Modeling flows through canopies with immersed boundary methods

Andreas Dörnbrack and Christian Kühnlein

Institut für Physik der Atmosphäre

DLR Oberpfaffenhofen

Piotr K. Smolarkiewicz

NCAR, Boulder CO,USA

Christoph Leifeld

DWD Offenbach

1 Motivation

2 Method

EULAG
Immersed Boundary Method
Generation of Forests
Analyses Methods

3 Results

4 Summary

5 Further Research

1 Motivation

Request by the German Weather Service:

Investigation of the wind and turbulence conditions for the take-off direction 21 of the regional airport Frankfurt/Hahn

Worst case scenario:

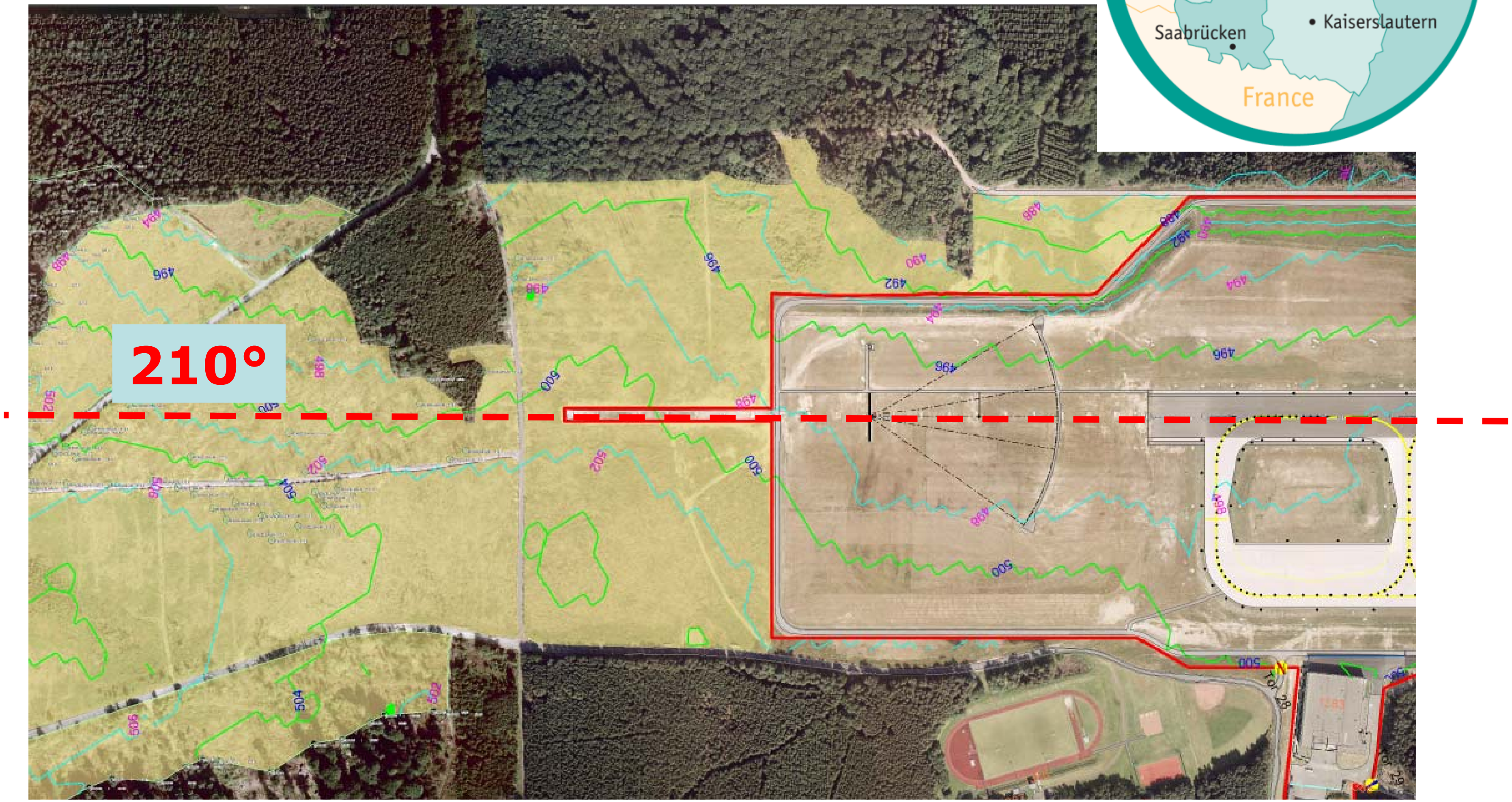
Wind 25 kt in 10 m altitude (from given directions), if possible gusts between 40 kt and 60 kt

1 Motivation

Airport Frankfurt/Hahn
Extended Runway 210°

1 Motivation

Airport Frankfurt/Hahn
Extended Runway 210°



1 Motivation

Airport Frankfurt/Hahn
Extended Runway 210°

1 Motivation

Tail Strike

1 Motivation

• there were no data available to estimate the potential of strong wind gusts on the aircraft

• there was no empirical knowledge to estimate the wind situation with the extended runway

• numerical simulations appeared to be the only tool that could give quantitative estimates of wind and turbulence structure in the lee of the forests

Create simplified set-up of the canopy structure at Frankfurt/Hahn

2 Method

Situation:

- atmospheric flow with mean wind speeds of ≈ 13 ms^{-1}

  → **nearly neutrally stratified flow**

- wind gusts of ± 25 ms^{-1}

  → **periodic boundary conditions for the simulation domain**

- canopies of different height, length and density

  → **forests = porous bodies → immersed boundary method**

Forest = Porous Body ??

Shaw and Schumann, 1992: treated the forest stand

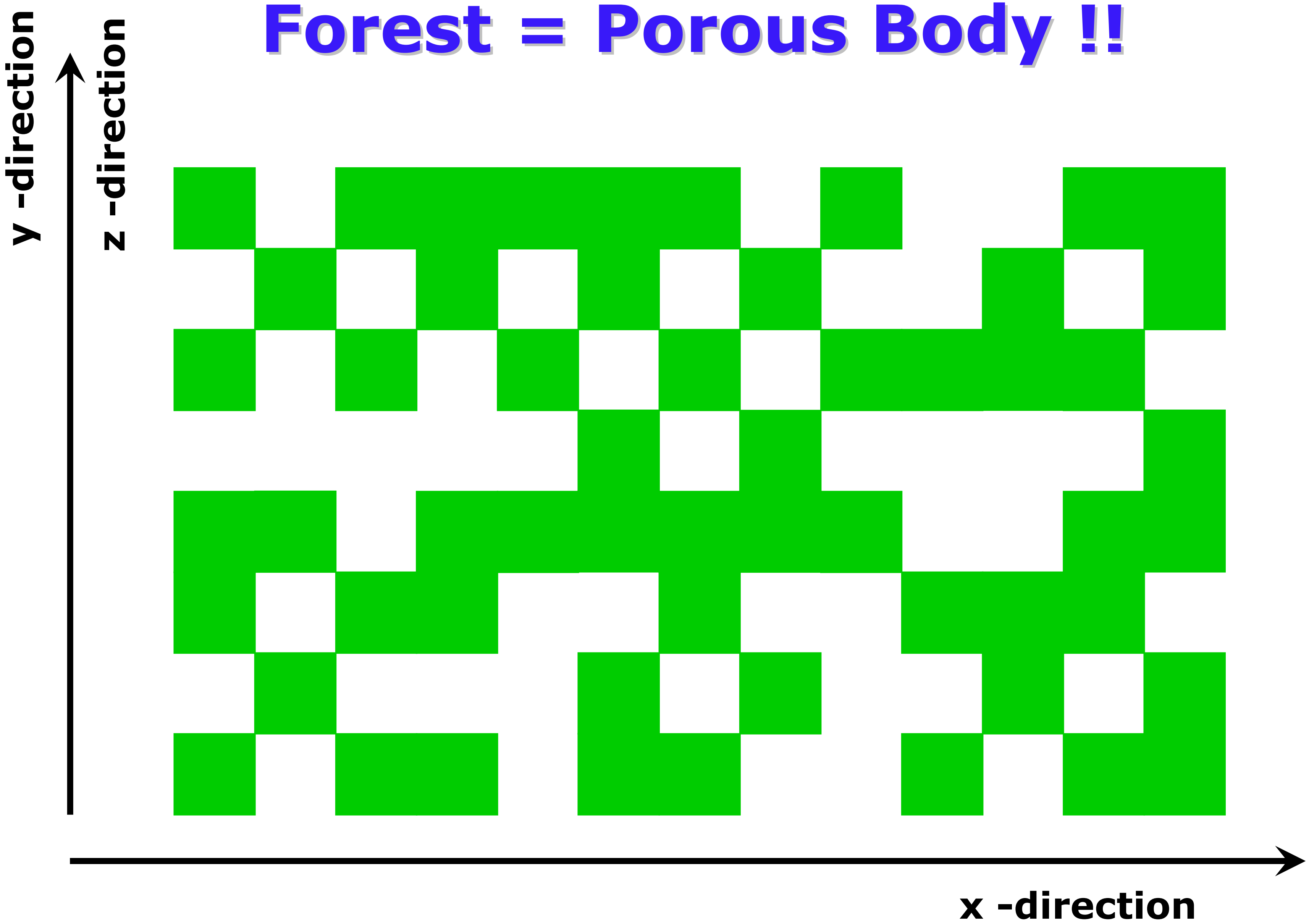
"as a porous body of horizontally uniform (leaf) area density $A(z)$ with constant drag coefficient C_D"

$$f_i(\mathrm{x}_C, t) = C_D\, A(z) \left| \mathrm{u} \right| u_i \qquad i = 1, 2, 3$$

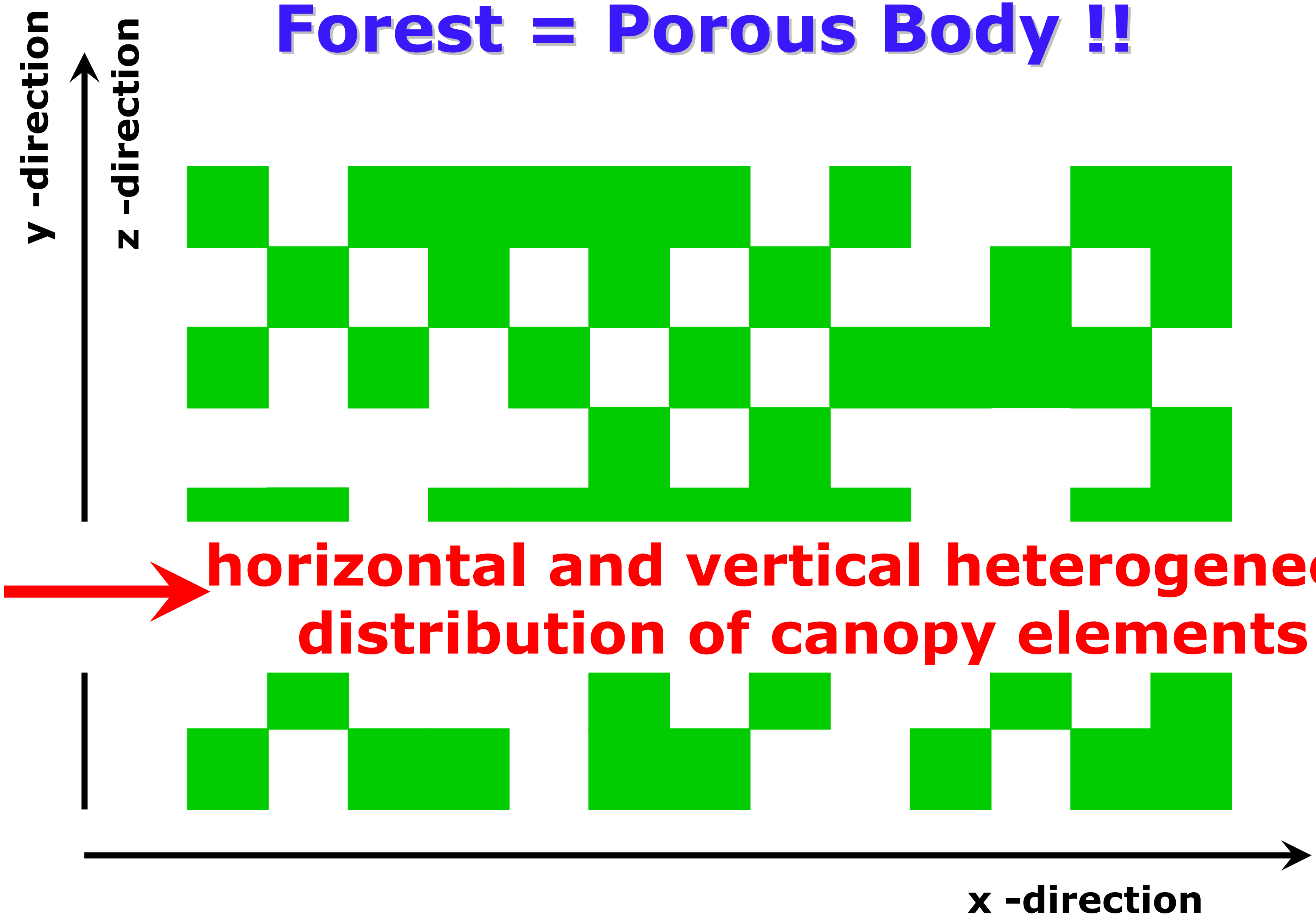
- sometimes called field-scale approach

- ongoing work for plant-scale approach to treat heterogeneous plants based on above equation

Forest = Porous Body !!

