Testing EULAG as a prospective dynamical core of the NWP model: results of the dry benchmarks

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Michael Baldauf (DWD) and Oliver Fuhrer (Meteo Swiss)

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    4. mountain waves (Pinty et al., 1995, Bonaventura, 2000),
- Summary

Introduction

Resolution of current numerical weather prediction (NWP) models approaches O(1km). Such relatively high resolutions is permitting for modeling convective atmospheric processes.

- Flows at scales O(1km) are highly turbulent and contain a large amount of energy near the grid scale. Therefore, conservative properties of the NWP models at such small scales are of the main importance.

- However increasing spatial resolution leads to improvement of modeling convective processes it also imposes serious problem with increasing steepness of the terrain. Too large steepness is one of the major constrains of the contemporary NWP models.

- Convection-Resolving Models requires also closer coupling between the dynamical core and the physical parameterizations.

Motivation

We are interested in development of new generation dynamical core, for future NWP models for very high resolutions, (as a part of research-development work of the COSMO consortium, COSMO: Consortium of Small Scale Modeling, grouping some of European national weather services)

EULAG (*EULarian semi-LAGrangian*) **nonhydrostatic anelastic** model developed at NCAR by P. Smolarkiewicz,W. Grabowski, J. Prusa and A. Wyszogrodzki. (Prusa and Smolarkiewicz, 2003; Smolarkiewicz and Prusa, 2005; Grabowski and Smolarkiewicz, 2002)

The characteristic features of the EULAG model:
- Conservative flux form of the governing equations
- Semi-implicit time integration scheme
- Finite volume discretization
- Capability of modeling flows over steep terrain.

COSMO (*Consortium of Small Scale Modeling*) – operational **nonhydrostatic compressible** model, base on hydro-thermodynamical equations in advection form

The characteristic features of COSMO model:
- Explicit time integration scheme
- Finite difference approximation in terrain following coordinates

