VELOCITY-AZIMUTH DISPLAY

ALGORITHM DESCRIPTION

NX-DR-03-007/30

1.0 PROLOGUE

1.1 FUNCTIONAL DESCRIPTION

The Velocity-Azimuth Display (VAD) algorithm is used to obtain the vertical profile of horizontal wind speed, direction, divergence and vertical velocity for the region of the atmosphere surrounding a Doppler radar. Velocity data at different azimuths collected from a Doppler radar scanning the atmosphere at a constant elevation angle about a vertical axis is used. This algorithm performs a harmonic analysis along with a best fit test on the Doppler velocities around the circumference of a circle at a specified slant range to obtain these parameters. The vertical wind velocity is obtained through a series of steps involving the relationship between horizontal wind speed and conservation of mass through a constant elevation surface above the radar.

1.2 SOURCE

The Velocity-Azimuth Display algorithm is the result of theoretical studies first performed by Lhermitte and Atlas (1961). Rabin and Zrnic (1980) applied the VAD technique to an environment void of precipitation (clear air) with excellent results. The algorithm described here was developed at the National Severe Storms Laboratory (NSSL) at Norman, Oklahoma, by Rabin and Zrnic.

REFERENCES

Lhermitte, R.M., and D. Atlas, 1961: Precipitation motion by pulse Doppler. Proc. Ninth Weather Radar Conf. Boston Amer. Meteor. Soc., 218-223.

Rabin, R.M., and D. Zrnic, 1980: Subsynoptic-scale vertical wind revealed by dual Doppler-radar and VAD analysis. J. Atmos.Sci., 37, 644-654.

Rabin, R.M., "Vertical Velocity Calculations from VAD", MEMO FOR THE RECORD, September 20, 1982. NSSL, Norman, Oklahoma.

1.3 PROCESSING ENVIRONMENT

This algorithm uses the data obtained by direct measurement of reflective power and Doppler velocity from a Doppler weather radar. The only preprocessing required before use is the elimination of bad data. The kinds of preprocessing include ground clutter elimination, anomalous propagation elimination, and elimination of data when echoes beyond the unambiguous range of the radar fold into the desired VAD analysis range. For clear air situations, Doppler velocities from adjacent sample volumes should be averaged to reduce scatter of Doppler velocity estimates caused by weak echoes. The velocity values should be thresholded on a signal-to-noise ratio (e.g.3dB) that is UCP adaptable.

2.0 INPUTS

2.1 IDENTIFICATION

AZIMUTH	=	Azimuthal position, in radians.
HEIGHT(Radar)	=	The radar height above sea level, in kilometers.
DENSITY (Atmospheric)	=	A set of density values for each altitude, in kg/km^3.
DENSITY (Atmos- pheric Gradient)	=	A set of density gradient values at each altitude, in kg/km^4.
ELEVATION	=	Elevation angle, in radians.
FIT TESTS	=	The number of times the fit test procedure is to be run (2).
REFLECTIVITY FACTOR (ZE)	=	The effective radar reflectivity factor of a SAMPLE VOLUME, in mm^6/m^3.
THRESHOLD (Begin in Azimuth)	=	Starting azimuth for VAD analysis, radians.
THRESHOLD (Data Points)	=	The minimum number of data points allowed for the Fourier least squares fitting, unitless (25).
THRESHOLD (End Azimuth)	=	Ending azimuth for VAD analysis, in radians.
THRESHOLD (Symmetry)	=	A value for determining symmetry, in km/hr (25.2).
THRESHOLD (Velocity)	=	A RMS velocity threshold (18), in km/hr.
TIME (Scan)	=	The beginning time of a scan, in hours. Precise to 1/3600 hr.
VAD (Analysis Ranges)	=	The set of specific slant range(s) for each elevation angle at which horizontal wind estimates are computed. The VAD Range is included in the VAD (Analysis Ranges), in 1/4 kilometers.