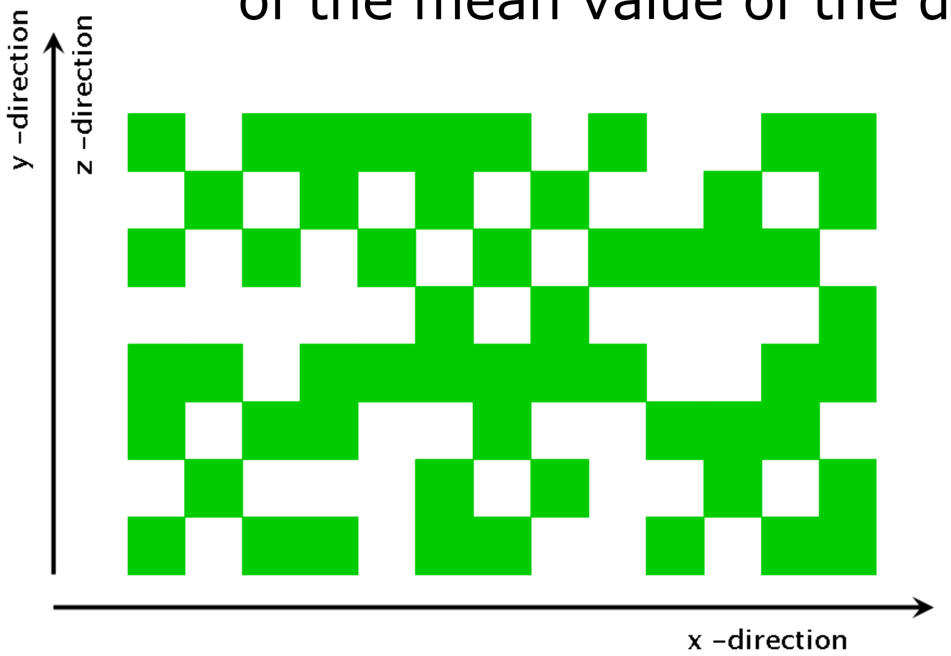


Forest = Porous Body !!



Forest Set-Up for LES

- Gaußian distribution of random numbers [-0.5,0.5]
- positive numbers: wood grid cells in the prescribed forest volume V_{Forest} = immersed boundaries
- porosity: $\chi = 1 - V_{Wood}/V_{Forest}$; can be controlled by ± shift of the mean value of the distribution



χ = 1 no forest elements

χ = 0 solid obstacle

EULAG - Anelastic Equations

$$\nabla \bullet (\rho_b v) = 0$$

$$\frac{dv}{dt} = -\nabla \pi' + g \frac{\theta'}{\theta_b} - f \times v' + F' - \beta v$$

$$\frac{d\theta'}{dt} = H - v \bullet \nabla \theta_e$$

$$\frac{de}{dt} = S(e) - \beta e$$

$$\psi' = \psi - \psi_e \qquad \psi = v, \theta, ... \qquad \pi' = \frac{p - p_e}{\rho_b}$$

EULAG - Immersed Boundary Method

Idea: "fluid sees a body through the forces of pressure and shear that exist along the body surface" (Goldstein et al. 1993)

→ the presence of a rigid body surface can be modeled with an external force field

→ prototype of feedback ($\alpha > 0$, $\beta > 0$):

$$f_i(\mathrm{x}_s, t) = -\alpha \int_0^t u_i(\mathrm{x}_s, t')\,dt' - \beta\, u_i(\mathrm{x}_s, t)$$

Stokes drag

Smolarkiewicz et al., 2007

$\alpha = 0.$ **$\beta^{-1} = 0.5\ \Delta t$**

EULAG - Immersed Boundary Method

$$\frac{d\Psi}{dt} = -\alpha \int_0^t \Psi(\tau) d\tau - \beta \Psi(t) + A \sin(\omega t)$$

Implicit numerical approximation in line with EULAG scheme:

$$\Psi^{n+1} = \hat{\Psi} + 0.5 \Delta t R^{n+1}; \quad \hat{\Psi} \equiv \Psi^n + 0.5 \Delta t R^n$$

$$R^n = -\alpha I^n(\Psi) - \beta \Psi^n + A \sin(\omega t^n)$$

$$I^n(\Psi) \equiv \Delta t \sum_{k=1}^{n} 0.5 \, (\Psi^{k-1} + \Psi^k)$$

Smolarkiewicz et al., 2007

EULAG - Immersed Boundary Method

$$\frac{d\Psi}{dt} = -\alpha \int_0^t \Psi(\tau)\,d\tau - \beta\,\Psi(t) + A\sin(\omega t)$$

$A = 1/(200\Delta t)$
$\Delta t = 10^{-3}$
$\alpha = 2\pi/(2\Delta t)$
$\omega = 2\pi/(200\Delta t)$

Smolarkiewicz et al. 2007:

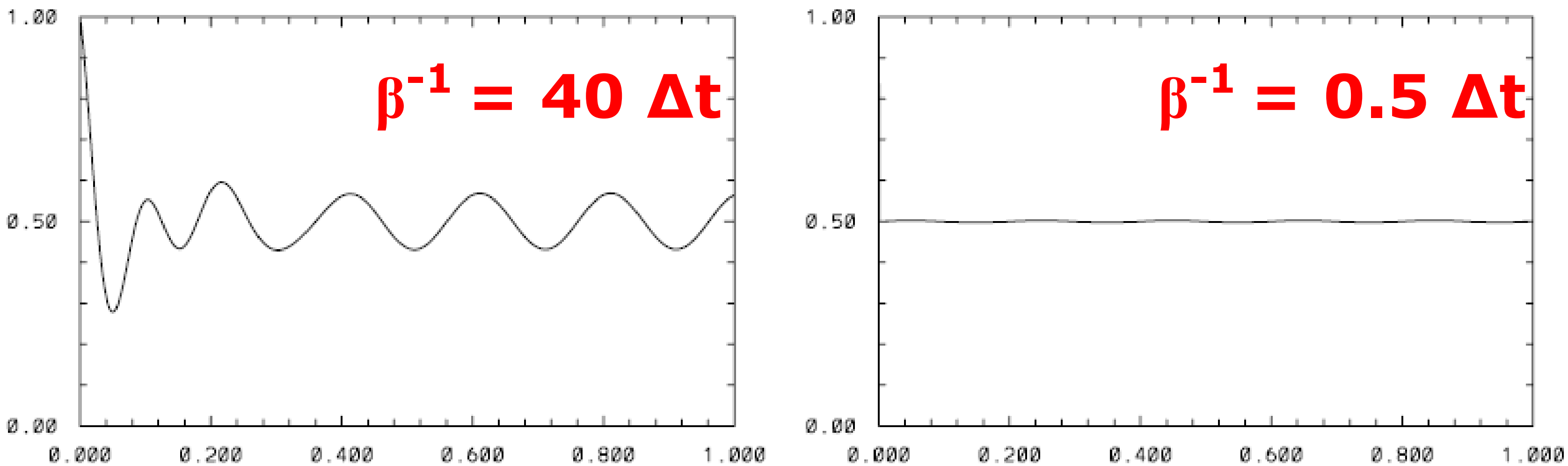


Fig. 11. Example of integrating (27) with an implicit second-order-accurate scheme (31). Here, $\delta t = 1/1000$, $\gamma = 2\pi/(2\delta t)$, $\omega = 2\pi/(200\delta t)$, $A = 1/(200\delta t)$, $\psi(t=0) = 1$, and $\beta^{-1} = 40\delta t$ (left plate) or $\beta^{-1} = 0.5\delta t$ (right plate); t and ψ are the abscissa and ordinate, respectively.

Numerical Simulations

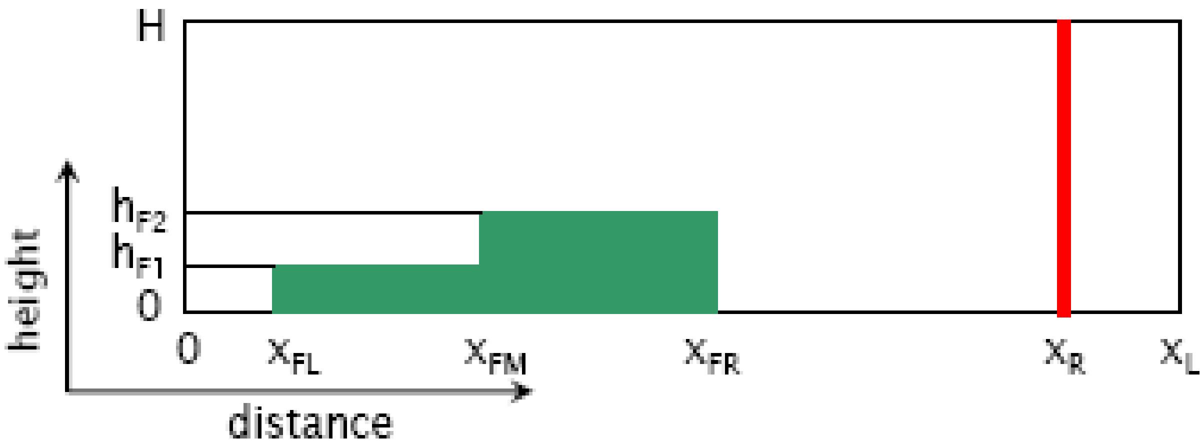
- open/cyclic boundary conditions in the horizontal directions
- rigid lid at the top height H = 200 or 300 m
- $\Delta x = \Delta y = 2$ m , $\Delta z = 1$ m, $\Delta t = 0.01$ s
- TKE closure with prescribed drag coefficient $C_D = 0.001$ at the lower surface
- Initial conditions:

$$u(x,y,z,t_0) = U_0,\ v(x,y,z,t_0) = w(x,y,z,t_0) = 0.$$

$$e(x,y,z,t_0) = 0.$$

- spin up time $t_{SPIN\text{-}UP} = 450$ s
- restart and simulation until $t_{END} = 600$ s;
- turbulence statistics for a 150 s period
- constant mass flux ensured by adaptive pressure gradient