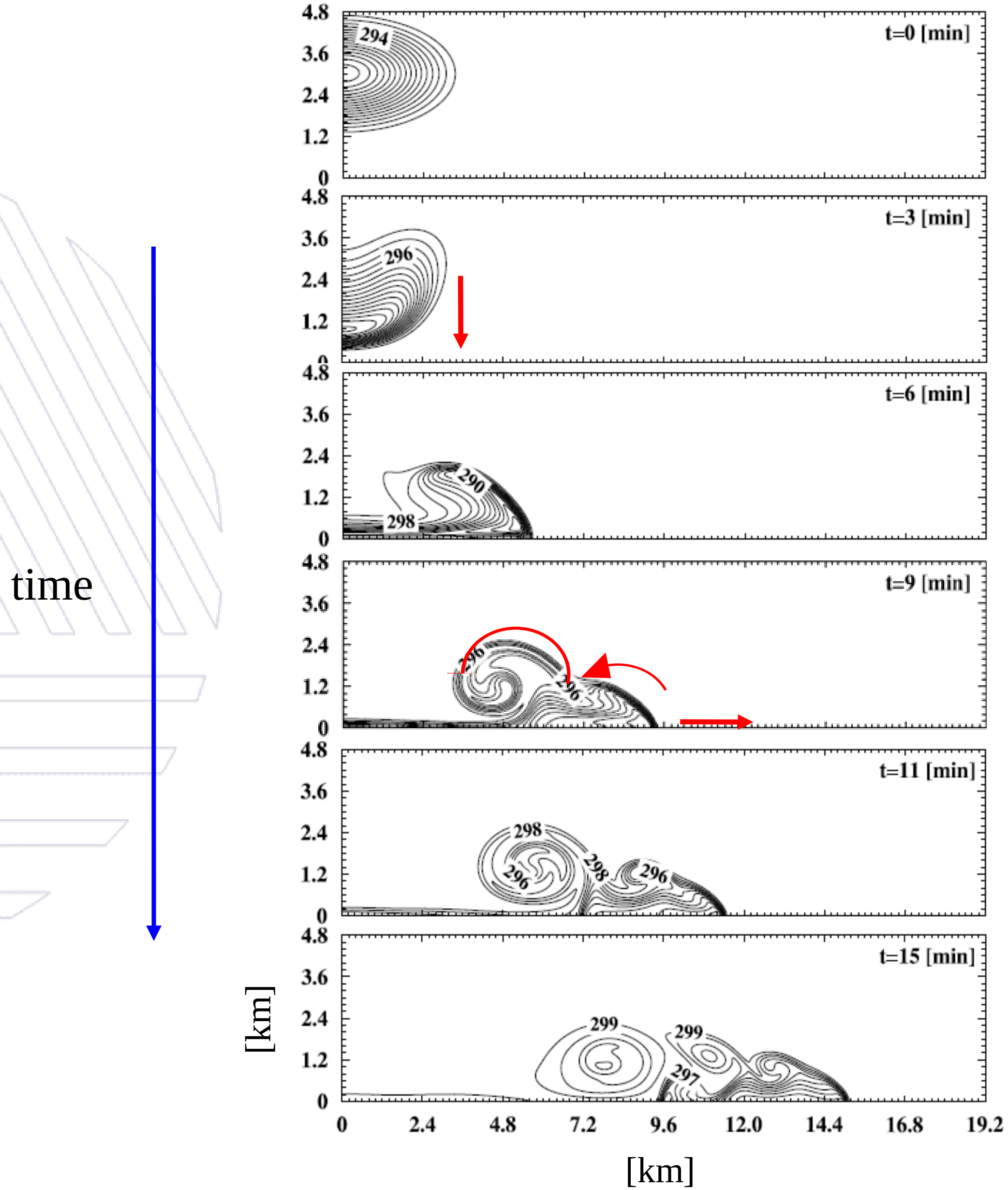
Cold density current

references:

Straka J. M, R. B. Wilhelmson, L. J. Wicker, J. R. Anderson and K. K. Droegemeier. Numerical solutions of a nonlinear density current – a benchmark solution and comparison. International Journal for Numerical Methods in Fluids, 17(1):1-22, ***1993***

