GENERAL MATRIX INVERSION TECHNIQUE FOR THE CALIBRATION OF ELECTRIC FIELD SENSOR ARRAYS ON AIRCRAFT PLATFORMS

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ABSTRACT: We have developed a calibration technique to determine the relationship between the electric fields measured at the aircraft and the external vector electric field. We have measured electric fields in and around thunderstorms and other clouds with several aircraft (e.g., ER-2, DC-8, Citation, Altus). Our calibration method is being used with all of our different aircraft/electric field sensing combinations and can be generalized to any reasonable combination of electric field measurements and aircraft. We determine a calibration matrix that represents the individual instrument responses to the external electric field. The aircraft geometry and configuration of field mills (FMs) uniquely define the matrix. The matrix can then be inverted to determine the external electric field from the FM outputs. A distinct advantage of the method is that if one or more FM needs to be eliminated from the equation (for example, due to a malfunction), it is a simple matter to reinvert the matrix without the malfunctioning FM. To demonstrate our calibration technique, we present data from one of our aircraft (Altus). We can also use our method to determine the "goodness-of-fit" of the matrix produced by our technique and use this information to further refine the calibration.

INTRODUCTION

Measurements of electric fields within clouds have been made by aircraft for many years [*e.g., Gunn et al.,* 1946*; Blakeslee et al.,* 1989*; Winn,* 1993]. One of the most difficult steps of measuring electric fields with aircraft is extracting the external electric field from the measurements of electric field at the aircraft [*e.g., Jones*, 1990*; Koshak et al.,* 1994]. The electric field as measured by an instrument (m_i) on the aircraft includes components from the external electric field (E_X, E_Y, E_Z), charge on the aircraft (E_Q), and various other processes (γ). This can be represented by:

$$m_i = M_{Xi}E_X + M_{Yi}E_Y + M_{Zi}E_Z + M_{Qi}E_Q + \gamma(?) \qquad (1)$$

where the various Ms are the responses of the electric field instrument to the external field and charge on the aircraft. Note that γ can depend on anything other than the external electric field or charge on the aircraft. If you expand (1) for all field measuring instruments on an aircraft, and neglect γ, the resultant system of equations can be represented by a matrix equation:

$$\underline{m} = \mathbf{M}\underline{E} \qquad (2)$$

where $\underline{m}$ is the vector field measuring instruments outputs, $\underline{E}$ is the vector external electric field (including the charge on the aircraft), **M** is the 6x4 calibration matrix (for the case where there are 6 field measuring instruments on the aircraft. Typically we use field mills (FMs) as our electric field measuring instruments.

To determine the external electric fields from the FM measurements, we must find the matrix **C** such that:

$$\underline{E} = \mathbf{C}\underline{m} \qquad (3)$$

where:

$$\mathbf{CM} = \mathbf{I} \qquad (4)$$

and **I** is the identity matrix.

Often, calibration methods attempt to determine the **C** matrix directly. Because the **C** matrix is not unique, this can be difficult. By solving for the unique **M** matrix, the calibration procedure can be simplified. We are using this procedure to calibrate the FM systems on several of our aircrafts (e.g., ER-2, DC-8, Citation, Altus).

CALIBRATION MATRIX PROCEDURE

We first find periods when the electric field at the aircraft is approximately known and is measurably different from zero for all four components (E_X, E_Y, E_Z, and E_Q). The most common situation is when the aircraft is performing roll and pitch maneuvers during fair weather conditions. The roll and pitch maneuvers will map the assumed vertical fair weather field into the aircraft frame E_X, E_Y, and E_Z components. To determine the E_Q coefficients, we need to have charge induced on the aircraft frame.