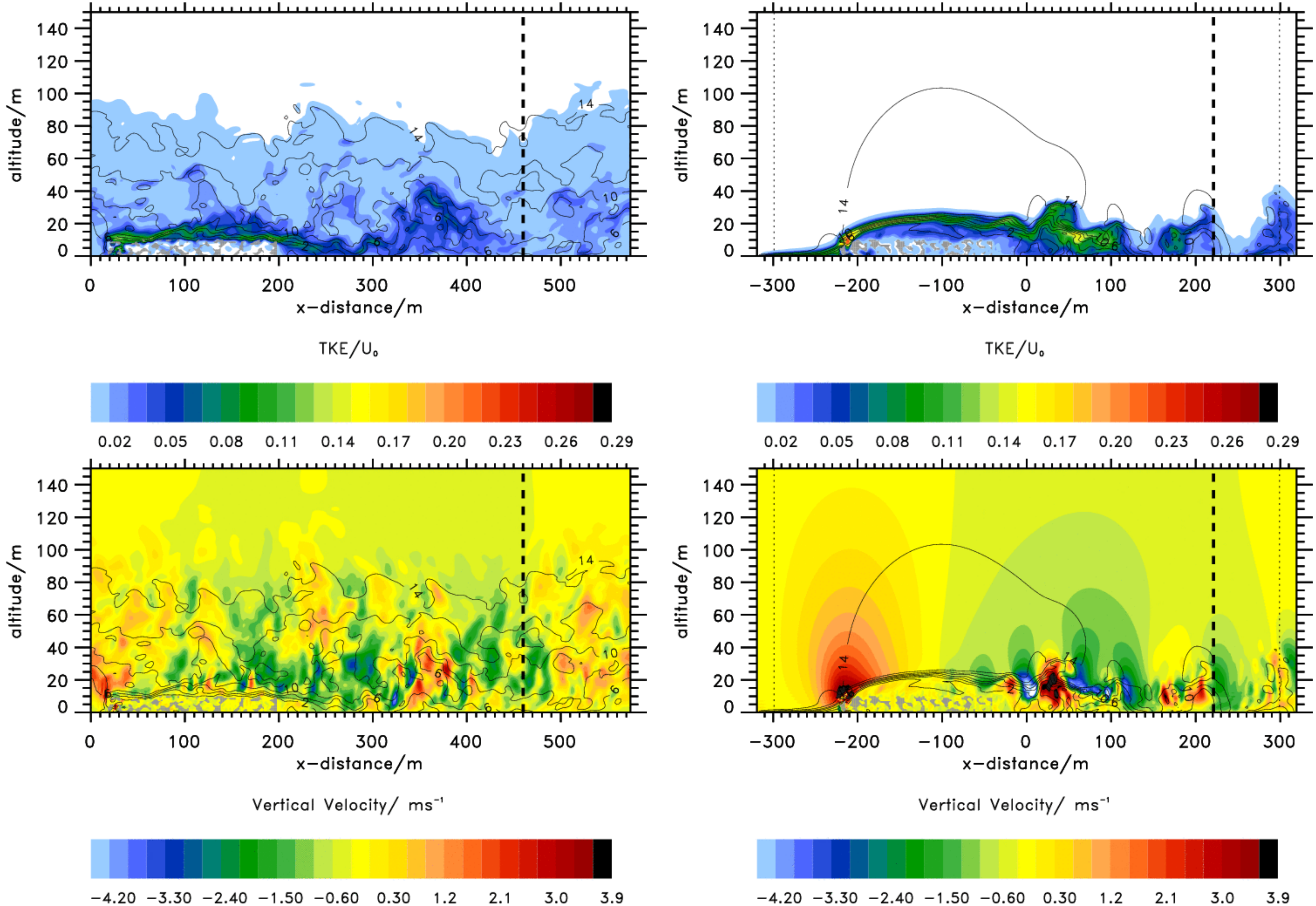
3 Results

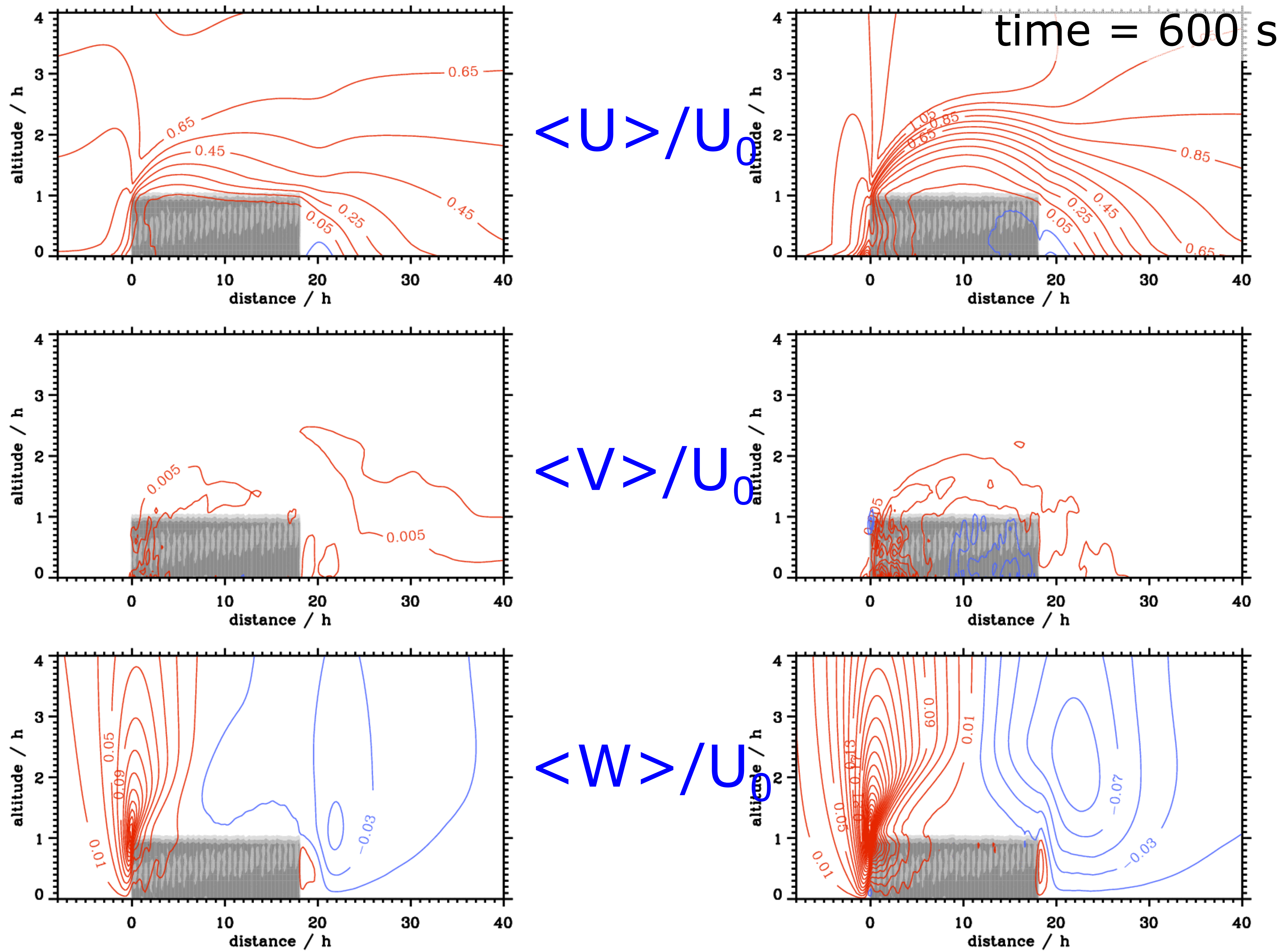


Modell-Run	x_{FL}/m	x_{FR}/m	x_R/m	x_L/m	h_{F1}/m	χ	BC in x-direction	n, m, l
HAHN-I_005	20	200	460	574	10 ± 2	0.174	periodical	288, 32, 201
HAHN-I_006	20	200	460	574	10 ± 2	0.933	periodical	288, 32, 201
HAHN-I_007	20	200	460	574	10 ± 2	0.565	periodical	288, 32, 201
HAHN-I_008	90	270	530	638	10 ± 2	0.554	open	320, 32, 201

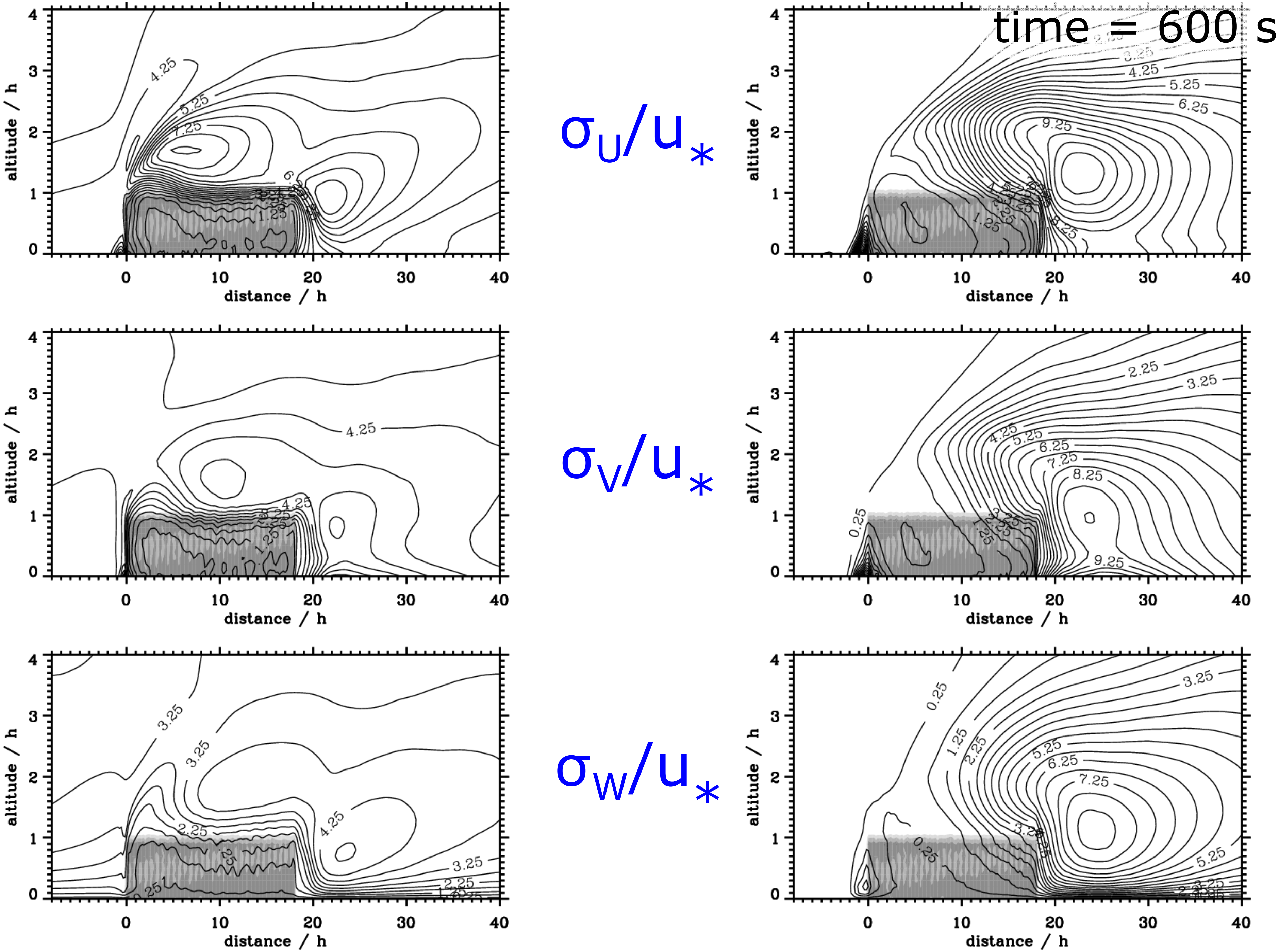
- cyclic vs open boundaries in x-direction
- influence of porosity χ

