Freie Universität Berlin

Sound-proof for small scales, compressible for large scales via scale-dependent time integration

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Thanks to ...

Ulrich Achatz	(Goethe-Universität, Frankfurt)
Didier Bresch	(Université de Savoie, Chambéry)
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Regime(s) of validity of sound-proof models

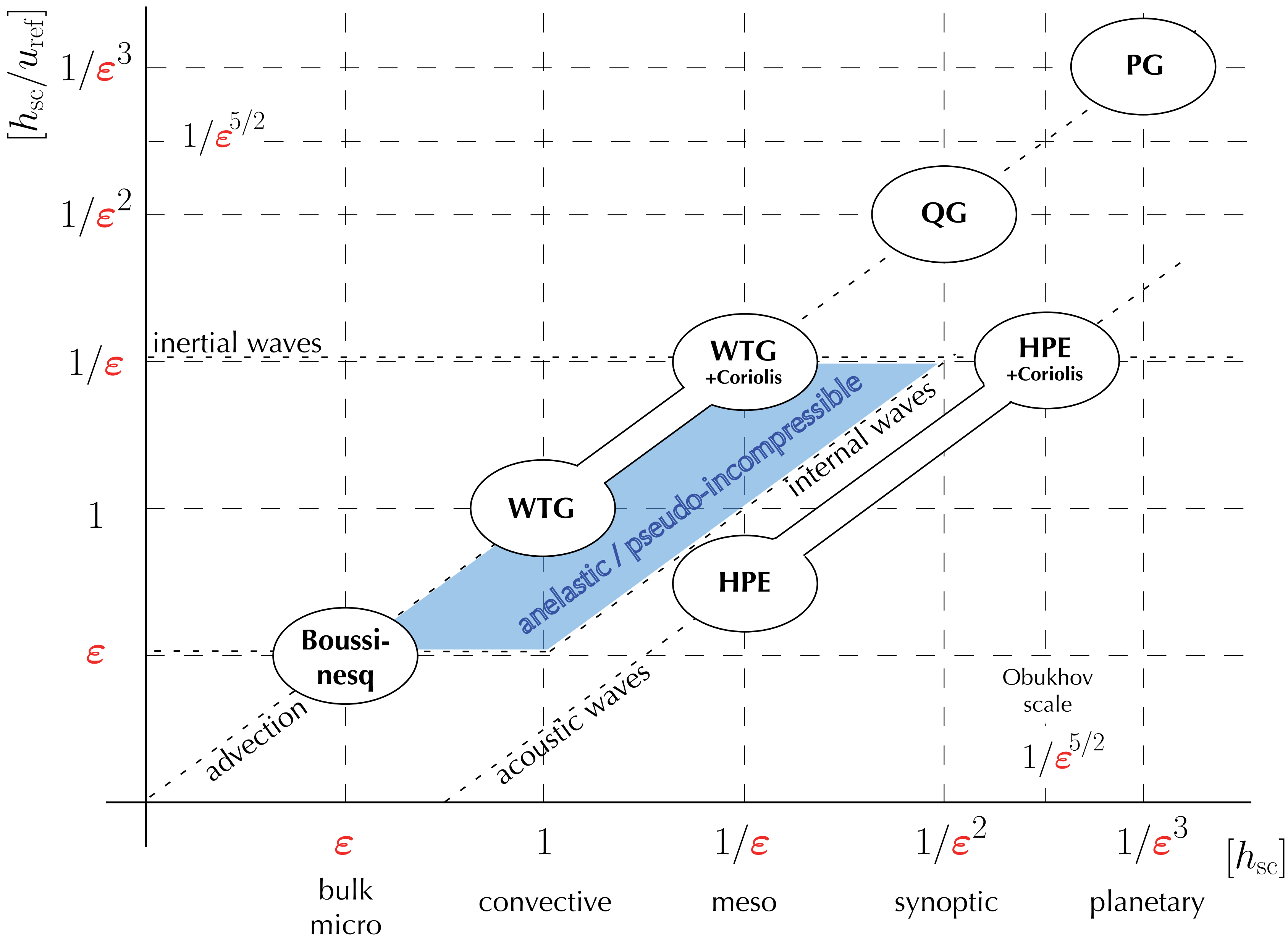
Motivation

Stratification limit in the design-regime

Wave-breaking regime with strong stratification

Scale-dependent time-integrator

Regimes of Validity ... Motivation



R.K., Ann. Rev. Fluid Mech, **42**, 2010

Regimes of Validity ... Motivation

Compressible flow equations

$$
\begin{aligned}
\rho_t + \nabla \cdot (\rho \boldsymbol{v}) &= 0 \\
(\rho \boldsymbol{u})_t + \nabla \cdot (\rho \boldsymbol{v} \circ \boldsymbol{u}) + P \nabla_{\|} \pi &= 0 \\
(\rho w)_t + \nabla \cdot (\rho \boldsymbol{v} w) + P \pi_z &= -\rho g \\
P_t + \nabla \cdot (P \boldsymbol{v}) &= 0
\end{aligned}
$$

drop term for:

anelastic (approx.)

pseudo-incompressible

hydrostatic-primitive

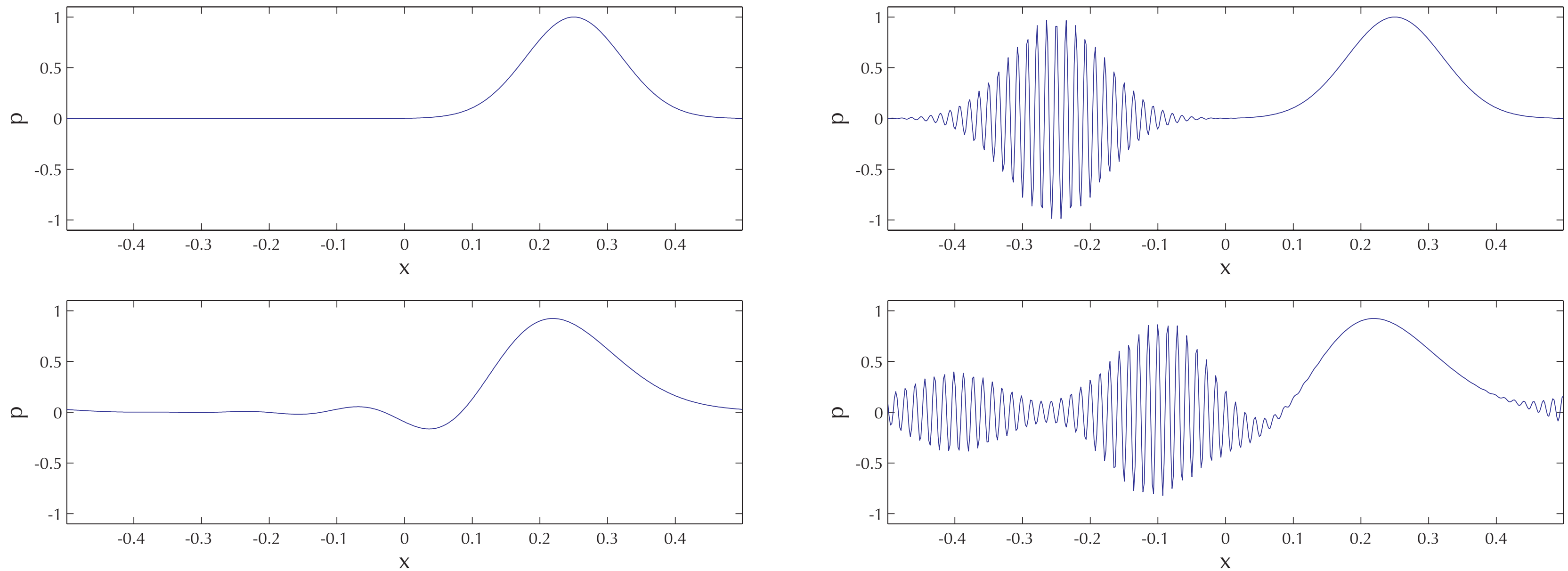
$$
P = p^{\frac{1}{\gamma}} = \rho\theta\,, \qquad \pi = p/\Gamma P\,, \qquad \Gamma = c_p/R\,, \qquad \boldsymbol{v} = \boldsymbol{u} + w\boldsymbol{k}\,, \quad (\boldsymbol{u} \cdot \boldsymbol{k} \equiv 0)
$$

Motivation ... Numerics

Why not simply solve the full compressible equations?

Linear acoustics, simple wave initial data, periodic domain

(integration: implicit midpoint rule, staggered grid, 512 ***grid pts.,*** $\mathrm{CFL} = 10$***)***

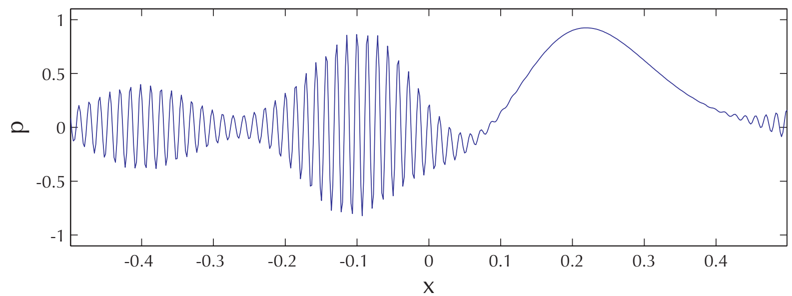


$t = 0$

$t = 3$

Motivation ... Numerics

Why not simply solve the full compressible equations?



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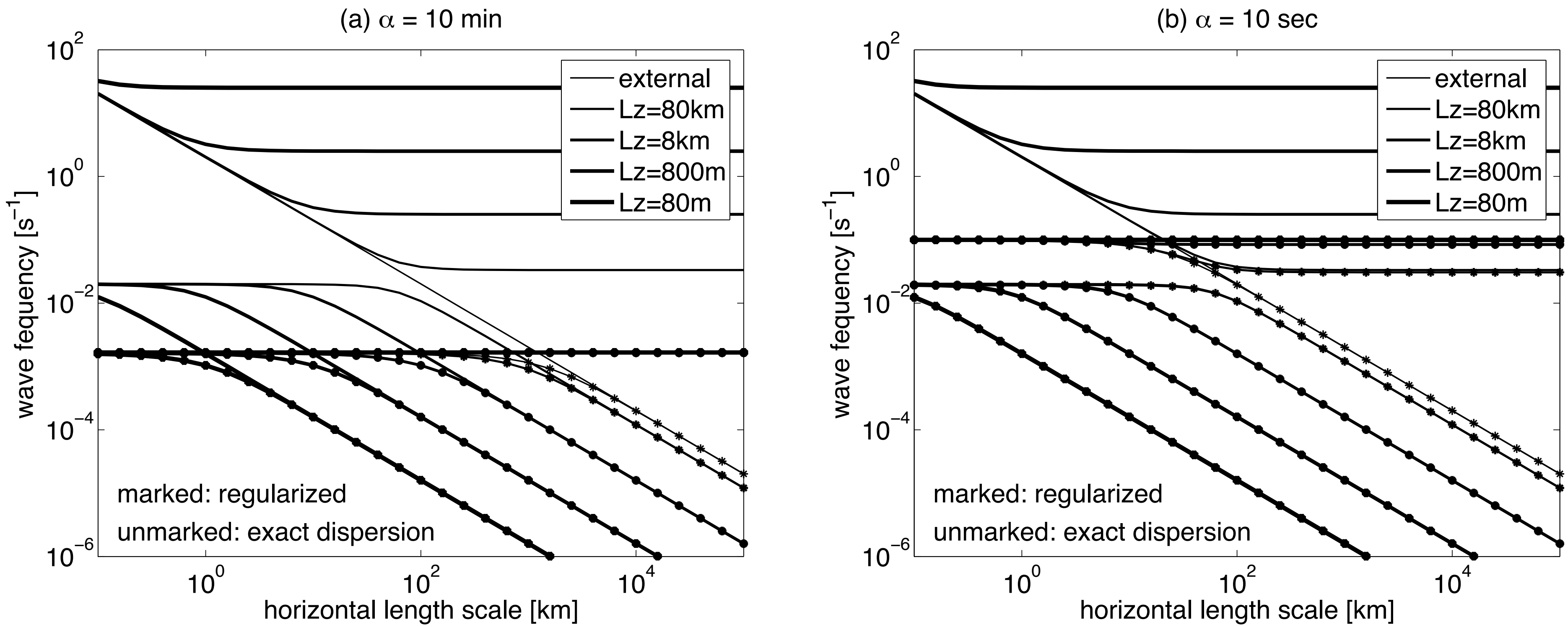

* adapted from Reich et al. (2007)

Goal

Compressible flow solver which

- properly handles long-wave dynamics
- defaults to **proper sound-proof limit** at small scales and for large time steps

Regime(s) of validity of sound-proof models

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Scale-dependent time-integrator

K., Achatz, Bresch, Knio, Smolarkiewicz, JAS, (2010)

Regimes of Validity ... Design Regime

Characteristic (inverse) time scales

		dimensional	dimensionless
advection	:	$\dfrac{u_{\rm ref}}{h_{\rm sc}}$	1
internal waves	:	$N = \sqrt{\dfrac{g}{\overline{\theta}}\dfrac{d\overline{\theta}}{dz}}$	$\dfrac{\sqrt{g h_{\rm sc}}}{u_{\rm ref}}\sqrt{\dfrac{h_{\rm sc}}{\overline{\theta}}\dfrac{d\overline{\theta}}{dz}} = \dfrac{1}{\varepsilon}\sqrt{\dfrac{h_{\rm sc}}{\overline{\theta}}\dfrac{d\overline{\theta}}{dz}}$
sound	:	$\dfrac{\sqrt{p_{\rm ref}/\rho_{\rm ref}}}{h_{\rm sc}} = \dfrac{\sqrt{g h_{\rm sc}}}{h_{\rm sc}}$	$\dfrac{\sqrt{g h_{\rm sc}}}{u_{\rm ref}} = \dfrac{1}{\varepsilon}$

Regimes of Validity ... Design Regime

Characteristic (inverse) time scales

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internal waves	:	$N = \sqrt{\dfrac{g}{\overline{\theta}}\dfrac{d\overline{\theta}}{dz}}$	$\dfrac{\sqrt{g h_{\text{sc}}}}{u_{\text{ref}}}\sqrt{\dfrac{h_{\text{sc}}}{\overline{\theta}}\dfrac{d\overline{\theta}}{dz}} = \sqrt{\dfrac{h_{\text{sc}}}{\overline{\theta}}\dfrac{d\widehat{\theta}}{dz}}$
sound	:	$\dfrac{\sqrt{p_{\text{ref}}/\rho_{\text{ref}}}}{h_{\text{sc}}} = \dfrac{\sqrt{g h_{\text{sc}}}}{h_{\text{sc}}}$	$\dfrac{\sqrt{g h_{\text{sc}}}}{u_{\text{ref}}} = \dfrac{1}{\varepsilon}$

Ogura & Phillips' regime* with two time scales

$$\overline{\theta} = 1 + \varepsilon^2 \widehat{\theta}(z) + \dots \qquad \Rightarrow \qquad \frac{h_{\text{sc}}}{\overline{\theta}}\frac{d\overline{\theta}}{dz} = O(\varepsilon^2)$$

* Ogura & Phillips (1962)

Regimes of Validity ... Design Regime

Characteristic (inverse) time scales

		dimensional	dimensionless
advection	:	$\dfrac{u_{\text{ref}}}{h_{\text{sc}}}$	1
internal waves	:	$N = \sqrt{\dfrac{g}{\overline{\theta}}\dfrac{d\overline{\theta}}{dz}}$	$\dfrac{\sqrt{gh_{\text{sc}}}}{u_{\text{ref}}}\sqrt{\dfrac{h_{\text{sc}}}{\overline{\theta}}\dfrac{d\overline{\theta}}{dz}} = \dfrac{1}{\varepsilon^{\nu}}\sqrt{\dfrac{h_{\text{sc}}}{\overline{\theta}}\dfrac{d\widehat{\theta}}{dz}}$
sound	:	$\dfrac{\sqrt{p_{\text{ref}}/\rho_{\text{ref}}}}{h_{\text{sc}}} = \dfrac{\sqrt{gh_{\text{sc}}}}{h_{\text{sc}}}$	$\dfrac{\sqrt{gh_{\text{sc}}}}{u_{\text{ref}}} = \dfrac{1}{\varepsilon}$

Realistic regime with three time scales

$$\overline{\theta} = 1 + \varepsilon^{\mu}\widehat{\theta}(z) + \dots \qquad \Rightarrow \qquad \frac{h_{\text{sc}}}{\overline{\theta}}\frac{d\overline{\theta}}{dz} = O(\varepsilon^{\mu}) \qquad (\nu = 1 - \mu/2)$$

Regimes of Validity ... Design Regime

Desirable:

1. **Sound-proof model** which
2. accurately represents the **(fast) internal waves**, and
3. remains accurate over **advective time scales**.