We next make a first guess at the "ideal" external electric field ($\mathbf{E}_{ideal}$) based on a simple electric field profile [*e.g.*, *Gish*, 1944] and the roll and pitch maneuvers. Figure 1 shows our first guess at the vector electric field during a set of roll and pitch maneuvers. Figure 2 shows the 6 FM outputs during the maneuvers. The first guess of the calibration matrix is determined by vector division performed in a least squares sense [*Anderson et al.*, 1999]:

$$\mathbf{M} = \mathbf{m}/\mathbf{E}_{ideal} \qquad (5)$$

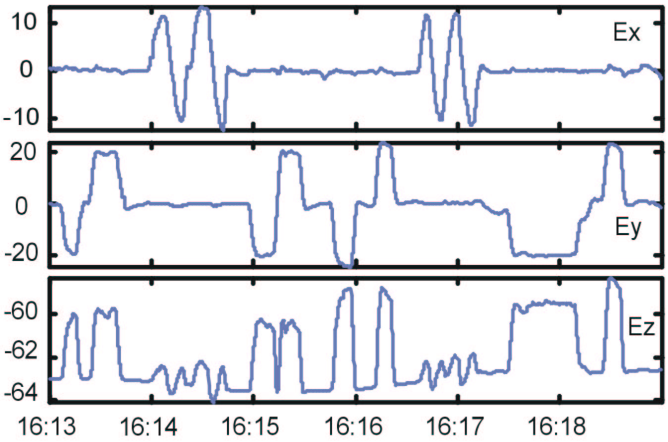

Figure 1. Ideal fields in the aircraft frame of reference during the roll/pitch maneuvers. The amplitude is in V/m and the time is in hours:minutes on day 02192.

where $\mathbf{m}$ is the 6xN measurements of FM output, $\mathbf{E}_{ideal}$ is the 4xN estimates of the electric field, and $\mathbf{M}$ is the 4x6 calibration matrix (N is the number of measurements of the electric field and FM output).

We then "correct" $\mathbf{M}$ based on any known symmetries or node lines in the FM placement on the aircraft. The corrected $\mathbf{M}$ matrix is then used to calculate a first guess at the "true" external electric field ($\mathbf{E}_{est}$). To calculate the fields from the FM outputs and the $\mathbf{M}$ matrix, we need to "invert" the $\mathbf{M}$ matrix. Since $\mathbf{M}$ is not a square matrix, we have to calculate a pseudoinverse of $\mathbf{M}$. We use the Moore-Penrose (MP) pseudoinverse [*Penrose*, 1955] to calculate the fields from the mill outputs:

$$\underline{E}_{est} = \text{pinv}(\mathbf{M})\underline{m} \qquad (6)$$

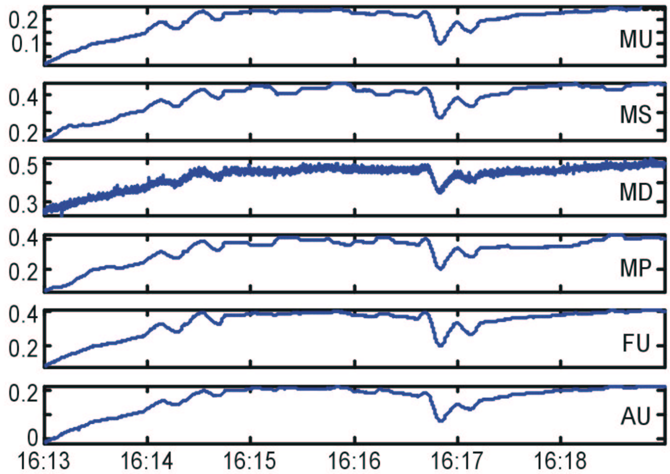

Figure 2. Raw FM output during the calibration maneuvers in this example. The amplitude is in kV/m and the time is in hours:minutes on day 02192.

The MP pseudoinverse is one of the infinite number of $\mathbf{C}$ matrices that are valid for (3) and (4).

Although the "ideal" electric field is a good first guess, the "true" electric field often has temporal and spatial variations that are not found in the "ideal" fields. For example, Figure 3 compares $\mathbf{E}_{ideal}$ with $\mathbf{E}_{est}$ during a period just before a calibration run. Note that the fields are much more variable in the "true" (red/dark) case. Attempting to force the $\mathbf{M}$ matrix to fit $\mathbf{E}_{ideal}$ when $\mathbf{E}_{est}$ is more like the "real" fields can introduce significant errors in the matrix determination process.