Cyclic vs Open Boundaries, $\chi \approx$ 0.55 time = 360 s

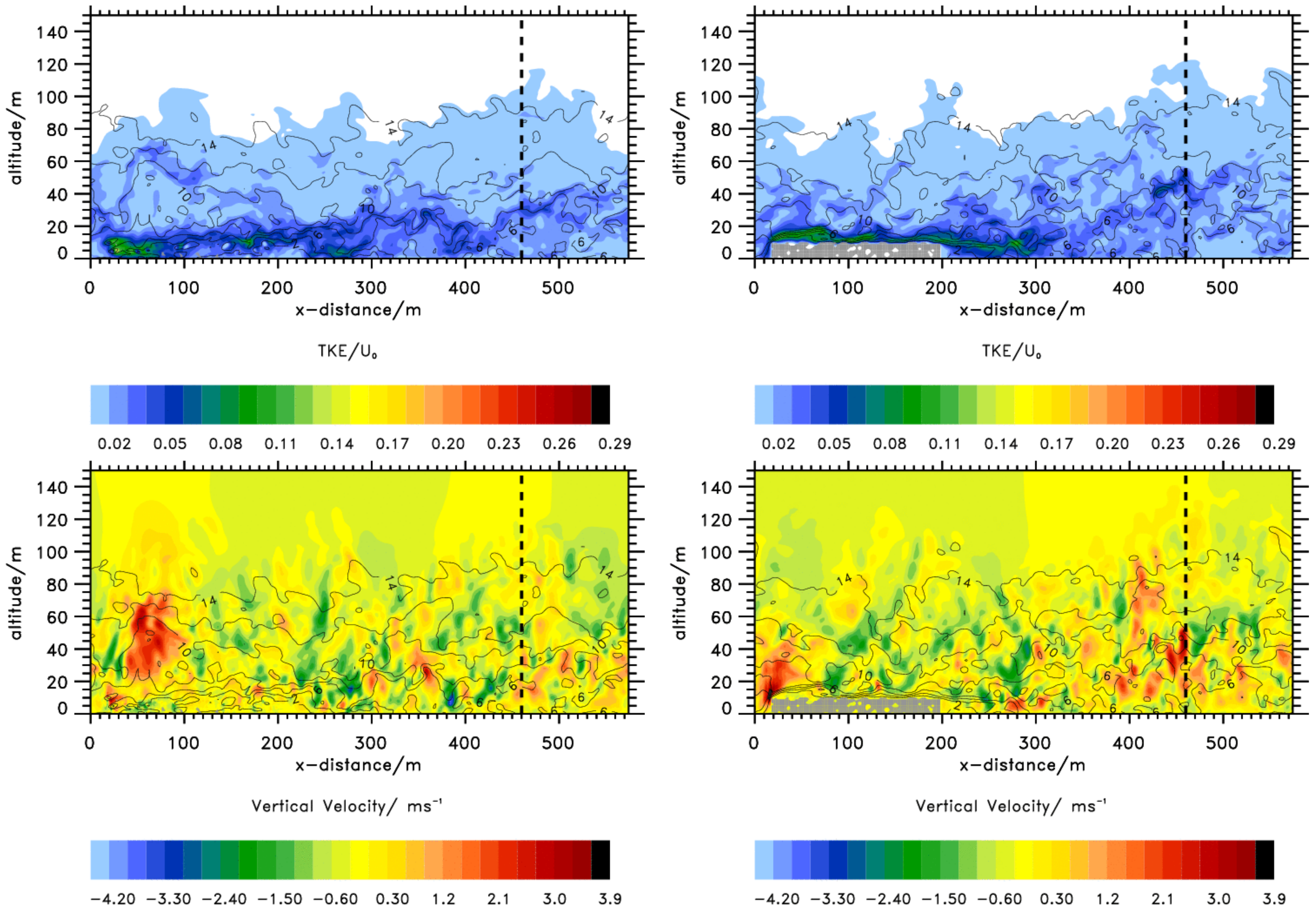


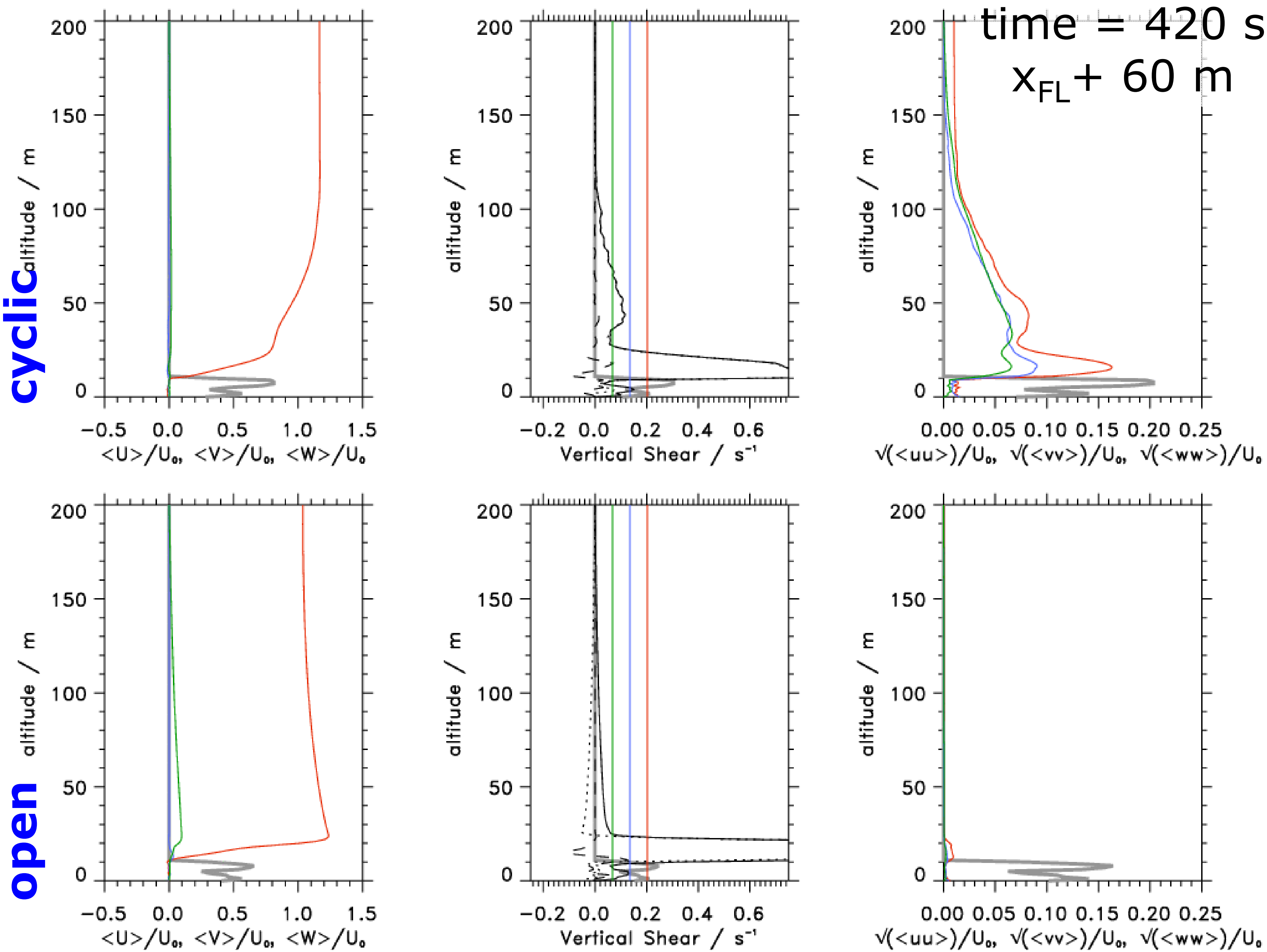


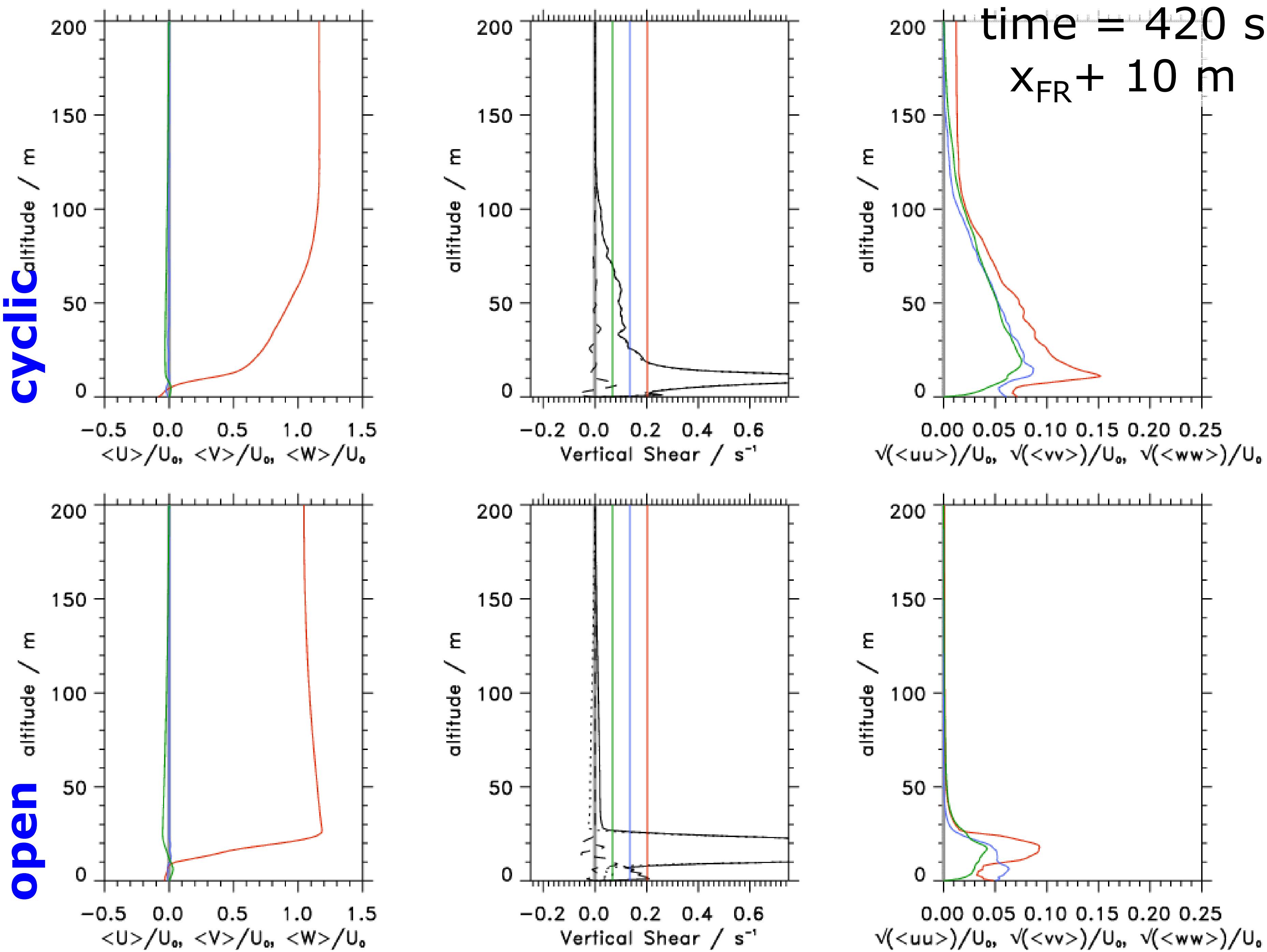
time = 600 s
$\langle U \rangle / U_0$
$\langle V \rangle / U_0$
$\langle W \rangle / U_0$
altitude / h
distance / h
0.65
0.45
0.25
0.05
1.05
0.85
0.005
0.09
0.01
0.13
-0.03
-0.07