VAD RANGE	=	The subset of VAD (Analysis Ranges) used to compute estimates of vertical velocity and divergence, in 1/4 kilometers.
VELOCITY (Doppler)	=	Doppler velocities in a SAMPLE VOLUME, in km/hr.

2.2 ACQUISITION

DENSITY (Atmospheric) and DENSITY (Atmospheric Gradient) are acquired through direct measurements or standard tables.

FIT TESTS is a system supplied parameter.

VELOCITY (Doppler) is acquired by direct Doppler radar measurement.

ELEVATIONs and AZIMUTHs are obtained by direct measurement of radar antenna pointing direction.

REFLECTIVITY FACTOR (ZE) is acquired directly from the Doppler radar hardware.

HEIGHT(Radar) is a scientifically determined parameter which is site specific.

THRESHOLD (End Azimuth), THRESHOLD (Begin Azimuth), THRESHOLD (Symmetry), THRESHOLD (Velocity), THRESHOLD (Data Points) and VAD RANGE are operator supplied parameters.

TIME (Scan) is acquired by direct measurement.

VAD Analysis Range is a system supplied parameter derived from the reporting altitudes of the VAD Winds product.

3.0 PROCEDURE

3.1 ALGORITHM

```
BEGIN ALGORITHM (VAD)

1.0  DO FOR ALL (ELEVATIONs)
         DO FOR ALL (VAD Analysis Range at each ELEVATION)
         1.1   DO FOR ALL (FIT TESTS)
                 COMPUTE (COEFFICIENT (Fourier, #1))
                 COMPUTE (COEFFICIENT (Fourier, #2))
                 COMPUTE (COEFFICIENT (Fourier, #3))
                 COMPUTE (DIRECTION (Wind))
                 COMPUTE (ROOT MEAN SQUARE)
                 COMPUTE (SPEED (Wind))
                 DO FOR ALL (AZIMUTHs between THRESHOLD (Begin
                            Azimuth) and THRESHOLD (End Azimuth))
                    COMPUTE (FIT)
                    IF (FIT is greater than zero)
                       THEN
                          IF (FIT minus the current VELOCITY
                             (Doppler) is greater than ROOT MEAN
                             SQUARE)
                             THEN (set current VELOCITY (Doppler)
                                  as missing)
                          END IF
                       ELSE
                          IF (current VELOCITY (Doppler) minus FIT
                             is greater than ROOT MEAN SQUARE)
                             THEN (set current VELOCITY (Doppler)
                                  as missing)
                          END IF
                     END IF

                     WRITE (COEFFICIENT (Fourier, #1))
                     WRITE (COEFFICIENT (Fourier, #2))
                     WRITE (COEFFICIENT (Fourier, #3))
                     WRITE (ROOT MEAN SQUARE)
                     WRITE (DIRECTION (Wind))
                     WRITE (FIT)
                     WRITE (NUMBER (Data Points))
                  END DO
               END DO
          1.2  COMPUTE (SYMMETRY)
          1.3  COMPUTE (HEIGHT (VAD Wind Estimate))

                IF (ROOT MEAN SQUARE is less than THRESHOLD
                   (Velocity) AND FIT is symmetric (SYMMETRY) and
                    NUMBER (Data Points) is greater than
                    THRESHOLD (Data Points))
               THEN COMPUTE (Speed Horizontal Wind))
                       WRITE (HEIGHT (VAD Wind Estimate))
                       WRITE (SPEED (Horizontal Wind))
```

```
                    WRITE (RANGE (Slant))
                    WRITE (ELEVATION)

                  IF (VAD Analysis Range equals VAD Range)
                    THEN
                      COMPUTE (VELOCITY (Precipitation Fall))
                      COMPUTE (DISTANCE (Earth Center))
                      COMPUTE (AREA (Surface))
                      COMPUTE (AREA (Surface Change))
                      COMPUTE (SPEED (Vertical Wind Change))
                      COMPUTE (SPEED (Vertical Wind))
                      COMPUTE (DIVERGENCE (Horizontal))
                      WRITE (DIVERGENCE(Horizontal))
                      WRITE (SPEED(Vertical Wind))
                  END IF
             END IF
        END DO
    END DO

END ALGORITHM (VAD)
```

3.2 COMPUTATION

3.2.1 NOTATION

ALT = ALTITUDE, the height above sea level, in km. Precise to 0.1 kilometers.

