PRECIPITATION ADJUSTMENT

ALGORITHM DESCRIPTION

NX-DR-03-020/29

1.0 PROLOGUE

1.1 FUNCTIONAL DESCRIPTION

The PRECIPITATION ADJUSTMENT algorithm determines the radar bias with respect to rain gages. The algorithm uses hourly gage reports from the Gage Data Acquisition support function and an ACCUMULATION SCAN (Hourly) output by the PRECIPITATION ACCUMULATION [19] algorithm to generate comparable sets of gage and radar samples for specified hourly accumulation periods. These sets are used to update the existing bias every hour.

The timing of execution of the bias estimation procedure must allow for two timing delays: One is the time required to execute the procedure itself. Since the best possible estimate of the bias is desired at the end of each clock hour, the TIME (Bias Estimation) is set as close as possible to the clock hour, but in no case earlier than ten minutes before the clock hour. In other words, the bias update procedure must execute within an elapsed time of less than ten minutes.

The second time delay is required to acquire data from the rain gages. This time delay is defined by the polling, transmission, and formatting delays in the off-site systems which actually acquire data from the gages and by the delay in posting the data to the gage data base by the Gage Data Acquisition support function. The ending TIME (Gage Accumulation) is set earlier than the TIME (Bias Estimation) by an amount of time sufficient for this gage data acquisition process. Thus, the bias estimate actually is computed for a one hour period from one ending TIME (Gage Accumulation) to the next. The PRECIPITATION ACCUMULATION algorithm is required to produce an hourly accumulation scan for each ending TIME (Gage Accumulation). The timing execution is illustrated on Figure 1.

Figure 1. Timing for bias estimation.

As soon as the TIME (Bias Estimation) is passed, the gage observation vector and radar observation vector are generated. The gage observation vector contains the hourly gage accumulations from the gage reports. The radar observation vector contains the radar accumulation values corresponding to each value in the gage observation vector. These are computed based on the hourly accumulation scan sample volume value directly over the gage and at the eight surrounding sample volumes from the hourly accumulation scan ending at the ending TIME (Gage Accumulation). If, because of system problems, no hourly accumulation scan exists for the ending TIME (Gage Accumulation), no gage-radar sets are generated.

The number of elements in these vectors specifies the available number of gage-radar sets. If either one or both of the hourly gage accumulation and corresponding radar accumulation are less than 0.6 mm, the values are not added to the gage observation vector or radar observation vector (no gage-radar set is generated for these pairs).

The algorithm then produces an updated estimate of the radar bias factor for the specified hourly accumulation period and also a forecast of the radar bias factor for the next hour. Both the updated and forecast values of the radar bias have an associated estimation error variance which is also computed.

The bias estimation procedure is an implementation of a discrete Kalman filter. It presumes that the mean multiplicative bias follows a random walk process, i.e., the bias is equally likely to increase or decrease over the next hour. Based on this model, the best forecast for the next hour is simply the best current estimate. If enough gage-radar sets were generated for the most recently completed ending TIME (Gage Accumulation), the forecast from the last execution of the procedure is updated based on those data. The radar-gage sets are also used during each execution to estimate a measurement error covariance matrix, which (speaking in loose terms) measures the significance of the radar-gage sets as an estimator of the radar bias.

If not enough gage-radar sets for the ending TIME (Gage Accumulation) were generated, the forecast bias from the last hour becomes the new current bias value and the new forecast bias value is set equal to the current bias value. The estimation error variance also increases each hour by an additive system noise variance.

When the bias estimate has not been calculated for more than one hour, the bias is drifted back to the reset BIAS value over a period equal to the maximum TIME (propagation). The drift is linear and begins with the second uncalculated hour.

1.2 SOURCE

The PRECIPITATION ADJUSTMENT algorithm was developed by Dr. Edward R. Johnson of the Georgia Institute of Technology in cooperation with the Radar Hydrology Group of the National Weather Service Hydrologic Research Laboratory. This algorithm is based on experience with a variety of Kalman filter applications. It was implemented for testing using D/RADEX data at the Pittsburgh Weather Service Forecast Office and using PROFS data in a post-analysis mode.

REFERENCES

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1.3 PROCESSING ENVIRONMENT

The bias estimation portion of the algorithm must be executed once each hour, using radar-gage sets which represent hourly accumulations for evenly spaced one hour time steps.

Therefore, the PRECIPITATION ADJUSTMENT algorithm requires one hour radar accumulations and gage accumulations for specific hourly periods. The time required to poll gages makes it impossible to obtain readings from all gages at exactly the beginning and end of the specified hourly period. Therefore, readings of gages before (if available), during, and after the specified hourly period from each gage are used to estimate the accumulation for exactly the specified hourly period. The specified hourly accumulation periods for the radar and gages will end the same time each hour at a particular site, but may vary from site to site depending on the time period normally required for gage data acquisition.

The rain gage data are acquired continuously by the Gage Data Acquisition support function which maintains a data base of gage reports. These reports are of two types: accumulator reports which give the total accumulation at a gage (mm) at a specified point in time and incremental reports which give the incremental accumulation at a gage (mm) for a specified duration at a specified time. The computation of hourly gage accumulation for the accumulator type reports is illustrated in Figure 2.

Figure 2 -Determination of an hourly gage accumulation for a specified hourly period from 3 gage readings of the accumulator type.

The computation of hourly gage accumulation for incremental reports is illustrated in Figure 3 where the vertical scale now is the precipitation rate (mm/hr), i.e., the accumulation from each report is an area on Figure 3.

Figure 3 -Determination of an hourly gage accumulation for a specified hourly period from incremental reports.

