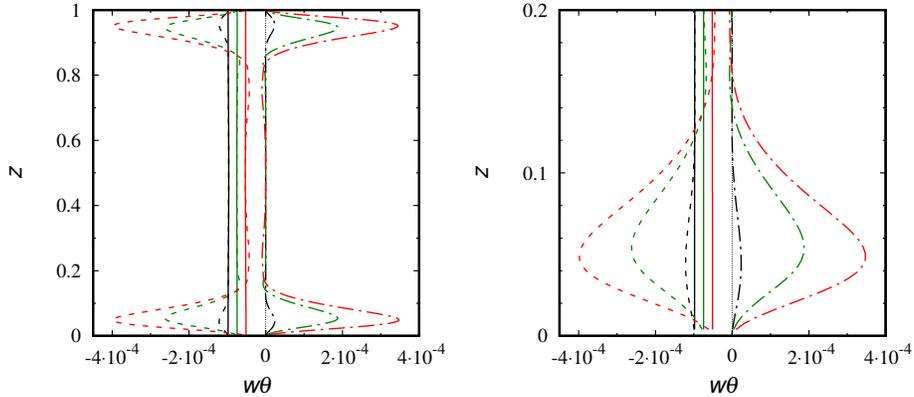


Figure 2: (left panel) Vertical component of the temperature flux from simulations Het25 (black), Het50 (green), and Het75 (red). Total flux is shown by solid curves, and contributions due to turbulence and due to molecular diffusion are shown by dot-dashed and dashed curves, respectively. (right panel) The same as in the left panel but for the lower part of the domain.

Figure 3 shows vertical profiles of the vertical-velocity skewness  $S_w = \langle w'^3 \rangle / \langle w'^2 \rangle^{3/2}$  and of the temperature skewness  $S_\theta = \langle \theta'^3 \rangle / \langle \theta'^2 \rangle^{3/2}$  in heterogeneous simulations. Both  $S_w$  and  $S_\theta$  are positive near the lower boundary (negative near the upper boundary by symmetry of the flow about  $z = 0.5$ ). A positive  $S_w$  indicates that positive (upward) vertical velocity has a lower fractional area coverage (more localized) than negative (downward) vertical velocity. Likewise, a positive  $S_\theta$  indicates a stronger localization of positive temperature fluctuations (about a horizontal-mean temperature) as compared to negative temperature fluctuations.

The situation is broadly similar to that in the convective boundary layer driven by the surface buoyancy flux. Both  $S_w$  and  $S_\theta$  are positive in the major part of the boundary layer, where highly localized positive temperature anomalies are collocated with positive vertical velocity, forming the flow structures known as convective plumes. It is these plumes that account for most of the upward temperature flux. Visualization of the  $w$  and  $\theta$  fields in our heterogeneous simulations (not shown) reveals strongly localized quasi-coherent flow structures characterized by high positive values of the vertical velocity and high positive fluctuations of temperature. It is these structures that generate positive turbulent temperature flux in the lower part of the flow (Fig. 2), i.e., precisely where both  $S_w$  and  $S_\theta$  are positive (Fig. 3). Similar arguments applied to the upper part of the flow (where both  $S_w$  and  $S_\theta$  are negative) explain a positive turbulent temperature flux there. Thus, the flow over heterogeneous surface features local convective instabilities and upward eddy heat transport, although the overall stratification remains stable and the heat is transported downward in the mean.

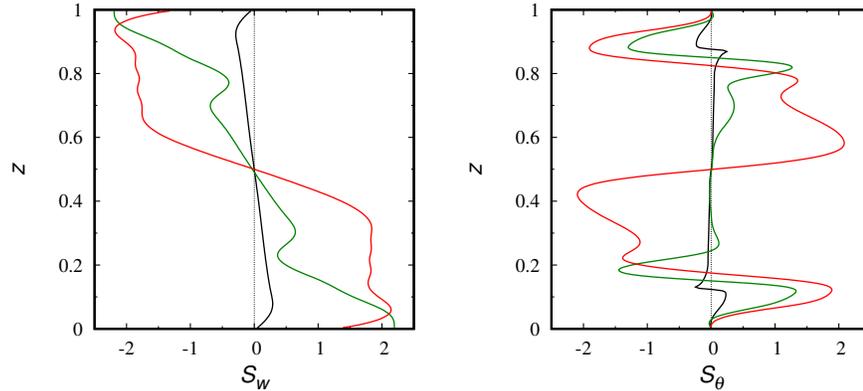


Figure 3: Vertical-velocity skewness (left panel) and temperature skewness (right panel) from simulations Het25 (black), Het50 (green), and Het75 (red).

## 4. Conclusions

Direct numerical simulations at bulk Reynolds number  $Re = 10^4$  and bulk Richardson number  $Ri = 0.25$  are performed to analyze the structure and mixing intensity in strongly stable boundary-layer flows over thermally homogeneous and heterogeneous surfaces. An idealized plane Couette flow set-up is used as a proxy for real-world flows. Some results from our DNS study are discussed in section 3. A more detailed account (including a comparison with large-eddy simulations of weakly stable boundary layers [7]) will be given in the symposium presentation and in the full-length paper.

The analysis in the present study was focused on the flow configuration where the crests of the surface temperature waves are normal to the mean flow (see section 2). Work is underway to analyze the effect of the surface heterogeneity orientation (e.g., temperature-wave crests normal vs. parallel to the mean flow) on the flow structure and mixing intensity. Results will be reported in subsequent publications.

## References

1. J. Komminaho, A. Lundbladh, and A. V. Johansson. Very large structures in plane turbulent Couette flow. *J. Fluid Mech.*, 320:259–285, 1996.
2. P. P. Sullivan, J. C. McWilliams, and C.-H. Moeng. Simulation of turbulent flow over idealized water waves. *J. Fluid Mech.*, 404:47–85, 2000.
3. P. P. Sullivan and J. C. McWilliams. Turbulent flow over water waves in the presence of stratification. *Phys. Fluids*, 14:1182–1195, 2002.
4. V. Avsarkisov, S. Hoyas, M. Oberlack, and J. P. García-Galache. Turbulent plane Couette flow at moderately high Reynolds number. *J. Fluid Mech.*, 751:R1, 2014.
5. E. Deusebio, C. P. Caulfield, and J. R. Taylor. The intermittency boundary in stratified plane Couette flow. *J. Fluid Mech.*, 781:298–329, 2015.
6. M. Lee and R. D. Moser. Extreme-scale motions in turbulent plane Couette flows. *J. Fluid Mech.*, 842:128–145, 2018.
7. D. V. Mironov and P. P. Sullivan. Second-moment budgets and mixing intensity in the stably stratified atmospheric boundary layer over thermally heterogeneous surfaces. *J. Atmos. Sci.*, 73:449–464, 2016.