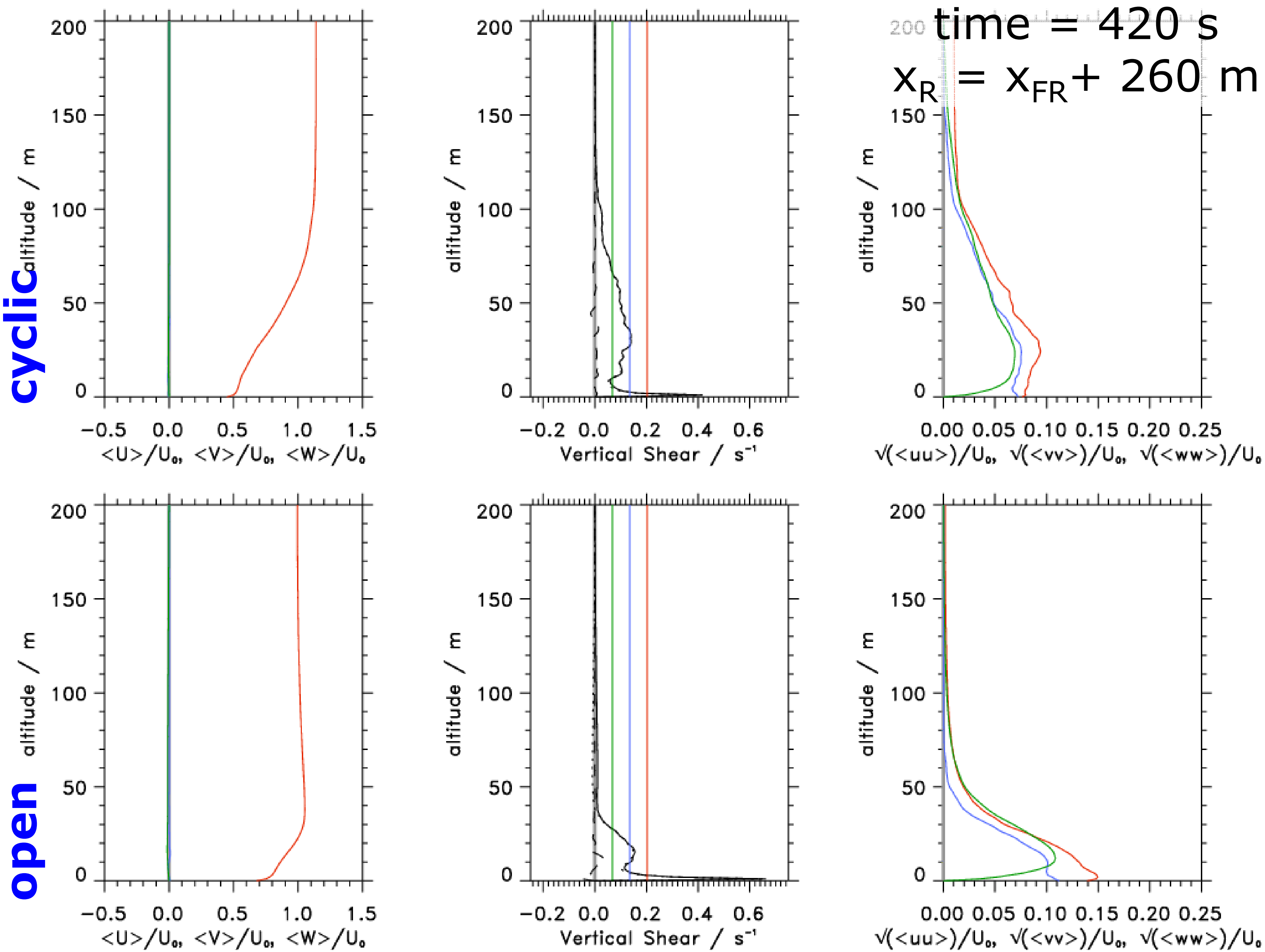

time = 600 s
σU/u*
σV/u*
σW/u*
altitude / h
distance / h

Influence of porosity χ time = 420 s




time = 420 s
x_{FL}+ 60 m
cyclic
open
altitude / m
<U>/U₀, <V>/U₀, <W>/U₀
Vertical Shear / s⁻¹
√(<uu>)/U₀, √(<vv>)/U₀, √(<ww>)/U₀


time = 420 s
x_FR + 10 m
cyclic
open
altitude / m
<U>/U₀, <V>/U₀, <W>/U₀
Vertical Shear / s⁻¹
√(<uu>)/U₀, √(<vv>)/U₀, √(<ww>)/U₀


time = 420 s
x_R = x_FR + 260 m
cyclic
open
altitude / m
<U>/U₀, <V>/U₀, <W>/U₀
Vertical Shear / s⁻¹
√(<uu>)/U₀, √(<vv>)/U₀, √(<ww>)/U₀

Maximum values at runway position

cyclic boundary condition in x-direction

thick forest: $\chi \approx 0.174$

Altitude Range	$<U> \pm \sigma_u$ /ms^{-1}	$<V> \pm \sigma_v$ /ms^{-1}	$<W> \pm \sigma_w$ /ms^{-1}	$<S>$ / s^{-1} ; kn /100 ft
0 … 50 m	11.7 ± 1.2	0.074 ± 0.862	-0.001 ± 0.828	0.426; 25.27
50 … 100 m	14.2 ± 0.9	0.026 ± 0.604	+0.008 ± 0.631	0.086; 5.11
100 … 200 m	14.5 ± 0.3	0.007 ± 0.150	+0.012 ± 0.297	0.017; 1.05

sparse forest: $\chi \approx 0.933$

Altitude Range	$<U> \pm \sigma_u$ /ms^{-1}	$<V> \pm \sigma_v$ /ms^{-1}	$<W> \pm \sigma_w$ /ms^{-1}	$<S>$ / s^{-1} ; kn /100 ft
0 … 50 m	11.7 ± 1.3	0.049 ± 1.025	-0.005 ± 0.960	0.379; 22.45
50 … 100 m	14.4 ± 0.9	0.066 ± 0.708	-0.038 ± 0.710	0.101; 6.01
100 … 200 m	14.5 ± 0.3	0.010 ± 0.131	-0.000 ± 0.228	0.014; 0.86

Sektor	**Wind mit 25kt**	**Intensität der Windscherung im Höhenbereich 50m – 100m**
A	120° (OSO)	LGT – MOD
	270° (W)	MOD
	300° (WNW)	MOD
B	120° (OSO)	MOD – STRONG
	270° (W)	MOD – STRONG
	300° (WNW)	MOD
C	120° (OSO)	LGT – MOD
	210° (SSW)	MOD
	240° (WSW)	LGT – MOD
	300° (WNW)	MOD – STRONG
D	120° (OSO)	MOD – STRONG
	180° (S)	MOD – STRONG
	210° (SSW)	MOD – STRONG
	300° (WNW)	MOD – STRONG

Verantwortliche Bearbeiter:

Klaus Sturm
Dipl. Meteorologe

Dr. Udo Busch
Dipl. Meteorologe

Dr. Christoph Leifeld
Dipl. Meteorologe

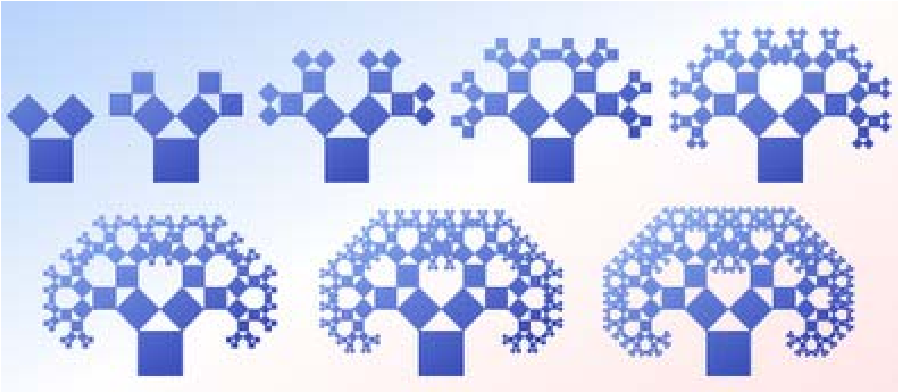
Offenbach am Main, den 31.07.2008

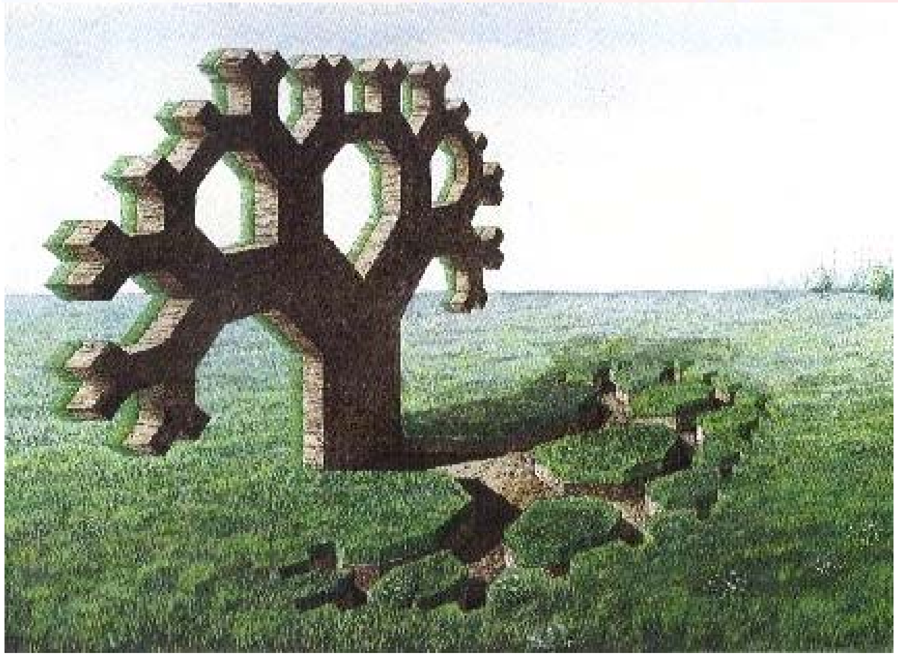
4 Summary

- the height of the turbulent boundary layer depends primarily on vertical depth of the canopy
- the porosity of the forest seems to have only a marginal impact on the BL height
- directly above the canopy layer and in the wake exist strong shear layers
- more extended canopies facilitate the detachment and vertical propagation of shear layers
- simulations with open boundaries in flow direction generates larger turbulent fluctuations above and in the lee of the canopy; the depth, however, is smaller compared to simulations with periodic boundary conditions

5 Further Research (together with J. Schröttl)

canopy structures with higher spatial resolution
and more realistic shape





array of eight 1m thick
2d-Pythagoras trees

$\Delta x = \Delta y = \Delta z = 0.05$ m
$\Delta t = 0.002$ s

n = m = 384 l=301

Statistical Results

Horizontally Coherent Canopy Stand
(h = 10 ± 2 m)

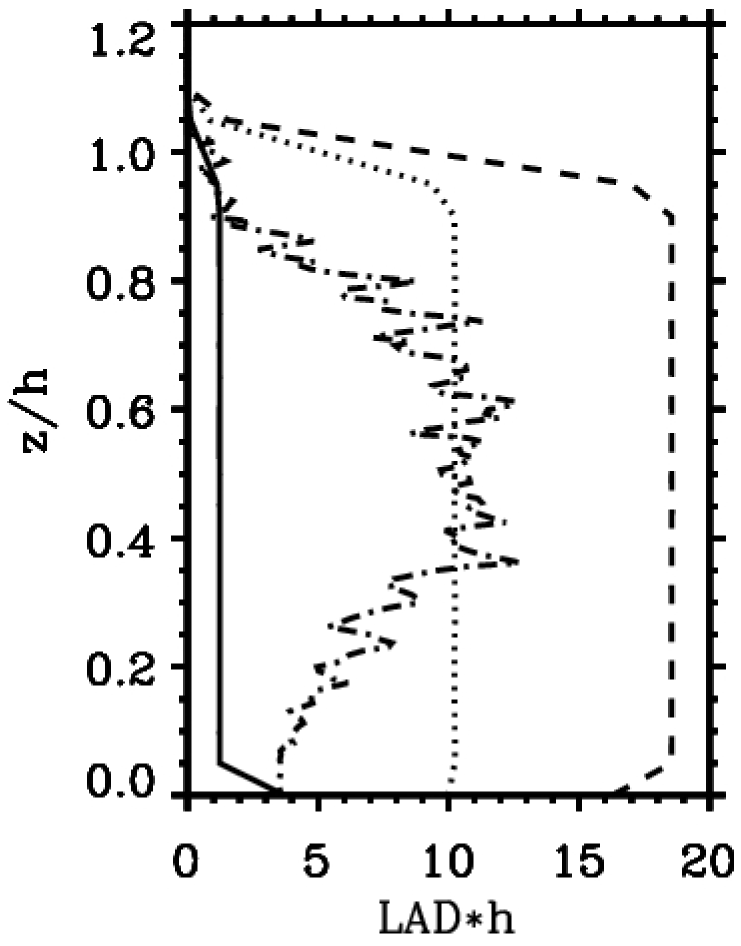
A: Vertically Uniform Leaf Area Density (LAD)

Porosity χ:			LAI
0.930	forest_101	————————	1.29
0.475	forest_102	....	10.23
0.053	forest_103	- - - - - - - - - -	18.48

Array of 8 quasi-2D Pythagoras trees (h ≈ 7 m)

0.950	trees_004	- · - · - · - · - · -	6.98

Vertical profiles of Leaf Area Density (LAD)