For several reasons the computation of hourly values from the gage reports is somewhat more complicated than the illustration of Figures 2 and 3. For accumulator reports, there is an upper limit (the THRESHOLD (Time Difference)) to the time difference between the ending TIME (Gage Accumulation) and the closest (in time) report. If no report is available within the THRESHOLD (Time Difference) of the ending TIME (Gage Accumulation) the estimate is considered too unreliable to be used, i.e., it is discarded.

A second problem arises from missing data. If no accumulator reports are available to bracket the ending TIME (Gage Accumulation) or if any of the hourly period has no incremental reports to cover it, then no estimate is possible.

It is possible that incremental reports will have variable durations, even overlapping durations, at a particular site. This complicates the gage precipitation estimates considerably.

Finally, the possibility of transmission errors, sensor errors, or other sources of error in the gage reports makes it necessary to include simple quality control procedures on the hourly gage estimates. First all hourly gage estimates below zero or above 400 mm are discarded. Then the mean and variance of the differences between hourly gage and radar estimates is computed and any gage-radar set with a difference greater than a threshold value (specified in terms of standard deviations) away from the mean difference is discarded.

Considering all of the above and the fact that precipitation must be detected (either by gage or radar) for a gage to appear in the gage-radar sets, it will be a relatively rare occurrence for all available gages to be included. Nevertheless, if more than 30 valid gage-radar sets are available, only the "first" 30 are used in the update procedure. The order is specified in the Gage Data Acquisition support function. This restriction serves to reduce the computational burden for the rare data-rich case.

Special care has been taken during algorithm development so that it will be computationally efficient and numerically stable. The algorithm should pose no numerical difficulties if all computations (including required matrix inversion) are in at least single precision (approximately a 6 decimal digit) floating point arithmetic.

The actual bias computation contained in steps 1.0 through 1.5 should be executed on a wall clock basis, regardless of the processing of other radar data, when the TIME (Bias Estimation) is passed. These parts of the algorithm need not be initiated <u>exactly</u> at the TIME (Bias Estimation), but any delays in initiating the bias computation must be accounted for in setting the TIME (Bias Estimation). Therefore the sum of the initiation delay and the actual bias computation time must not exceed the 10 minute limit established above. If an alternative initiation process tied to scan completion or some other criterion rather than the wall clock is adopted, but the same 10 minute limit can be assured, then the alternative will be acceptable.

2.0 INPUTS

2.1 IDENTIFICATION

ending TIME (Gage Accumulation)	=	The ending time of the ACCUMULATION SCAN (Hourly) and the computed hourly gage values from which the gage-radar sets are generated. Will be the same time each hour for a particular radar site.
TIME (Bias Estimation)	=	The time each hour which signals the initiation of the BIAS computation process. Precise to 1/16 hour.
ending TIME (Accumulation)	=	The ending time of an ACCUMULATION SCAN (Hourly)
THRESHOLD (Number of Sets)	=	The minimum number of GAGE-RADAR SETs for which for which covariance estimation is allowed (2-10).
reset BIAS	=	The reset (long term mean) multiplicative factor that adjusts for the radar bias when compared to surface precipitation measurements. Precise to 0.01.
TIME (reset BIAS)	=	The time required to complete the drift from a computed bias estimate to the reset BIAS. Precise to 0.5 hours.
reset MEAN SQUARE Error	=	The reset (long term mean) Mean Square Error (MSE) value used when the MSE has been propagated for an extended time. Precise to 0.01.
maximum MEAN SQUARE ERROR	=	The maximum MEAN SQUARE ERROR allowed.
GAGE REPORTs (Accumulator)	=	Reported values of accumulation at each gage and time of occurrence.
GAGE REPORTs (Incremental)	=	Reported values of incremental accumulation, increment duration, and time of occurrence.
THRESHOLD (Time Difference)	=	Maximum time to the closest GAGE REPORT (Accumulator) for a valid estimate of accumulation at the ending TIME (Gage Accumulation).
CATEGORY(Precipitation)	=	The precipitation category currently in effect.

CATEGORY	MEANING
0	No precipitation was detected during the past hour
1	Significant precipitation was detected during the past hour
2	Light precipitation was detected during the past hour

PRECIPITATION STATUS MESSAGE	=	An alphanumeric message which includes radar ID, TIME(Stamp), current radar status, current operational mode, current scan strategy, TIME (Last Precipitation Detected), CATEGORY (Precipitation), number of gages in a data base, and time since last update to the gage data base.
FLAG (Zero Hourly Accumulation)	=	A set or cleared flag indicating, if set, that all current ACCUMULATION SCAN (Hourly) values can be assumed to be zero .
FLAG (Zero Scan-to-Scan)	=	A set or cleared flag indicating, if set, that all current ACCUMULATION SCAN(Scan-to-Scan) values can be assumed to be zero.
FLAG (No Hourly Accumulation)	=	A set or cleared flag indicating, if set, that no hourly accumulations were computed for the hour ending at the current ending TIME (Accumulation).
SYSTEM NOISE	=	The estimated mean square error of the drift in the radar BIAS from one hour to the next.
VARIANCE ADJUSTMENT FACTOR	=	An adjustment factor used to insure that the MATRIX (Measurement Covariance) is positive -definite; a dimensionless number less than 1.0 and greater than or equal to 0.
THRESHOLD (Gage Discard)	=	Threshold value of the normalized gage-radar difference to discard the gage data value. A dimensionless number. A precision of at least 0.1 is required.
DISTANCE MATRIX	=	The distance from each gage to every other gage, in kilometers.

