Preliminary Results of Directly Assimilating Wind Speed and Direction
Based on WRFDA 3D-Var System

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1. Introduction

At present, the assimilation of \( u \) and \( v \) wind vector components calculated from wind speed (SP) and wind direction (DIR) observations (obs) is used as the only scheme for assimilating wind observations into the Weather Research and Forecasting Data Assimilation (WRFDA) three-dimensional variational data assimilation (3D-Var) system. The observational error of the \( u \) and \( v \) components is assigned based on the SP obs, which does not consider the observational error of DIR obs. Obviously, such a practice means that wind direction data is considered as a perfect and true atmosphere state when \( u \) and \( v \) components are calculated from SP and DIR obs. However, DIR obs has a obvious observational error, especially when SP is small. In this situation, as long as the observation minus background (OMB) of the \( u \) or \( v \) component is smaller than \( n \)-times (\( n=5 \) in default) of SP observational error, \( u \) or \( v \) will all be assimilated no matter how large the OMB of direction may be. This assumption can be dangerous, so instead, the wind data selection and rejection will be more reasonable if DIR observational error can be considered. In order to clarify the issue, a new method of directly assimilating SP and DIR was introduced in WRFDA 3D-Var system. The rest of this paper briefly describes the theory, code development and preliminary results.

2. Observation operator

2.1 Observation operator

The WRFDA 3D-Var system originates from the 3D-Var system in MM5 developed by Barker et al. (2004) based on multivariate incremental formulation (Courtier et al. 1994). The simplified form of a prescribed cost function can be expressed as:

\[
x_j = x_j + \left( B^{-1} + H^T R^{-1} H \right)^{-1} H^T R^{-1} \left( y_j - H(x_j) \right)
\]  

(1)

where \( B \) is the background error covariance matrix, \( R \) is the observation error covariance matrix, \( H \) is the linearized version of nonlinear operator \( H \), and superscript \( T \) means the adjoint of this matrix. \( y_j - H(x_j) \) is the innovation vector (IV).

According to the Eq. (1), the tangent linear operator (TL) and adjoint operator (AD) are needed. The nonlinear observation operators for SP and DIR are expressed as Eq. (2),

\[
\begin{align*}
\text{speed} & = \sqrt{u^2 + v^2} \\
\text{direction} & = n\pi + \arctan \left( \frac{u}{v} \right)
\end{align*}
\]  

(2)
According to Eq. (2), the TL and AD were developed in WRFDA 3D-Var system. It can be expressed as:

**Tangent linear process**

\[
\begin{pmatrix}
\frac{\delta u}{\delta \nu} \\
\frac{\delta v}{\delta \nu} \\
\frac{\delta sp}{\delta \nu} \\
\frac{\delta dir}{\delta \nu}
\end{pmatrix}^n = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\frac{u}{\sqrt{u^2+v^2}} & \frac{v}{\sqrt{u^2+v^2}} & -\frac{u}{u^2+v^2} & 0 \\
\frac{u}{u^2+v^2} & \frac{v}{u^2+v^2} & 0 & 0
\end{pmatrix}^n \begin{pmatrix}
\frac{\delta u}{\delta \nu} \\
\frac{\delta v}{\delta \nu} \\
\frac{\delta sp}{\delta \nu} \\
\frac{\delta dir}{\delta \nu}
\end{pmatrix}^{n-1}
\]

**Adjoint process**

\[
\begin{pmatrix}
\frac{\delta u}{\delta \nu} \\
\frac{\delta v}{\delta \nu} \\
\frac{\delta sp}{\delta \nu} \\
\frac{\delta dir}{\delta \nu}
\end{pmatrix}^n = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\frac{u}{\sqrt{u^2+v^2}} & \frac{v}{\sqrt{u^2+v^2}} & -\frac{u}{u^2+v^2} & 0 \\
\frac{u}{u^2+v^2} & \frac{v}{u^2+v^2} & 0 & 0
\end{pmatrix}^n \begin{pmatrix}
\frac{\delta u}{\delta \nu} \\
\frac{\delta v}{\delta \nu} \\
\frac{\delta sp}{\delta \nu} \\
\frac{\delta dir}{\delta \nu}
\end{pmatrix}^{n-1}
\]

2.2 Innovation vector

Innovation vector (IV) is defined as OMB. SP and DIR obs in grid coordinate, which is the same coordinate as the background, are first calculated as earth-relative. The background SP and DIR are derived from the model wind vector components. Herein, the definitive range of IV dir is defined as [-180, 180], which guarantees the DIR analysis always falls within an area of smaller absolute value (i.e., 360° ± OMB) when an absolute value of OMB is larger than 180°.

3. Sensitivity experiments

3.1 Experiment scheme

The TAMDAR observations with speeds smaller than 15 ms⁻¹ for 12 UTC June 1 are used (DIR observational error is larger when SP is small). In order to present the advantage of assimilating SP and DIR, we perturbed the obs with two methods: one is to add a constant perturbation of 10°, 30°, 90° to each DIR obs respectively (‘Cons_pert’); the other is to add Gaussian distribution with a standard deviation of 10°,30°,90° to each DIR obs (‘Gaus_pert’). Fig.1 presents the distribution of Gauss-pattern perturbation of N~(0,30) for 3263 wind obs. In the following parts, we take the experiments with a constant perturbation of 30° as an example. The wind observation will be rejected when DIR OMB is larger than 50° or twice of the observational error (Fig.2), which is from our past study (Gao et al. 2011). The ‘CTL’ means the experiment assimilating the original data, ‘Cons_pert_30’ is the experiment ‘Cons_pert’ with a standard deviation of 30°. Exp.name_uv and Exp.name_wind means an assimilation experiment by assimilating u and v components and SP and DIR respectively.

