16th WRF Users Workshop

WRF Dynamics – Best Practices

Vertical coordinate and Treatment of Terrain

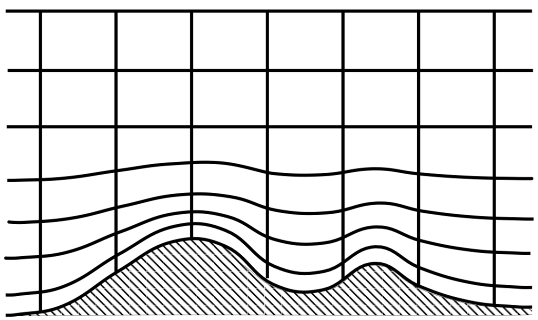
Gravity Wave Absorbing layer

Modifications for Moist LES Simulations

Joe Klemp, NCAR/MMM

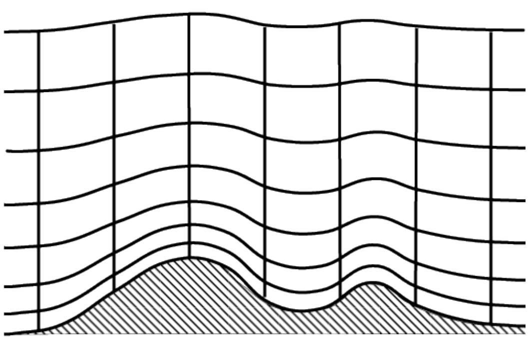
Terrain-Following Vertical Coordinate

Height based



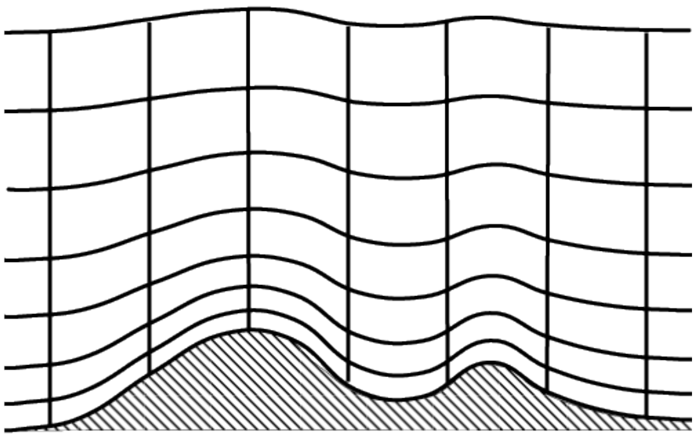
- Coordinate surfaces are fixed in time and space
- Typically employs a rigid lid upper boundary
- Diabatic heating occurs at constant volume
- Does not permit external (free-surface) wave modes

Pressure based (sigma)



- Coordinate surfaces move in response to pressure changes
- Typically employs a constant pressure upper boundary
- Diabatic heating occurs at constant pressure
- Allows external (free-surface) wave modes
- Full dynamical equations revert to the a hydrostatic sigma coordinate system with a simple switch (*non-hydostatic=.false.*)

WRF Terrain-Following Sigma (mass) Coordinate



Hydrostatic dry pressure: p_d

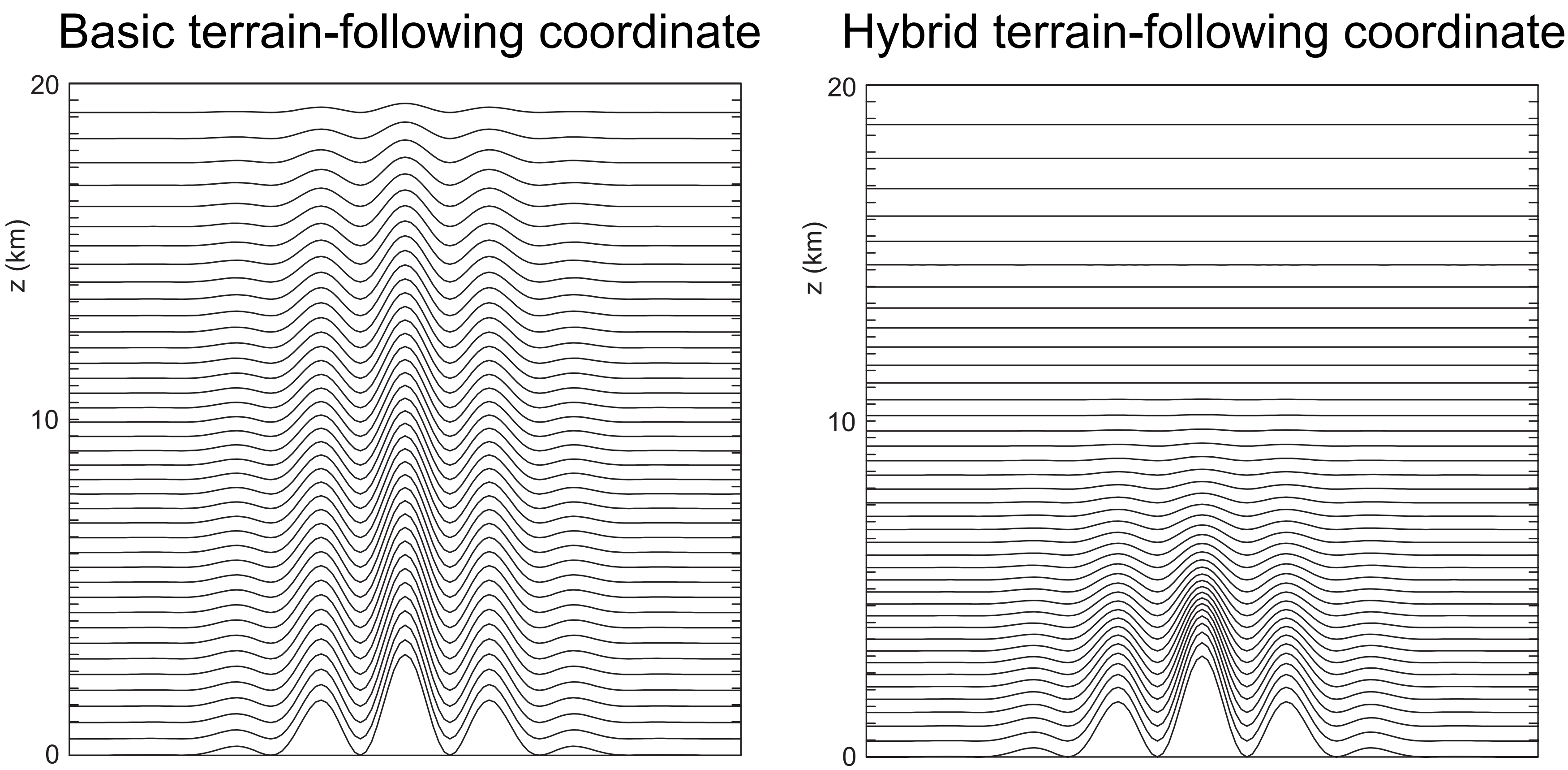
Vertical coordinate: $\eta = \dfrac{p_d - p_t}{p_s - p_t}$