2.2 ACQUISITION

The following are unit adaptation parameters:

THRESHOLD (Time Difference)
ending TIME (Gage Accumulation)
THRESHOLD (Number of Sets)
SYSTEM NOISE
VARIANCE ADJUSTMENT FACTOR
reset BIAS
TIME (reset BIAS)
reset MEAN SQUARE ERROR
maximum MEAN SQUARE ERROR
THRESHOLD (Gage Discard)
TIME (Bias Estimation)

The following are obtained from the data base maintained by the Gage Data Acquisition support algorithm:

GAGE REPORT (Accumulator)
GAGE REPORT (Incremental)
DISTANCE (Between Gages)

PRECIPITATION STATUS MESSAGE and the CATEGORY(Precipitation) is obtained from the Precipitation Detection support function.

3.0 PROCEDURE

3.1 ALGORITHM

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BEGIN ALGORITHM (PRECIPITATION ADJUSTMENT)

1.01  IF (Time is past TIME (Bias Estimation) AND the previous TIME (Bias
         Update) is before TIME (Bias Estimation))
      THEN
   1.1   Replace the previous BIAS with the forecast BIAS.  Replace the
         previous MEAN SQUARE ERROR with  the forecast MEAN SQUARE ERROR.
         Replace the value  of the previous TIME (Bias Update) with the
         current time.
   1.2   IF (ACCUMULATION SCAN (Hourly) with an ending TIME (Accumulation)
            equal to ending TIME (Gage Accumulation) exists)
         THEN
   l.2.l    DO FOR ALL (Gages)
               IF (Any GAGE REPORTs (Accumulator) are available for this gage)
               THEN
                  COMPUTE  (TIMES (Closest Accumulator))
                  COMPUTE  (maximum TIME (Gage Interpolation))
                  IF (Maximum TIME (Gage Interpolation) is less than or equal
                     to THRESHOLD (Time Difference))
                  THEN
                     COMPUTE  (accumulator GAGE ACCUMULATION  (Estimated
                              Hourly))
                  ELSE
                     There is no estimate for this gage.
                  END IF
               ELSE
                  COMPUTE  (DURATIONS (Increment Report Overlap))
                  COMPUTE  (FRACTION(Incremental Gage Report Overlap))
   1.2.1.1        Sort GAGE REPORTS (Incremental) into descending order of
                  FRACTION (Incremental Gage Report Overlap). For those cases
                  where FRACTION(Incremental Gage Report Overlap) is equal,
                  secondary sort is in descending order of DURATION
                  (Incremental Report Overlap)
                  COMPUTE  (FLAG (Zero Precipitation for  Minute))
                  COMPUTE  (incremental GAGE ACCUMULATION (Estimated  Hourly))
               END IF
            END DO
   1.2.2    DO FOR ALL  (GAGE ACCUMULATIONs (Estimated Hourly))
               IF (Gage accumulation is less than 0 or more than 400 mm)
               THEN
                  Delete GAGE ACCUMULATION (Estimated Hourly)
               ELSE
                  COMPUTE  (RADAR OBSERVATION)
               END IF
            END DO
   1.2.3    IF (The number of GAGE ACCUMULATIONs (Estimated Hourly) is greater
               than THRESHOLD (Number of Sets))
            THEN
               COMPUTE  (MEAN GAGE RADAR DIFFERENCE)
               COMPUTE  (STANDARD DEVIATION GAGE-RADAR DIFFERENCE)
               DO FOR ALL  (GAGE ACCUMULATIONs (Estimated Hourly))
                  COMPUTE  (GAGE-RADAR DIFFERENCE (Normalized))
                  IF (GAGE-RADAR DIFFERENCE (Normalized) is greater than
                     THRESHOLD (Gage Discard))
                  THEN
                      Delete GAGE ACCUMULATION (Estimated Hourly).
                  END IF
               END DO
            END IF
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1.2.4    DO FOR ALL  (GAGE ACCUMULATIONs (Estimated Hourly))
            IF (RADAR OBSERVATION AND GAGE ACCUMULATION (Estimated Hourly)
               both greater than or equal to 0.6 mm)
            THEN
               (Include RADAR OBSERVATION in the VECTOR (Radar
               Observation). Include GAGE ACCUMULATION (Estimated Hourly)
               in the VECTOR (Gage Observation).  RADAR OBSERVATION and
               GAGE ACCUMULATION (Estimated Hourly) are thus paired into a
               GAGE-RADAR SET.  Keep track of which gage is stored in each
               element of VECTOR(Gage Observation).  Keep count of the
               number of GAGE-RADAR SETS)
            END IF
         END DO
         IF (All RADAR OBSERVATIONs are equal to 0)
         THEN
            (The number of GAGE-RADAR sets is set to 0)
         END IF
      ELSE
         (The number of GAGE-RADAR SETs is set to 0)
      END IF
1.3   IF (The number of GAGE-RADAR SETs is greater than or equal to
         THRESHOLD (Number of Sets))
      THEN
1.3.1    COMPUTE  (INNOVATIONS)
         IF (The maximum INNOVATIONS value is less than or equal to 1 mm)
         THEN
            Reset (current BIAS) with (previous BIAS)
            Replace (current MEAN SQUARE ERROR) with (previous MEAN SQUARE
            ERROR)
         ELSE
 1.3.2      COMPUTE  (INNOVATIONS(Variance Parameter))
 1.3.3      COMPUTE  (INNOVATIONS(Decay Parameter))
 1.3.4      COMPUTE  (INNOVATIONS(Second Variance Parameter))
 1.3.5      COMPUTE  (OBSERVATION(Variance Parameter))
 1.3.6      COMPUTE  (OBSERVATION(Decay Parameter))
 1.3.7      COMPUTE  (OBSERVATION(Second Variance Parameter))
 1.3.8      COMPUTE (VARIANCE INFLATOR)
 1.3.9      IF (Number of GAGE-RADAR SETs greater than 30)
            THEN (Delete all but first 30 entries)
            END IF
 1.3.10     COMPUTE  (MATRIX (Measurement Covariance))
 1.3.11     COMPUTE  (MATRIX (Gain))
 1.3.12     COMPUTE  (current BIAS)
 1.3.13     COMPUTE  (current MEAN SQUARE ERROR)
            Replace previous TIME (Bias Compute) with the ending TIME (Gage
            Accumulation).
            Set (last_gage_based BIAS) equal to (forecast BIAS)
         END IF
 1.3.14  COMPUTE  (forecast BIAS)
 1.3.15  COMPUTE  (forecast MEAN SQUARE ERROR)