CFl = COEFFICIENT (Fourier, #1), the Fourier series coefficient #1 that allows for the best possible approximation of a function, in km/hr. Precise to 0.1 km/hr.

CF2 = COEFFICIENT (Fourier, #2), the Fourier series coefficient #2 that allows for the best possible approximation of a function, in km/hr. Precise to 0.1 km/hr.

CF3 = COEFFICIENT (Fourier, #3), the Fourier series coefficient #3 that allows for the best possible approximation of a function, in km/hr. Precise to 0.1 km/hr.

j = INDEX (Azimuth), the index for AZIMUTH.

VD = VELOCITY (Doppler), Doppler velocities in a SAMPLE VOLUME, in km/hr. Precise to 0.1 km/hr.

AZ = AZIMUTH, azimuthal position, in radians. Precise to 10^{-3} radians.

DW = DIRECTION (Wind), the horizontal direction from which the wind is flowing, in radians. Precise to 10^{-3} radians.

RMS = ROOT MEAN SQUARE, the square root of mean squared deviations of Doppler velocities from a Fourier least-squares fit curve, in km/hr. Precise to 0.1 km/hr.

SHW = SPEED (Horizontal Wind), horizontal wind speed, in km/hr. Precise to 0.1 km/hr.

SPW = SPEED (Wind), the horizontal wind speed not corrected for the elevation angle of measurement, in km/hr. Precise to 0.1 km/hr.

FIT = FIT, the Doppler velocity value at a given azimuth angle on the Fourier least squares fitted curve, in km/hr. Precise to 0.1 km/hr.

PHI# = ELEVATION, elevation angle, in radians. Precise to 10^{-3} radians.

HVAD = HEIGHT (VAD Wind Estimate), the height above the ground level from which data was collected in kilometers. Precise to 0.1 kilometers.

RS = RANGE (Slant), the slant range to the center of a SAMPLE VOLUME, in kilometers. Precise to 0.1 kilometers.

RE = RADIUS (Earth), the radius of the Earth (6371), in kilometers. Precise to 10^{-4} kilometers.

VPF = VELOCITY (Precipitation Fall), the precipitation fall velocity, in km/hr. Precise to 0.1 km/hr.

ZE = REFLECTIVITY FACTOR (ZE), the effective radar reflectivity factor of a SAMPLE VOLUME, in mm^6/m^3.

HR = HEIGHT (Radar), the radar height above sea level, in kilometers. Precise to 0.1 kilometers.

DIV = DIVERGENCE (Horizontal), horizontal divergence, in 1/hr. Precise to 10^{-2} hr^{-1}.

DEC = DISTANCE (Earth Center), the distance from the Earth center, in km. Precise to 0.1 kilometers.

i = INDEX (Elevation), the index for ELEVATION.

AS = AREA (Surface), the surface area in the plane above the radar bounded by the VAD analysis range, in km^2.

ASC = AREA (Surface Change), the rate of change of AREA (Surface) with respect to height, in km^2/km or kilometers. Precise to 0.1 kilometers.

SVC = SPEED (Vertical Wind Change), the rate of change of the vertical wind speed at a specific altitude, in (km/hr)/km or 1/hr.

PI# = PI, a mathematical constant (3.1416), unitless.

RHO# = DENSITY (Atmospheric), a set of density values for each altitude, in kg/km^3. Precise to 10^6 kg/km^3.