g (column mass per unit area): $\mu = p_s - p_t$

Layer mass per unit area: $\rho_d \Delta z = -\dfrac{1}{g}\Delta p_d = -\dfrac{\mu}{g}\Delta\eta$

Conserved prognostic variables: $\mu, \quad U = \mu u, \quad V = \mu v, \quad W = \mu w, \quad \Theta = \mu\theta$

WRF Hybrid Pressure Vertical Coordinate



WRF Hybrid Pressure Vertical Coordinate

Based on dry hydrostatic pressure:

$$p_d = B(\eta)(p_s - p_t) + [\eta - B(\eta)](p_0 - \bar{p}_t) + p_t$$

$B(\eta)$: Relative weighting between terrain-following and pure dry hydrostatic pressure coordinate:

$$\eta = \frac{p_d - p_t}{p_s - p_t} \quad \text{for } B(\eta) = \eta \quad \text{(basic terrain following)}$$

$$\eta = \frac{p_d - p_t}{p_0 - \bar{p}_t} \quad \text{for } B(\eta) = 0 \quad \text{(pure pressure)}$$

Coordinate metric: $\mu_d(x, y, \eta, t) = \dfrac{\partial p_d}{\partial \eta} = B_\eta \, p_c + (1 - B_\eta)(p_0 - \bar{p}_t)$

$p_c = p_s - p_t$ ~ mass in each vertical column

= μ_d for $B(\eta) = \eta$ (basic terrain following)

WRF Hybrid Pressure Vertical Coordinate

Continuity equation for dry hydrostatic pressure:

$$\frac{\partial \mu_d}{\partial t} + (\boldsymbol{\nabla} \cdot \boldsymbol{V})_\eta = 0$$

$$-\int_1^0 \frac{\partial}{\partial t}\left(\frac{\partial p_d}{\partial \eta}\right) d\eta = -\int_1^0 B_\eta \frac{\partial p_c}{\partial t} d\eta = \boxed{\frac{\partial p_c}{\partial t} = \int_1^0 \boldsymbol{\nabla} \cdot \boldsymbol{V}_H \, d\eta}$$

