An Upper-Boundary Gravity-Wave Absorbing Layer for NWP Applications

Joe Klemp, Jimy Dudhia, & Alex Hassiotis
National Center for Atmospheric Research & Penn State University
Preventing Artificial Reflection of Vertically Propagating Gravity-Wave Energy

- Radiation upper boundary conditions assume simplified conditions not well suited for real-data simulations.

- Rayleigh damping upper absorbing layer widely used for idealized simulations, but requires a known reference state.

- Horizontal diffusion absorbing layer often too weak due to linear stability constraints, and vertical diffusion alters the background environment.
Implicit Rayleigh Absorbing Layer in Split-Explicit Time Integration

Height-coordinate equations \((V_H, w, \pi, \theta)\):

\[
V_H^{\tau+\Delta\tau} = V_H^\tau - \Delta\tau \left( c_p \theta^t \nabla_H \pi'^\tau - F_{V_H}^t \right)
\]

\[
\pi_1 = \pi^\tau - \Delta\tau \left\{ C^t \left[ \nabla \cdot \rho^t \theta^t V_H^{\tau+\Delta\tau} + \frac{1}{2} \partial_z (\rho^t \theta^t w^\tau) \right] - F_{\pi}^t \right\}
\]

\[
\begin{align*}
  w^* &= w^\tau - \Delta\tau \left( c_p \theta^t \partial_z \pi'^{\tau*} - g \frac{\theta'}{\theta} - F_{w}^t \right) \\
  \pi^* &= \pi_1 - \frac{1}{2} \Delta\tau C^t \partial_z (\rho^t \theta^t w^*)
\end{align*}
\]

\[
w^{\tau+\Delta\tau} = w^* - R_w \Delta\tau w^{\tau+\Delta\tau}
\]

\[
\theta'^{\tau+\Delta\tau} = \theta^\tau - \Delta\tau \left( \partial_z \theta^t w^{\tau+\Delta\tau} - F_{\theta}^t \right)
\]

\[
\pi'^{\tau+\Delta\tau} = \pi_1 - \frac{1}{2} \Delta\tau C^t \partial_z (\rho^t \theta^t w^{\tau+\Delta\tau})
\]
Combining the intermediate steps, the $w$ equation becomes:

$$\frac{\partial w}{\partial T} + c_p \theta^t \frac{\partial}{\partial z} \overline{\pi'^z} - g \frac{\theta'^t}{\theta} + R_w w^{\tau+\Delta \tau} - \frac{\Delta T^2 c_t^2}{4 \rho^t \theta^t} \frac{\partial}{\partial z^2} \left\{ \rho^t \theta^t R_w w^{\tau+\Delta \tau} \right\} = F_w^t$$

- Normal Rayleigh damping term that vanishes in hydrostatic limit
- Additional damping term that remains effective in hydrostatic limit
Steady 2-D Linear Wave Equation
Implicit Rayleigh Absorbing Layer

\[ w(x, \tilde{z}) = \hat{w}(\tilde{z}) e^{ikx} \quad \tilde{z} = \frac{Nz}{U} \]

Wave equation within absorbing layer:

\[ \frac{\partial}{\partial \tilde{z}^2} \left\{ (1 - F^2 - i\frac{1}{4}\alpha^2 \beta) \hat{w} \right\} + [1 - K^2 (1 - i\beta)] \hat{w} = 0 \]

where:

\[ K = \frac{kU}{N}, \quad F = \frac{f}{kU}, \quad \alpha = kc\Delta, \quad \beta = \frac{R_w}{kU} \]

Wave equation below absorbing layer:

\[ \hat{w}_l(\tilde{z}) = C_1 e^{i\Lambda_z (\tilde{z} - \tilde{z}_d)} + C_2 e^{-i\Lambda_z (\tilde{z} - \tilde{z}_d)}, \quad \Lambda_z = \left( \frac{1 - K^2}{1 - F^2} \right)^{1/2} = \frac{N\lambda_z}{U} \]

Solve by matching \( p \) and \( w \) at bottom of absorbing layer:

\[ r = \left| \frac{C_2}{C_1} \right| = \left| \frac{\Lambda_z \hat{w}(\tilde{z}_d) + i\partial_{\tilde{z}} \hat{w}(\tilde{z}_d)}{\Lambda_z \hat{w}(\tilde{z}_d) - i\partial_{\tilde{z}} \hat{w}(\tilde{z}_d)} \right| \]
Reflection Coefficient for Implicit Rayleigh Absorbing Layer

\[ \beta_w(\tilde{z}) = \beta_{max} \sin^2 \left( \frac{\pi (\tilde{z} - \tilde{z}_d)}{2 (\tilde{z}_t - \tilde{z}_d)} \right) \]

Implicit Damping in Adjustment Step

\[ K = F = \alpha = 0.1 \]
Linear Mountain-Wave Simulation with Implicit Rayleigh Absorbing Layer

Vertical Velocity for 10 m Bell Mountain with $a = 10$ km half-width

Linear Analytic Solution  Numerical Simulation at $Ut/a=108$
Mesoscale & Microscale Meteorological Division / NCAR

Vertical Momentum-Flux Profiles

\[ D_R = 10 \text{ km} \]
\[ D_R = 5 \text{ km} \]
Reflection Coefficient for $D_R/\lambda_z=1$

Default Value of damping coefficient in WRF/ARW V3.0 is $R_w = 0.2 \, \text{s}^{-1}$
Idealized Squall-Line Simulation

No absorbing layer

Absorbing layer with $R_w = 0.2 \text{ s}^{-1}$

1 h

5 h
WRF Forecast over Colorado Front Range

Model Initialized 04 Dec 2007 00 UTC

NCAR Mesa Site
WRF Forecast over Colorado Front Range

Model Initialized 04 Dec 2007 00 UTC

100 mb top, 5 km implicit Rayleigh damping layer

100 mb top, no upper damping layer

$\text{Model Initialized 04 Dec 2007 00 UTC}$

$\text{WRF Forecast over Colorado Front Range}$