      ELSE
1.3.17   COMPUTE  (TIME (Difference #2))
1.3.18   IF (TIME (Difference #2) is greater than TIME (reset BIAS))
         THEN
1.3.18.1    Replace the current BIAS with the reset BIAS.
1.3.18.2    Replace the current MEAN SQUARE ERROR with the reset MEAN ARE
            ERROR.
1.3.18.3    Replace the forecast BIAS with reset BIAS.
1.3.18.4    Replace the forecast MEAN SQUARE ERROR with reset MEAN SQUARE
            ERROR.
         ELSE IF  (TIME(Difference #2) is less than or equal to one hour)
         THEN
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   1.3.18.5    Replace the current BIAS with forecast BIAS.
   1.3.18.6    Replace the MEAN SQUARE ERROR with forecast MEAN SQUARE ERROR.
   1.3.18.7    COMPUTE  (MEAN SQUARE ERROR (Propagated Forecast))
            ELSE  (TIME(Difference #2) is greater than one hour and less than
                  or equal to TIME(reset BIAS))
            THEN
   1.3.18.8    COMPUTE  drifted BIAS
   1.3.18.9    Replace the current BIAS with the drifted BIAS
   1.3.18.10   Replace the forecast BIAS with the drifted BIAS
   1.3.18.11   Replace the MEAN SQUARE ERROR with forecast MEAN SQUARE ERROR.
   1.3.18.12   COMPUTE  (MEAN SQUARE ERROR (Propagated Forecast))
            END IF
         END IF
   1.4   WRITE (BIAS)
   1.5   WRITE (MEAN SQUARE ERROR)
      END IF

END ALGORITHM (PRECIPITATION ADJUSTMENT)
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3.2 COMPUTATION

3.2.1 NOTATION

FZM	=	FLAG (Zero Precipitation for Minute), a flag used to indicate whether or not precipitation occurred within the minute. If the flag is set, no precipitation has occurred within that minute.
FIX	=	INTEGER (Fixed), a function that converts the real argument to an integer number by truncation.
FRAC	=	FRACTION (Incremental Gage Report Overlap), the fraction of incremental gage report that overlaps the hourly accumulation period, unitless.
MIGRbeg	=	beginning MINUTE (Incremental Gage Report), the minute at which the overlap of the incremental gage report begins during the hourly accumulation period, in minutes.
MIGRend	=	ending MINUTE (Incremental Gage Report), the minute at which the overlap of the incremental gage report ends during the hourly accumulation period, in minutes.
TGAend	=	ending TIME (Gage Accumulation), the ending time of the ACCUMULATION SCAN (Hourly) and the computed hourly gage values from which the gage-radar sets are generated. Will be the same time each hour for a particular radar site.
TGAbeg	=	beginning TIME (Gage Accumulation), one hour before the ending TIME (Gage Accumulation).
Tabeg	=	beginning TIME (Accumulation), the beginning time of an ACCUMULATION SCAN (Hourly). Precise to 1/1200 hr.
Taend	=	ending TIME (Accumulation), the ending time of an ACCUMULATION SCAN (Hourly). Precise to 1/1200 hr.
MSEmax	=	maximum MEAN SQUARE ERROR, the maximum Mean Square Error (M.S.E.) allowed (0.8).
ROB	=	RADAR OBSERVATION, the radar observation used as part of the gage-radar set by a Kalman filtering procedure. Based on the hourly radar accumulations at the nine sample volumes closest to the gage, in mm.
Bpre	=	previous BIAS, the previous multiplicative factor that adjusts for the radar bias when compared to surface precipitation measurements.
MSEpre	=	previous MEAN SQUARE ERROR, the Mean Square Error (M.S.E.) of the previous BIAS value.
Bcur	=	current BIAS, the current multiplicative factor that adjusts for the radar bias when compared to surface precipitation measurements.
MSEcur	=	current MEAN SQUARE ERROR, the Mean Square Error (M.S.E.) of the current BIAS value.
Bfor	=	forecasted BIAS, the forecasted multiplicative factor that adjusts for the radar bias when compared to surface precipitation measurements.

MSEfor = forecasted MEAN SQUARE ERROR, the Mean Square Error (M.S.E.) of the forecasted BIAS value.

MSEprofor = MEAN SQUARE ERROR (Propagated Forecast), the sum of the previous Mean Square Error and SYSTEM NOISE.

VGO = VECTOR (Gage Observation), the computed hourly gage accumulations for the hour ending with the ending TIME (Gage Accumulation) obtained from the gage reports, in millimeters.

GRA = GAGE REPORT (Accumulator), reported values of accumulation in mm at each gage and time of occurrence. Precise to 0.01 mm.

GRI = GAGE REPORT (Incremental), reported values of incremental accumulation in mm, increment duration and time occurrence. Precise to 0.01 mm.