1. Cold density current

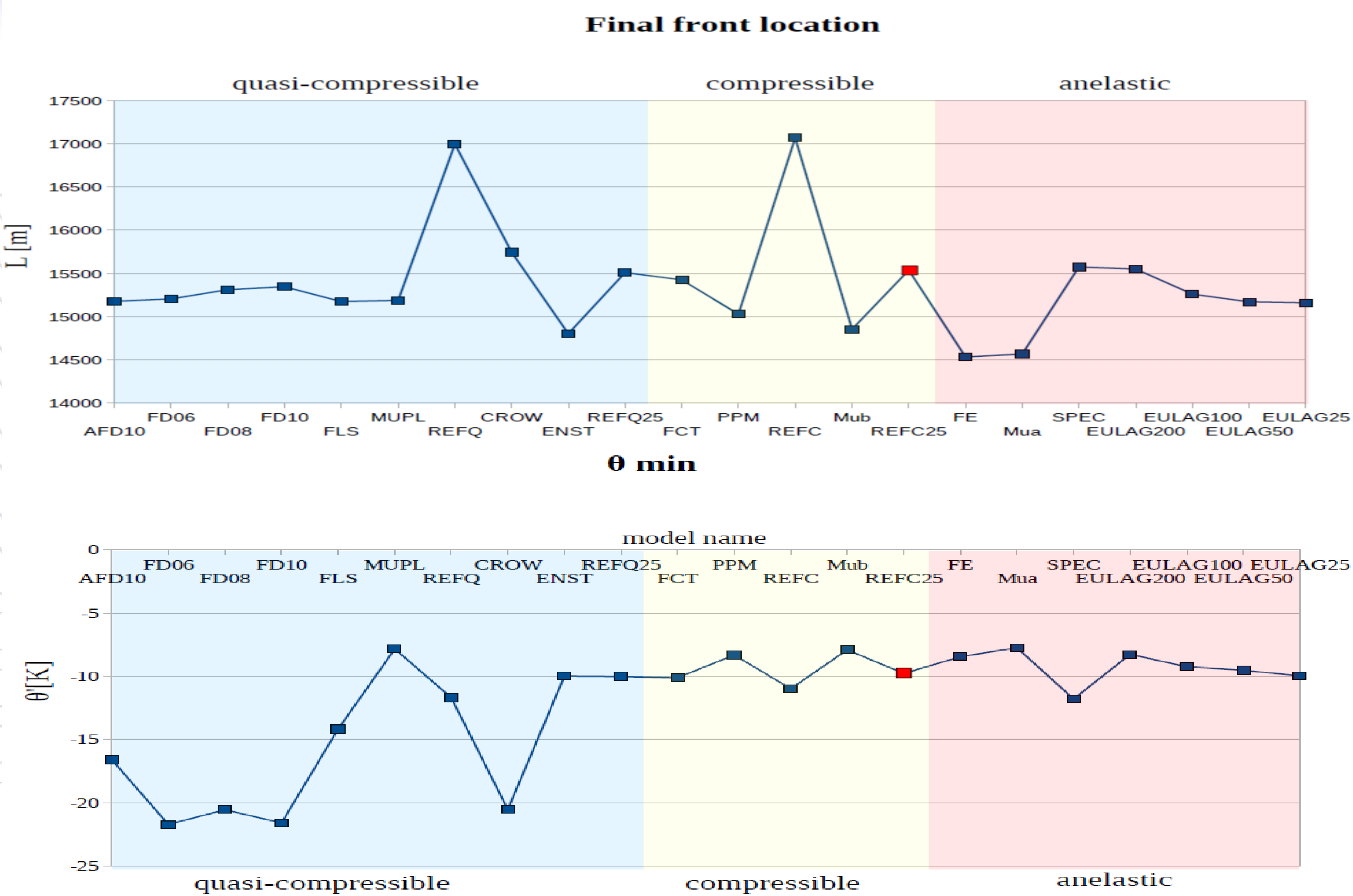
Potential temperature



Experiment configuration:

- isentropic atmosphere, $\theta(z)$=const (300K)
- periodic lateral boundaries
- free-slip bottom b.c.
- constant subgrid mixing, $K=75m^2/s$
- domain size 51.2km x 6.4km
- bubble min. temperature -15K
- bubble size 8km x 4km
- no initial flow
- integration time 15min

1. Cold density current – comparison with other models



- REFC25 (reference compressible)

1. Cold density current – comparison with other models

Relative differences of Straka's + Eulag solutions vs. benchmark

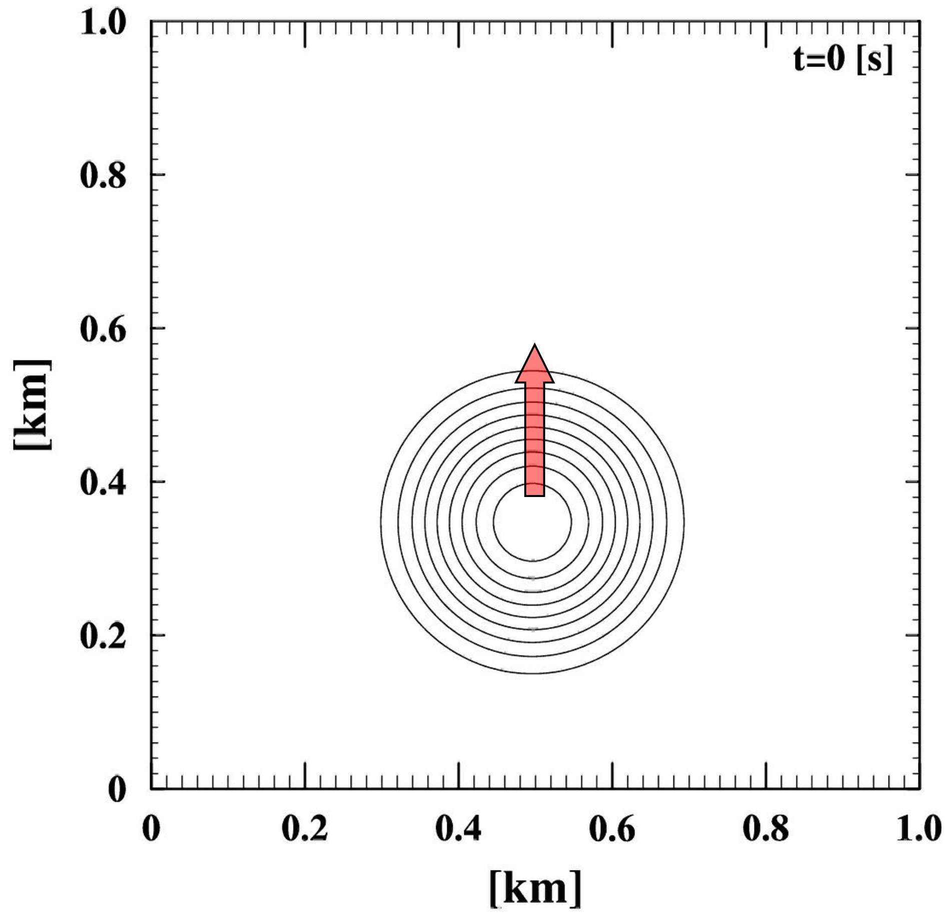
model name	front location L [m]	error L [%]	θ max	θ min	error θ min [%]
AFD10	15175,97	-2,38	0,61	-16,59	41,07