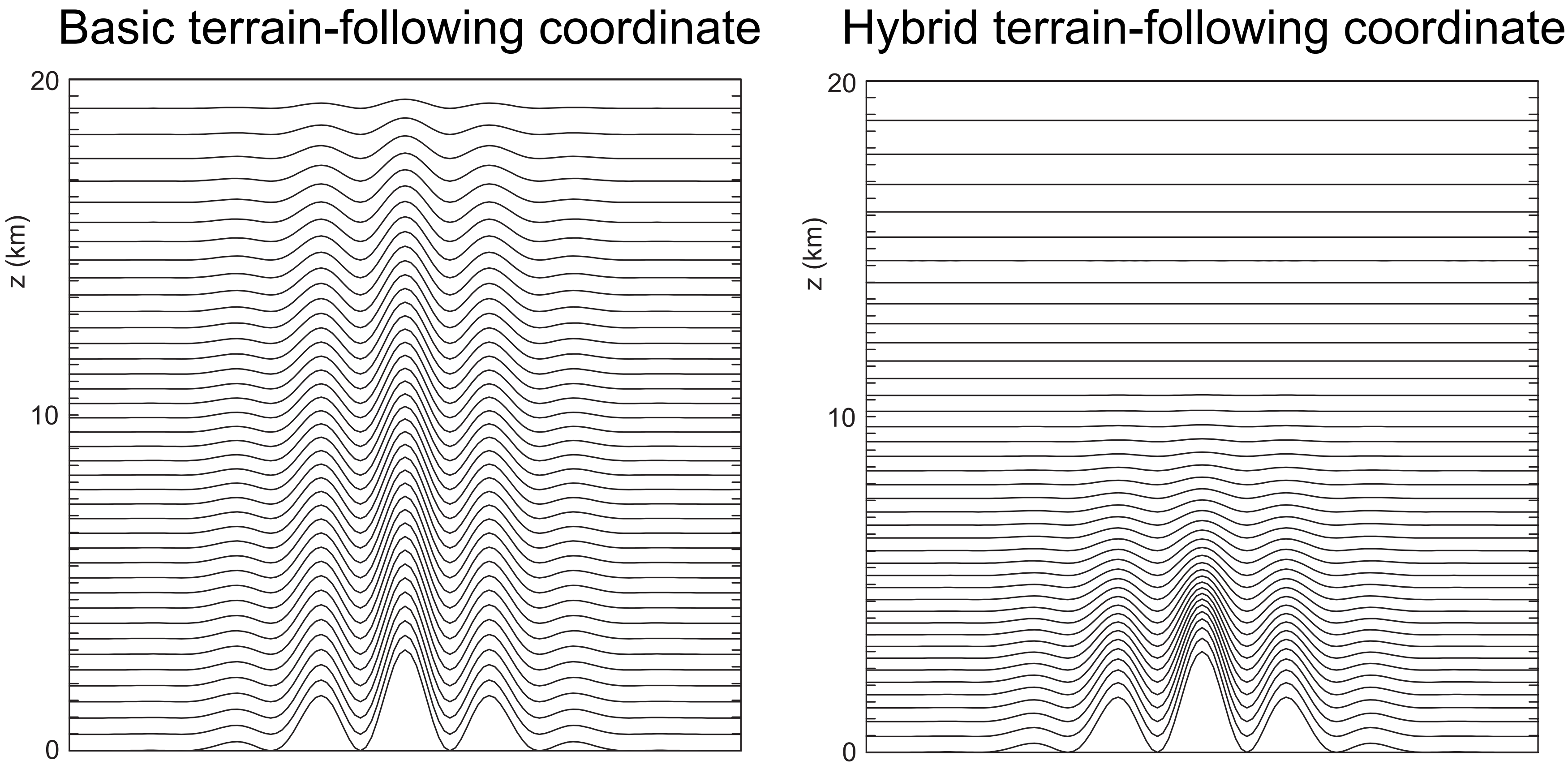
$$\boxed{\Omega = -\int_1^\eta \left(B_\eta \frac{\partial p_c}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{V}_H\right) d\eta}$$

Identical to equations solved for basic terrain following coordinate for $p_c = \mu_d$ and $B(\eta) = \eta$

Recover μ_d from: $\mu_d = C_1(\eta) p_c(x,y) + C_2(\eta)$

where $C_1(\eta) = B_\eta$ and $C_2(\eta) = (1 - B_\eta)(p_0 - \bar{p}_t)$ depend only on η