GA = GAGE ACCUMULATION (Estimated Hourly), the estimated hourly accumulation in mm at a gage for the one hour period ending at TAend.

Gainc = Incremental GAGE ACCUMULATION (Estimated Hourly), the estimated hourly accumulation in mm at a gage for the one hour period ending at TAend computed from incremental gage reports.

Gaacc = Accumulator GAGE ACCUMULATION (Estimated Hourly), the estimated hourly accumulation in mm at a gage for the one hour period ending at TAend computed from accumulator gage reports.

TG = TIME (Gage Report), the time of a particular accumulator gage report.

TZA = TIME (Closest Accumulator), the set of times TGbefbeg, TGaftbeg, TGbefend, TGaftend defining the times of occurrence of GAGE REPORTs (Accumulator) closest in time to TGAbeg and TGAend.

TGbefbeg = before-beginning TIME (Gage Report), the TIME (Gage Report) which is before and closest to TGAbeg.

TGaftbeg = after-beginning TIME (Gage Report), the TIME (Gage Report) which is after and closest to TGAbeg.

TGbefend = before-ending TIME (Gage Report), the TIME (Gage Report) which is before and closest to TGAend.

TGaftend = after-ending TIME (Gage Report), the TIME (Gage Report) which is after and closest to TGAend.

TGImax = Maximum TIME (Gage Interpolation), the maximum time difference in hours from the TIMEs (Gage Report) closest to TAbeg and TAend, respectively.

TBCpre = previous TIME (Bias Compute), the time that the BIAS was last computed using gage values.

TIA = TIME (Increment Report Accuracy), twice the accuracy of the Beginning TIME (Increment Report) or ending TIME (Increment

Report), whichever is the least accurate, in hours to the nearest 1/1200 hr.

Tibeg	=	Beginning TIME (Increment Report), the time of the start of a particular incremental gage report.
TIend	=	Ending TIME (Increment Report), the time of the end of a particular incremental gage report.
DIO	=	DURATION (Incremental Report Overlap), the duration of the portion of a particular incremental gage report that overlaps the hourly accumulation period, in hours to the nearest 1/1200 hour.
T2	=	TIME (Difference #2), the length of time, in hours to the nearest 1/60 hour, since the BIAS has been calculated using gage data.
GR	=	GAGE-RADAR DIFFERENCE, the difference in accumulation between a GAGE ACCUMULATION (Estimated Hourly) and the corresponding radar accumulation, in millimeters.
MGR	=	MEAN GAGE RADAR DIFFERENCE, the mean of the gage-radar differences in precipitation accumulation, in millimeters. Precise to 0.1 mm.
SGR	=	STANDARD DEVIATION GAGE-RADAR DIFFERENCE, the standard deviation of the gage-radar difference in precipitation accumulation, in millimeters. Precise to 0.1 mm.
GRN	=	GAGE-RADAR DIFFERENCE (Normalized), the normalized difference in accumulation between an hourly gage report and the corresponding radar accumulation, dimensionless.
GRS	=	GAGE-RADAR SET, the combination of associated pairs of hourly radar accumulations and hourly gage accumulations.
VRO	=	VECTOR (Radar Observation), the hourly radar accumulation for the hour ending at the ending TIME (Gage Accumulation). Each value is based on the hourly accumulation scan values of the sample volume directly over the gage and the eight surrounding sample volumes, in mm.
MXC	=	MATRIX (Covariance Measurement), a symmetric measurement error covariance matrix.
DG	=	DISTANCE (Between Gages), the distance between gages in the VECTOR(Gage Observation), in kilometers. Precise to 0.1 km.
INN	=	INNOVATIONS, a vector of the differences between the VECTOR (Gage Observation) and the VECTOR (Radar Observation) multiplied by the previous BIAS.
INNavg	=	average INNOVATIONS, the average of all INNOVATIONS, in mm.
MXG	=	MATRIX (Gain), a vector (1 x N matrix) giving the gain values - the multiplicative factors giving the influence of each innovation on the updated bias.
IV	=	INNOVATIONS (Variance Parameter), a variance parameter of an isotropic exponential-form covariance function of the

difference between the adjusted radar accumulation and the gage accumulation, in mm^2.

ISV = INNOVATIONS (Second Variance Parameter), a second variance parameter which allows for variance at zero distance in addition to the exponential decay portion of an isotropic exponential-form covariance function of the difference between the adjusted radar accumulation and the gage accumulation, in mm^2.

IDP = INNOVATIONS (Decay Parameter), a decay parameter of an isotropic exponential-form covariance function of the difference between the adjusted radar accumulation and the gage accumulation, in 1/km. A precision of at least 4 decimal digits over a range of at least $10^{10}$to 10^{-10} power is required.

OV = OBSERVATION (Variance Parameter), a variance parameter of the function that improves the mathematical properties of a matrix composed of the outer product of the VECTOR (Radar Observation), in mm^2.

OSV = OBSERVATION (Second Variance Parameter), a second variance parameter of the function that improves the mathematical properties of a matrix composed of the outer product of the VECTOR (Radar Observation), in mm^2.

OD = OBSERVATION (Decay Parameter), a decay parameter of a function that improves the mathematical properties of a matrix composed of the outer product of the VECTOR (Radar Observation), in 1/km. A precision of at least 4 decimal digits over a range of at least 10^{10} to 10^{-10} power is required.

FMD = FLAG (Missing Data), a flag indicating missing incremental gage report data during the hourly accumulation period.

VI = VARIANCE INFLATOR, a value that reduces the measurement error estimates to account for imperfect prediction of the radar bias itself.