FD06	15205,56	-2,18	1,37	-21,7	54,97
FD08	15312,25	-1,47	1,4	-20,54	52,41
FD10	15344,9	-1,25	1,16	-21,58	54,72
FLS	15174,2	-2,39	0,68	-14,16	30,99
MUPL	15188,39	-2,3	0	-7,82	-24,91
REFQ	16998,44	8,59	0,61	-11,67	16,27
CROW	15745,98	1,32	0,1	-20,52	52,36
ENST	14801,46	-4,97	0,06	-9,97	1,95
REFQ25	15509,17	-0,18	0	-10	2,26
FCT	15426,02	-0,72	0	-10,11	3,29
PPM	15027,97	-3,39	0,02	-8,31	-17,6
REFC	17069,85	8,98	0,56	-10,97	10,88
Mub	14853,97	-4,6	0,44	-7,9	-23,69
REFC25	**15537,44**	**0**	**0**	**-9,77**	**0**
FE	14532,76	-6,91	0,19	-8,44	-15,84
Mua	14566,62	-6,66	0,19	-7,73	-26,38
SPEC	15574,6	0,24	0,7	-11,78	17,05
EULAG200	15550	0,08	0	-8,26	-18,33
EULAG100	15260	-1,82	0	-9,24	-5,78
EULAG50	15170	-2,42	0	-9,52	-2,67
EULAG25	15160	-2,49	0	-9,96	1,87
		>3%			**>10%**