Regimes of Validity ... Design Regime

$$
\begin{aligned}
\tilde{\theta}_\tau &+ \frac{1}{\varepsilon^{\nu}}\,\tilde{w}\,\frac{d\widehat{\theta}}{dz} && && = -\tilde{\boldsymbol{v}}\cdot\nabla\tilde{\theta} \\
\tilde{\boldsymbol{v}}_\tau &+ \frac{1}{\varepsilon^{\nu}}\,\frac{\tilde{\theta}}{\overline{\overline{\theta}}}\,\boldsymbol{k} && + \frac{1}{\varepsilon}\,\overline{\theta}\nabla\tilde{\pi} && = -\tilde{\boldsymbol{v}}\cdot\nabla\tilde{\boldsymbol{v}} - \varepsilon^{1-\nu}\tilde{\theta}\nabla\tilde{\pi} \;\; \cdot \\
\tilde{\pi}_\tau & && + \frac{1}{\varepsilon}\left(\gamma\Gamma\overline{\pi}\nabla\cdot\tilde{\boldsymbol{v}} + \tilde{w}\frac{d\overline{\pi}}{dz}\right) && = -\tilde{\boldsymbol{v}}\cdot\nabla\tilde{\pi} - \gamma\Gamma\tilde{\pi}\nabla\cdot\tilde{\boldsymbol{v}}
\end{aligned}
$$

For the linear variable coefficient system:

- ✓ Conservation of weighted quadratic energy
- ✓ Control of time derivatives by initial data $(\tau = O(1))$

... consider internal wave scalings for $\tau = O(\varepsilon^{\nu})$:

$$
\vartheta = \frac{\tau}{\varepsilon^{\nu}}\,, \qquad \pi^* = \varepsilon^{\nu-1}\tilde{\pi}\,,
$$

Regimes of Validity ... Design Regime

Fast linear compressible / pseudo-incompressible modes

$$
\begin{aligned}
\tilde{\theta}_{\vartheta} \; + \; \tilde{w}\,\frac{d\overline{\theta}}{dz} \qquad\qquad\qquad\qquad &= \; 0 \\
\tilde{\boldsymbol{v}}_{\vartheta} \; + \; \frac{\tilde{\theta}}{\overline{\theta}^{\varepsilon}}\,\boldsymbol{k} + \overline{\theta}^{\varepsilon}\nabla\pi^{*} \qquad\qquad &= \; 0 \\
\varepsilon^{\mu}\,\pi^{*}_{\vartheta} + \left(\gamma\Gamma\overline{\pi}^{\varepsilon}\nabla\cdot\tilde{\boldsymbol{v}} + \tilde{w}\frac{d\overline{\pi}^{\varepsilon}}{dz}\right) &= \; 0
\end{aligned}
$$

Vertical mode expansion (separation of variables)

$$
\begin{pmatrix} \tilde{\theta} \\ \tilde{\boldsymbol{u}} \\ \tilde{w} \\ \pi^{*} \end{pmatrix} (\vartheta, \boldsymbol{x}, z) = \begin{pmatrix} \Theta^{*} \\ \boldsymbol{U}^{*} \\ W^{*} \\ \Pi^{*} \end{pmatrix} (z) \;\; \exp\left(i\left[\boldsymbol{\omega}\vartheta - \boldsymbol{\lambda}\cdot\boldsymbol{x}\right]\right)
$$

Regimes of Validity ... Design Regime

Relation between compressible and pseudo-incompressible vertical modes

$$-\frac{d}{dz}\left(\frac{1}{1-\varepsilon^{\mu}\frac{\omega^2/\lambda^2}{\overline{c}^{\varepsilon\,2}}}\frac{1}{\overline{\theta}^{\varepsilon}\overline{P}^{\varepsilon}}\frac{dW^*}{dz}\right)+\frac{\lambda^2}{\overline{\theta}^{\varepsilon}\overline{P}^{\varepsilon}}W^* = \frac{1}{\omega^2}\frac{\lambda^2 N^2}{\overline{\theta}^{\varepsilon}\overline{P}^{\varepsilon}}W^*$$

$\varepsilon^{\mu} = 0$: pseudo-incompressible case

regular Sturm-Liouville problem for internal wave modes *(rigid lid)*

$\varepsilon^{\mu} > 0$: compressible case

nonlinear Sturm-Liouville problem ...

$\frac{\omega^2/\lambda^2}{\overline{c}^{\varepsilon\,2}} = O(1)$: perturbations of pseudo-incompressible modes & EVals

Regimes of Validity ... Design Regime

$$-\frac{d}{dz}\left(\frac{1}{1-\varepsilon^{\mu}\frac{\omega^2/\lambda^2}{\bar{c}^{\varepsilon\,2}}}\frac{1}{\overline{\theta}^{\varepsilon}\,\overline{P}^{\varepsilon}}\frac{dW^*}{dz}\right)+\frac{\lambda^2}{\overline{\theta}^{\varepsilon}\,\overline{P}^{\varepsilon}}W^* = \frac{1}{\omega^2}\frac{\lambda^2 N^2}{\overline{\theta}^{\varepsilon}\,\overline{P}^{\varepsilon}}W^*$$

Internal wave modes $\left(\frac{\omega^2/\lambda^2}{\bar{c}^{\varepsilon\,2}} = O(1)\right)$

- pseudo-incompressible modes/EVals $=$ compressible modes/EVals + $O(\varepsilon^{\mu})$ †
- phase errors remain small ***over advection time scales*** for $\mu > \frac{2}{3}$

The anelastic and pseudo-incompressible models remain relevant for stratifications

$$\frac{1}{\overline{\theta}}\frac{d\overline{\theta}}{dz} < O(\varepsilon^{2/3}) \qquad \Rightarrow \qquad \Delta\theta\big|_0^{h_{\rm sc}} \lesssim 40\ \mathrm{K}$$

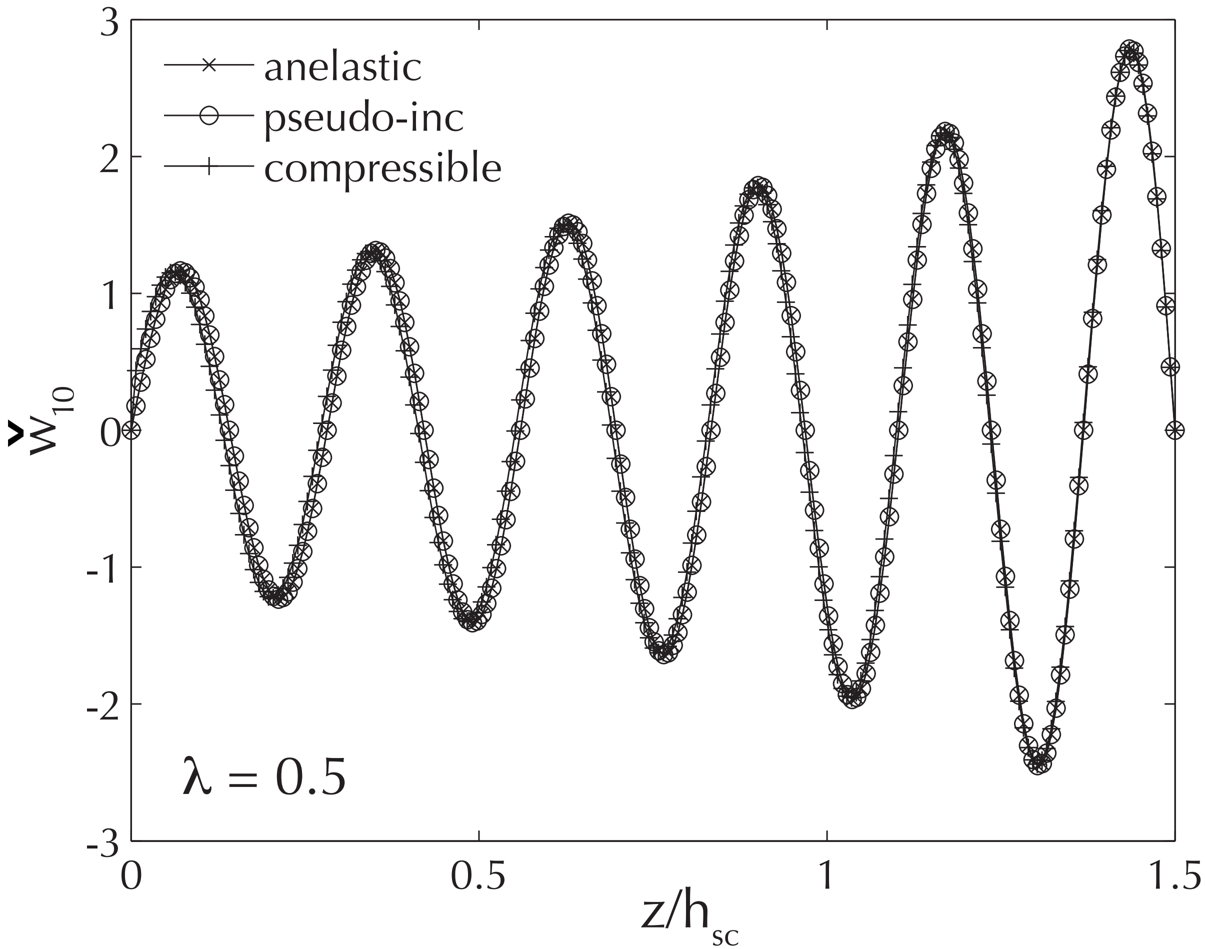
not merely up to $O(\varepsilon^2)$ as in Ogura-Phillips (1962)

R.K., Achatz, Bresch, Knio, Smolarkiewicz, JAS, (2010)

† **rigorous proof with D. Bresch**

Regimes of Validity ... Design Regime

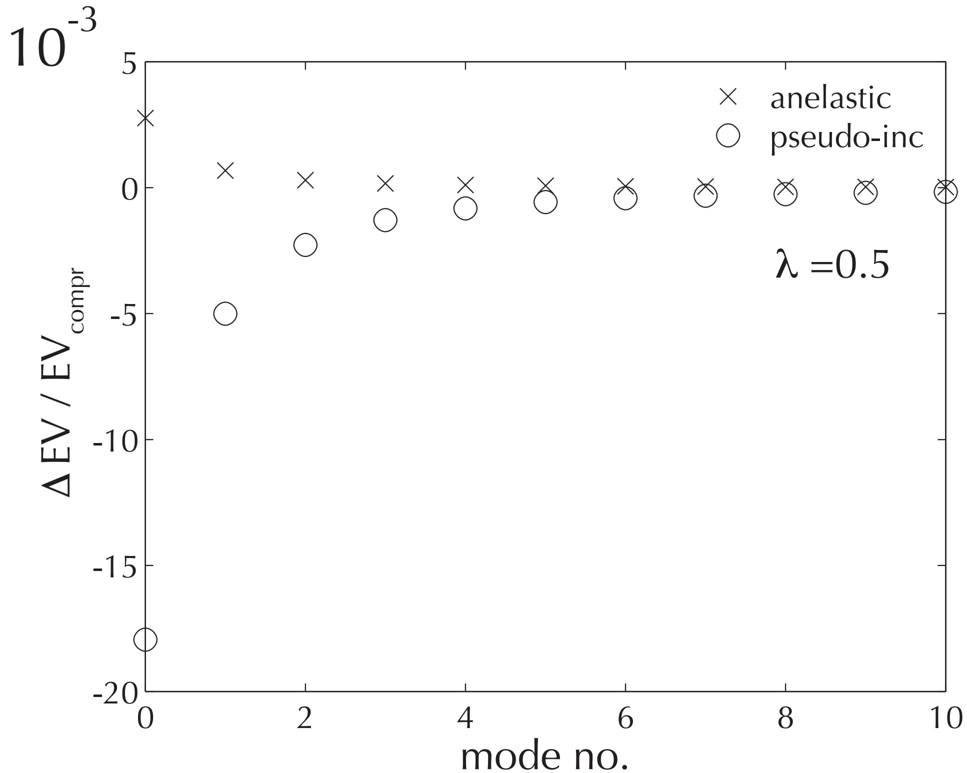
A typical vertical structure function $(L \sim \pi h_{\rm sc} \sim 30 \text{ km})$

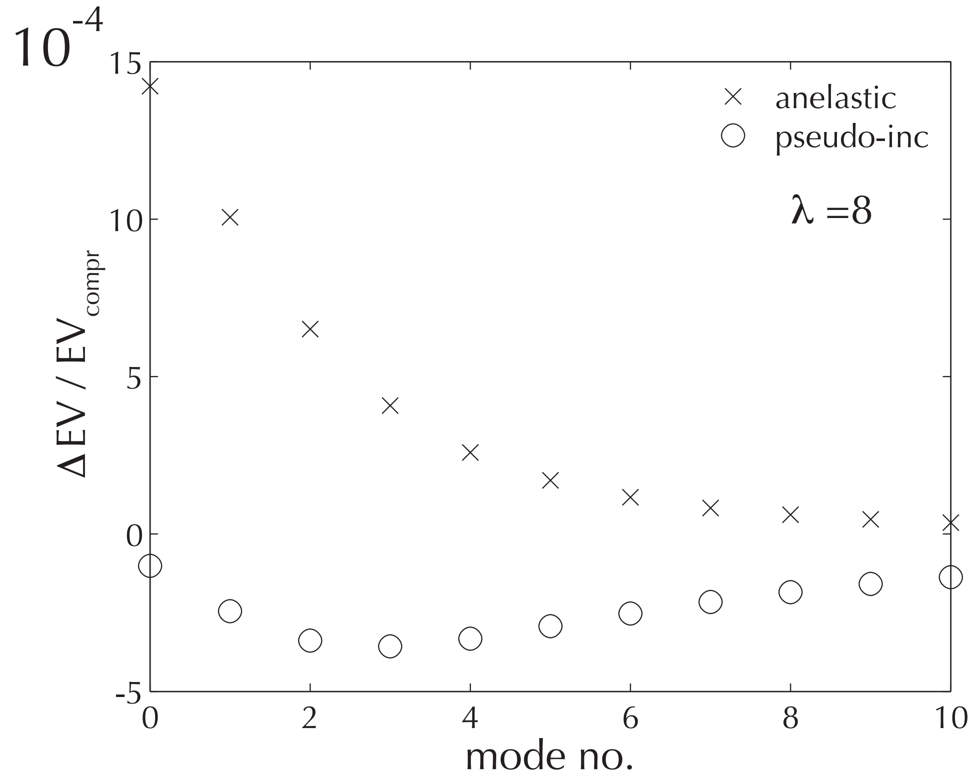


Regimes of Validity ... Design Regime

Relative eigenvalue errors

$$\frac{EV_{\text{sproof}} - EV_{\text{compr}}}{EV_{\text{compr}}}$$





R.K., Achatz, Bresch, Knio, Smolarkiewicz, JAS, (2010) thanks to Dr. V. LeDoux, Ghent, for the SL-solver MATSLISE!