WRF Hybrid Pressure Vertical Coordinate



WRF Hybrid Pressure Vertical Coordinate

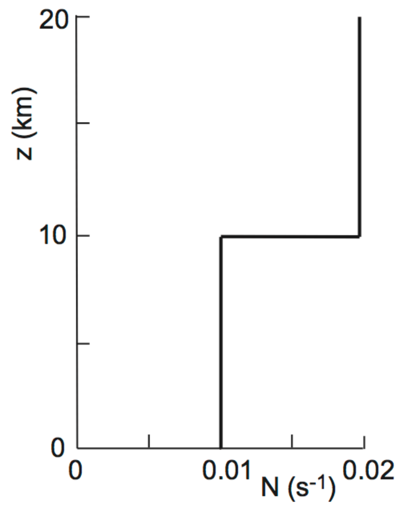
$$H = 2000 \text{ m}$$

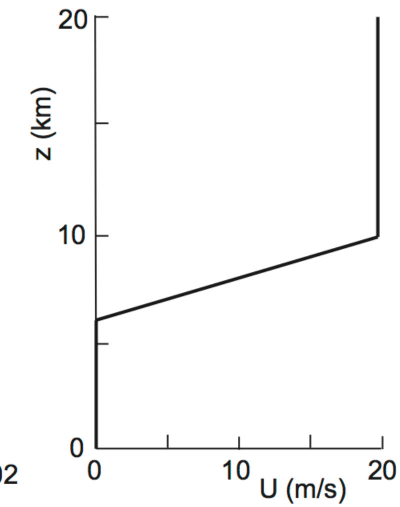
$$U_m = 20 \text{ m/s}$$

$$\Delta x = 1000 \text{ m}$$

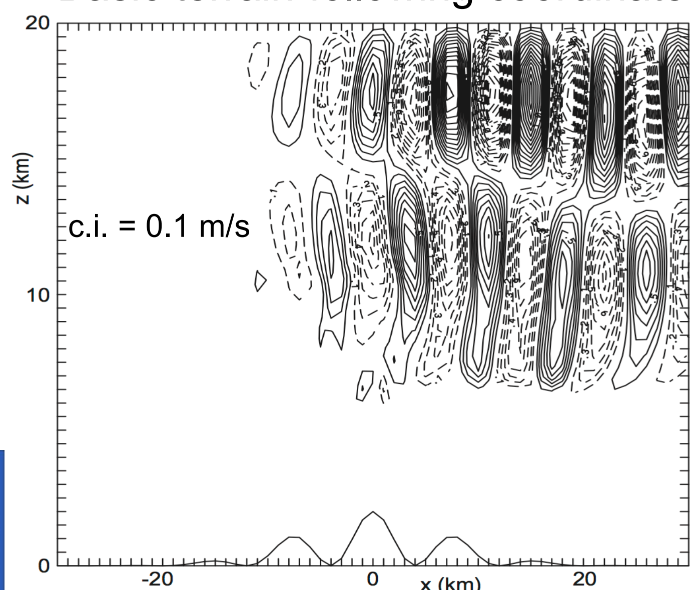
$$\Delta z = 500 \text{ m}$$

$$\Delta t = 12 \text{ s, ns} = 6$$

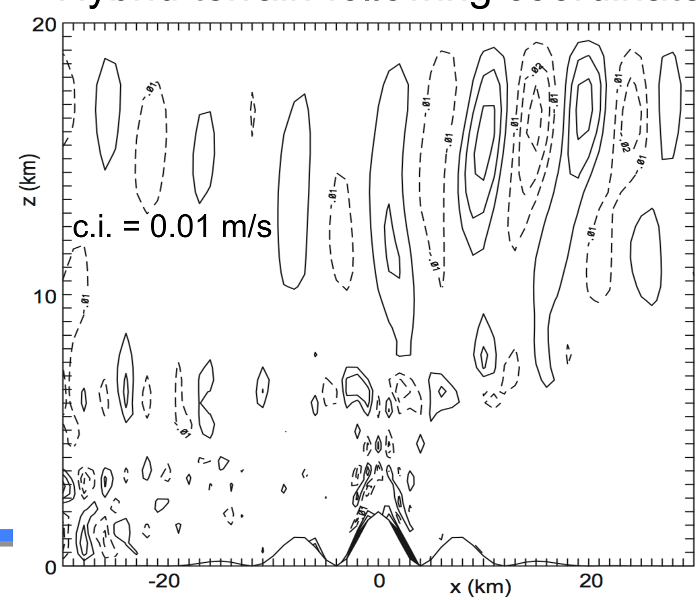




Basic terrain-following coordinate



Hybrid terrain-following coordinate



Gravity-Wave Absorbing layer

A constant pressure surface or a rigid lid upper domain boundary is totally reflective to upward propagating gravity-wave energy.

A absorbing layer in the upper portion of a model domain can absorb gravity-wave energy and prevent artificial reflections

Alternative approaches for an absorbing layer:

- Second order eddy viscosity (*damp_opt* = *1*)
  - Not effective in damping larger horizontal wavelengths
  - May alter upper level atmospheric structure
  - Not recommended
- Explicit Rayleigh damping on prognostic variables (*damp_opt* = *2*)
  - Can work well on idealized simulations of perturbations about a mean state
  - Not suitable for real data simulations
- Implicit Rayleigh damping on vertical velocity (*damp_opt* = *3*)
  - Works well for both idealized and real-data simulations
  - Can only be used in the nonhydrostatic equations (*non-hydrostatic* = *.true.*)

Gravity-Wave Absorbing layer

Implicit Rayleigh *w* Damping Layer
for Split-Explicit Nonhydrostatic NWP Models

Modification to small time step:

- Step horizontal momentum, continuity, and potential temperature equations to new time level: $U^{\tau+\Delta\tau}$ $\mu^{\tau+\Delta\tau}$ $\Omega^{\tau+\Delta\tau}$ $\Theta^{\tau+\Delta\tau}$
- Step vertical momentum and geopotential equations (implicit in the vertical): $W^{*\tau+\Delta\tau}$ $\phi^{*\tau+\Delta\tau}$
- Apply implicit Rayleigh damping on *W* as an adjustment step: $W^{\tau+\Delta\tau} = W^{*\tau+\Delta\tau} - \Delta\tau R_w(\eta) W^{\tau+\Delta\tau}$
- Update final value of geopotential at new time level: $\phi^{\tau+\Delta\tau}$

Implicit Rayleigh Gravity-Wave Absorbing layer

Applying implicit Rayleigh damping directly in *W* equation:

$$\frac{\partial W}{\partial t} + g\overline{\left(\mu_d - \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\right)}^{\tau} + R_w W^{\tau+\Delta\tau} = F_W^t$$

Applying implicit Rayleigh damping as an adjustment step:

$$\frac{\partial W}{\partial t} + g\overline{\left(\mu_d - \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\right)}^{\tau} + R_w W^{\tau+\Delta\tau} - \frac{g^2\Delta\tau^2}{4}\frac{\alpha}{\alpha_d}\frac{\partial}{\partial \eta}\left[\frac{c^2}{\mu_d \alpha_d^2}\frac{\partial}{\partial \eta}\left(\frac{R_w W^{\tau+\Delta\tau}}{\mu_d}\right)\right] = F_W^t$$