![Figure 1. The distribution of Gauss-pattern wind direction perturbation meeting N~(0,30)](image)

3.2 Comparison between CTL_uv and CTL_wind

For comparing the performance of each sensitivity experiment, we choose the experiment CTL_wind as the bench-mark. The reason why CTL_wind was selected is that assimilating SP and DIR
(ass_\_wind) is better than assimilating u and v components (ass_\_uv) through comparing the 6 h forecast with GFS analysis (Fig.3). The positive improvement of the forecast can be obtained above 800 hPa from CTL_\_wind since it rejected some bad obs, although it assimilates similar data amount to ass_\_uv. However, below 800 hPa, the assimilation of SP and DIR rejected more data because the OMB of direction was considered as Table 1. Actually, the 'bad' data just means OMB is larger than the defined threshold value and is not absolutely bad. Some obs rejected are still good enough to provide some weather information where the background is bad, especially below 800 hPa. Thus, the suitable threshold value of OMB need be researched further.

![Graph 1](image1.png)

Figure 2. The observational errors of TAMDAR wind speed (left) and direction (right) by speeds

![Graph 2](image2.png)

Figure 3. The difference of RMS of 6-h forecast of CTL_\_uv and CTL_\_wind against GFS analysis Positive means that assimilating SP and DIR is better than assimilating u and v components

<table>
<thead>
<tr>
<th>Exp. \ OMB</th>
<th>&gt;100°</th>
<th>70°-100°</th>
<th>40°-70°</th>
<th>20°-40°</th>
<th>&lt;20°</th>
</tr>
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<tr>
<td>CTL__uv</td>
<td>57</td>
<td>76</td>
<td>264</td>
<td>702</td>
<td>2089</td>
</tr>
<tr>
<td>CTL__wind</td>
<td>0</td>
<td>0</td>
<td>117</td>
<td>680</td>
<td>2084</td>
</tr>
</tbody>
</table>

### 3.3 Analysis verification

Figure 4 presents the RMS profiles of analyses of Cons_pert_30 against the bench-mark with RMS difference between ass_\_uv and ass_\_wind. The average improvement for SP is up to 0.5 ms\(^{-1}\), and that is with 10° for DIR, which incorporates more effects for v component. The reason is given in Fig. 5. An average of near 20 (40) obs from 4\(^{th}\) to 8\(^{th}\) (12\(^{th}\) to 17\(^{th}\) ) level are rejected when SP are smaller (larger) than 3 (10) ms\(^{-1}\), which is responsible for smaller DIR OMB. Obviously, the rejected obs make some negative effects for the analysis.

The similar conclusion was drawn from 'Gaus_pert_30' except smaller RMS magnitude (not shown). It is related to Gauss-pattern perturbation distribution because the perturbation smaller than
30° is taking 68.8%. It means 68.8% obs in Gaus_pert_30 are added, which results in fewer 'bad' obs in Gaus_pert_30 than in Cons_pert_30.

Figure 4. The RMS of analysis of ass_uv and ass_wind against CTL_wind analysis in Cons_pert with a standard deviation of 30° and the difference (red circle-line) of RMS of ass_uv and ass_wind. Positive means that assimilating SP and DIR is better than assimilating u and v components

Figure 5. The profile of OMB of DIR and obs number assimilated in Cons_pert with a constant perturbation of 30° when SP<3m/s (left) and SP>10m/s (right)

3.4 Forecast verification

Figure 6 presents the verification of 6 h and 12 h forecast for Cons_pert_30. We can see the
improvement increases with height, which can be up to 0.6 ms$^{-1}$ for the 6 h forecast of Cons_pert_30, and 0.4 ms$^{-1}$ for 12 h forecast is good as expected. However, little negative effects can be seen at lower levels like in Fig. 3. It means that some of obs in lower levels that were rejected still can provide information, and the background in lower levels is poor quality in some ways.

Figure 6. The difference of RMS of 6h (left) and 12h (right) forecast of ass_uv and ass_wind against GFS analysis in Cons_pert with a constant perturbation of 30°. The unit is ms$^{-1}$ for U, V and SP, and degree (°) for DIR. Positive means that assimilating SP and DIR is better than assimilating u and v components. The RMS difference of DIR is divided by 10.

Figure 7 presents the obs number assimilated in CTL and Cons_pert. The more obs there are, the larger the standard deviation of the perturbation will be. So, the obs number assimilated should decrease with the standard deviation. We can see the obs number assimilated is steady for ass_uv from a standard deviation of 10° to 90°. Almost all of the bad obs (+90) are assimilated for ass_uv. Although there is no quality guarantee on the few remaining obs, we do partly solve the problem by assimilating SP and DIR.

Figure 7. The observation number assimilated in CTL and Cons_pert. +10, +30 and +90 means the Cons_pert experiment with a constant perturbation of 10°, 30°, 90° respectively.

4. Conclusion

A series of assimilation comparison experiments for u and v components, as well as assimilating wind speed and direction are presented. The goal of this study is to get more reasonable wind assimilation results after considering the wind direction observational error. Through the sensitivity experiments, we can see that assimilating wind speed and direction rejects some bad data effectively.
Additional positive forecast skill of 6 and 12 h forecasts can be obtained by assimilating wind speed and direction when compared to GFS analysis. However, some negative forecast skill can also be seen at lowest levels near surface because many obs were rejected there, which should have provided some useful information. This suggests that the background in the lower levels is of poor quality in some respects, and this issue needs further research.

5. References