Regime(s) of validity of sound-proof models

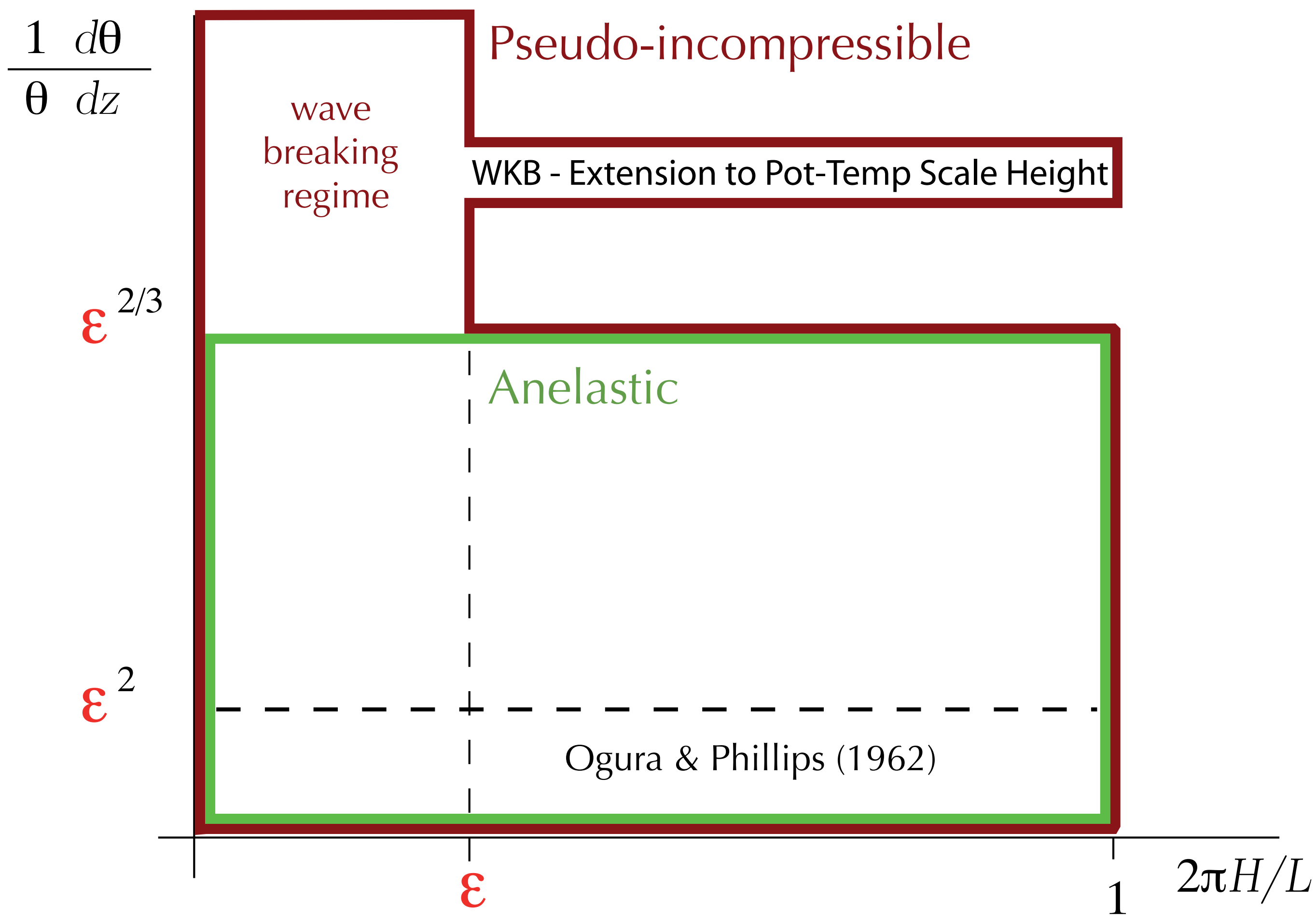
Motivation

Stratification limit in the design-regime

Wave-breaking regime with strong stratification

Scale-dependent time-integrator

Regimes ... Summary

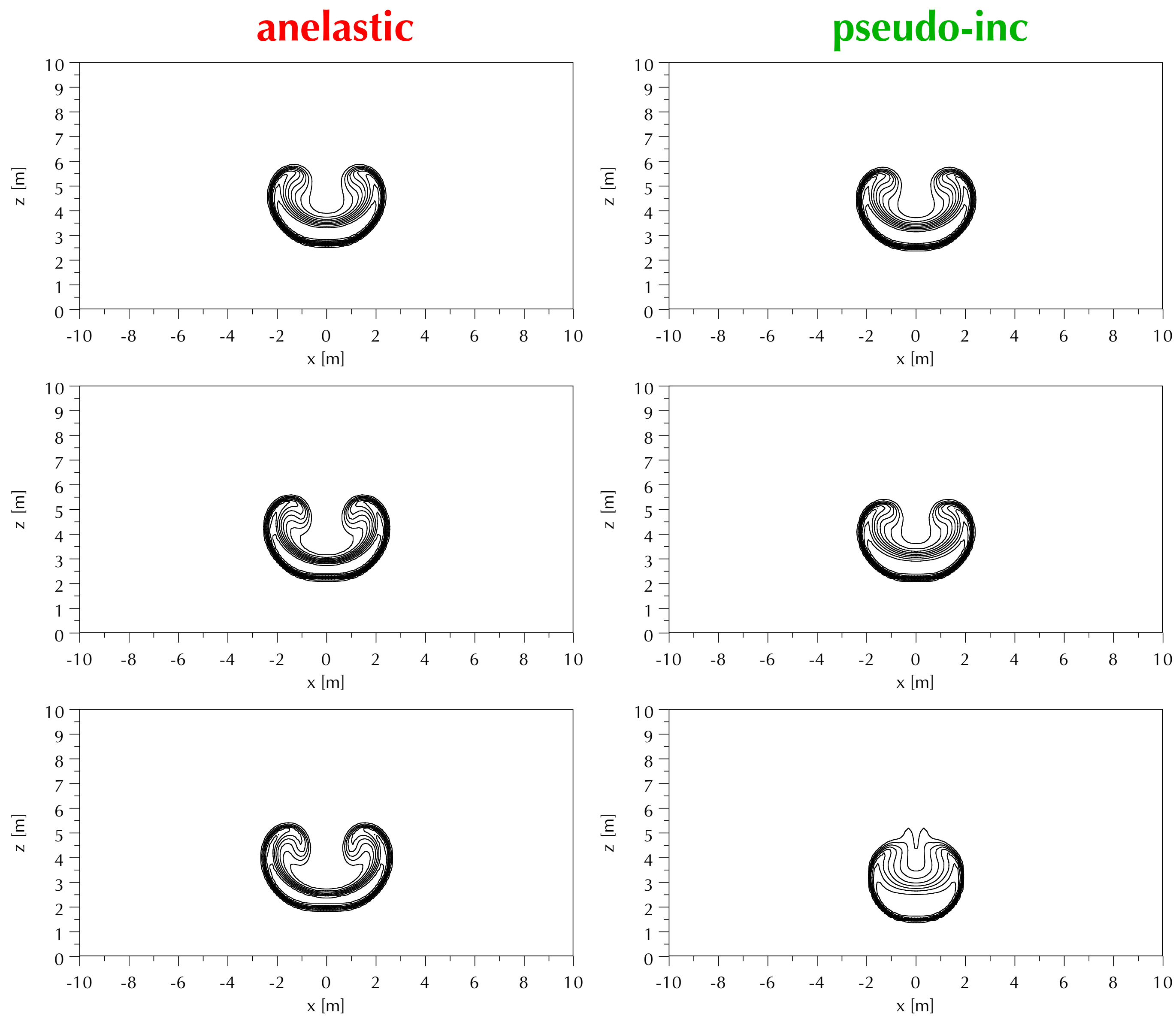


The pseudo-incompressible model wins by a small margin

see Ulrich's talk

Regimes ... very small scales

Cold air blobs at very small scales



$$\theta_1/\theta_2 = 0.9$$

$$\theta_1/\theta_2 = 0.5$$

$$\theta_1/\theta_2 = 0.1$$

see Ulrich's talk

Regime(s) of validity of sound-proof models

- Motivation
- Stratification limit in the design-regime
- Wave-breaking regime with strong stratification

Scale-dependent time-integrator

Numerics

Why not simply solve the full compressible equations?

Competing approaches: **model codes**

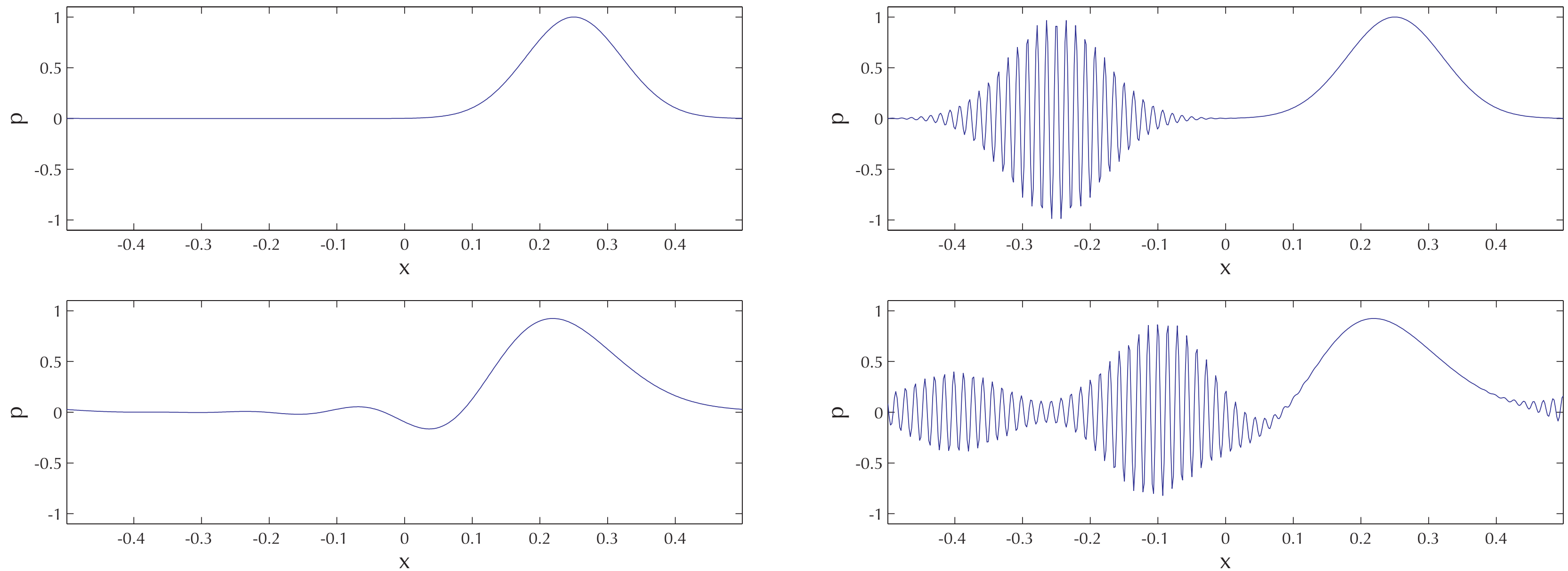
- Split-explicit / multi-rate methods, e.g.,
  - Runge-Kutta (slow) + forward-backward (fast), e.g.,
    Wicker & Skamarock, MWR, (98), ... ; *MM5, LM, WRF ...*
  - Multirate infinitesimal schemes, peer methods
    Wensch et al., BIT, (09); *ASAM, ...*
- Semi-implicit / linearly implicit schemes
  - explicit advection, damped 2nd or 1st-order schemes for fast modes, e.g.,
    Robert, Japan Met. J., (69), ... ; *UKMO, ...*
  - linearly implicit Rosenbrock-type methods, e.g.,
    Reisner et al., MWR, (05), ...; *ASAM, LANL Hurricane model, ...*
- Fully implicit integration

Numerics

Why not simply solve the full compressible equations?

Simple wave initial data, periodic domain

(integration: implicit midpoint rule, staggered grid,** 512 **grid pts.,** $\mathrm{CFL} = 10$**)



$t = 0$

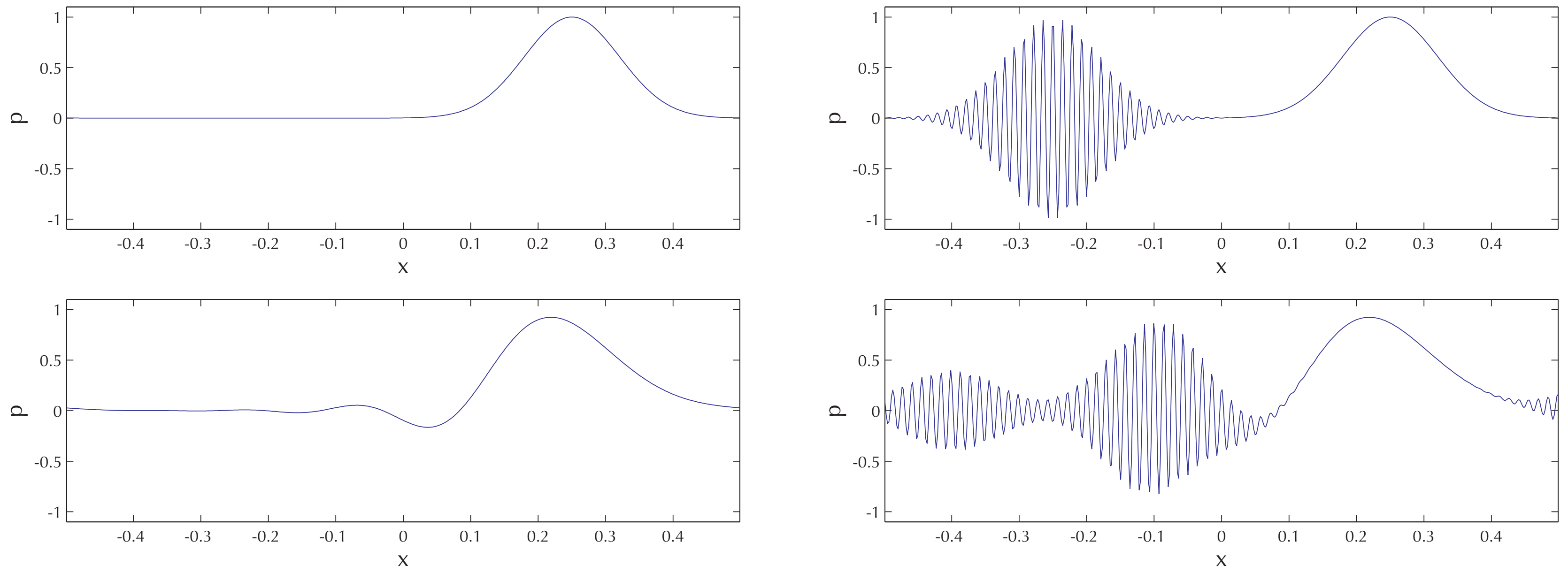
$t = 3$

Numerics

Why not simply solve the full compressible equations?

Simple wave initial data, periodic domain

(integration: implicit midpoint rule, staggered grid, 512 *grid pts.,* $\mathrm{CFL} = 10$*)*



$t = 0$

$t = 3$

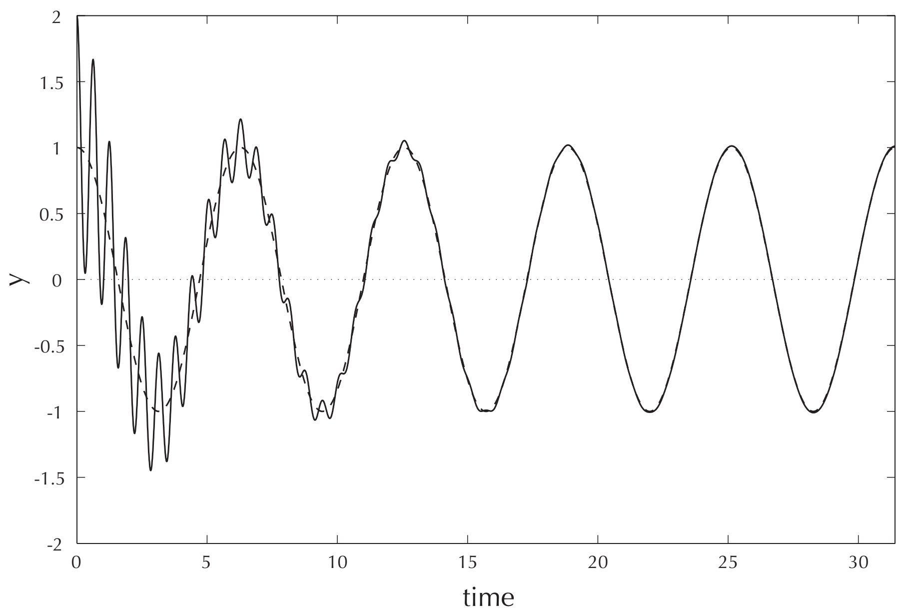
Ideas:

- Slave short waves $(c\Delta t/\ell > 1)$ to long waves $(c\Delta t/\ell \leq 1)$
- with pseudo-incompressible limit behavior

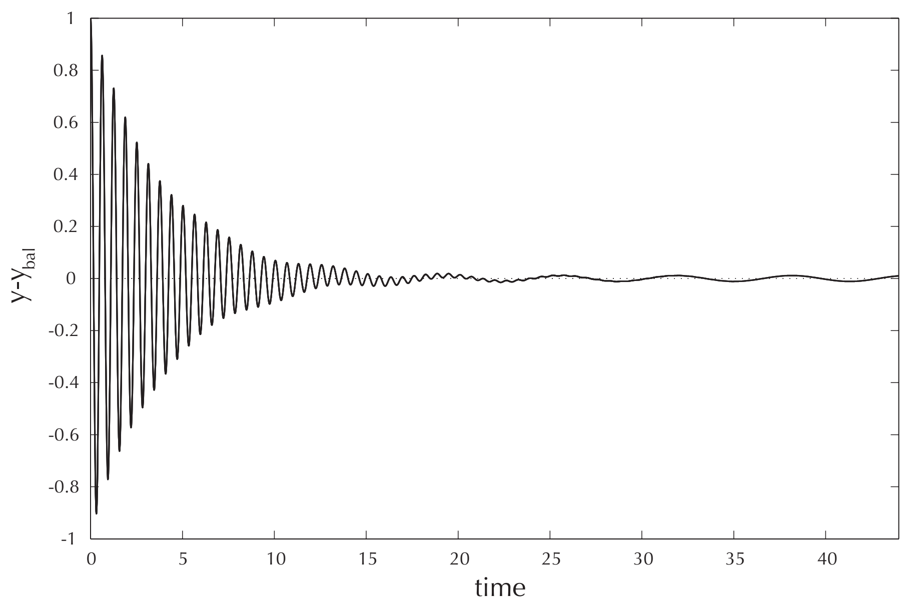
"super-implicit" scheme
non-standard multi grid
projection method