Normal Rayleigh damping term that vanishes in hydrostatic limit

Additional damping term that remains effective in hydrostatic limit

Implicit Rayleigh Gravity-Wave Absorbing layer

Characteristics of implicit w Rayleigh absorbing layer

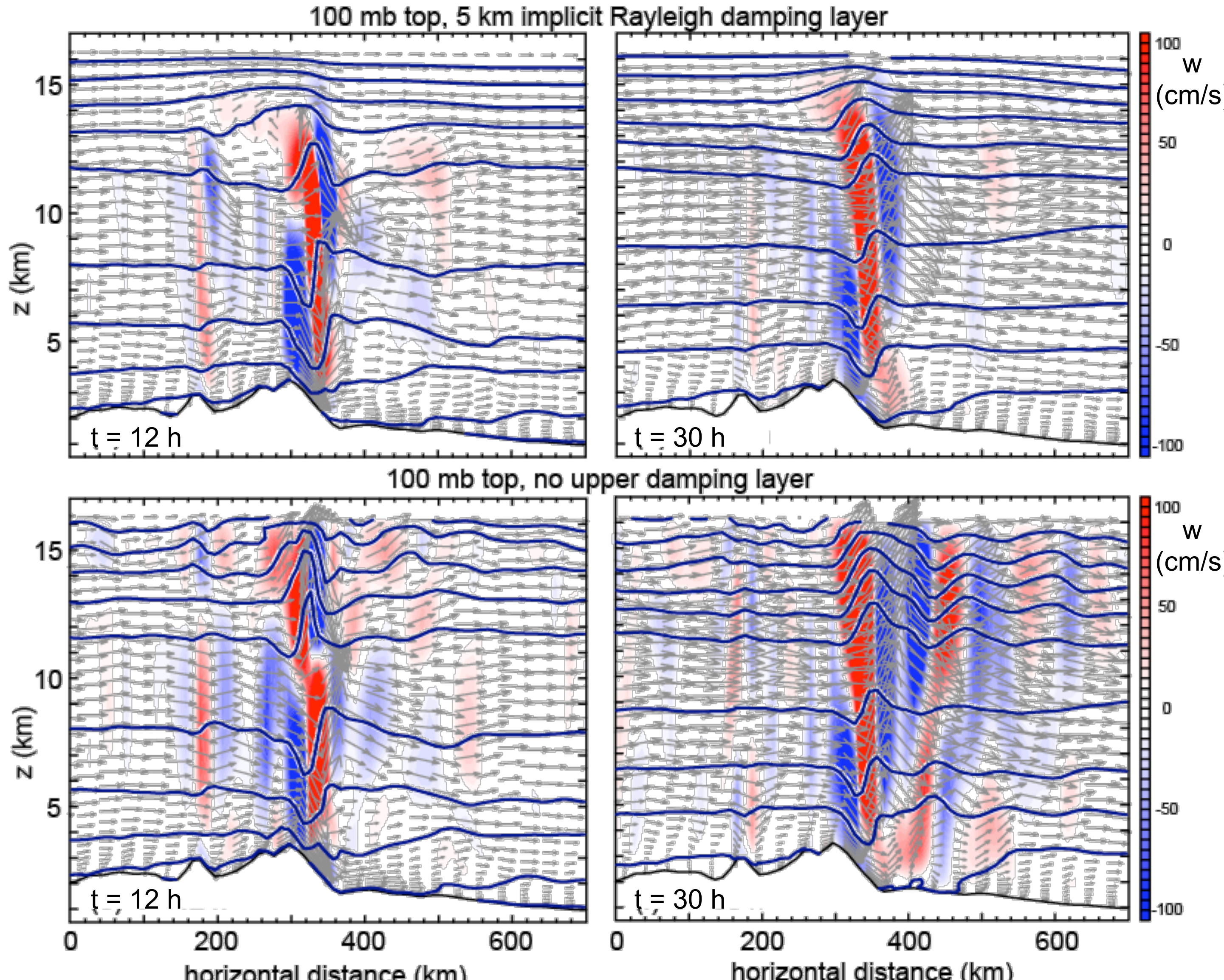
$$W^{\tau+\Delta\tau} = W^{*\tau+\Delta\tau} - \Delta\tau R_w(\eta) W^{\tau+\Delta\tau}$$

$$R_w(\eta) = \begin{cases} \gamma_r \sin^2\left[\frac{\pi}{2}\left(1 - \frac{z_{top}-z}{z_d}\right)\right] & \text{for } z \geq (z_{top} - z_d); \\ 0 & \text{otherwise,} \end{cases}$$

	Name list	default	recommended
Implicit Rayleigh w absorbing layer	*damp_opt*	0	3
z_d - thickness of damping layer (m)	*zdamp*	5000.	~1 vertical λ
γ_r - damping coefficient (t^{-1})	*damp_coeff*	0	0.2