VAF = VARIANCE ADJUSTMENT FACTOR, an adjustment factor used to insure that MATRIX (Measurement Covariance) is positive-definite; a dimensionless number less than 1.0 and greater than or equal to 0. (Estimated to be 0.5).

SYS = SYSTEM NOISE, the estimated mean square error of the drift in the radar BIAS from one hour to the next (0.5).

TBUpre = previous TIME (Bias Update), the time at which the BIAS was last updated, to at least the nearest 1/60 hour.

TSavgcur = average current TIME (Scan), the average TIME (Scan) of the current RATE SCAN (or implied zero RATE SCAN if FLAG (Zero Rate) is set) to at least 1/1200 hour.

TSavgpre = average previous TIME (Scan), the average TIME (Scan) of the last good RATE SCAN (or implied zero RATE SCAN if FLAG (Zero Rate) is set) to at least 1/1200 hour.

TBres = TIME (reset BIAS), time allowed since last computed BIAS before drifting back to the reset BIAS, in hours (default 12).

Bres = Reset BIAS, (default 1.00).

Bcom = last_gage_based BIAS, the last computed multiplicative factor that adjusts for the radar bias based on surface precipitation measurements.

NOTE: All times require a precision of at least 1/1200 hr unless otherwise stated. All variables with units of mm and mm^2 require a precision of at least 0.1 mm and 0.1 mm^2, unless otherwise stated. All computations must be done in at least single precision (approximately a 6 decimal digit) floating point arithmetic.

3.2.2 SYMBOLIC FORMULAS

COMPUTE (TIMEs (Closest Accumulator))

$$TGbefbeg = TGAbeg - \min_{TG} (TGAbeg - TG)$$

such that $TG \le TGAbeg$ (i.e., the value of TG that minimizes (TGAbeg -TG) subject to the given constraint)

$$TGaftbeg = \min_{TG} (TG - TGAbeg) + TGAbeg$$

such that $TGAbeg \le TG$

$$TGbefend = TGAend - \min_{TG} (TGAend - TG)$$

such that $TG \le TGAend$

$$TGaftend = \min_{TG} (TG - TGAend) + TGAend$$

such that $TGAend \le TG$

COMPUTE (Maximum TIME (Gage Interpolation))

TGImax = Max(Min(TGAbeg-TGbefbeg,TGaftbeg-TGAbeg),

Min(TGAend-TGbefend,TGaftend-TGAend))

Note: If any of the values TGbefbeg, TGaftbeg, TGbefend, TGaftend are undefined because no gage reports exist which satisfy the given constraints in their computation, then set TGImax to a value greater than THRESHOLD (Time Difference).

COMPUTE (Accumulator GAGE ACCUMULATION (Estimated Hourly))

GAacc =

$$\left[GRAbefend + \frac{(TGAend - TGbefend)\,(GRAaftend - GRAbefend)}{(TGaftend - TGbefend)} \right]$$

$$- \left[GRAbefbeg + \frac{(TGAbeg - TGbefbeg)\,(GRAaftbeg - GRAbefbeg)}{(TGaftbeg - TGbefbeg)} \right]$$

Note: (1) The notation GRAbefend refers to the value of GRA at the time TGbefend, etc.

(2) If TGaftend is identical to TGbefend then the second term on the right hand side above is zero.

(3) If TGaftbeg is identical to TGbefbeg then the fourth term on the right hand side above is zero.

COMPUTE (DURATIONs (Increment Report Overlap))

IF ((TIend$_n$ - TIbeg$_n$) $\le$ 1.25)
THEN
DIO$_n$ = MAX(0,(MIN(TGAend,TIend$_n$) - MAX(TGAbeg, TIbeg$_n$)))
ELSE
DIO$_n$ = 0
END IF

where n is over all available incremental gage reports for a particular gage.

```
COMPUTE (FRACTION (Incremental Gage Report Overlap))

      DO FOR ALL (n GAGE REPORTS (Incremental))
            FRAC_n = DIO_n / (TIend_n - TIbeg_n)
      END DO

COMPUTE (FLAG (Zero Precipitation for minute))

      DO FOR ALL (Minutes from 1 to 60)
            Clear Flag (Zero Precipitation for minute)
      END DO
      DO FOR ALL (n GAGE REPORTS (Incremental) in sorted order)
            IF    (DIO_n > 0.0)
            THEN
                  MIGRbeg_n = FIX ((MAX (TGAbeg, TIbeg_n-TGAbeg)*60 + 0.5) + 1
                  MIGRend_n = FIX ((MIN (TGAend, TIend_n-TGAbeg)*60 + 0.5)
                  DO FOR ALL(Minutes from MIGRbeg_n to MIGRend_n)
                        IF    (GRI_n = 0.0)
                              Set FLAG (Zero Precipitation for minute)
                        END IF
                  END DO
            END IF
      END DO

COMPUTE (incremental GAGE ACCUMULATION (Estimated Hourly))

      NUSED = 0
      GAinc = 0
      DO FOR ALL (Minutes from 1 to 60)
            IUSED_m = 0
      END DO
      DO FOR ALL (n GAGE REPORTS (Incremental) in sorted order)
            MIGRbeg_n = FIX (MAX (TGAbeg, TIbeg_n) - TGAbeg) * 60 + 0.5) + 1
            MIGRend_n = FIX (MIN (TGAend, TIend_n) - TGAbeg) * 60 + 0.5)
            NPOS = 0
            JUSED = 0
            DO FOR ALL (Minutes from MIGRbeg_n to MIGRend_n)
                  IF    (FZM_m is clear)
                  THEN
                        IF    (GRI_n > 0.0)
                        THEN
                              NPOS = NPOS + 1
                        END IF
                        IF    (IUSED_m = 0.0)
                        THEN
                              JUSED = JUSED + 1
                        END IF
                  END IF
                  IF    (IUSED_m = 0)
                  THEN
                        IUSED_m = 1
                        NUSED =  NUSED + 1
                  END IF
            END DO
            IF (NPOS > 0) GAinc = GAinc + GRI_n * FRAC_n * JUSED/NPOS
      END DO
      IF    (NUSED ≥ 55)
      THEN
            GAinc = GAinc * 60/NUSED
      ELSE
            GAinc = -99
```