DRHO# = DENSITY (Atmospheric Gradient), a set of density gradient values at each altitude, in kg/km4. Precise to 10^{6}kg/km^{4} .

SVW = SPEED (Vertical Wind), the vertical wind speed (positive upward), in km/hr. Precise to 10^{-3}km/hr.

ND = NUMBER (Data Points), the number of non-missing data points between THRESHOLD (Begin Azimuth) and THRESHOLD (End Azimuth) that went into the Fourier least squares fitting.

SYMM = SYMMETRY, a flag value that is positive if the absolute value of COEFFICIENT(Fourier, #1) is less than the THRESHOLD(Symmetry) and the absolute value of COEFFICIENT(Fourier, #1) minus SPEED(Wind) is less than or equal to zero.

Note: Precision is equivalent to the units unless otherwise stated.

3.2.2 SYMBOLIC FORMULAS

COMPUTE (COEFFICIENTS (Fourier))

$$CF1 = [(1/ND) \sum_j VD_j] - 2\,REL\{[(Ql-(Q2)(Ql)^{COM})/(1-|Q2|^2)]Q4\}$$

$$CF2 = REL\{(Ql-(Q2)(Ql)^{COM})/(1-|Q2|^2)\}$$

$$CF3 = IM\{(Ql-(Q2)(Ql)^{COM})/(1-|Q2|^2)\}$$

where IM is the imaginary part of a complex variable, REL is the real part and

$$Q1 = \{[(1/ND) \sum_j VD_j] - (Q3)/[2(Q4)^{COM}]\}/\{Q4-[1/(4\,(Q4)^{COM})]\}$$

$$Q2 = \{Q4^{COM}-[(Q5)/(2(Q4)^{COM})]\}/\{Q4-[1/(4(Q4)^{COM})]\}$$

$$Q3 = (1/ND) \sum_j VD_j\,[\cos(AZ_j) - (-1)^{0.5} \sin(AZ_j)]$$

$$Q4 = [1/2ND] \sum_j [\cos(AZ_j) + (-1)^{0.5} \sin(AZ_j)]$$

$$Q5 = [1/2ND] \sum_j [\cos(2AZ_j) - (-1)^{0.5} \sin(2AZ_j)]$$

Where COM denotes a complex conjugate, ND is the number of non-missing data points between THRESHOLD (Begin Azimuth) and THRESHOLD (End Azimuth) that went into the Fourier least squares fitting, and j varies between THRESHOLD (Begin Azimuth) and THRESHOLD (End Azimuth).

COMPUTE (DIRECTION (Wind))

$$DW = [\pi - \tan^{-1}(CF3/CF2)] \text{ for } \tan^{-1} = 0 \text{ to } \pi$$

*if DW is less than 0:

$$DW(<0) = DW + 2\pi$$

NOTE: $\pi = 3.1416$

COMPUTE (ROOT MEAN SQUARE)

$$RMS = \{1/ND \sum_{j} [-\cos(AZ_j - DW)(CF2^2 + CF3^2)^{1/2}$$

$$+ CFl - VD_j]^2\}^{1/2}$$

where ND is the number of data points on the VAD circumference and j (the summation index) varies between THRESHOLD (Begin Azimuth) and THRESHOLD (End Azimuth).

COMPUTE (SPEED (Wind))

$$SPW = [(CF2)^2 + (CF3)^2]^{0.5}$$

COMPUTE (FIT)

$$FIT_j = [-\cos(AZ_j - DW)(SPW)] + CFl$$

COMPUTE (SYMMETRY)

Positive (symmetric) if |CFl| is less than THRESHOLD (Symmetry) AND |CFl| - SPW is less than or equal to zero.