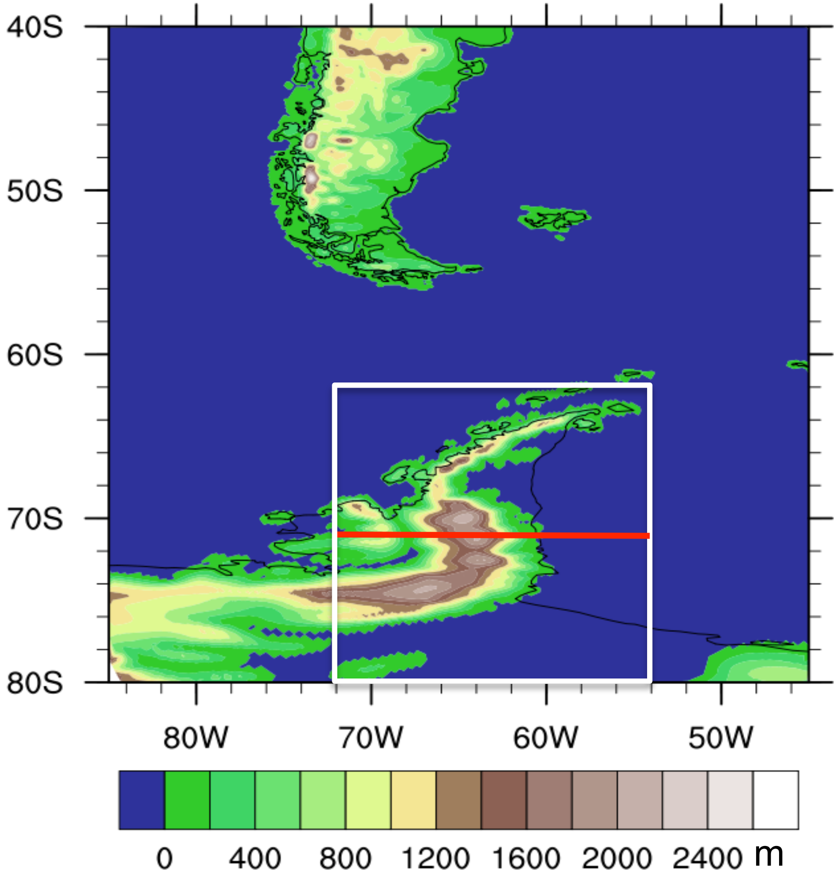
(For more detail, see Klemp et al. 2008, MWR, p. 3987)

Gravity-Wave Absorbing layer



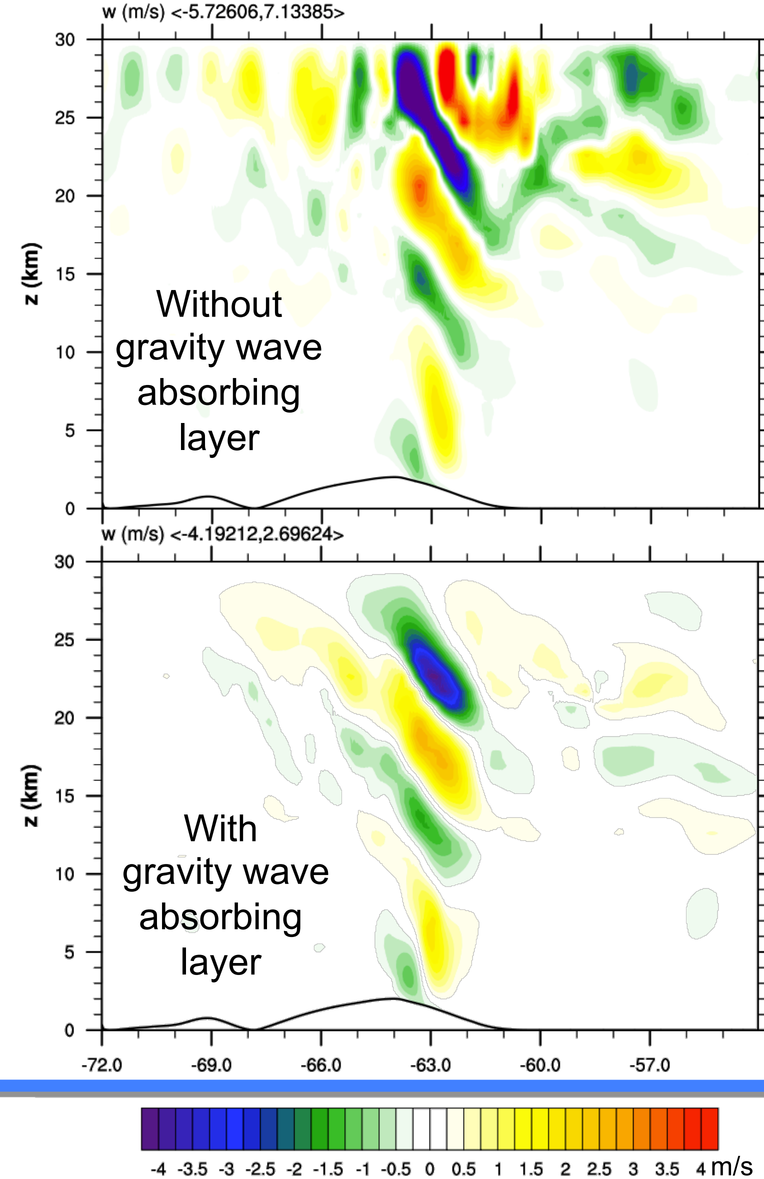
Gravity-Wave Absorbing layer

Antarctic Peninsula



15 km mesh, z_d = 8 km

Vertical Velocity



Improvement of WRF Dynamics for LES Applications

Artificial behavior in WRF LES simulations of low clouds documented by Yamaguchi and Feingold (*JAMES*, 2012)

- Strong sensitivity to size of acoustic time step
- Small-scale numerical noise in fields

Similar behavior recently obtained in WRF LES research led by PNNL for DYCOMMS II and ARM SGP stratocumulus cases.