Numerics

$$\varepsilon \ddot{y} + \varepsilon \kappa \dot{y} + y = \cos(t)\,, \qquad \begin{cases} y(0) &= 1 + a \\ \dot{y}(0) &= 0 \end{cases}, \qquad (\varepsilon = 0.01)$$



$y(t)$



$y(t) - \cos(t)$

Numerics

$$\varepsilon\ddot{y} + \varepsilon\kappa\dot{y} + y = \cos(t)$$

Slow-time asymptotics for $\varepsilon \ll 1$:

$$y(t) = y^{(0)}(t) + \varepsilon y^{(1)}(t) + \dots\,, \qquad \begin{aligned} y^{(0)}(t) &= \cos(t) \\ y^{(1)}(t) &= -(\ddot{y}^{(0)} + \kappa\dot{y}^{(0)})(t) \end{aligned}$$

Associated "super-implicit" discretization *(extreme BDF)*:

$$\begin{aligned} y^{n+1} &= \cos(t^{n+1}) - \varepsilon\left[(\delta_t + \kappa)\ \dot{y}\right]^{*,n+1} \\ \dot{y}^{n+1} &= \frac{1}{\Delta t}\left(y^{n+1} - y^n + \frac{1}{2}\left(y^{n+1} - 2y^n + y^{n-1}\right)\right) \end{aligned}$$

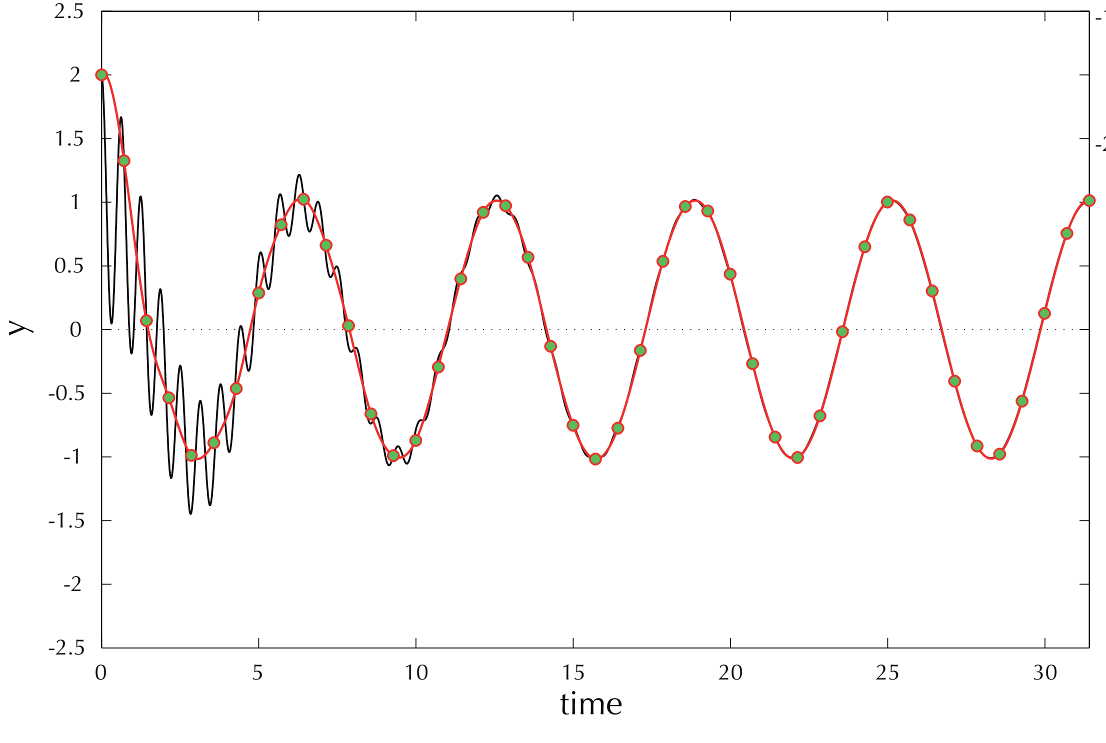
where

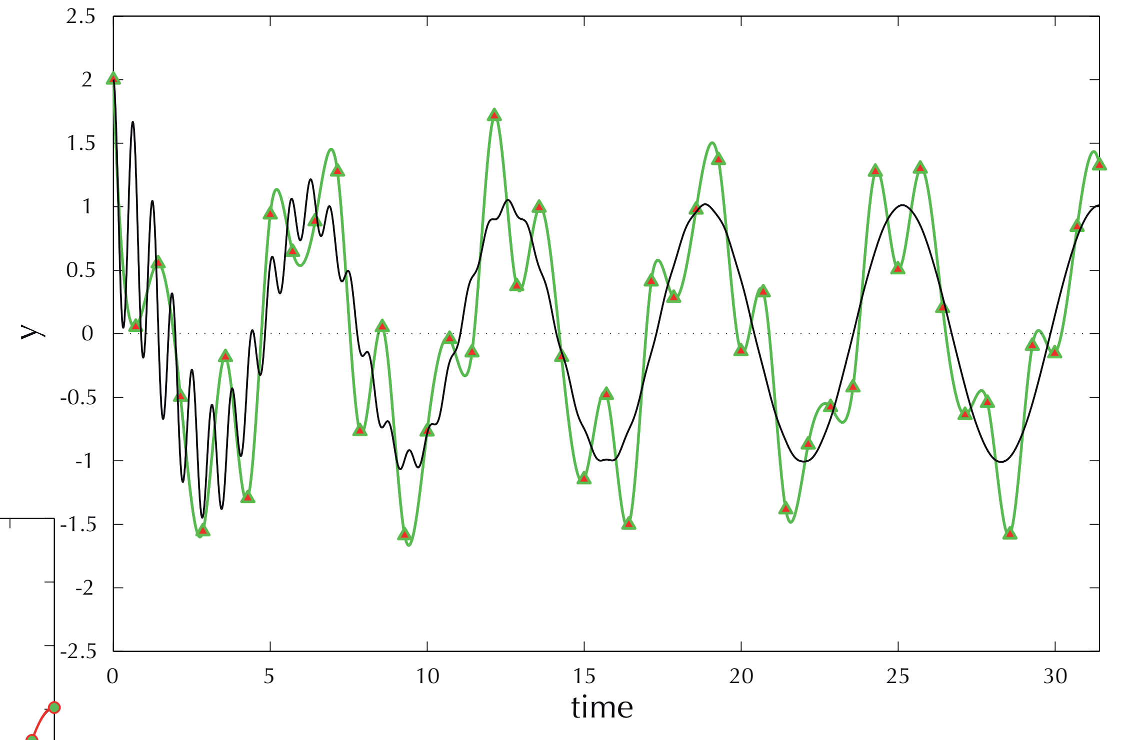
$$\begin{aligned} u^{*,n+1} &= 2u^n - u^{n-1} \\ (\delta_t u)^{*,n+1} &= \frac{1}{\Delta t}\left(u^n - u^{n-1} + \frac{3}{2}(u^n - 2u^{n-1} + u^{n-2})\right) \end{aligned}$$

Neta & Fukushima, AIAA-2000-4029; Neta, Comp. & Math., 50(2005)

Numerics

Implicit midpoint rule $\Delta t = 7\sqrt{\varepsilon}$

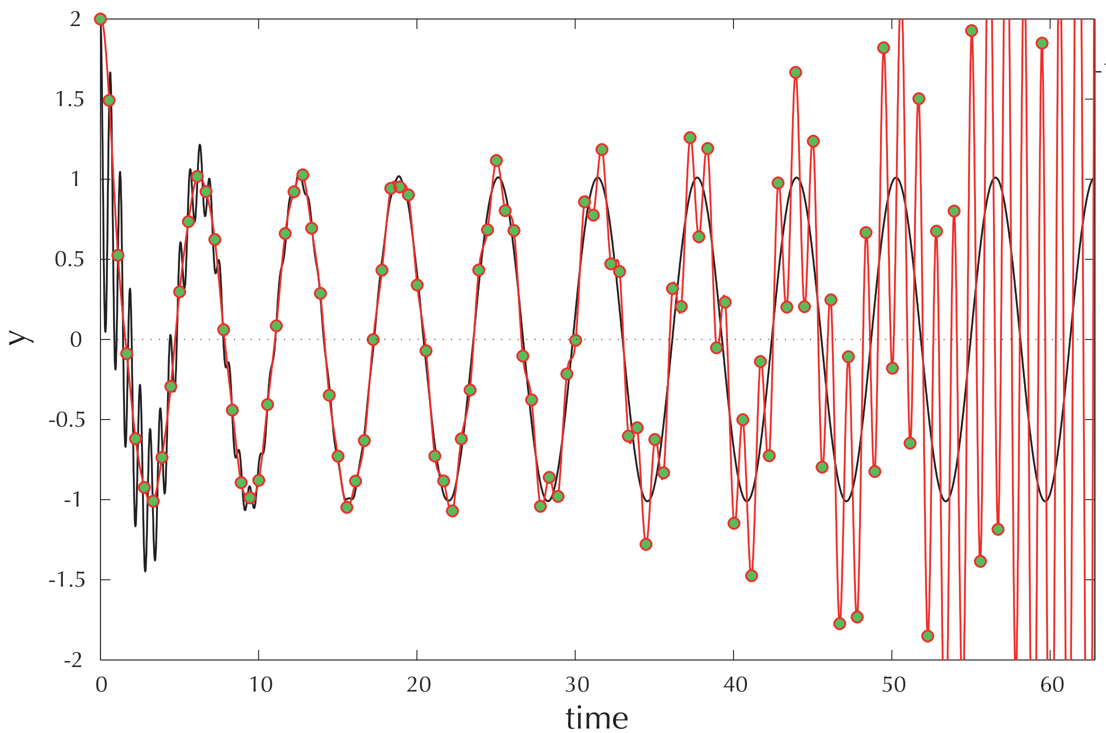


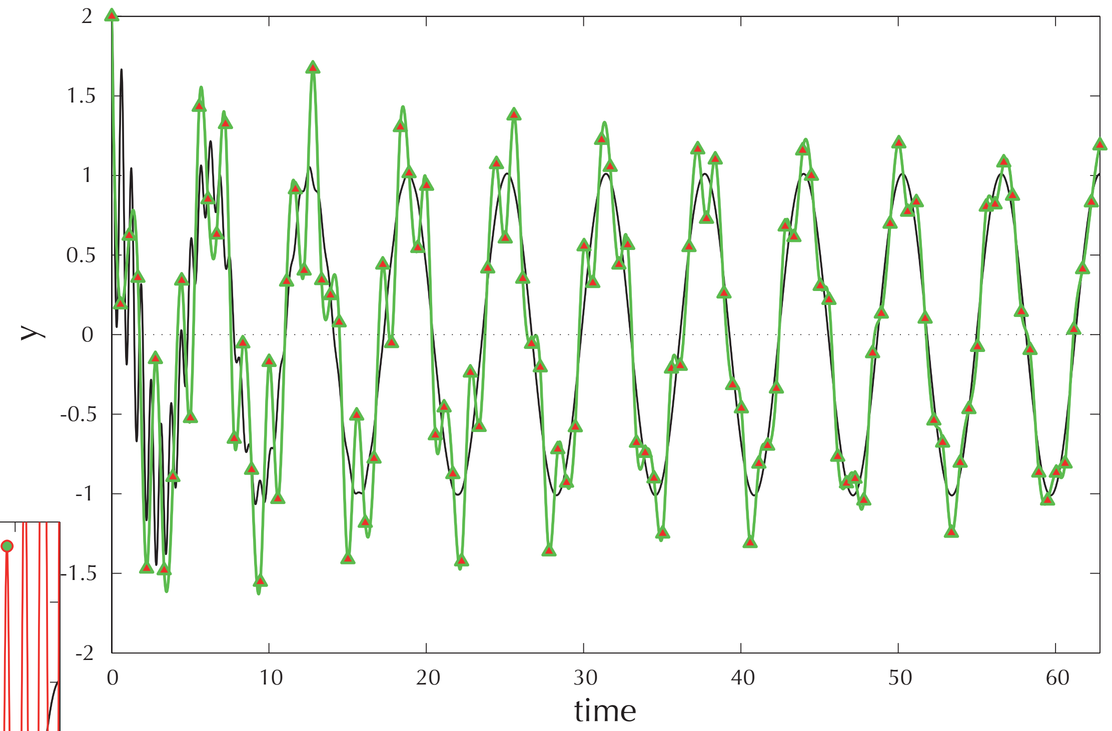


$\Delta t = 7\sqrt{\varepsilon}$ Super-implicit scheme

Numerics

Implicit midpoint rule $\Delta t = 5.55\sqrt{\varepsilon}$



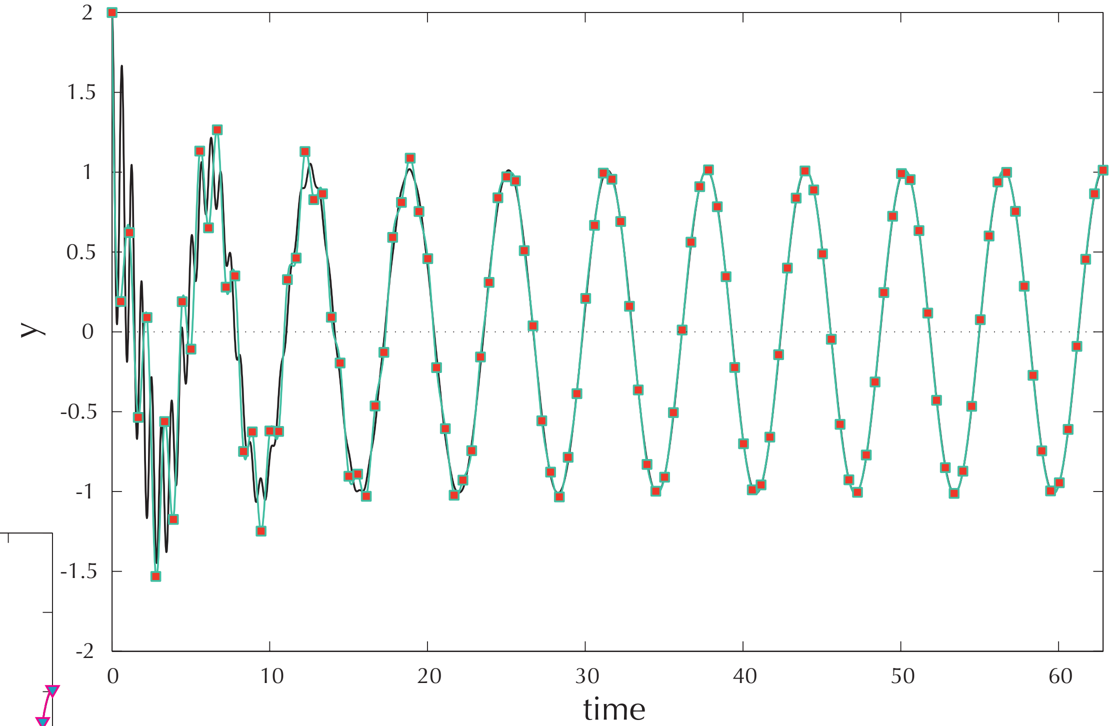


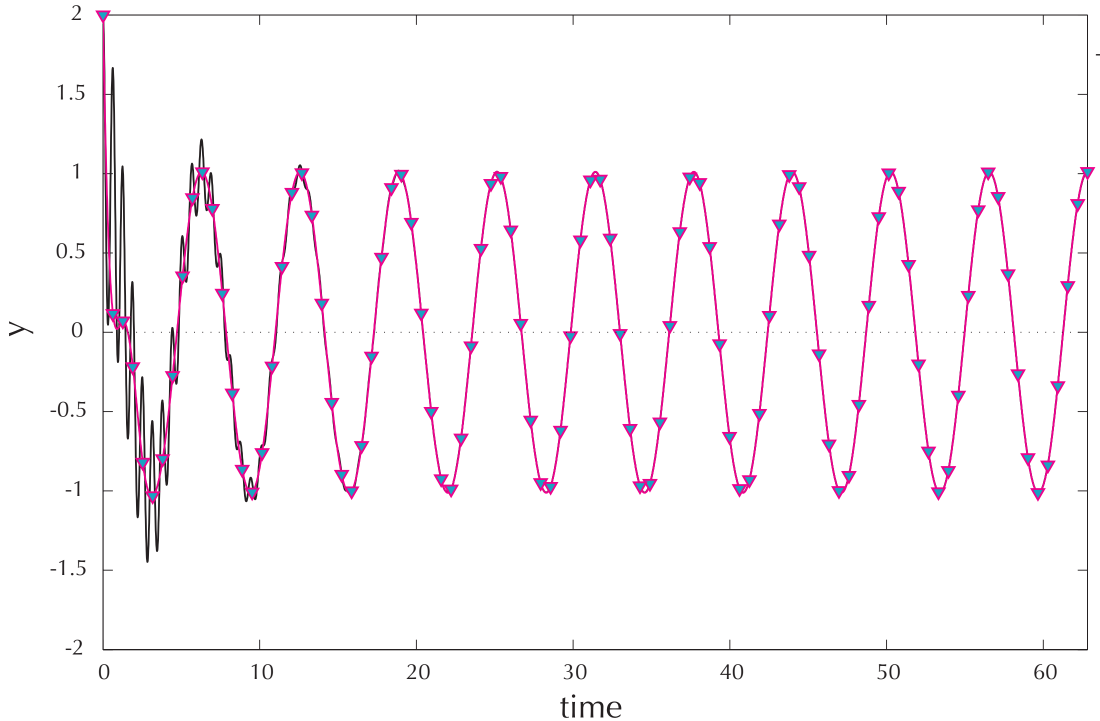
$\Delta t = 5.55\sqrt{\varepsilon}$ Super-implicit scheme