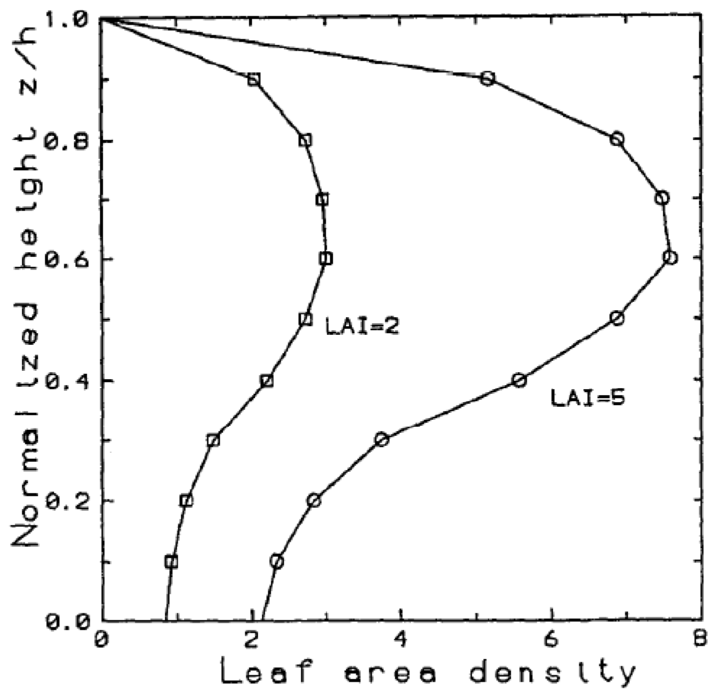
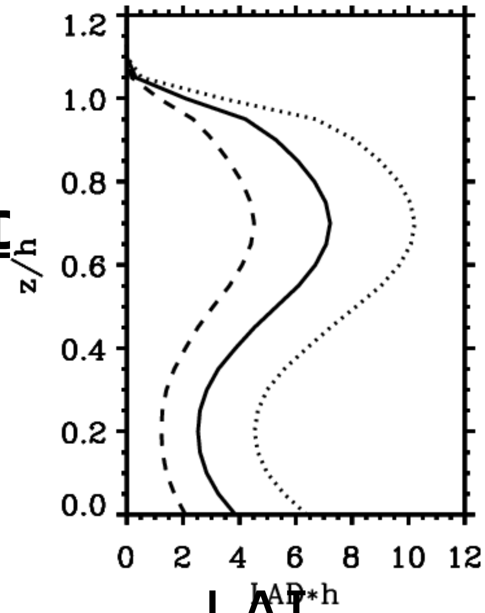


Fig. 2. Vertical profiles of leaf area density (a) for two values of LAI. Leaf area density is normal by multiplication by the canopy height.

Shaw and Schumann, 1992

Statistical Results

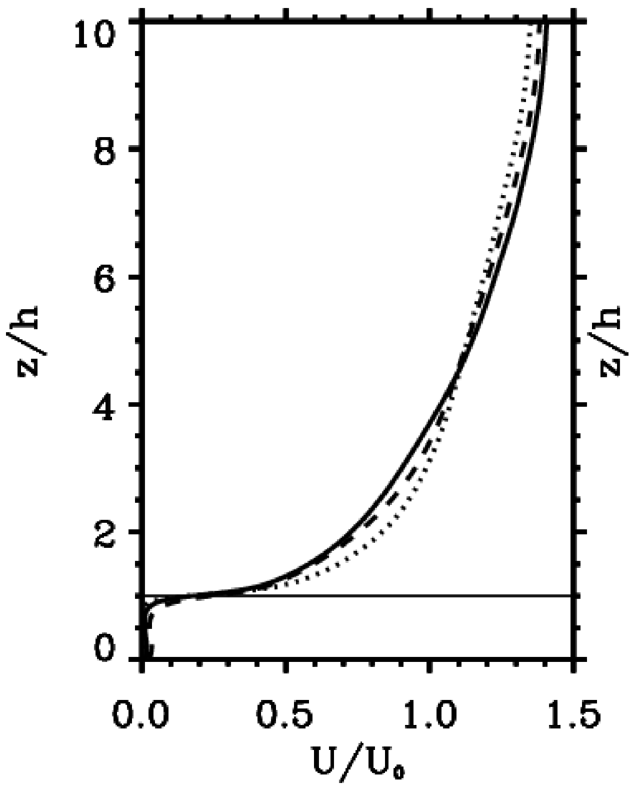
Horizontally Coherent Canopy Stand
(h = 10 ± 2 m)



B: Vertically Varying Leaf Area Density (LAD)

Porosity χ:			LAI
0.625	forest_105	···········	7.32
0.758	forest_104	———	4.72
0.861	forest_106	- - - - -	2.71

Mean Velocity Components



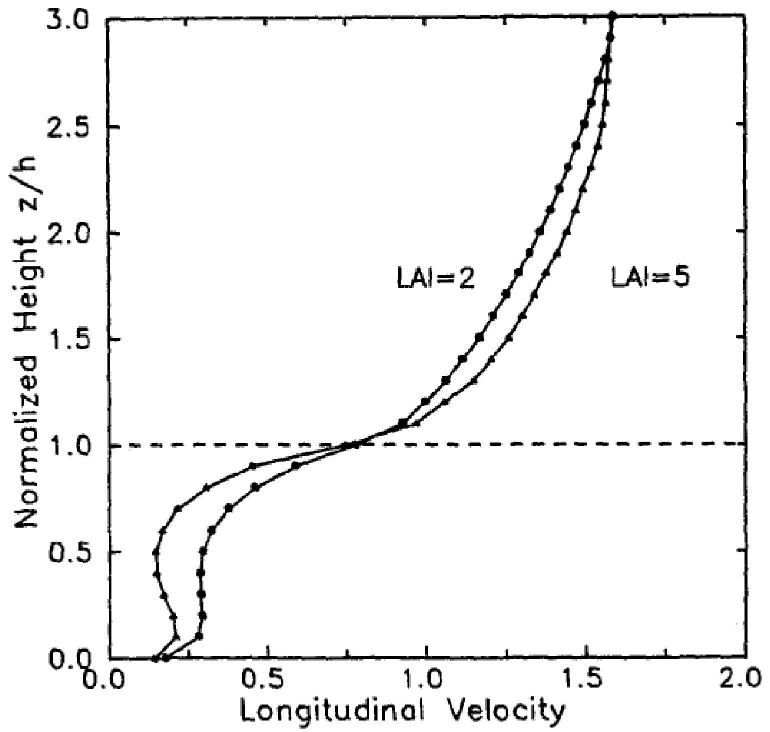
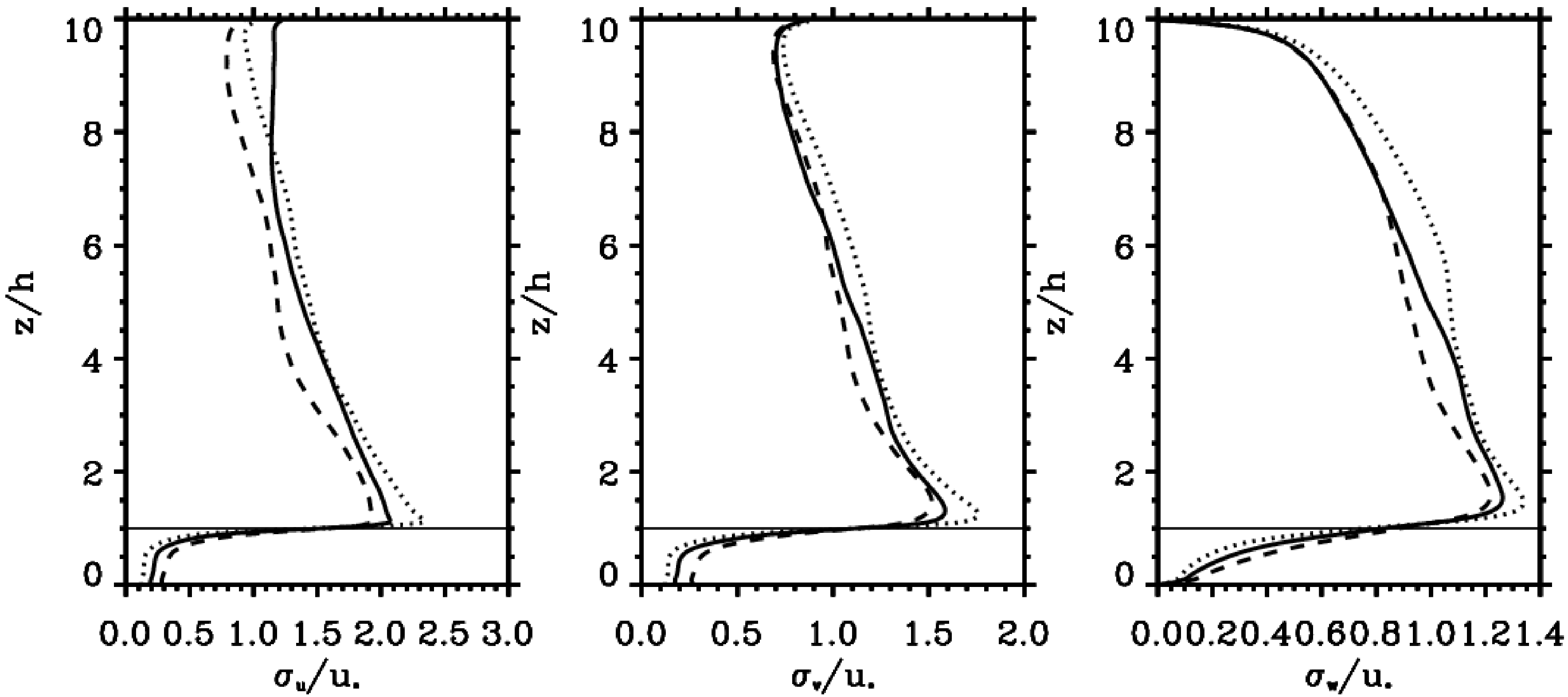


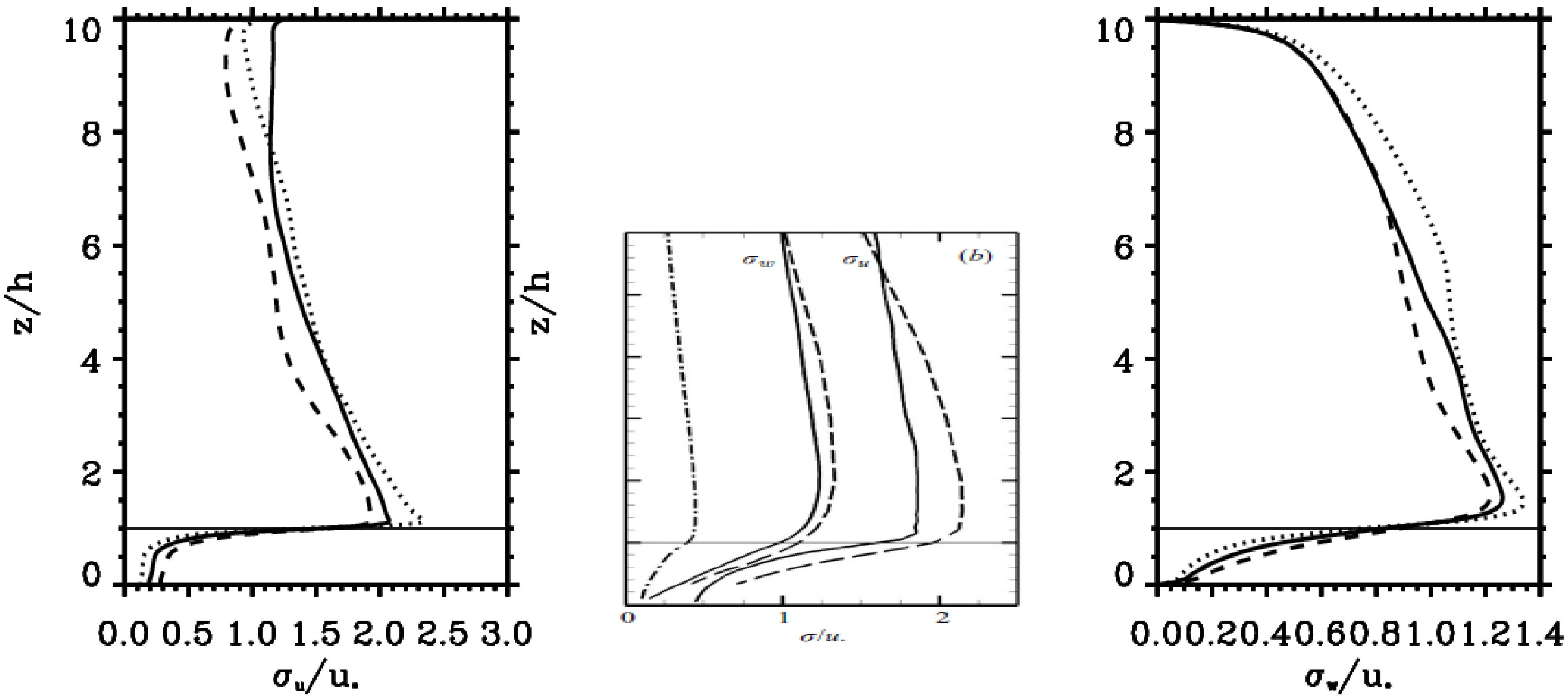
Fig. 3. Spatial mean longitudinal velocity profiles for two values of LAI under weakly unstable conditions. Velocity is normalized by the vertically averaged longitudinal velocity.

