CATALYZING CRAIK-LEIBOVICH INSTABILITIES BY BREAKING WAVES

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Abstract. Gravity waves on the sea surface reflect coupling between the atmosphere and ocean. They play important dynamical roles in the transfer of momentum, heat, and scalars, but how they interact with background oceanic (and atmospheric) turbulence is still not well understood. This is especially true under high winds where waves foster Langmuir circulations but also break intermittently. Results from high-resolution large eddy simulations (LESs) that include stochastic breaking waves and the Craik-Leibovich “vortex force” are analyzed. Under certain sea states turbulence (anisotropic seed vorticity) generated by breaking waves initiates Craik-Leibovich instabilities that lead to coherent structures, viz., Langmuir cells and intermittent downwelling jets both of which impact ocean mixing. We suggest a mechanism as to how breaking waves couple with the vortex force that leads to the formation of these flow structures.

1. Introduction

Gravity waves at the sea surface impact momentum and scalar exchange in the marine boundary layers. Waves are particularly important in the upper ocean boundary layer (OBL) as they break intermittently and thus sporadically inject momentum into the water column leading to the growth of currents [1; 2]. This momentum (and energy) transfer from winds to waves to currents is beautifully captured in photographs taken from aircraft under high wind conditions (figure 1). Also, when waves and currents are averaged over multiple periods, a net wave-current interaction fosters the development of Langmuir circulations [3; 4]. The elegant and delicate mathematical steps and theoretical arguments leading to these conservative wave-current interactions appear as “vortex forces” in the equations of motion [5; 3; 6; 7]. There has long been intuitive speculation that wave breaking and Langmuir circulations interact [8; 9; 4], but observing and elucidating this coupling is difficult in complex sea states owing to the intermittent nature and widely varying scales associated with breaking waves. Also, Stokes drift which contributes to the vortex force is a Lagrangian property of the wavefield and not readily deduced from fixed-point Eulerian measurements.

Recently we have developed idealized large-eddy simulation (LES) models of the turbulent OBL which include both phase-averaged wave-current interactions and intermittent wave breaking [10; 11; 12; 13]. These turbulence resolving models can be used to examine possible couplings between wave breaking and Langmuir circulations. In our LES model, the wave-averaged dynamical effects on currents are the result of an asymptotic theory pioneered by Craik and Leibovich [5; 3] and later extended by McWilliams et al. [6; 7]. Effects of intermittent breaking are included as a probability density function (PDF) of parameterized stochastic impulses that depend on time and space. The PDF of breaking varies with wind speed and wave age and obeys the conservation rule that when averaged over long time and large area reduces to a traditional bulk formula for constant wind stress $\tau$. Wave effects appear in our LES model equations for currents,
U \sim 20\text{m/s}

Figure 1: Photograph of breaking waves in the Gulf of Tehuantepec at a wind speed of approximately $20\text{ms}^{-1}$. The image is from a low-level flight of the NCAR C-130. Courtesy W. Melville and C. Friehe.

scalars, and subgrid-scale turbulent kinetic energy (TKE). Schematically, the LES equation for resolved-scale momentum $\rho \mathbf{u}$ with wave effects is

$$\frac{\partial \mathbf{u}}{\partial t} = \ldots + \mathbf{u}^\text{St} \times \mathbf{\omega} + \sum_{i=1}^{n} A^{(i)},$$

where spatially filtered variables are indicated by $(\phantom{\text{th}})$. In (1), the resolved velocity is $\mathbf{u}$, the vorticity is $\mathbf{\omega} = \nabla \times \mathbf{u}$, $\mathbf{u}^\text{St}$ is the Stokes drift associated with the wave field, and $A^{(i)}$ are compact (in time and space) accelerations designed to mimic wave breaking. At any time step of the simulation the number of active impulses $n$ varies depending on the PDF of breaking waves. Dots in (1) denote all other terms in conventional LES of geophysical boundary layers, viz., advection, buoyancy, Coriolis, pressure gradients, and subgrid-scale flux divergence, for example see [14]. The vertical profile of Stokes drift is a primary input to LES of the OBL with wave-averaged effects when currents are generated by uniform stress. Typically $u^\text{St}(z)$ is estimated from a single monochromatic wavenumber component $k$ selected to match the peak in the wave height spectrum; then the Stokes drift has depth variation $u^\text{St} \sim e^{2kz}$ [13; 15; 16; 17]. In these previous LES the vortex force has been shown to have a significant influence on the turbulent eddies and their vertical fluxes in the OBL through the generation of Langmuir circulations [5; 13].