Numerics

Blended scheme $\Delta t = 5.55\sqrt{\varepsilon}$

$$\Delta y\big|_{\mathrm{BL}} = \eta\, \Delta y\big|_{\mathrm{IMP}} + (1-\eta)\, \Delta y\big|_{\mathrm{SupI}}$$





$\Delta t = 5.55\sqrt{\varepsilon}$ BDF2 – for comparison

Numerics

Compressible flow equations:

$$
\begin{aligned}
\boldsymbol{\rho_t} + \nabla\cdot(\rho\boldsymbol{v}) &= 0 \\
(\rho\boldsymbol{v})_t + \nabla\cdot(\rho\boldsymbol{v}\circ\boldsymbol{v}) + P\nabla\pi &= -\rho g\boldsymbol{k} \\
\boldsymbol{P_t} + \nabla\cdot(P\boldsymbol{v}) &= 0
\end{aligned}
$$

$$
P = p^{\frac{1}{\gamma}} = \rho\theta\,, \qquad \pi = p/\Gamma P\,, \qquad \Gamma = c_p/R
$$

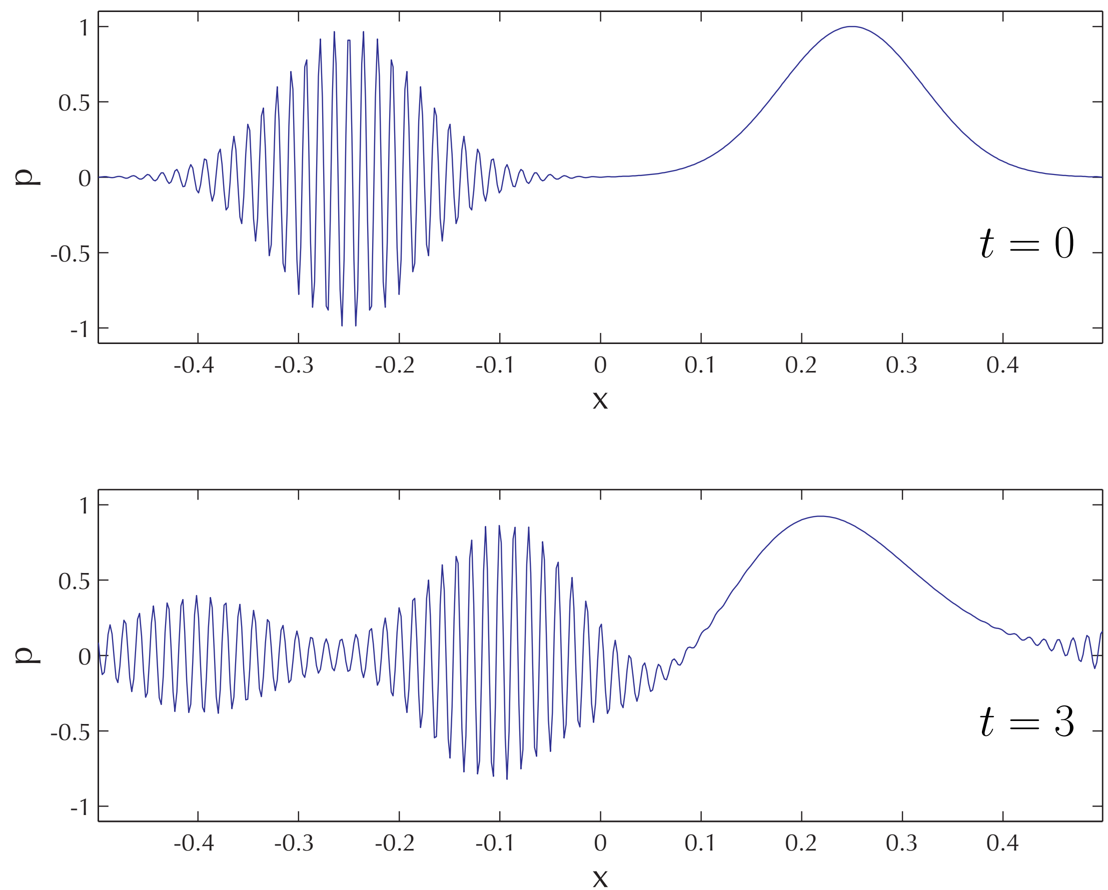
Numerics

For starters: **1D Linear acoustics:**

$$u_t + p_x = 0$$
$$p_t + c^2 u_x = 0$$

Desired:

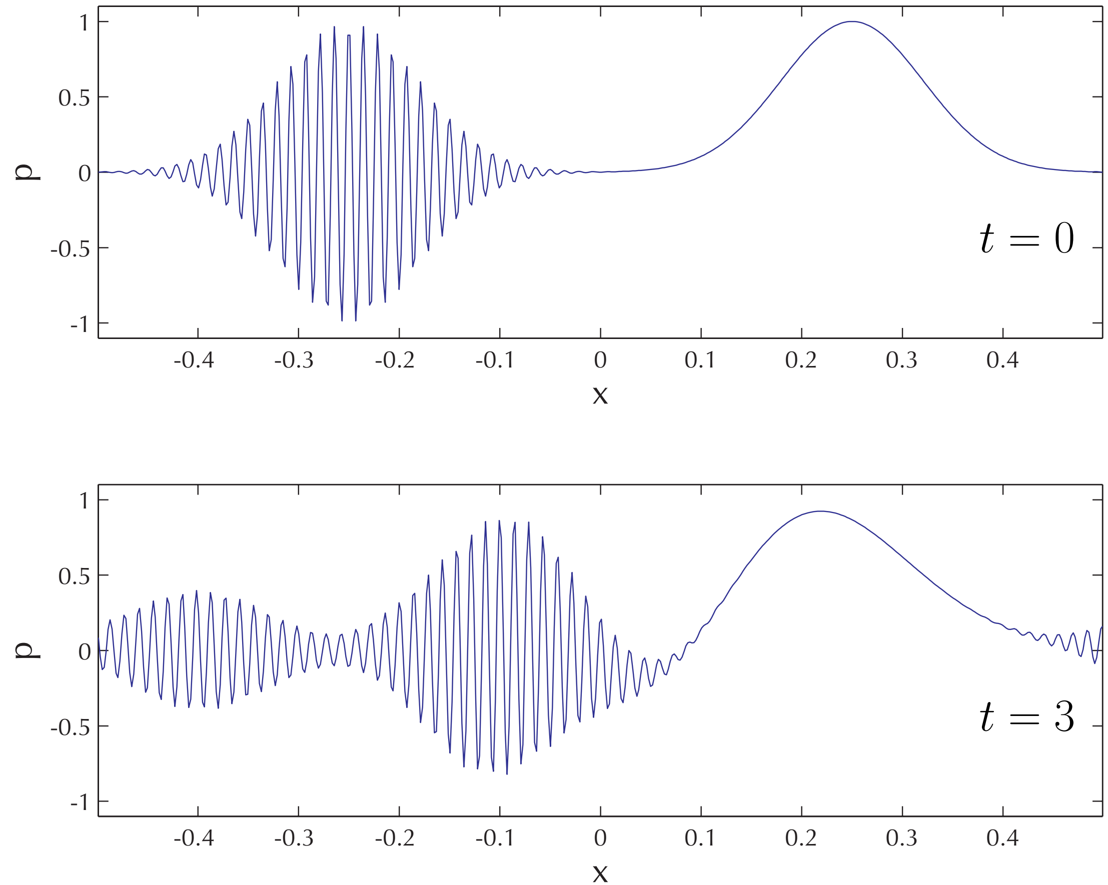
- remove underresolved modes
- minimize dispersion for marginally resolved modes



Numerics

1D Linear acoustics:

$$\begin{aligned} u_t + p_x &= 0 \\ p_t + c^2 u_x &= 0 \end{aligned}$$



Desired:

- remove underresolved modes
- minimize dispersion for marginally resolved modes

Strategy:

scale-dependent IMP-SupI-Blended scheme via **multi grid**

Implicit mid-point rule for linear acoustics

$$\frac{u^{n+1}-u^n}{\Delta t}+\frac{\partial}{\partial x}p^{n+\frac{1}{2}}=0\,,\qquad \frac{p^{n+1}-p^n}{\Delta t}+c^2\,\frac{\partial}{\partial x}u^{n+\frac{1}{2}}=0$$

with

$$X^{n+\frac{1}{2}}=\frac{1}{2}\left(X^{n+1}+X^n\right)$$

Implicit problem for half-time fluxes

$$u^{n+\frac{1}{2}}=u^n-\frac{\Delta t}{2}\frac{\partial}{\partial x}p^{n+\frac{1}{2}}\,,\qquad p^{n+\frac{1}{2}}=p^n-\frac{c^2\Delta t}{2}\frac{\partial}{\partial x}u^{n+\frac{1}{2}}$$

Eliminate $u^{n+\frac{1}{2}}$

$$\left(1-\frac{c^2\Delta t^2}{4}\frac{\partial^2}{\partial x^2}\right)\,p^{n+\frac{1}{2}}=p^n-\frac{c^2\Delta t}{2}\frac{\partial}{\partial x}u^n$$

Numerics

Implicit mid-point rule $\Rightarrow$ super-implicit

$$u^{n+\frac{1}{2}} = u^n - \frac{\Delta t}{2}\frac{\partial}{\partial x}p^{n+\frac{1}{2}}$$

$$\underline{p^{n+\frac{1}{2}}} = \underline{p^n} - \frac{c^2\Delta t}{2}\frac{\partial}{\partial x}u^{n+\frac{1}{2}}$$

key step:

$$\begin{aligned} u^{n+\frac{1}{2}} &= u^n - \frac{\Delta t}{2}\frac{\partial}{\partial x}p^{n+\frac{1}{2}} \\ &= \quad - \frac{c^2\Delta t}{2}\frac{\partial}{\partial x}u^{n+\frac{1}{2}} - \frac{\Delta t}{2}\left(\frac{\partial p}{\partial t}\right)^{\mathbf{BD},n+\frac{1}{2}} \end{aligned}$$

Pressure **"projection"** equation

$$\frac{c^2\Delta t}{2}\frac{\partial^2}{\partial x^2}p^{n+\frac{1}{2}} = c^2\frac{\partial}{\partial x}u^n + \left(\frac{\partial p}{\partial t}\right)^{\mathbf{BD},n+\frac{1}{2}}$$

Numerics

Scale-dependence via multi-grid

$$p = \sum_{j=1}^{J} p^{(j)}$$

where

$$p^{(j)} = (1 - P \circ R)\ R^{j-1} p \qquad \text{with} \qquad \begin{array}{ll} R : & \text{MG restriction} \\ P : & \text{MG prolongation} \end{array}$$

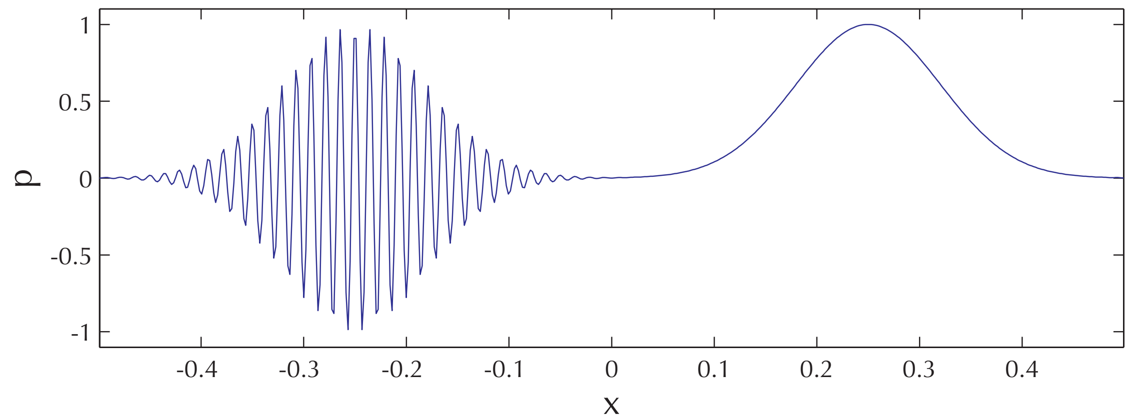
scale-dependent blending

$$u^{n+\frac{1}{2}} = u^n - \frac{\Delta t}{2} \frac{\partial}{\partial x} p^{n+\frac{1}{2}}$$

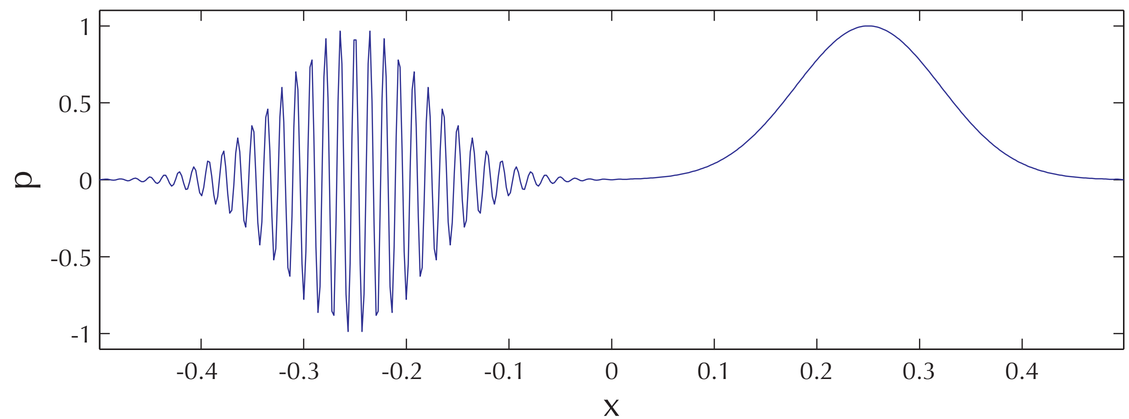
$$\sum_j \boldsymbol{\eta}^{(j)} {p^{(j)}}^{n+\frac{1}{2}} = \sum_j \boldsymbol{\eta}^{(j)} {p^{(j)}}^{n} - \frac{c^2 \Delta t}{2} \frac{\partial}{\partial x} u^{n+\frac{1}{2}} - \sum_j (1 - \boldsymbol{\eta}^{(j)}) \frac{\Delta t}{2} \left(\frac{\partial p^{(j)}}{\partial t} \right)^{\mathbf{BD}, n+\frac{1}{2}}$$