Shaw and Schumann, 1992

Turbulent Velocity Variances



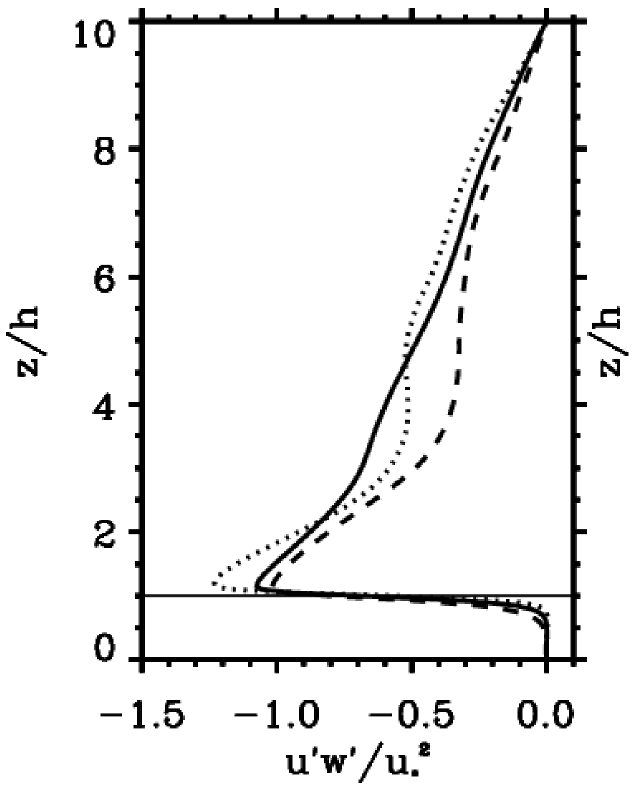
Turbulent Velocity Variances



LES: Finnigan et al. JFM 2009

Wind tunnel: Brunet et al. BLM 1994

Turbulent Momentum Fluxes



Shaw and Schumann, 1992

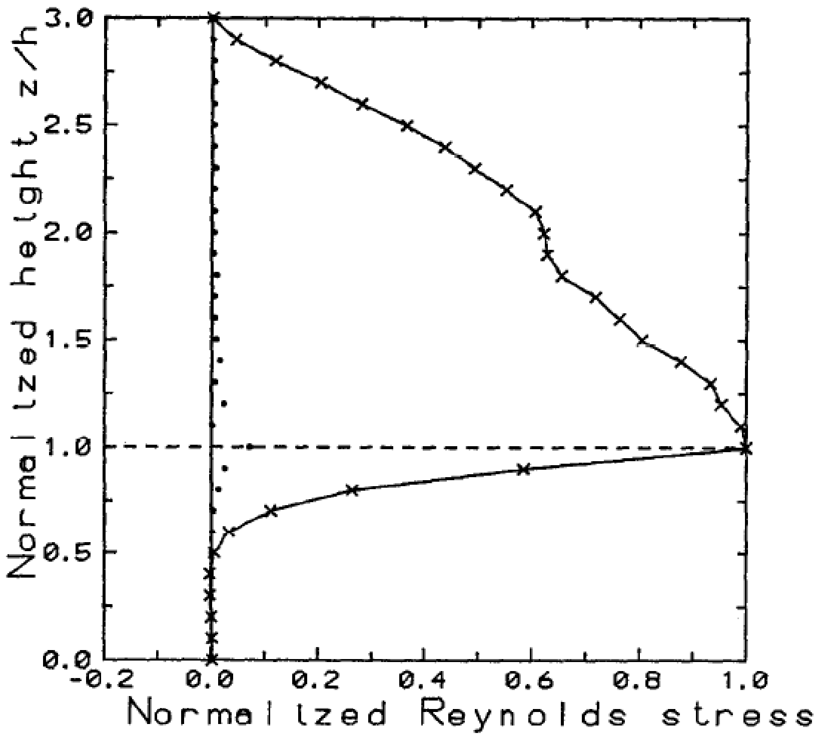
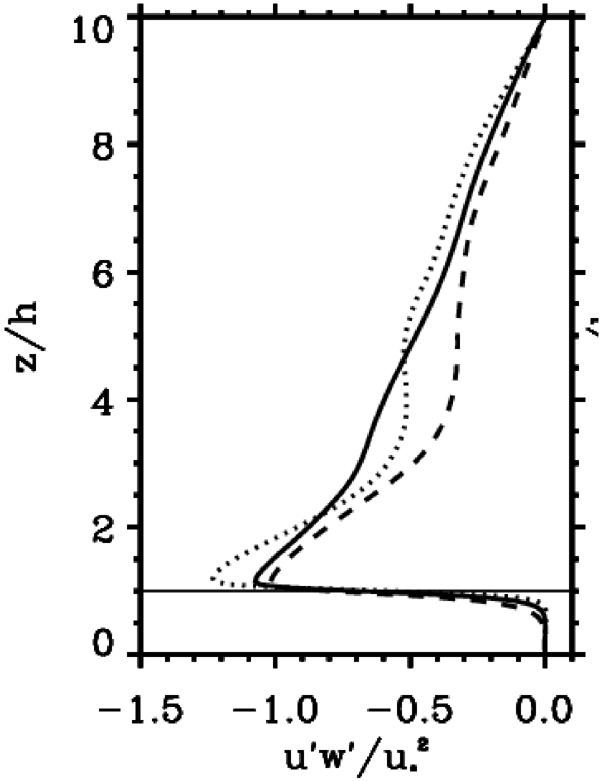


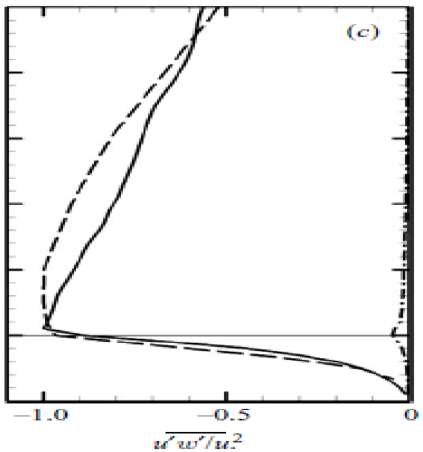
Fig. 4. Vertical profile of the spatial mean Reynolds stress for a LAI of 5 and weakly unstable conditions, and normalized to its value at the top of the canopy. The solid line is the sum of resolved and subgrid-scale components of the Reynolds stress. The dots are the SGS component.

