Research on Atmospheric Data Assimilation Techniques for Antarctic Applications:
Schemes and Preliminary Results

Qingnong Xiao¹, Chengsi Liu², and Kekuan Chu¹

1. College of Marine Science, University of South Florida
2. Institute of Atmospheric Physics/LASG, Beijing, China
Outline

• Motivations

• Technical schemes

• Preliminary results

• Summary
Advanced data assimilation

- 3D-Var
  ✓ All data can be assimilated simultaneously, with a pre-defined background error covariance matrix.
  ✓ Equivalent to OI, but it can ingest non-conventional data.
  ✓ It is possible to add constraints to the cost function to control spurious noise.

- 4D-Var
  ✓ It is a non-sequential data assimilation technique, fitting observations in the whole assimilation window (optimal trajectory).
  ✓ It is applied in many operational centers.
  ✓ However, there are disadvantages compared with EnKF technique (TL and AD are difficult to code; background error covariance is evolved only within assimilation window and it is usually static at analysis time).

- Ensemble Kalman filter
  ✓ It is a hot topic in recent years, and research shows promising results.
  ✓ It is easy to design and code, and can include any physical process as needed.
  ✓ One of the prominent advantages is its flow-dependent background error covariance.
Advanced data assimilation

- Ensemble-based variational data assimilation, En3/4D-Var
  - It is proposed by Liu et al. (2008; 2009):
    
    
  
  - It is variational approach, minimizing a cost function to find the optimal analysis state.
  
  - It adopts the technique of EnKF to include the flow-dependent background error covariance from ensemble forecast.
  
  - It can be implemented in the existing variational data assimilation system without significant changes of the system setup.
  
  - Preliminary results from WRF En3/4D-Var are satisfactory.
En3D-Var (Lorenc 2003)

3D-Var
(incremental, precondition)

\[ J = \frac{1}{2} (x_a - x_b)^T B^{-1} (x_a - x_b) + \frac{1}{2} (H(x_a) - x_y)^T O^{-1} (H(x_a) - x_y) \]

\[ \delta x = x_a - x_b \quad B = U U^T \quad \delta x = U w \]

\[ J(w) = \frac{1}{2} w^T w + \frac{1}{2} (H U w + d)^T O^{-1} (H U w + d) \]

\[ \nabla w J = w + U H^T O^{-1} (H U w + d) \]

\[ W_a = W_a^{N-1} - \rho d \]

\[ \nabla J \neq 0 \]

\[ \nabla J = 0 \]

\[ x_a = x_b + U w_a \]

En3D-Var

\[ \delta x = x_a - x_b \quad B = X_b' X_b \quad \delta x = X_b' w' \]

\[ J(w') = \frac{1}{2} w'^T w' + \frac{1}{2} (H X_b' w' + d)^T O^{-1} (H X_b' w' + d) \]

\[ \nabla w' J = w' + X_b' H^T O^{-1} (H X_b' w' + d) \]

\[ W_a = W_a^{N-1} - \rho d \]

\[ \nabla J \neq 0 \]

\[ \nabla J = 0 \]

\[ x_a = x_b + X_b' w'_a \]
En4D-Var

\[ J = \frac{1}{2} (x_a - x_b)^T B^{-1} (x_a - x_b) + \sum_{i=0}^{N} \frac{1}{2} (HM_{0-i}(x_a) - y_i)^T O^{-1} (HM_{0-i}(x_a) - y_i) \]

En4D-Var (generalized En3D-Var)

\[ MX_b' = \frac{1}{\sqrt{N-1}} (MX_{bi} - MX_b) \]

\[ J(w) = \frac{1}{2} w^T w + \frac{1}{2} \sum_{i=0}^{N} (HM_{0-i}X_b'w + d_i)^T O^{-1} (HM_{0-i}X_b'w + d_i) \]

\[ \nabla_w J = w + \sum_{i=0}^{N} X_b'^T M_{i=0}^T H_i^T O^{-1} (HM_{0-i}X_b'w + d_i) \]

\[ W_{a'} = W_{a''} - \rho d \]

\[ \nabla J \neq 0 \]

\[ \nabla J = 0 \]

\[ x_a = x_b + X_b' w_a' \]
En4D-Var

\[ J = \frac{1}{2} (x_a - x_b)^T B^{-1} (x_a - x_b) + \sum_{i=0}^{N} \frac{1}{2} (HM_{0-i}(x_a) - y_i)^T O^{-1} (HM_{0-i}(x_a) - y_i) \]

En4D-Var (generalized En3D-Var)

\[ M_{X_b} = \frac{1}{\sqrt{N-1}} (MX_{bi} - MX_b) \]

\[ J(w) = \frac{1}{2} w^T w + \frac{1}{2} \sum_{i=0}^{N} (HM_{0-i}X_b w + d_i)^T O^{-1} (HM_{0-i}X_b w + d_i) \]

\[ \nabla_w J = w + \sum_{i=0}^{N} X_b^T M_{i=0}^T H_i^T O^{-1} (HM_{0-i}X_b w + d_i) \]

\[ W_{a}^N = w_{a}^{N-1} - \rho d \]

\[ \nabla J \neq 0 \]

\[ \nabla J = 0 \]

\[ x_a = x_b + X_b w_a' \]
En4D-Var

\[ J = \frac{1}{2} (x_a - x_b)^T B^{-1} (x_a - x_b) + \frac{1}{2} \sum_{i=0}^{N} (HM_{0-i} (x_a) - y_i)^T O^{-1} (HM_{0-i} (x_a) - y_i) \]