Numerics

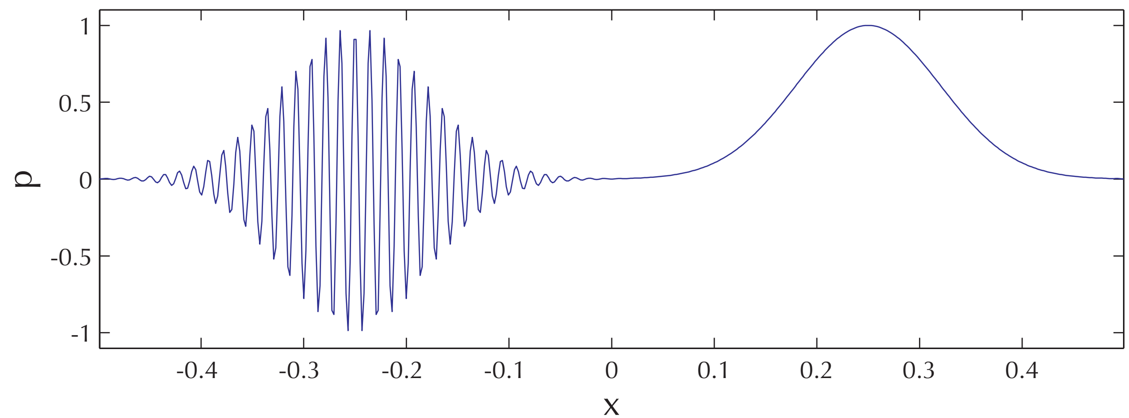
implicit midpoint



new scheme



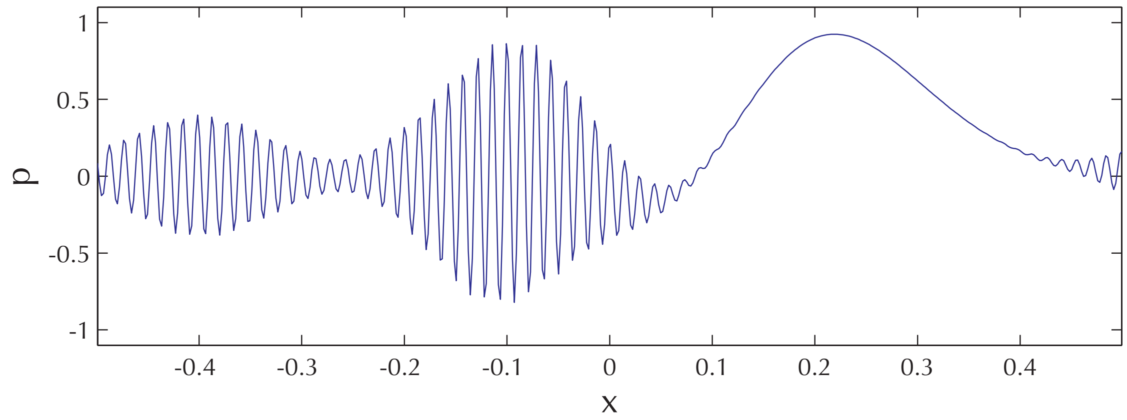
BDF2



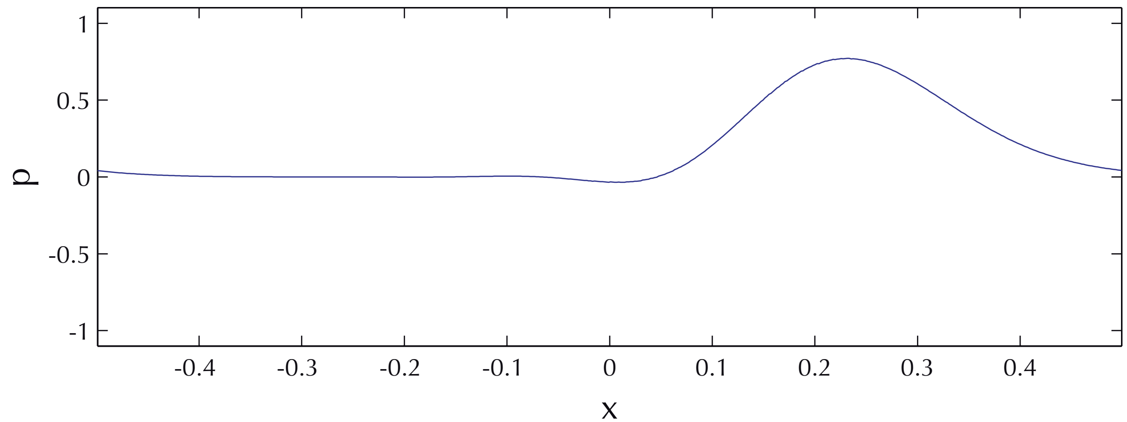
$t = 0$

Numerics

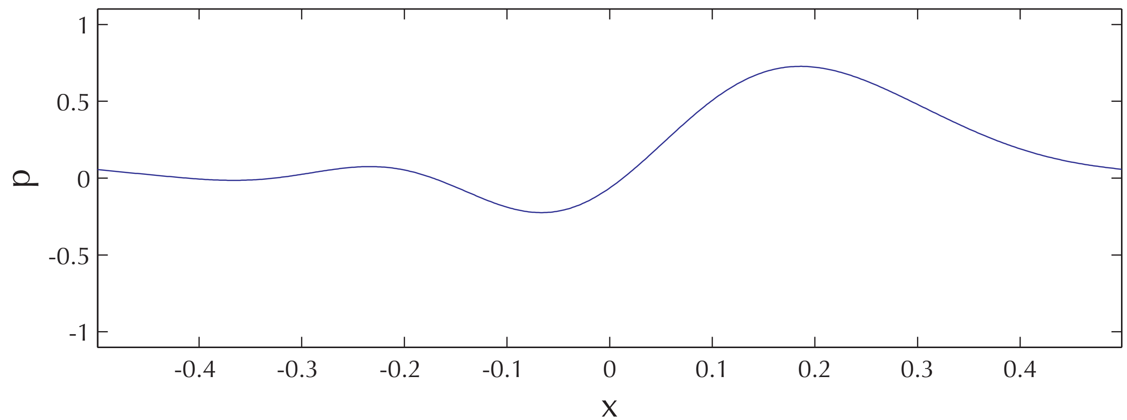
implicit midpoint



new scheme



BDF2

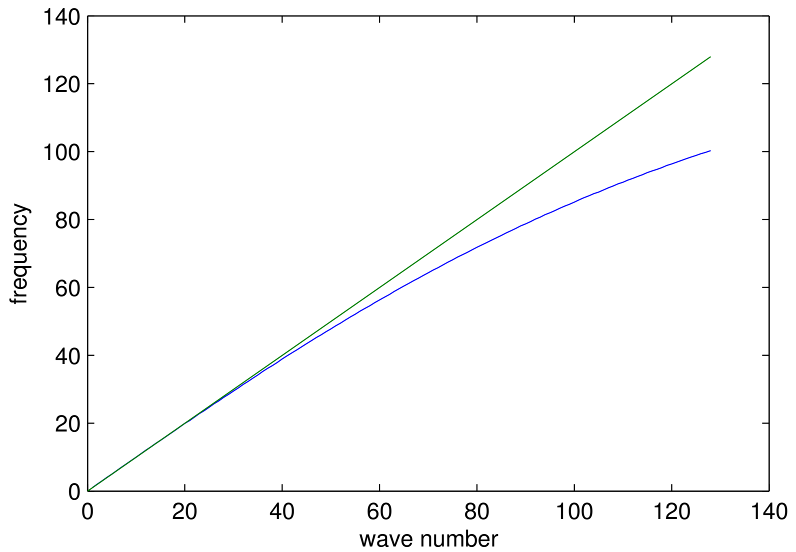


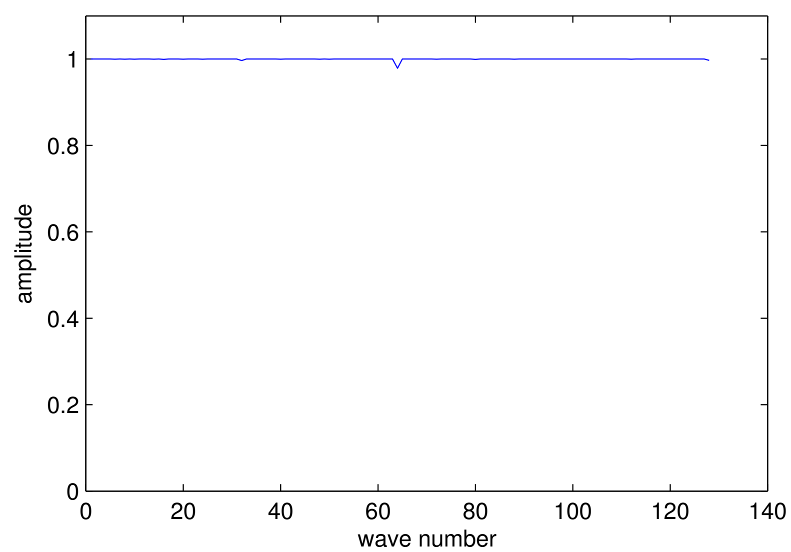
$t = 3$

Model Equations – Dispersion Relation and Amplitude

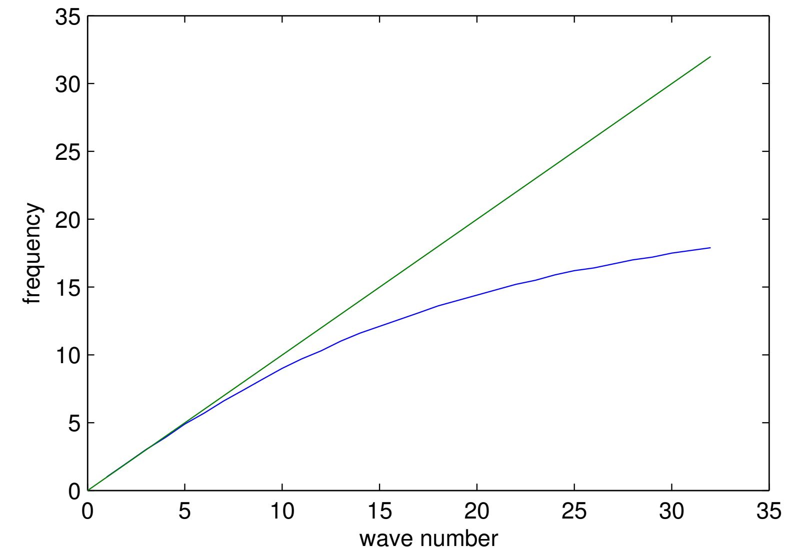
Implicit midpoint rule:

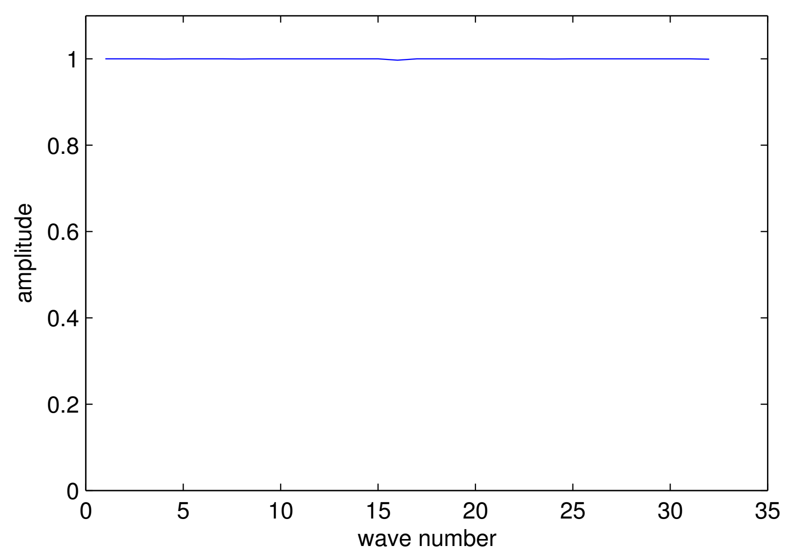
CFL=1





CFL=10

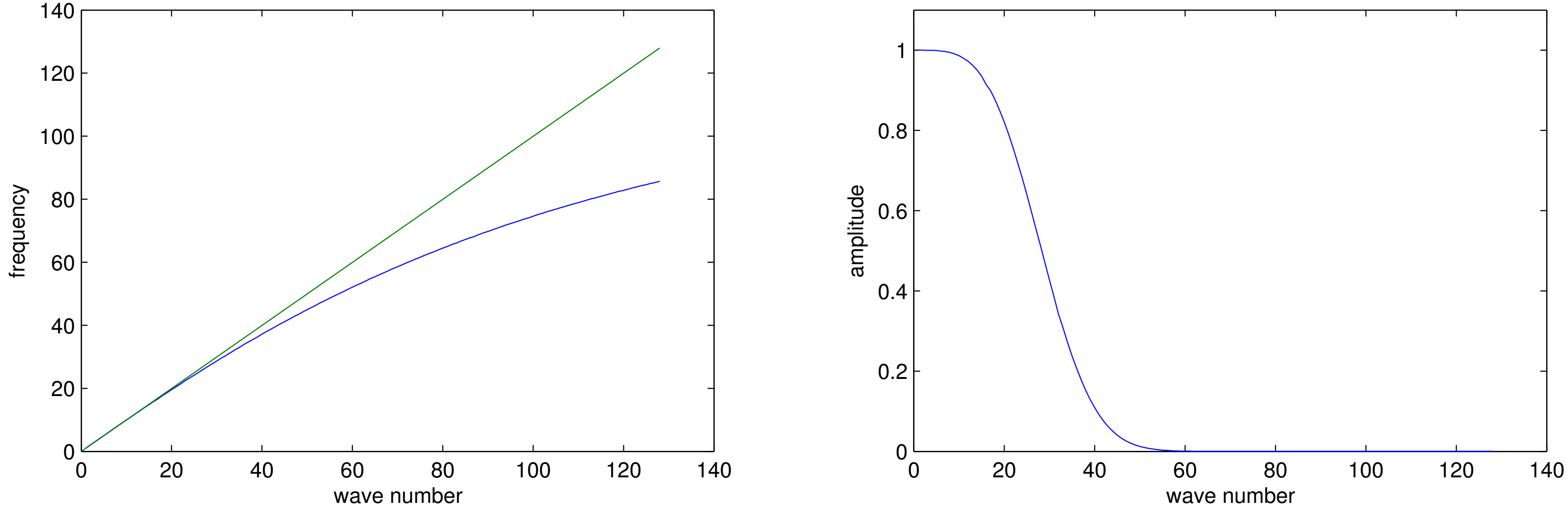




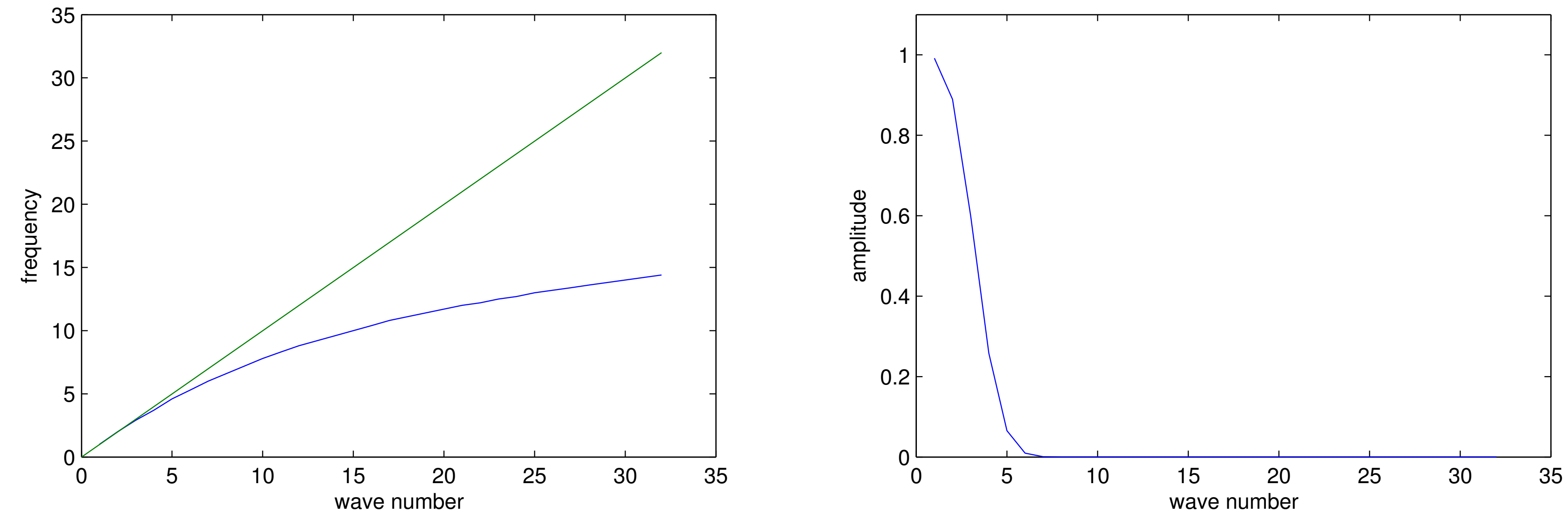
Model Equations – Dispersion Relation and Amplitude

BDF-2:

CFL=1



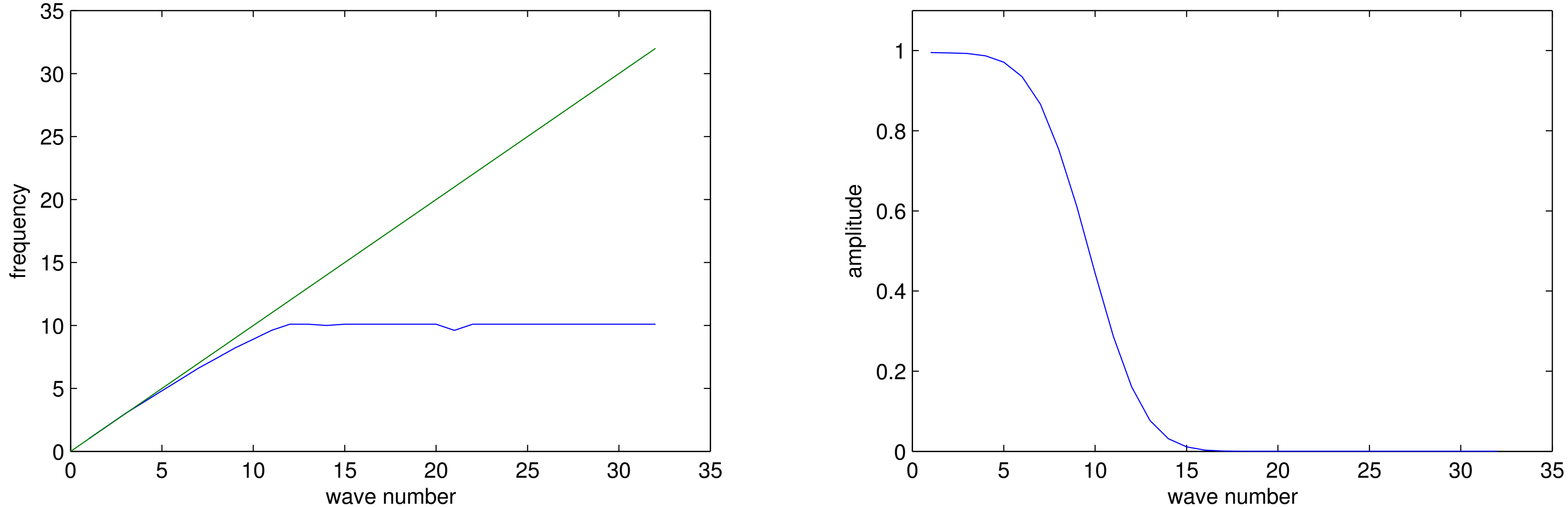
CFL=10



Dispersion Relation and Amplitude

Blended Scheme

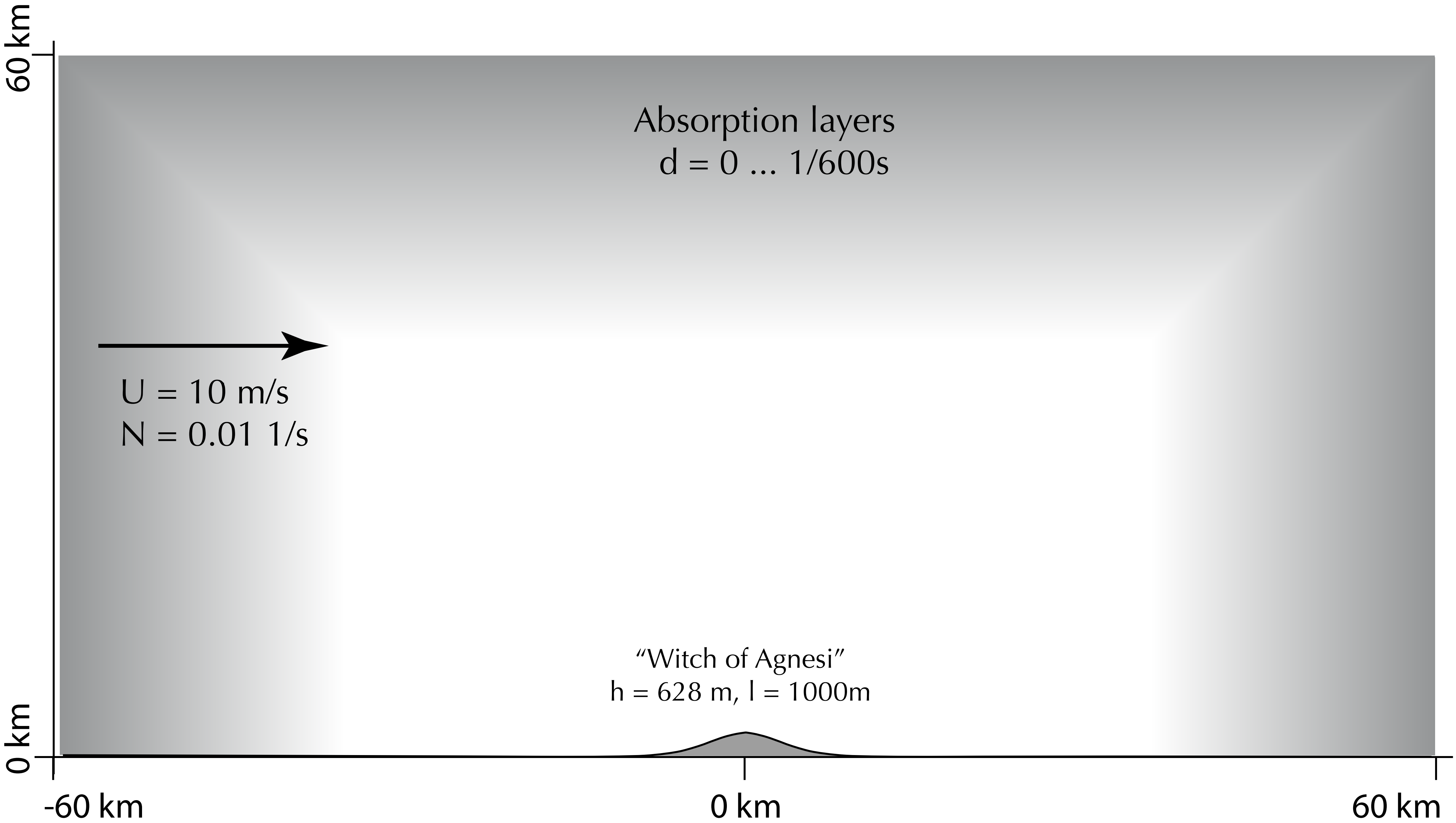
CFL=10



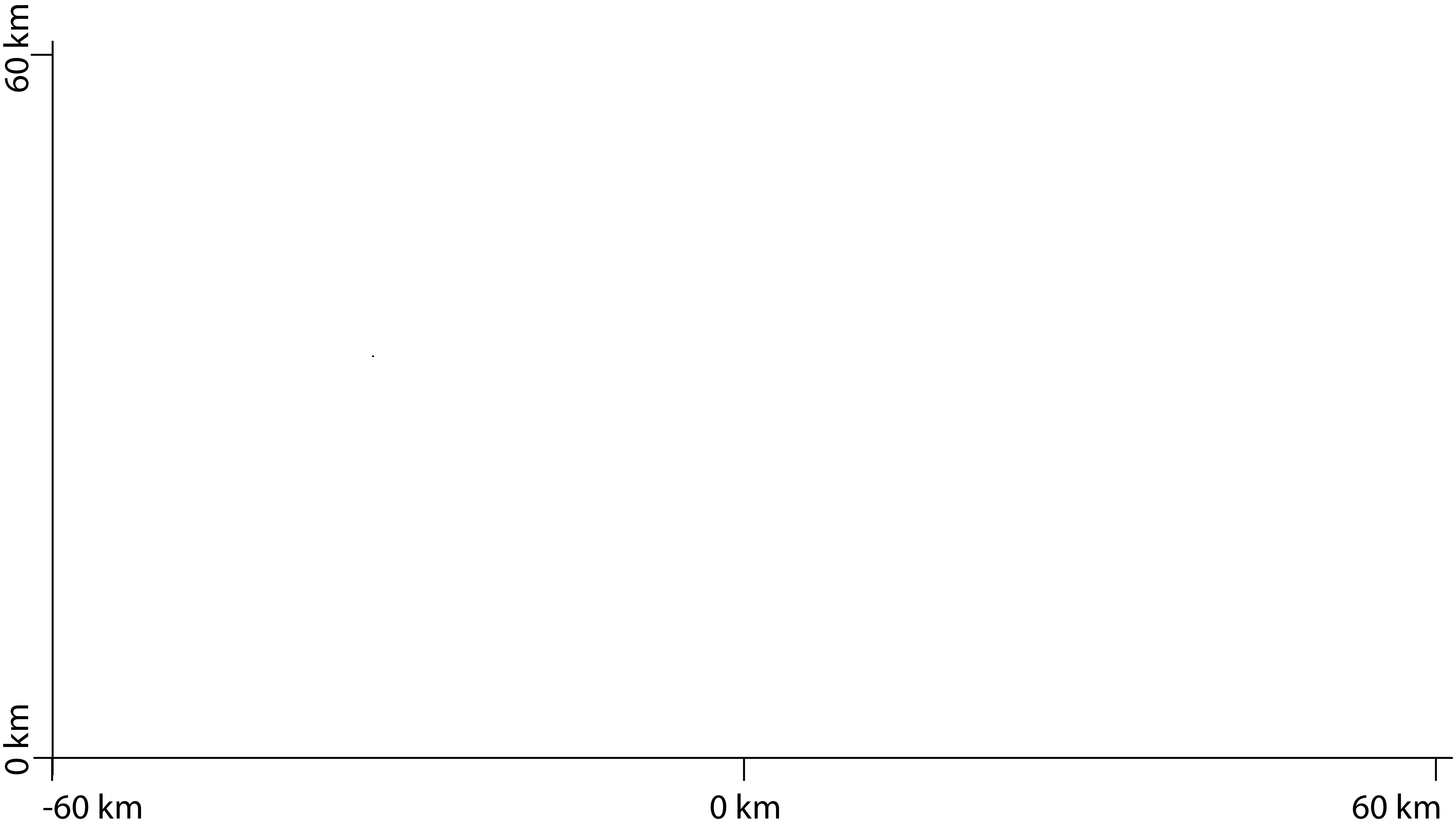
Preliminary results with implicit midpoint

(without IMP-SupI–blending)

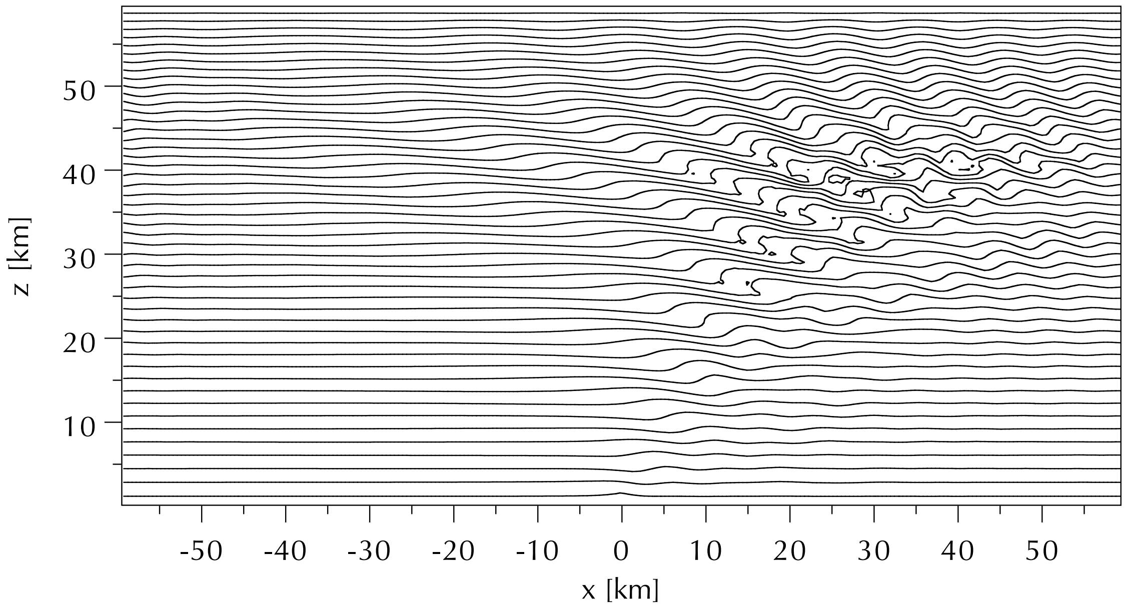
Breaking wave-test for anelastic models (Smolarkiewicz & Margolin (1997))



Breaking wave-test for anelastic models (Smolarkiewicz & Margolin (1997))



Breaking wave-test for anelastic models (Smolarkiewicz & Margolin (1997))

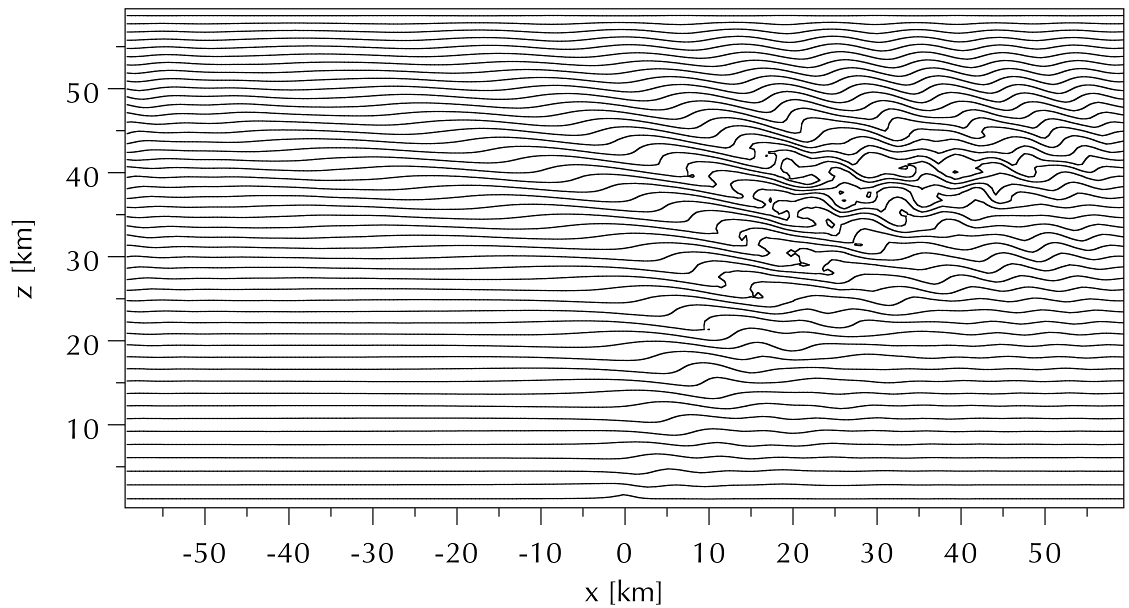


pseudo-incompressible

3 hours

sharpened van Leer's limiter

$\Delta t \, \nabla \cdot (P\boldsymbol{v}) < 10^{-3}$



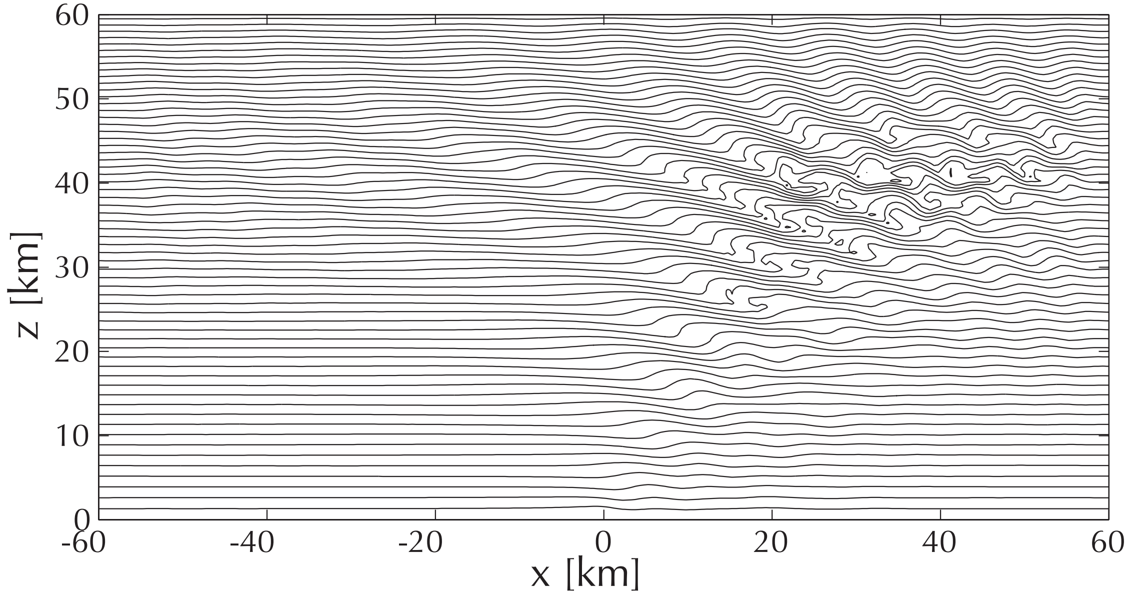
anelastic

3 hours

sharpened van Leer's limiter

$\Delta t \, \nabla \cdot (P\boldsymbol{v}) < 10^{-3}$

Breaking wave-test for anelastic models (Smolarkiewicz & Margolin (1997))



pseudo-incompressible

3 hours

sharpened van Leer's limiter

$\Delta t \, \nabla \cdot (P\boldsymbol{v}) < 10^{-4}$

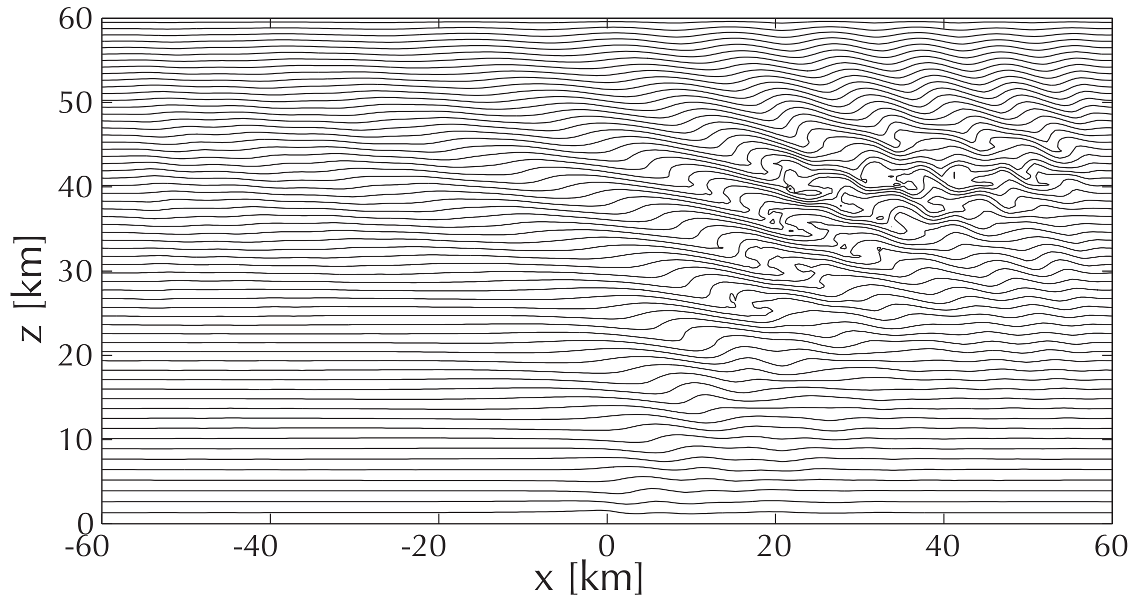
Compressible Euler eqs.