Turbulent Momentum Fluxes

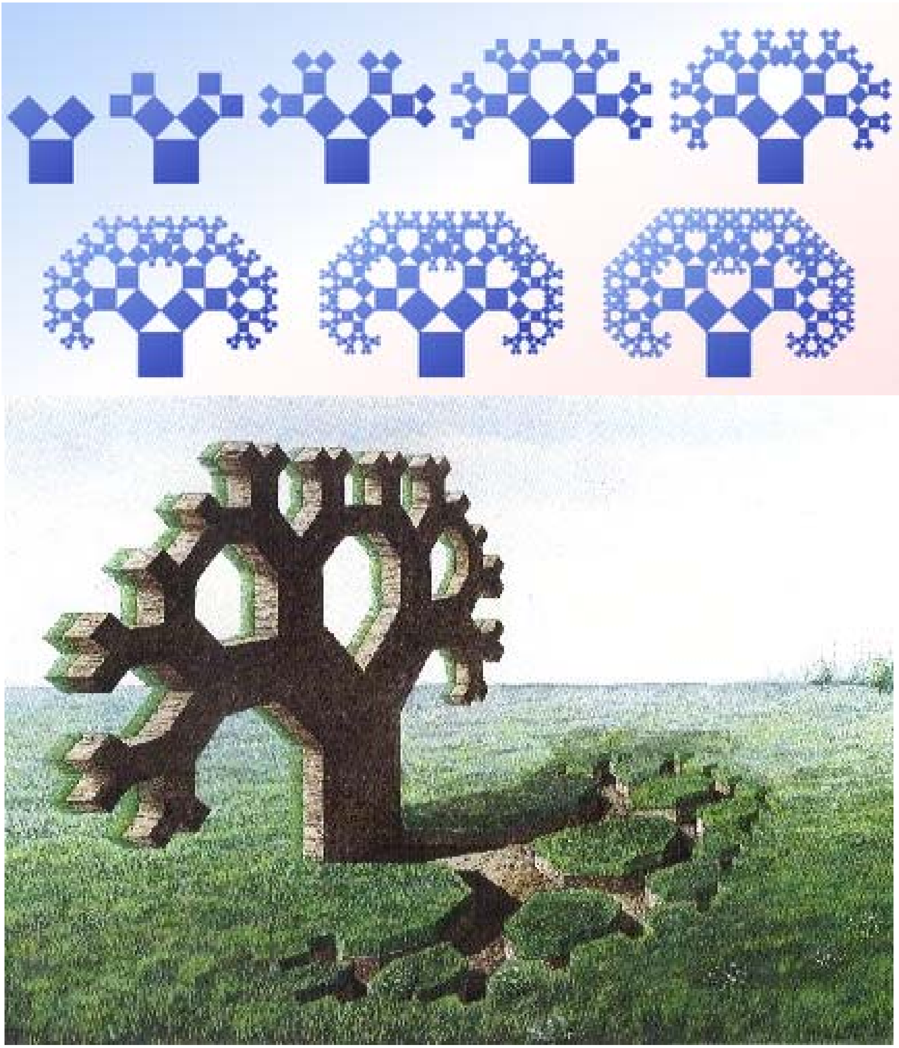


LES: Finnigan et al. JFM 2009

Wind tunnel: Brunet et al. BLM 19

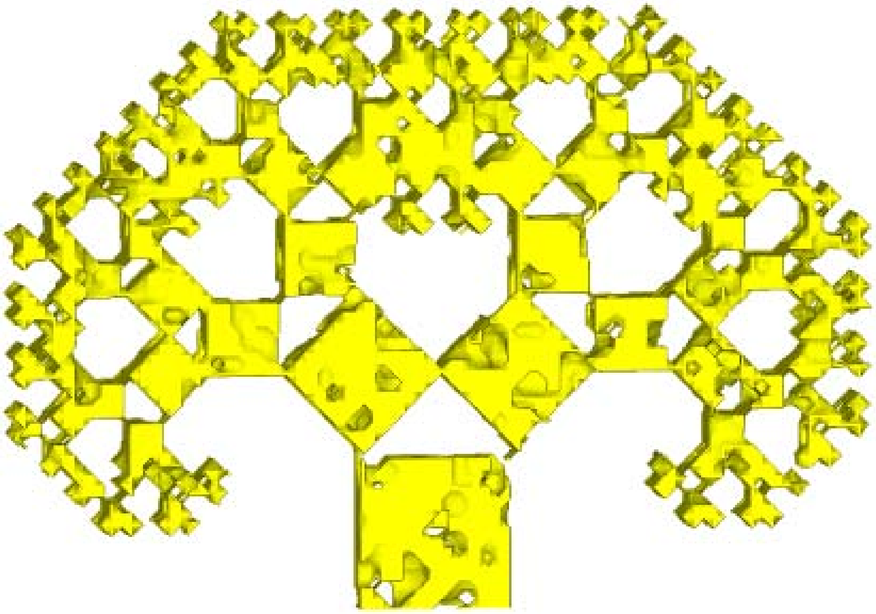


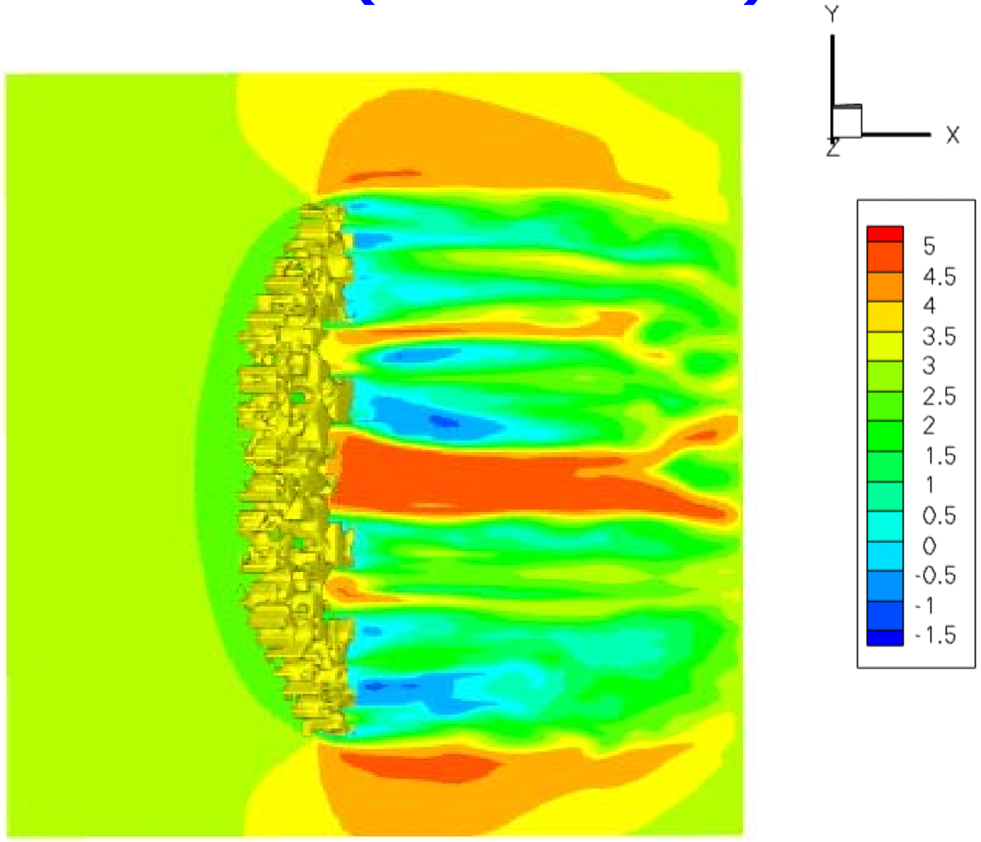
Outlook:
Simulations around a single 3D fractal tree



Outlook:
Simulations around a single 3D fractal tree

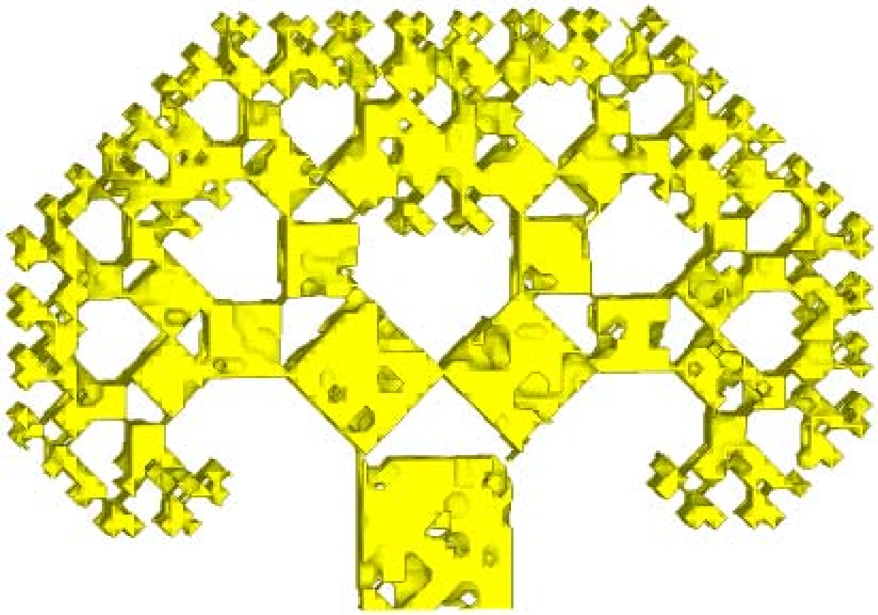
Josef Schröttl (LMU Munich)

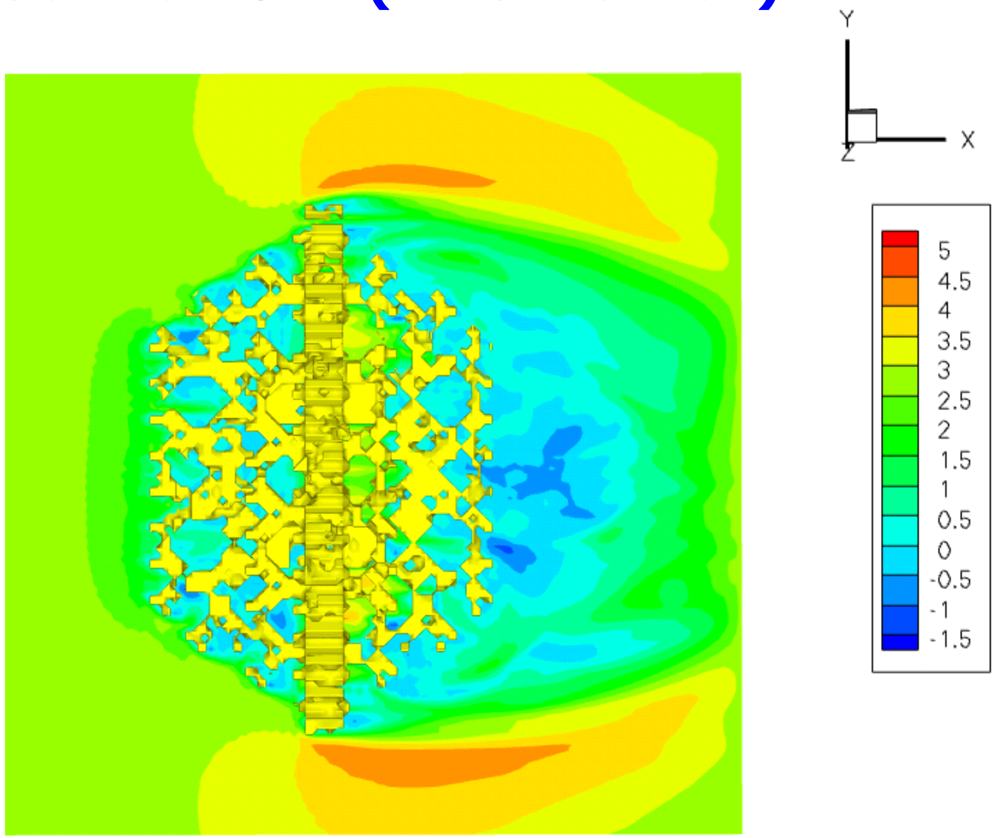




Outlook:
Simulations around a single 3D fractal tree

Josef Schröttl (LMU Munich)

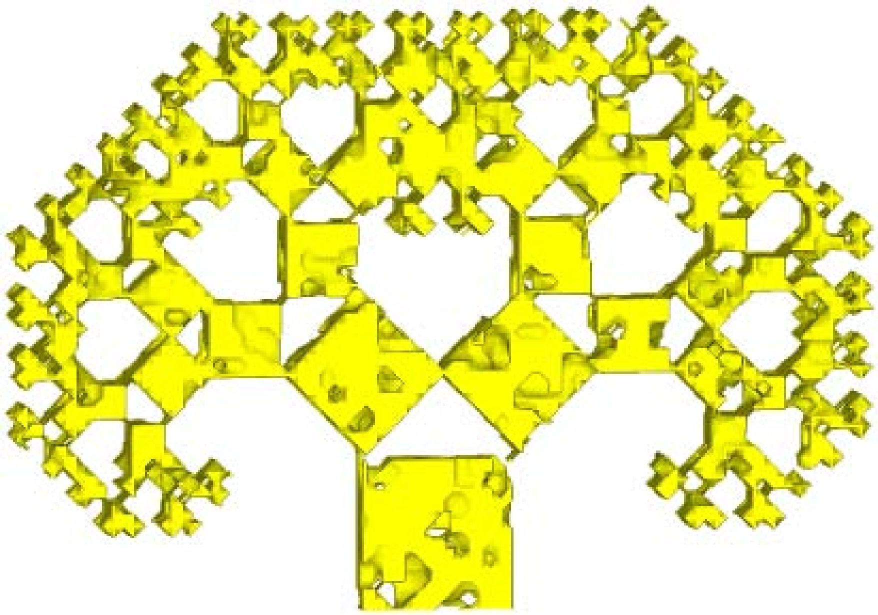


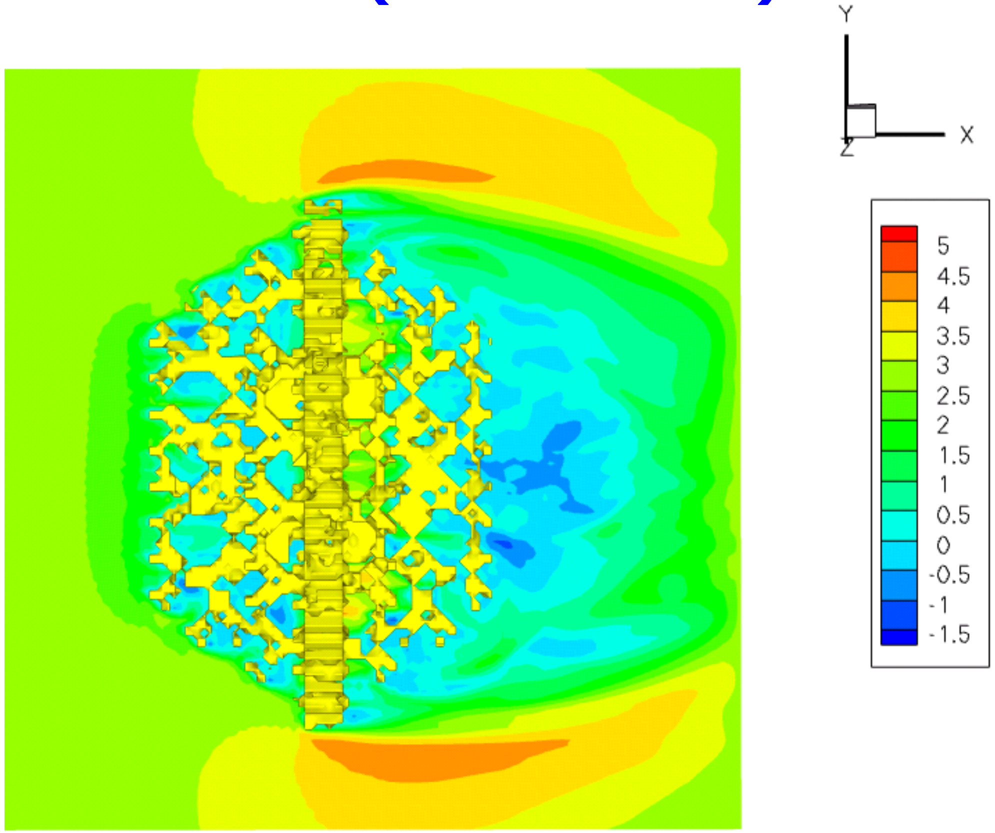


Slice of u Velocity Contours - intermediate height

Outlook:
Simulations around a single 3D fractal tree

Josef Schröttl (LMU Munich)





Slice of u Velocity Contours - intermediate height