- Sensitivity isolated to environments having a strong (nearly discontinuous) drop in moisture near the top of the PBL.
- Artificial behavior removed by including effects of water vapor variation during the acoustic time steps.
- See WRF Workshop talk 7.1 on Thursday afternoon

Xiao et al.: Modifications to WRF's dynamical core to improve the treatment of moisture for large-eddy simulations.

Time Integration: Acoustic Step Using $\Theta = \mu_d \theta$

$U^{\tau+\Delta\tau}$	$\dfrac{\partial U}{\partial t} + \left(\mu_d \alpha \dfrac{\partial p}{\partial x} + \dfrac{\alpha}{\alpha_d} \dfrac{\partial p}{\partial \eta} \dfrac{\partial \phi}{\partial x} \right)^{\tau} = R_U^t$
$\mu_d^{\tau+\Delta\tau},\ \Omega^{\tau+\Delta\tau}$	$\dfrac{\partial \mu_d}{\partial t} + \dfrac{\partial U}{\partial x}^{\tau+\Delta\tau} + \dfrac{\partial \Omega}{\partial \eta}^{\tau+\Delta\tau} = 0$
$\Theta^{\tau+\Delta\tau}$	$\dfrac{\partial \Theta}{\partial t} + \left(\dfrac{\partial U \theta^t}{\partial x} + \dfrac{\partial \Omega \theta^t}{\partial \eta} \right)^{\tau+\Delta\tau} = R_\Theta^t$
$W^{\tau+\Delta\tau},\ \phi^{\tau+\Delta\tau}$	$\left\{ \begin{array}{r} \dfrac{\partial W}{\partial t} + g \overline{\left(\mu_d - \dfrac{\alpha}{\alpha_d} \dfrac{\partial p}{\partial \eta} \right)}^{\tau} = R_W^t \\ \mu_d^t \dfrac{\partial \phi}{\partial t} + U^{\tau+\Delta\tau} \dfrac{\partial \phi}{\partial x}^t + \Omega^{\tau+\Delta\tau} \dfrac{\partial \phi}{\partial \eta}^t - g \overline{W}^{\tau} = R_\phi^t \end{array} \right.$
$\alpha_d^{\tau+\Delta\tau}$	$\alpha_d^{\tau+\Delta\tau} = -\dfrac{1}{\mu_d^{\tau+\Delta\tau}} \dfrac{\partial \phi}{\partial \eta}^{\tau+\Delta\tau}$
$p^{\tau+\Delta\tau}$	$p^{\tau+\Delta\tau} = p_0 \left(\dfrac{R_d \Theta^{\tau+\Delta\tau} [1 + (R_v/R_d) q_v^t]}{p_0 \mu_d^{\tau+\Delta\tau} \alpha_d^{\tau+\Delta\tau}} \right)^{\gamma}$

Time Integration: Acoustic Step Using $\Theta_m = \mu_d \theta \left(1 + \frac{R_v}{R_d} q_v\right)$

$$U^{\tau+\Delta\tau} \qquad \frac{\partial U}{\partial t} + \left(\mu_d \alpha \frac{\partial p}{\partial x} + \frac{\alpha}{\alpha_d} \frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial x}\right)^{\tau} = R_U^t$$

$$\mu_d^{\tau+\Delta\tau},\ \Omega^{\tau+\Delta\tau} \qquad \frac{\partial \mu_d}{\partial t} + \frac{\partial U^{\tau+\Delta\tau}}{\partial x} + \frac{\partial \Omega^{\tau+\Delta\tau}}{\partial \eta} = 0$$

$$\Theta_m^{\tau+\Delta\tau} \qquad \frac{\partial \Theta_m}{\partial t} + \left(\frac{\partial U \theta_m^t}{\partial x} + \frac{\partial \Omega \theta_m^t}{\partial \eta}\right)^{\tau+\Delta\tau} = R_{\Theta_m}^t$$

$$W^{\tau+\Delta\tau},\ \phi^{\tau+\Delta\tau} \quad \begin{cases} \dfrac{\partial W}{\partial t} + g\overline{\left(\mu_d - \dfrac{\alpha}{\alpha_d} \dfrac{\partial p}{\partial \eta}\right)}^{\tau} = R_W^t \\[2ex] \mu_d^t \dfrac{\partial \phi}{\partial t} + U^{\tau+\Delta\tau} \dfrac{\partial \phi}{\partial x}^t + \Omega^{\tau+\Delta\tau} \dfrac{\partial \phi}{\partial \eta}^t - g\overline{W}^{\tau} = R_\phi^t \end{cases}$$

$$\alpha_d^{\tau+\Delta\tau} \qquad \alpha_d^{\tau+\Delta\tau} = -\frac{1}{\mu_d^{\tau+\Delta\tau}} \frac{\partial \phi}{\partial \eta}^{\tau+\Delta\tau}$$

$$p^{\tau+\Delta\tau} \qquad p^{\tau+\Delta\tau} = p_0 \left(\frac{R_d \Theta_m^{\tau+\Delta\tau}}{p_0 \mu_d^{\tau+\Delta\tau} \alpha_d^{\tau+\Delta\tau}}\right)^{\gamma}$$