3 hours

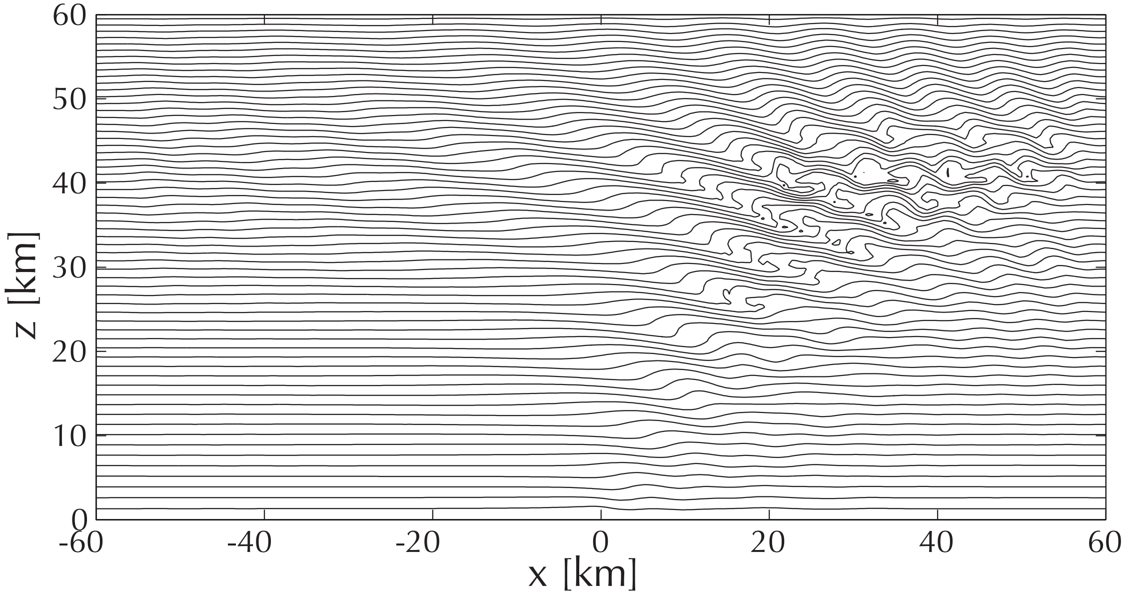
sharpened van Leer's limiter

$\Delta t \cdot \text{residual} < 10^{-4}$

$\text{CFL}_{\text{adv}} = 1.0$



Breaking wave-test for anelastic models (Smolarkiewicz & Margolin (1997))



pseudo-incompressible

3 hours

sharpened van Leer's limiter

$\Delta t\, \nabla \cdot (P\boldsymbol{v}) < 10^{-4}$

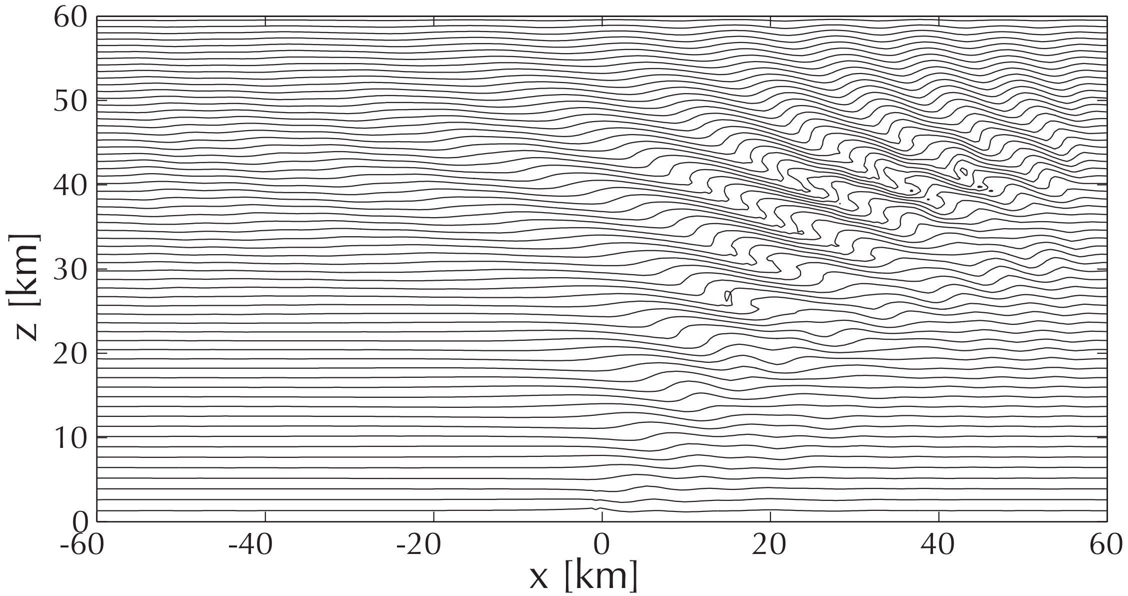
Compressible Euler eqs.

3 hours

sharpened van Leer's limiter

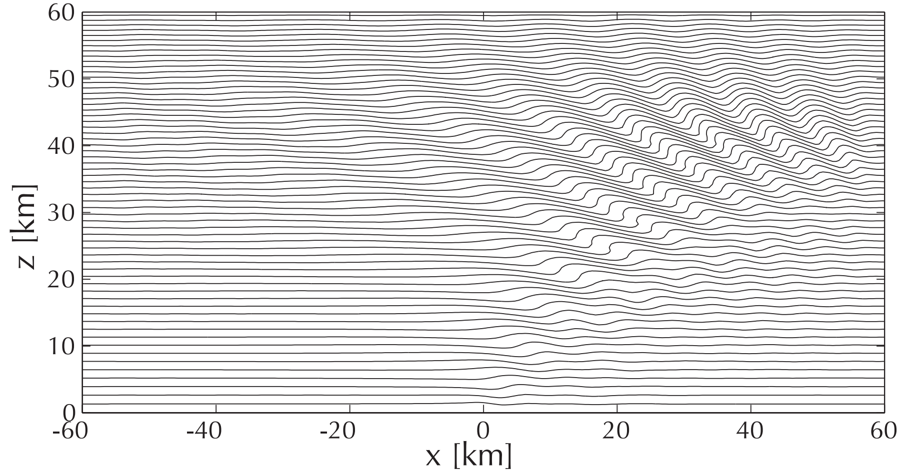
$\Delta t \cdot \mathrm{residual} < 10^{-4}$

$\mathbf{CFL_{ac} = 2.0}$

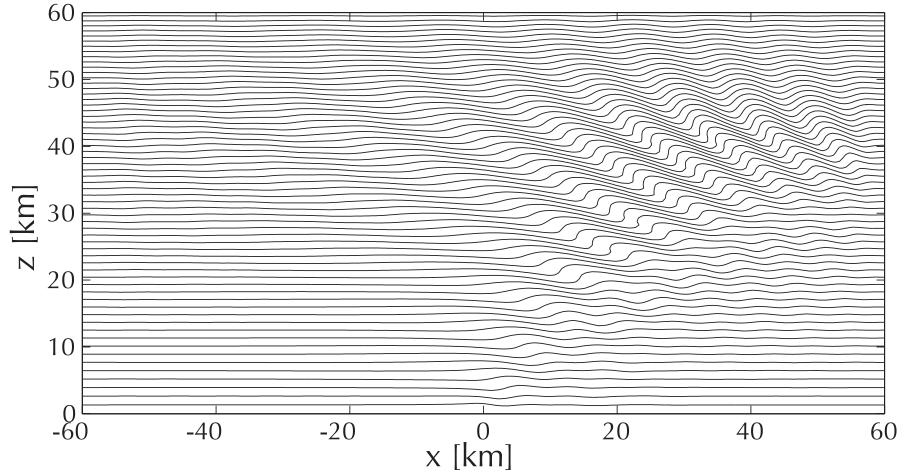


Results at time $t = 2\,h$

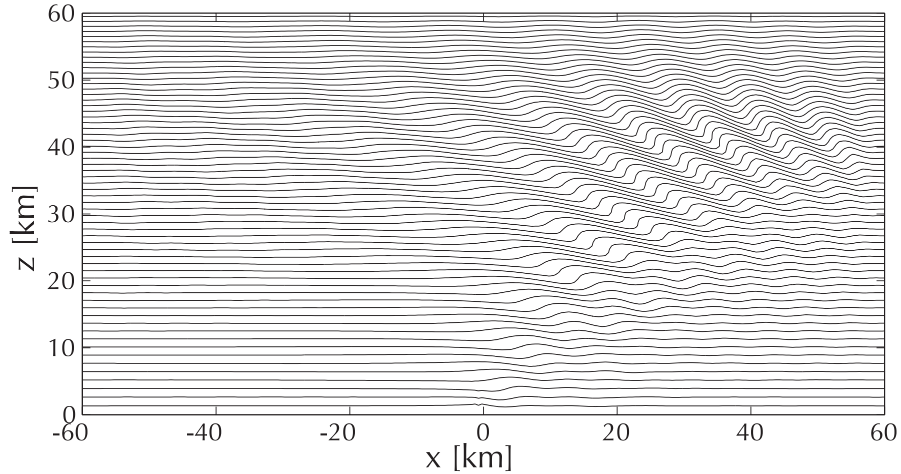
pseudo-incompressible



compressible, $\mathrm{CFL}_{\mathrm{adv}} = 1$



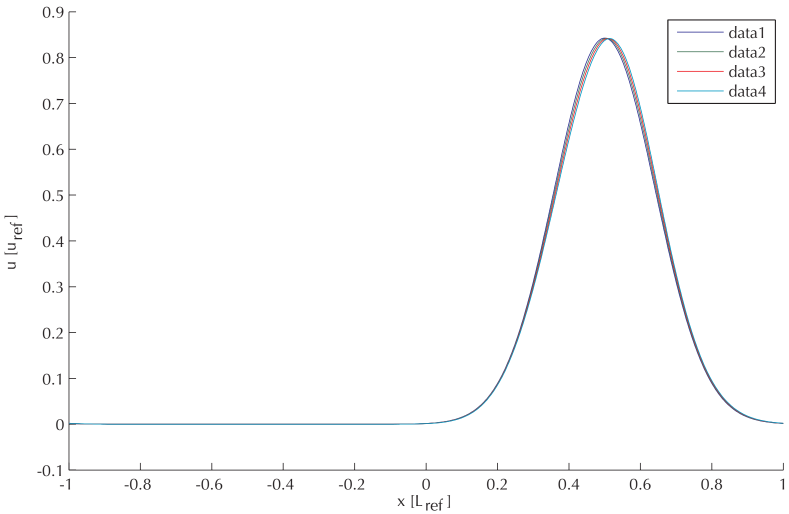
compressible, $\mathrm{CFL}_{\mathrm{ac}} = 2$

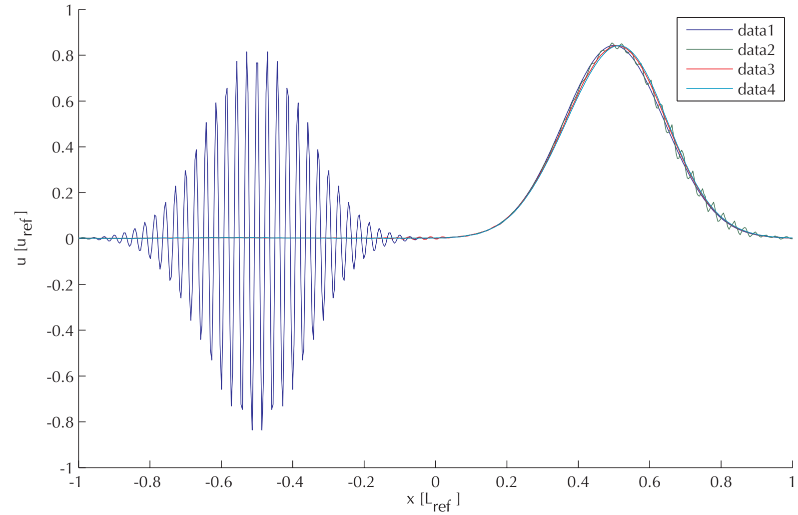


1D Acoustic test revisited, compressible Euler $(\mathrm{Mach} = 10^{-4})$

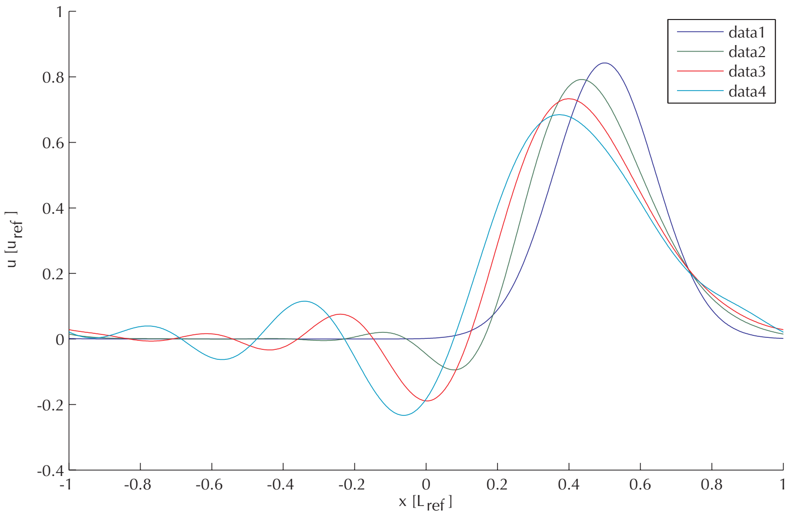
single pulse data

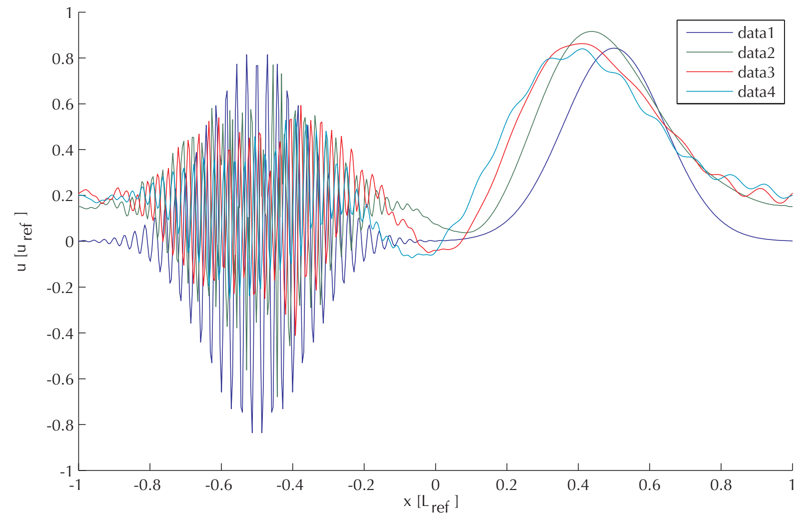
multiscale data





$\mathrm{CFL_{ac}} = 1.0$





$\mathrm{CFL_{ac}} = 10.0$

Summary

Regime(s) of validity of sound-proof models

- Motivation
- Stratification limit in the design-regime
- Wave-breaking regime with strong stratification

Scale-dependent time-integrator