The next step is to correct the $\mathbf{E}_{est}$ calculated for any known errors. For example, the $\underline{E}_{Xest}$ field calculated from the $\mathbf{M}$ matrix is the "true" $\underline{E}_{Xtrue}$ field with contributions from the other four components:

$$\underline{E}_{Xest} = \varepsilon_1\underline{E}_{Xtrue} + \varepsilon_2\underline{E}_{Ytrue} + \varepsilon_3\underline{E}_{Ztrue} + \varepsilon_4\underline{E}_{Qtrue} \qquad (7)$$

as long as:

$$\varepsilon_1 >> \varepsilon_2, \varepsilon_3, \varepsilon_4 \qquad (8)$$

and we can identify the other components in the estimated field, we can subtract the fractional components of $\underline{E}_{Yest}$, $\underline{E}_{Zest}$, and $\underline{E}_{Qest}$ from $\underline{E}_{Xest}$ to produce an $\underline{E}_{Xest}$ that is very close to $\underline{E}_{Xtrue}$. We perform the same operations to the $\underline{E}_{Yest}$, $\underline{E}_{Zest}$, and $\underline{E}_{Qest}$ as we did for the $\underline{E}_{Xest}$ to produce an $\mathbf{E}_{est}$ that is very close to $\mathbf{E}_{true}$.

We then substitute the values of $\mathbf{E}_{est}$ into (3) for $\mathbf{E}_{ideal}$ to determine a more refined **M** matrix. We correct this new **M** matrix again for any known symmetries and field nodes in the FM placement on the aircraft and calculate a new $\mathbf{E}_{est}$. We again correct $\mathbf{E}_{est}$ for errors and the process starts over again. We repeat this process until the **M** matrix converges. For reasonable combinations of FM and aircraft, this process usually takes less than five iterations.

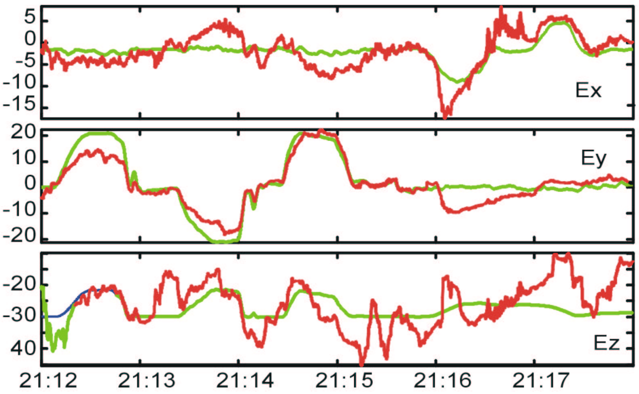

Figure 3. Ideal vs. realistic fields during a flight of the North Dakota Citation. The red/dark plots are the realistic fields while the green/light plots are the ideal fields. The amplitude is in V/m and the time is in hours:minutes on day 01176.

After the **M** matrix has converged, there is still one calibration step left. The steps above create an **M** matrix that is relatively calibrated, that is, the components of the electric field are correctly proportioned to $\mathbf{E}_{true}$. However, they may differ by a multiplicative constant from $\mathbf{E}_{true}$. We need to compare our fields calculated with the final **M** matrix to a known electric field. This is typically done by flying the aircraft near a calibrated ground based FM and comparing the relative amplitudes of the fields at the ground and at the aircraft. The ratio of the two fields produces the factors needed in the final absolute calibration of the **M** matrix.

RESULTS

To test the validity of the **M** matrix, we can see how the fields calculated from the use of the **M** matrix compare to what one would expect in a given situation. Figure 4 show **M** matrix based fields during an overflight of a thunderstorm near Florida. If the storm was a simple dipole, one would expect the $\underline{E}_X$ fields to go negative, then cross zero at the time the $\underline{E}_Z$ field peaked (positive values) and then decay back to near zero. The fields in Figure 4 are a reasonable approximation of this pattern.