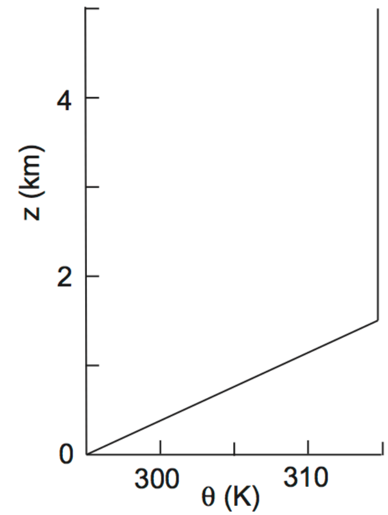
Idealized Test Case with 2-D WRF Prototype

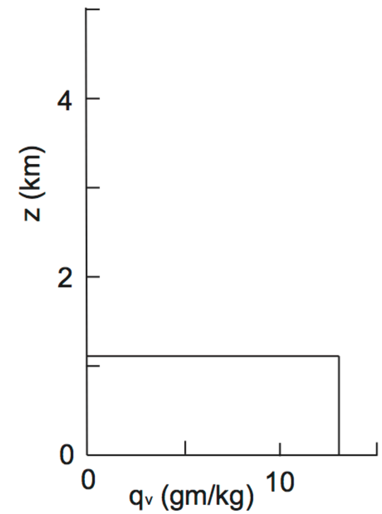
$$U_m = 12 \text{ m/s}$$

$$\Delta x = 100 \text{ m}$$

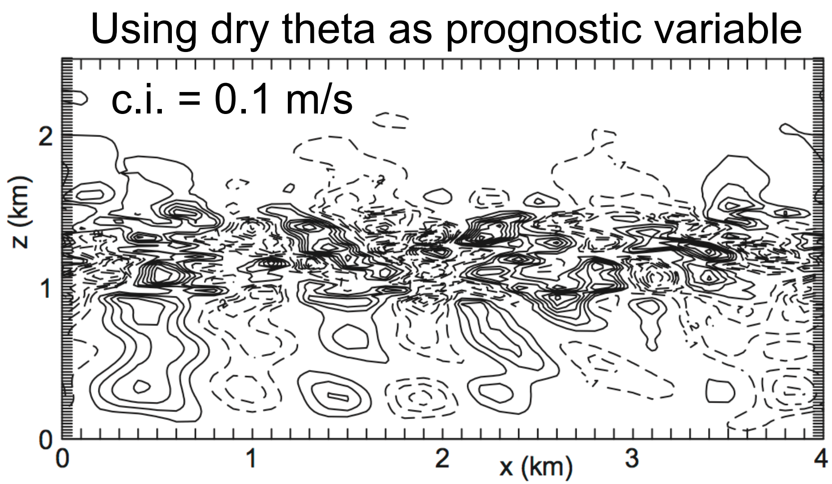
$$\Delta z = 20 \text{ m}$$

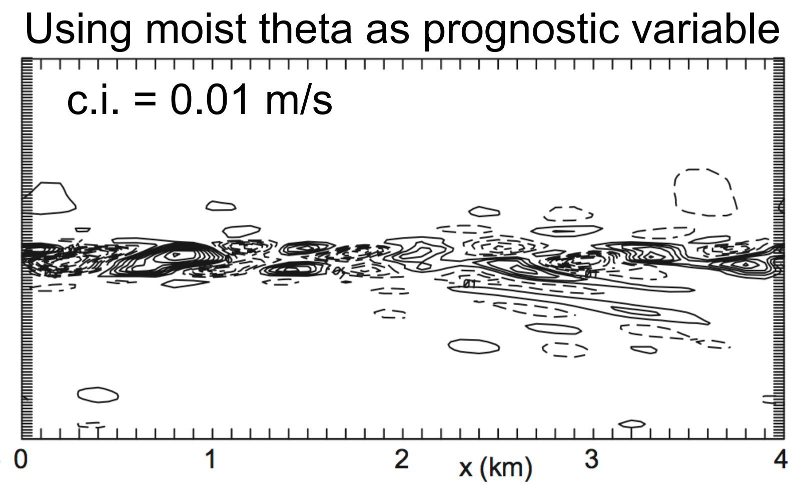
$$\Delta t = 1 \text{ s, ns} = 6$$





Vertical velocity fields at 1 h





Modified WRF Dynamics with Θ_m as Prognostic Variable

use_theta_m = 1 (default = 0)

Released in WRF version 3.7, but presently does not work with grid nesting

Should be fixed in the next WRF release update

After further testing, this formulation will become the standard for all simulations