Our current LES model of the OBL builds on past work and incorporates additional wave effects. The most important are: (1) the vertical profile of Stokes drift, which appears in the vortex force, is obtained by integration of empirical wave height spectra [18] over all wavenumbers $k$; and, (2) the assumption of constant stress is replaced by stochastic forcing. As a result of these and other improvements, the LES model accounts for varying sea state expressed in terms of a bulk wave age $c_p/u_\ast$ where $c_p$ is the phase speed of the peak in the wave height
Figure 2: Flow visualization of the $u-$ current near the water surface ($z = -1.0\text{m}$) from an LES with stochastic wave breaking and vortex force at a wind speed of 30ms$^{-1}$ [10]. The horizontal ($\Delta x, \Delta y$) resolution of this calculation ($500 \times 500$ gridpoints) is 1.5m. The vertical spacing (128 gridpoints) varies from 1.0m at the surface to 1.2m at the bottom of the computational domain. The positive (red) contours are locations of breaking waves. The color bar is in units of ms$^{-1}$ and the winds are from left to right.

spectrum and $u_{sta}$ is the atmospheric friction velocity. A typical LES flow field from recent calculations at a wind speed $U_a = 30\text{ms}^{-1}$ approaching wind-wave equilibrium $c_p/u_{sta} = 30$ is shown in figure 2. This snapshot of the $u-$current is rich in detail; a spectrum of breaking waves in various stages of development is visible, and the range of breaker scales varies from small breakers $O(1)\text{m}$ to large breakers $O(50 - 75)\text{m}$. We note the flow structures shown in figure 2 are not present in comparable simulations driven by constant stress. Our focus is on analyzing the vortex force $V_f = u^{St} \times \delta\mathbf{r}$ from LES solutions for varying wave age when $\tau$ is replaced by $A^{(i)}$, i.e., in the presence of intermittent wave breaking as depicted in figure 2.

2. Results

For a wave field propagating solely in the alongwind direction ($x-$direction) $u^{St} = u^{St}(z)\hat{i}$ and then the vortex force has only two non-zero components

$$V_f = u^{St}(0\hat{i}, -\bar{v}_z \hat{j}, \bar{v}_x \hat{k})$$  \hspace{1cm} (2)

where the spanwise and vertical vorticity

$$\bar{v}_y = \left( \frac{\partial v}{\partial z} - \frac{\partial w}{\partial x} \right) \text{ and } \bar{v}_x = \left( \frac{\partial v}{\partial x} - \frac{\partial w}{\partial y} \right).$$  \hspace{1cm} (3)

With an externally imposed Stokes profile the vortex force is simply the resolved vorticity field weighted by an exponentially decaying function of $z$. In our model
Figure 3: Vertical profiles of root-mean-square (rms) values of the vortex force in the presence of breaking waves from LES at a wind speed $U_a = 15\text{ms}^{-1}$. Panels a) and b) are $(V_{fy}, V_{fz})$ components, respectively. Wave age $c_p/\nu_a = [19, 23, 30]$ (red, blue, green) curves respectively. No breaking is solid black curve. The vortex force is normalized by the water friction velocity and mixed layer depth $\nu_a^2/|h|$. There is no feedback mechanism allowing breakers to modify Stokes drift, but breaking does interact with the vortex force; i.e., in (1) each breaker impulse $\mathbf{A}^{(i)}$ generates $\mathbf{v}$ capable of coupling to the vortex force as indicated by (2). For winds and waves approaching equilibrium our PDF of breaking emphasizes small scales, whereas for strongly forced growing seas is weighted more heavily towards the peak in the wave height spectrum [10]. The scale and magnitude of the vorticity generated by breaking thus depend on wave age [19] and wind speed [20] which potentially makes the vortex force intermittent and sea state dependent.

In figures 3 and 4 we show vertical profiles of the root-mean-square (rms) and flatness (or kurtosis) of $\mathbf{V}_f$. (Statistics at each $z$ are obtained by averaging across $x-y$ planes and over 20 3D volumes.) First we notice that overall the strength of breaking impacts the statistical moments. In all cases the fluctuations in vortex force are large compared to the average surface stress, hence both components...
of $V_f$ are dynamically important compared to shear forces as expected for Langmuir turbulence [17; 13]. Formally the vertical component $V_{fz}$ is analogous to a buoyancy force in the equations of motion with its mean value absorbed in the hydrostatic balance [3], but the fluctuations in the vertical component of vortex force are important. Surprisingly, near the water surface the rms intensity of $V_{fy}$ decreases in the presence of breaking while the vertical component $V_{fz}$ is largely unaffected. A spectral analysis of the vortex force (not presented) shows that these trends hold across spatial scale. Previously we reported [10] that the main impact of breaking is to weaken the coherence of Langmuir cells accompanied by small reductions in the spanwise variance $v^2$ compared to simulations driven by constant stress alone. This translates into lower values of $V'_{fy}$ leaving $V'_{fz}$ unaffected. This is expected based on the definitions (2) and (3). Digging deeper into the statistics we notice that breaking alters the flatness of $V_f$ as shown in figure 4. The large departures from a Gaussian distribution (flatness values = 3) indicate that the vortex force has taken on a spotty spatially intermittent character likely mimicking the intermittency of the breaker distribution as shown in figure 2. The intermittent character of the vortex force is further quantified in the PDFs shown in figure 5. The PDF of $V_{fy}$ is symmetrical, non-Gaussian, and exhibits long tails in the presence of strong breaking. Meanwhile the PDF of the vertical vortex force is clearly asymmetrical, being biased towards positive values in the presence of vigorous breaking.