To further test the validity of the **M** matrix, we can use the calculated E values and the **M** matrix to estimate what the FM outputs "should" be. The difference between the actual FM outputs and what the **M** matrix calculates can be converted to a "goodness-of-fit" (GOF) parameter:

$$GOF = \Sigma_{mills}(\mathbf{ME} - \underline{m})^2 \qquad (9)$$

substituting for **E** we get:

$$GOF = \Sigma_{mills}(\mathbf{M}(\mathbf{C}\underline{m}) - \underline{m})^2 \qquad (10)$$

and substituting for **C** we get:

$$GOF = \Sigma_{mills}(\mathbf{M}(\mathrm{pinv}(\mathbf{M})\underline{m}) - \underline{m})^2 \qquad (11)$$

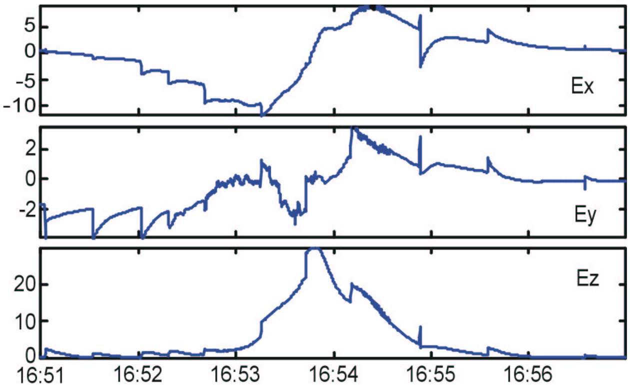

Figure 4. Calibrated electric fields during overflight of a Florida thunderstorm. The amplitude is in kV/m and the time is in hours:minutes on day 02225.

which depends only on the mill outputs (which are known) and the calibration matrix **M**. Ideally, the GOF value would be zero. A plot of the GOF for the initial **M** matrix and current **M** matrix for the Altus aircraft/FM combination during an overpass (Figure 4) of a thunderstorm is shown in Figure 5. Note that the GOF for the current **M** matrix is smaller than the one for the initial matrix indicating the current **M** matrix is a better fit to the FM data. By attempting to minimize the GOF by varying the components of **M**, we can further refine the **M** matrix solution.

One of the properties of the MP pseudoinverse is that one can use weighting factor matrices to bias the pseudoinverse to favor or ignore some of the FM measurements. This can be very useful when a measurement is suspect for some reason. For example, in Figure 2, the MD instrument has significantly greater noise than the other five instruments. By using a weighting factor matrix in the pseudoinverse, the effect of the excess MD instrument noise can be reduced or even eliminated. As long as **M** is the correct matrix for the aircraft/FM combination and the weighting factor matrix is reasonable, the resulting $\mathbf{E}_{est}$ will be the same as calculated with the pseudoinverse of **M** directly.

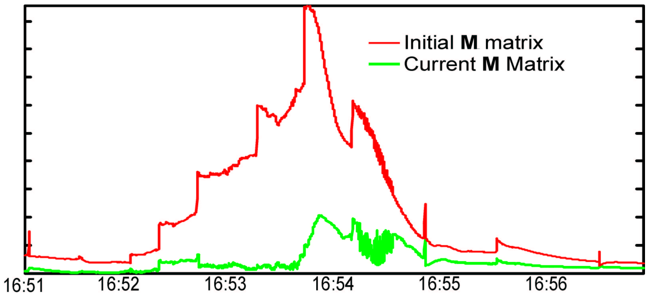

Figure 5. GOF values for the initial (red/dark) and current (green/light) **M** matrix. Amplitude units are arbitrary and the time units are hours:minutes on day 02225.

CONCLUSION

We have developed a method of determining the relationship between the external electric fields and the outputs of electric field meters for a generalized aircraft platform. As long as the FMs are not placed in a pathological fashion and there are more FMs than electric field components, the method converges after a few iterations. By determining the **M** matrix (the matrix that produces the FM outputs from the field values) instead of the **C** matrix (the matrix that produces the field values from the FM outputs), we can use the properties of the **M** matrix to simplify the calibration process and also add the ability to emphasize or eliminate FM outputs on the fly.

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