En4D-Var (generalized En3D-Var)

\[ \text{MX}_b = \frac{1}{\sqrt{N-1}} (\text{MX}_{bi} - \text{MX}_b) \]

\[ J(w) = \frac{1}{2} w^T w + \frac{1}{2} \sum_{i=0}^{N} (HM_{0-i} X_b w + d_i)^T O^{-1} (HM_{0-i} X_b w + d_i) \]

\[ \nabla_w J = w + \sum_{i=0}^{N} X_b^T M_{i=0}^T H_i O^{-1} (HM_{0-i} X_b w + d_i) \]

\[ w'_a = w'_a - \rho d \]

\[ \nabla J \neq 0 \]

adjoint Tangent linear

\[ \nabla J = 0 \]

\[ x_a = x_b + X_b w'_a \]
En4D-Var

\[ J = \frac{1}{2} (x_a - x_b)^T B^{-1} (x_a - x_b) + \sum_{i=0}^{N} \frac{1}{2} (HM_{0-i}(x_a) - y_i)^T O^{-1} (HM_{0-i}(x_a) - y_i) \]

En4D-Var (generalized En3D-Var)

\[ \nabla_w J = w + \sum_{i=0}^{N} \frac{X_b^T M_i^T H_i O^{-1} (HM_{0-i} X_b w + d_i) - \rho d}{\sqrt{N-1}} \]

\[ \nabla J \neq 0 \] adjoint

\[ \nabla J = 0 \] Tangent linear

\[ x_a = x_b + X_b w_a' \]
En4D-Var

En4D-Var (generalized En3D-Var)

\[ J = \frac{1}{2} (x_a - x_b)^T B^{-1} (x_a - x_b) + \sum_{i=1}^{N} \frac{1}{2} (HM_{0,i}(x_a) - y_i)^T O^{-1}(HM_{0,i}(x_a) - y_i) \]

\[ MX_b' \approx \frac{1}{\sqrt{N-1}} (MX_{bi} - MX_b) \]

\[ J(w) = \frac{1}{2} w^T w + \frac{1}{2} \sum_{i=0}^{N} (HM_{0,i}X_b'w + d_i)^T O^{-1}(HM_{0,i}X_b'w + d_i) \]

\[ \nabla_w J = w + \sum_{i=0}^{N} X_b' X_{i=0}^T H_{i=0}^T O^{-1}(HM_{0,i}X_b'w + d_i) \]

\[ \nabla J \neq 0 \]

\[ \nabla J = 0 \]

\[ x_a = x_b + X_b'w_a' \]

adjoint Tangent linear

En4D-Var (Opt.2)

\[ HMX_b' \approx \frac{1}{\sqrt{N-1}} (HMX_{bi} - HMX_b) \]

\[ J(w) = \frac{1}{2} w^T w + \frac{1}{2} \sum_{i=0}^{N} (HMX_b'w + d_i)^T O^{-1}(HMX_b'w + d_i) \]

\[ \nabla_w J = w + \sum_{i=0}^{N} (HM_{0,i}X_b'w + d_i)^T O^{-1}(HM_{0,i}X_b'w + d_i) \]

\[ \nabla J \neq 0 \]

\[ \nabla J = 0 \]

\[ x_a = x_b + X_b'w_a' \]
En4D-Var

En4D-Var (generalized En3D-Var)

\[ J = \frac{1}{2} (x_a - x_b)^T B^{-1} (x_a - x_b) + \sum_{i=0}^{N-1} \frac{1}{2} (HM_{0,i}(x_a) - y_i)^T O^{-1}(HM_{0,i}(x_a) - y_i) \]

\[ MX_b' \approx \frac{1}{\sqrt{N-1}} (MX_{bi} - MX_b) \]

\[ J(w) = \frac{1}{2} w^T w + \frac{1}{2} \sum_{i=0}^{N-1} (HM_{0,i}X_b'w + d_i)^T O^{-1}(HM_{0,i}X_b'w + d_i) \]

\[ \nabla_w J = w + \sum_{i=0}^{N} (HM_{0,i}X_b')^T O^{-1} (HM_{0,i}X_b'w + d_i) - \rho \nabla J \]

\[ \nabla J \neq 0 \]

Tangent linear

adjoint

\[ x_a = x_b + X_b'w_a' \]

\[ \nabla J = 0 \]