END IF

where:

NUSED = Running total number of minutes in the gage accumulation hour

IUSED = Used minute array, if the value is equal to one, the minute has been used in the calculations, and if it is zero, it has not been used

NPOS = Number of minutes in the incremental gage report with greater than zero precipitation

JUSED = Number of minutes within the gage report used to calculate the gage incremental accumulation

n = n is over all available incremental gage reports for a particular gage

m = m refers to minutes, and ranges from 1 to 60.

COMPUTE (RADAR OBSERVATION)

$Q5_n = ASH_n(TGAbeg, TGAend)$

where the index n is over the 9 radar sample volumes closest to the mth raingage.

IF (MAX ($Q5_n$) is greater than GA_m AND MIN ($Q5_n$) is less than GA_m)
THEN
$ROB_m = GA_m$
ELSE
$ROB_m = Q5_n$ such that $|Q5_n - GA_m|$ is a minimum for the mth gage.
END IF

This allows for two major possibilities:

(1) misnavigation of the radar sample volumes or the raingage location.

(2) rainfall drifting from the elevation observed by the radar to the ground.

COMPUTE (MEAN GAGE-RADAR DIFFERENCE)

$$MGR = \frac{1}{N} \sum \left(GA_n - ROB_n\right)$$

where (GA - ROB) = GR and N is the number of existing GAGE ACCUMULATIONs (Estimated Hourly) for which either or both GA_n or ROB_n is greater than 0.6 mm.

COMPUTE (STANDARD DEVIATION GAGE-RADAR DIFFERENCE)

$$SGR = \left[\frac{1}{N-1} \sum_n \left(GA_n - ROB_n - MGR\right)^2 \right]^{1/2}$$

where N is the number of existing GAGE ACCUMULATIONs (Estimated Hourly) for which either or both GA_n or ROB_n is greater than 0.6 mm.

COMPUTE (GAGE-RADAR DIFFERENCE (Normalized))

$$GRN = \frac{\left| GA - ROB - MGR \right|}{SGR}$$

<u>COMPUTE</u> (INNOVATIONS)

$INN_n = VGO_n - (VRO_n\ (Bpre))$

where n is set to 1 through the number of gage-radar pairs.

<u>COMPUTE</u> (INNOVATIONS (Variance Parameter))

Step 1 - Compute INNavg and IV

$$INNavg = \frac{1}{N} \sum INN_n$$

$$IV = \frac{1}{N} \sum \left(INN_n - INNavg\right)^2$$

where N is the number of gage-radar sets.

Step 2 - Form the lists $Q2_m$ and $Q1_m$ from 1 to M where,

$$M = \frac{N\ (N-1)}{2}$$

$Q1_m$ = set of values DG_{nL}, the distance from gage n to gage L in VECTOR (Gage Observation), where $n \neq L$, $n > L$

$Q2_m$ = set of values $(INN_n - INNavg) \times (INN_L - INNavg)$ where $n \neq L$ (corresponding to Q1), $n > L$

In other words, the upper triangular portion of the matrix of values DG_{nL}, not including diagonal terms.

Then take natural log of Q2:

$$Q2 = \begin{cases} Ln(Q2),\ Q2 > 0.0 \\ \\ -10.00,\ Q2 \leq 0.0 \end{cases}$$

Step 3 - Estimate IDP and ISV

<u>COMPUTE</u> (INNOVATIONS (Decay Parameter))

$$IDP = \frac{\sum\ Q1_m\ Q2_m - \left[\left(\sum Q1_m \right) \left(\sum Q2_m \right) \right] / M}{\sum\ \left(Q1_m \right)^2 - \left[\sum \left(Q1_m \right) \right]^2 / M}$$

where summations are from m=1 to m=M

<u>COMPUTE</u> (INNOVATIONS (Second Variance Parameter))

$$ISV = \exp \left[\frac{\sum Q2_m}{M} - IDP\ \frac{\sum Q1_m}{M} \right]$$

where summations are from 1 to M

Step 4 - Validity test

If IV $\geq$ ISV and IDP < 0 then this computation is completed. Otherwise, continue with Step 5 below.

Step 5 - Re-estimate parameters

a) Add to lists Q1 and Q2 all values where n = L, using same procedure as described in Step 2 above. The new value of M is:

$$M = \frac{N\ (N + 1)}{2}$$

In other words, add the diagonal elements to the lists.

b) Re-estimate IDP and ISV as in Step 3 above.

Step 6 - Final Validity Test

If IV < ISV or IDP $\geq$ 0 then set ISV = 0

<u>COMPUTE</u> (OBSERVATIONS (Variance Parameters))

Step 1 - Form the list $Q2_m$ and $Q1_m$ from 1 to M where,

$$M = \frac{N\ (N - 1)}{2}$$

$Q1_m$ = set of values DG_{nL}, the distance from gage n to gage L in VECTOR (Gage Observation), where n $\neq$ L, n > L

$Q2_m$ = set of values (VRO_n) (VRO_L)

where n $\neq$ L (corresponding to $Q1_m$), n > L

Then take natural log of $Q2_m$:

$$Q2 = \begin{cases} Ln\left(Q2_m\right),\ Q2_m > 0.0 \\ \\ -10.0,\ Q2 \leq 0.0 \end{cases}$$

Step 2 - Estimate OV

$$OV = \frac{1}{N}\left[\ \Sigma\ \left(VRO_n\right)^2\ \right]$$

where summation is for n from 1 to N, number of GAGE-RADAR sets.