COMPUTE (SPEED (Horizontal Wind))

$$SHW = (CF2^2 + CF3^2)^{1/2} / \cos(\phi)$$

COMPUTE (HEIGHT (VAD Wind Estimate))

$$HVAD = \{RS^2 + 2[(4/3)(RE)(RS)(\sin(\phi))]\}/\{2(4/3\ RE)\}$$

COMPUTE (VELOCITY (Precipitation Fall))

For liquid precipitation,

$$VPF = 9.54\ (ZE^{0.114})\ [1.01091 + (HVAD + HR)(0.02863) +$$

$$(HVAD + HR)^2\ 0.00259]$$

For snow,

$$VPF = 2.94\ ZE^{0.063}$$

For clear air,

VPF = 0

COMPUTE (DISTANCE (Earth Center))

$DEC_i = RE - E + E\,[1 + (RS/E)^2 + 2RS/E \sin \phi_i]^{0.5}$

where: E = 4/3 RE

COMPUTE (AREA (Surface))

$AS_i = 2\pi\ (DEC_i)^2[1-\cos Q6_i]$

where:

$$Q6_i = 4/3 \cos^{-1} \left[\frac{E^2 + (DEC_i + RE/3)^2 - RS^2}{2E(DEC_i + RE/3)} \right]$$

NOTE: $DEC_o = RE$

COMPUTE (AREA (Surface Change))

$$ASC_i = \frac{2AS_i}{DEC_i} - \left\{ \frac{B(DEC_i)^2 \sin(Q6_i)}{(RE)\sin(3/4Q6_i)} \left[1 + \frac{RS^2 - E^2}{(DEC_i + RE/3)^2} \right] \right\}$$

COMPUTE (SPEED (Vertical Wind Change))

$$SVC_i^* = -3/2\ \frac{(B)(RS)}{RE} \left[DEC_i + RE/3\right] \frac{CF1_i}{AS_i}$$

$$- \left[\frac{1}{AS_i}\ ASC_i + \frac{1}{RHO\#_i}\ DRHO\#_i \right] SVW_{i-1}$$

(assume $SVW_{i-1} = 0$ for i = 1)

* : Use last good value of SVW_{i-1} if current value is not available.

COMPUTE (SPEED (Vertical Wind))

$SVW_i = SVW_{i-1} + (SVC_i)(DEC_i - DEC_{i-1})$

Assuming: $SVW_0 = 0$

COMPUTE (DIVERGENCE (Horizontal))

$DIV = 2(CFl_{ih})/[(RS)(COS\ \phi_i)]$

$+[2VPF \tan \phi_i /(RS)]$

Where:

$CFl_{ih} = CFl_i - (SVW_i) \sin \phi_i$

*: Use last good value of SVW_{i-1} if current value is not available.

4.0 OUTPUTS

4.1 IDENTIFICATION

DW is a table of horizontal wind direction as a function of height for this observation time.

SHW is a table of horizontal wind speed as a function of height for this observation time.

DIV is a table of horizontal divergence as a function of height for this observation time.

SVW is a table of vertical wind speed (positive upward) as a function of height for this observation time.

4.2 DISTRIBUTION

Outputs from the VAD algorithm can serve as inputs to a wind shear algorithm and a turbulence algorithm.

5.0 INFERENCES

5.1 LIMITATIONS

This version of the VAD algorithm does not account for deformation (e.g., fronts). The successful operation of this algorithm rests on the assumption that there are enough echoes to receive coherent Doppler velocity estimates. When operating in the clear air mode the VAD algorithm averages three sample volumes in range to reduce the scatter of Doppler velocities due to weak echo returns. There are many equations relating equivalent reflectivity to precipitation fall velocity. An incorrect relationship will cause errors in the estimates of horizontal divergence and vertical wind speed.

Compromises may need to be made in determining the optimum slant range for the VAD. At short ranges ground clutter can completely saturate data and at long ranges the resolution of the Doppler radar decreases.

5.2 FUTURE DEVELOPEMENTS

The only future development regarding the VAD algorithm is the determination of the values for velocity threshold and slant range. This will probably be done during the NEXRAD algorithm testing phase.