En4D-Var (Opt.2)

\[ HMX_b' \approx \frac{1}{\sqrt{N-1}} (HMX_{bi} - HMX_b) \]

\[ J(w) = \frac{1}{2} w^T w + \frac{1}{2} \sum_{i=0}^{N-1} (HMX_b'w + d_i)^T O^{-1}(HMX_b'w + d_i) \]

\[ \nabla_w J = w + \sum_{i=0}^{N} (HM_{0,i}X_b')^T O^{-1} (HM_{0,i}X_b'w + d_i) \]

\[ \nabla J \neq 0 \]

\[ x_a = x_b + X_b'w_a' \]

\[ \nabla J = 0 \]
En4D-Var

\[ J = \frac{1}{2} (x_a - x_b)^T B^{-1} (x_a - x_b) + \sum_{i=0}^{N-1} \frac{1}{2} (\text{HMX}_0 w(x_a) - y_i)^T O^{-1} (\text{HMX}_0 w(x_a) - y_i) \]

En4D-Var (generalized En3D-Var)

\[ \text{MX}_b \approx \frac{1}{\sqrt{N-1}} (\text{MX}_b - \text{MX}_b) \]

\[ J(w) = \frac{1}{2} w^T w + \frac{1}{2} \sum_{i=0}^{N-1} (\text{HMX}_b w + d_i)^T O^{-1} (\text{HMX}_b w + d_i) \]

En4D-Var (Opt.2)

\[ \text{HMX}_b \approx \frac{1}{\sqrt{N-1}} (\text{HMX}_b - \text{HMX}_b) \]

\[ J(w) = \frac{1}{2} w^T w + \frac{1}{2} \sum_{i=0}^{N-1} (\text{HMX}_b w + d_i)^T O^{-1} (\text{HMX}_b w + d_i) \]

\[ \nabla_w J = w + \sum_{i=0}^{N-1} \nabla_w \text{MX}_b w(x_a) \]

\[ \nabla_w J = w + \sum_{i=0}^{N-1} (\text{HMX}_b w + d_i)^T O^{-1} (\text{HMX}_b w + d_i) \]

\[ \nabla J \neq 0 \]

adjoint  Tangent linear

\[ \nabla J = 0 \]

\[ x_a = x_b + X_b w_a' \]

\[ x_a = x_b + X_b w_a' \]

\[ \nabla J = 0 \]
Some characteristics of En4D-Var

- En4D-Var uses the flow-dependent B matrix from ensemble forecast.

- It avoids tangent linear and adjoint models in its formulation (in Opt.2).

- It couples incremental approach with preconditioning using ensemble perturbation matrix.

- But sampling errors are introduced to En4D-Var (in Opt.2).
Experimental designs for OPP project

- Experiments with various data assimilation techniques for the Antarctic weather predictions through case studies and a month long verification.

- The data assimilation techniques to be tested are WRF 3D-Var, 4D-Var, En3D-Var and En4D-Var.

- The case selected is a cyclone penetrating the Western Antarctic Ice Sheet (WAIS) from 1200 UTC 3 through 1200 UTC 6 October 2007.

- Verification will be performed for the whole month of October 2007.

- Some preliminary results from case study using WRF 3D-Var has been finished.
Model Domains and Physics

**Model Grids:**
Two-way nesting
Outer grid: 220*290 (45 km)
Inner grid: 442*418 (15 km)
43 vertical levels

**Model Physics:**
WSM 5-class scheme
RRTM long wave radiation scheme
Goddard short wave radiation scheme
Mellor-Yamada-Janjic TKE PBL scheme
Kain-Fritsch (new Eta) cumulus scheme
Background Error Covariance

National Meteorological Center method (Parrish and Derber 1992)

The differences between 24- and 12-h forecasts in October 2007 were taken as background errors to calculate the background error covariance.
The Antarctic cyclone analysis (FNL)
Initial state at 1200 UTC 3 Oct.

FNL

CNTL

ASSIM1 (12-h 3DVAR cycling)

ASSIM2 (24-h 3DVAR cycling)
24-hr forecast at 1200 UTC 4 Oct.
48-hr forecast at 1200 UTC 5 Oct.
Intensity change

![Graph showing intensity change over forecast hours. The graph plots CSLP (hpa) against forecast hour (hr). Different lines represent different assimilation methods: FNL, CNTL, ASSIM1, and ASSIM2.]
Summary and On-going Work

- Data assimilation schemes for the OPP project are designed. We will test 3DVAR, 4DVAR, En3DVAR and En4DVAR for the Antarctic applications.

- One-month runs from 0000 UTC 01 till 0000 UTC 31 October 2007, two times a day at 0000 UTC and 1200 UTC from the NCEP FNL analysis with AMPS domain configuration has been conducted. Background error covariance has been generated.

- WRF 3DVAR experiments for the Antarctic cyclone case on 3-6 October 2007 has been started. WRF 4DVAR and En3/4DVAR experiments will follow.

- We will perform one-month verifications for the designed data assimilation techniques in the future.
Thank you!

Questions and comments are welcome.
Proof-of-concept test with shallow water model

Evolution of domain-average RMSE
Single observation test
(single T observation at 850hpa at 24-12Z Jan.)

WRF-Var  En4D-Var without localization  En4D-Var with localization

Increments of wind vector and temperature at 1000hpa