Step 3 - Estimate OSV and OD

<u>COMPUTE</u> (OBSERVATIONS (Decay Parameter))

$$OD = \frac{\Sigma\ Q1_m\ Q2_m - \left\lfloor \left(\Sigma\ Q1_m\right)\left(\Sigma\ Q2_m\right)\right\rfloor / M}{\Sigma\left(Q1_m\right)^2 - \left[\ \Sigma\left(Q1_m\right)\right]^2 / M}$$

where, summations are from 1 to M.

COMPUTE (OBSERVATIONS (Second Variance Parameter))

$$OSV = \exp\left[\frac{\sum Q2_m}{M} - OD\frac{\sum Q1_m}{M}\right]$$

where, summations are from 1 to M.

Step 4 - Validity test.

If OV $\geq$ OSV and OD < 0 then this computation is completed. Otherwise, continue with step 5 below.

Step 5 - Re-estimate Parameters

a) Add to lists Q1 and Q2 all values where n = L, using same procedures as described in Step 1 above. The new value of M is:

$$M = \frac{N\,(N + 1)}{2}$$

b) Re-estimate OD and OSV as in Step 3 above.

Step 6 - Final validity test.

If OV < OSV or OD $\geq$ 0 then set OSV = 0.

COMPUTE (VARIANCE INFLATOR)

$$VI = MIN\left[MSEpre,\ VAF\left(\frac{IV}{OV}\right)\right]$$

COMPUTE (MATRIX (Measurement Covariance))

If ISV = 0 set OSV = 0

If OSV = 0 set ISV = 0.

MXC_{nL} = ISV exp[(IDP)(DG_{nL})] + $Q3_{nL}$(IV - ISV) - VI[OSV exp[OD DG_{nL}] + $Q3_{nL}$(OV - OSV)]

where indices n and L range from 1 to N (N=Number of gage-radar sets (N x N symmetric matrix)).

$$Q3_{nL} = \begin{cases} 1,\ if\ n = L \\ 0,\ if\ n \neq L \end{cases}$$

finally, if any MXC_{nL} < 0, set it to 0.

COMPUTE (MATRIX (Gain))

Step 1 - Construct intermediate matrix Q4 (N x N).

$Q4_{nL}$ = (MSEpre)(VRO_n)(VRO_L) + MXC_{nL}

where n=1 to N, L=1 to N.

Step 2 - Invert matrix Q4. Note that Q4 is symmetric and positive definite by its construction.

The notation $Q4_{nL}^{-1}$ is the nLth element of the inverse of

$$Q4, \text{ not } \left(\frac{1}{Q4_{nL}}\right)$$

Step 3 - Gain computation. Note that the gain is actually a (1 x N) matrix with elements MXG_n.

$$MXGn = MSEpre \sum_{L=1}^{N} \left[\left(VRO_L\right) \left(Q4_{nL}^{-1}\right) \right]$$

for n=1 to N.

<u>COMPUTE</u> (BIAS)

Bcur = Bpre + E MXG_n INN_n

where, summation is from 1 to N.

<u>COMPUTE</u> (MEAN SQUARE ERROR)

MSEcur = (1 - E MXG_n ROB_n) MSEpre

where, summation is from 1 to N.

<u>COMPUTE</u>(forecast BIAS)

Bfor = Bcur

<u>COMPUTE</u> (forecast MEAN SQUARE ERROR)

MSEfor = Min (MSEmax, MSEcur + SYS)

<u>COMPUTE</u> (TIME (Difference #2))

T2 = $|TBU_{pre} - TBC_{pre}|$

<u>COMPUTE</u> (MEAN SQUARE ERROR (Propagated Forecast))

MSEprofor = Min (MSEmax, MSEpre + SYS)

<u>COMPUTE</u> (drifted BIAS)

drifted BIAS = (((TBres - T2 - 1) * Bcom) + ((T2 - 1) * Bres)) / TBres

Note. The drift to a reset BIAS actually begins on the second hour after a BIAS was last computed. Then it is completed (TBres + 1) hours after the BIAS was last computed.

4.0 OUTPUTS

4.1 IDENTIFICATION

The current values of the BIAS and MEAN SQUARE ERROR and the ending TIME (Gage Accumulation). Also, the GAGE-RADAR SET of paired gage and radar-based hourly precipitation estimates.

4.2 DISTRIBUTION

The ending TIME (Gage Accumulation) is specified for the PRECIPITATION ACCUMULATION [019] algorithm.

All other outputs are intended for input to the PRECIPITATION PRODUCTS [021] algorithm.

5.0 INFERENCES

5.1 LIMITATIONS

n order to update the bias estimate using gage data at any hour, this algorithm requires that at least THRESHOLD (Number of Sets) gages have precipitation during the hour. Several times this many gages must report data in order for the bias adjustment to be computed with sufficient frequency, since precipitation does not usually cover the entire area of radar coverage. Frozen hydrometers may result in inaccurate gage measurements.

5.2 FUTURE DEVELOPMENTS

More sophisticated filtering procedures, particularly to estimate the system noise, are under investigation at this time. Refinements for snow situations will also be investigated in the future.