3. Discussion

These statistical distributions highlight that breaking and vortex force can couple in our LES. Our interpretation of how this coupling occurs is based on laboratory observations of breaking waves [2] and the Craik-Leibovich instability mechanism CL2 [3]. In figure 6 we show the time and space evolution of the flowfields induced by an idealized 2D breaker in a laboratory wave tank. The aftermath of this breaking event is clear. A coherent spanwise vortex is created by the overturning breaker, and as time advances it propagates downstream and descends deeper into the water. The lifetime of this vortex is long, more than 50 wave periods. Based on these measurements we conclude that the vorticity field generated by
Figure 6: Streamlines illustrating the mean flow development in the aftermath of a 2D breaking wave from laboratory measurements of Melville et al. [2]. The color code shows the magnitude of the mean velocity. The main feature is a coherent vortex with axis aligned with the y–spanwise direction. It slowly propagates downstream and deepens as time advances. From top-to-bottom the non-dimensional time $t = [3, 10.5, 26.5, 34.5, 42.5, 50, 58]$. $t$ and streamwise distance $x$ are made dimensionless by the pre-breaking wave period and wavelength.
Vortex generated by a breaking wave

Figure 7: Sketch illustrating the CL2 instability mechanism for breaker-induced generation of Langmuir circulations, adapted from [3]. The initial forward impulse from a 3D breaking wave generates a surface current perturbation \( u(y,z) \), spanwise vorticity \( \omega_y \) beneath the water surface, and vertical vorticity \( \omega_z \) at the spanwise edges of the breaker. \( \omega_y \) is amplified by the vortex force \( V_{fy} \) and enhances the CL2 instability. Positive signed \( \omega_y \) generated by breaking biases the vertical component \( V_{fz} \) towards positive values.

breaking contains a coherent anisotropic flow structure. In figure 7 the essential ingredients of the CL2 mechanism are presented. We have modified the original Craik-Leibovich interpretation to include the idealized single breaking wave depicted in figure 6.

A scenario of the CL2 instability process instigated by breaking is as follows: Initially breaking supplies a forward impulse \( u(y,z) \) that leaves behind a vorticity field with preferred orientation and sign \((\omega_y, \omega_z, \mathbf{k})\). Along the centerline of the perturbation \( y = 0 \), \( \omega_y \rightarrow \omega_y \), \( \mathbf{j} \) and the sign of the vortex rotation is positive as in figure 6. However the breaking process is taken to be 3D in our simulations, i.e., breakers span a finite length \( \lambda \) in the \( y \)-direction. As vortex lines are continuous, at the edges \( y = \pm \lambda/2 \) the spanwise breaker vortex must bend vertically upward towards the water surface as depicted in figure 7; thus a 3D breaking wave generates significant amounts of vertical vorticity \( \pm \omega_z \) as well as spanwise vorticity \( \omega_y \). Given this seed perturbation the CL2 mechanism then becomes activated.

The breaker induced vorticity components directly contribute to the vortex force as indicated by (2). We immediately notice with \( \omega_y > 0 \) a positive signed vertical vortex force \( V_{fz} \) is generated along the centerline \( y = 0 \). The PDF distribution in figure 5b reflects this. It is strongly asymmetrical with a bias towards positive values in the presence of large scale breaking. Vertically oriented vorticity is crucial to the CL2 mechanics. Since the sign of the vertical vorticity rotation is opposite at the spanwise edges of each breaker, equal strength but opposing vortex forces \( V_{fy} = u^H \omega_z \) are generated at \( y = \pm \lambda/2 \). Our idealization of the CL2 process is symmetrical about the plane \( y = 0 \) which is reflected in the symmetrical PDF of \( V_{fy} \) shown in figure 5. Opposing vortex forces \( V_{fy} \) promote the CL2 instability. As time advances from the initial breaker impulse the two opposing spanwise vortex
forces laterally squeeze the interior flow enhancing the $u-$perturbation. This further amplifies the initial vertical vorticity, reinforces the vortex force $V_{f_y}$ and the instability grows. The vertical gradient in Stokes drift, acting over multiple wave periods, then tilts the vertical vorticity downwind creating streamwise vorticity $\omega_x \hat{i}$ in a rapid-distortion process as discussed by Leibovich [3] and Teixeira & Belcher [9].

We interrogated the LES solutions reported in [10] looking for evidence for the CL2 mechanics discussed in figure 7. Classic streamwise oriented Langmuir circulations are found in the presence of breaking waves similar to those reported by other investigators [13; 15] However, we also notice that if breaking occurs at large scales and generates significant amounts of vorticity, then the convergence resulting from powerful spanwise vortex forces $\pm V_{f_y}$ leads to vigorous downwelling “jets”. The lifecycle of a breaker induced “jet” is shown in figure 8. The lateral convergence of vertical vorticity generated by a 20m breaker under the action of the vortex force is clearly evident in this example. We found that these coherent structures, which persist well below the water surface, co-exist with traditional Langmuir cells. Overall our LES results suggest that breaker vorticity couples with the CL2 instability mechanism and unique coherent structures emerge from the interactions between waves and turbulence.
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