Rising thermal

references:

Giraldo, F. X. and M. Resteli. A study of spectral element and discontinuous Galerkin methods for the Navier-Stokes equations in nonhydrostatic mesoscale atmospheric modelling: equation sets and test cases. J. Comp. Phys., 227:3849-3877, ***2008***

2. Rising thermal



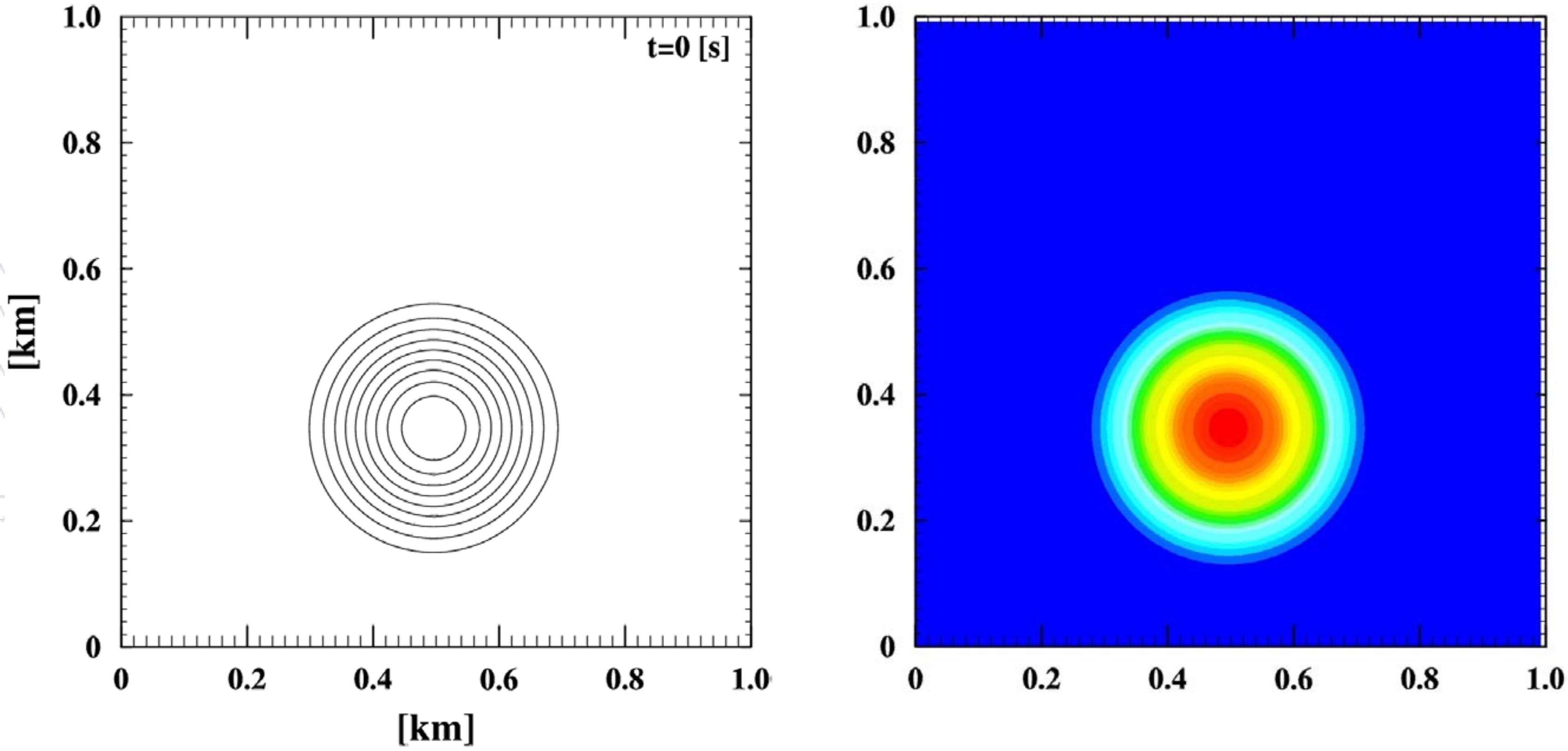
Setup overview:

- domain size 1km x 1 km
- resolution 16 x16m, 8x8m, 4x4m
- free-slip b.c.
- rigid boundaries
- isentropic (θ=const)
- no external flow
- inviscid
- max. temperature 0.5K

$$\frac{\theta_c}{2}\left[1+\cos\left(\frac{\pi_c r}{r_c}\right)\right]$$

No analytic solution

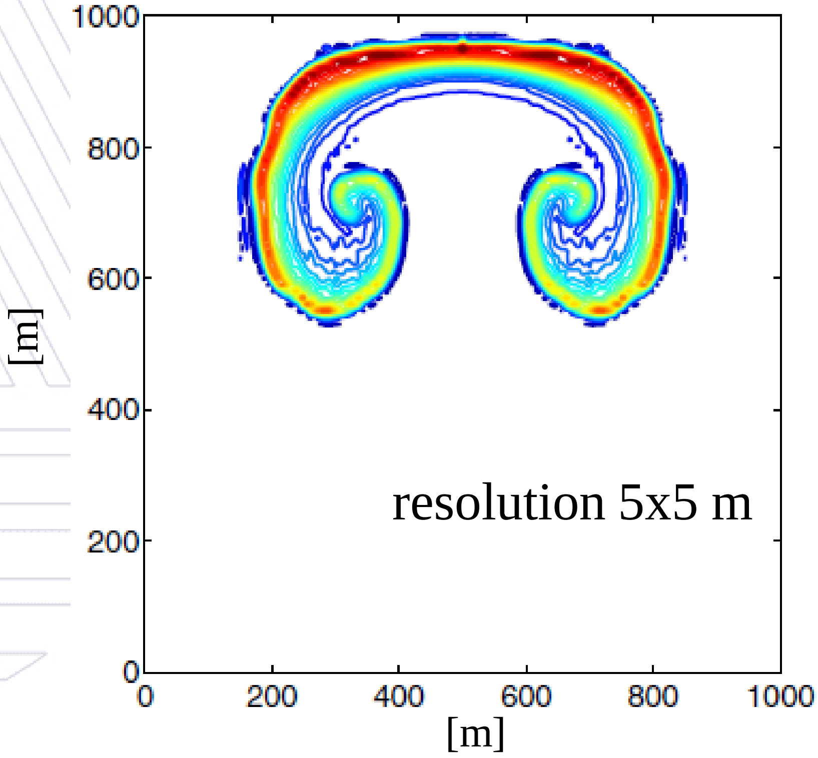
2. Rising thermal



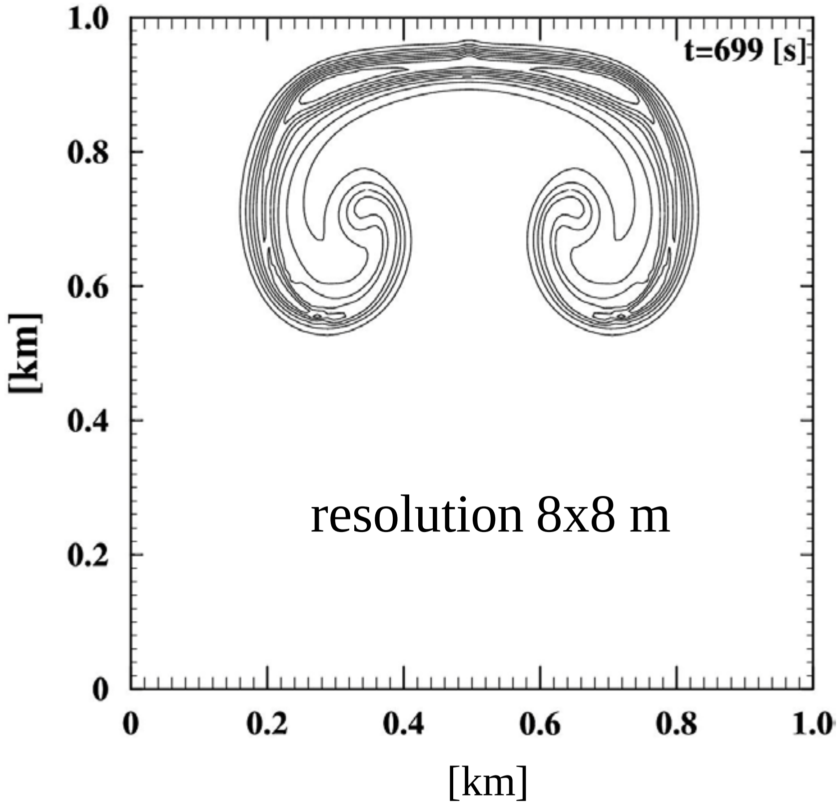
2. Rising thermal

Comparison of results

Giraldo and Reseti (2008) (fully compressible)



EULAG (2009) (anelastic)



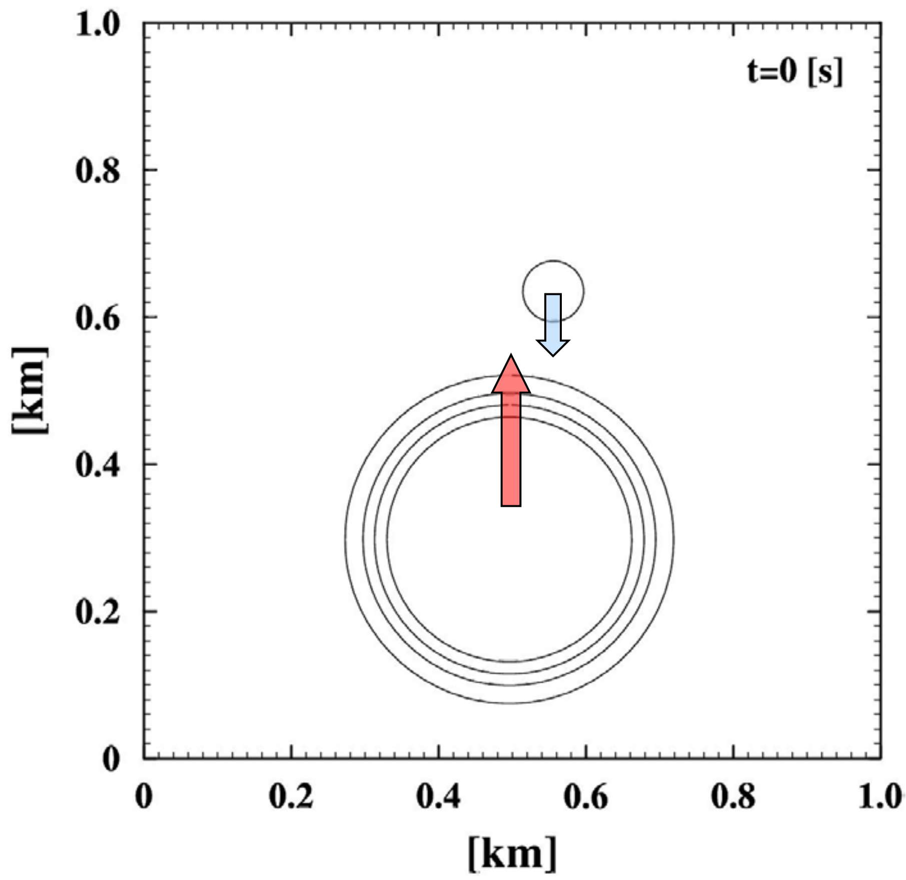
Potential temperature perturbation after 700s

Interacting bubbles

reference:

Robert, A., *Bubble convection experiments with a semi-implicit formulation of the Euler equations. J. Atmos. Sci., 50: 1865-1873,* ***1993***

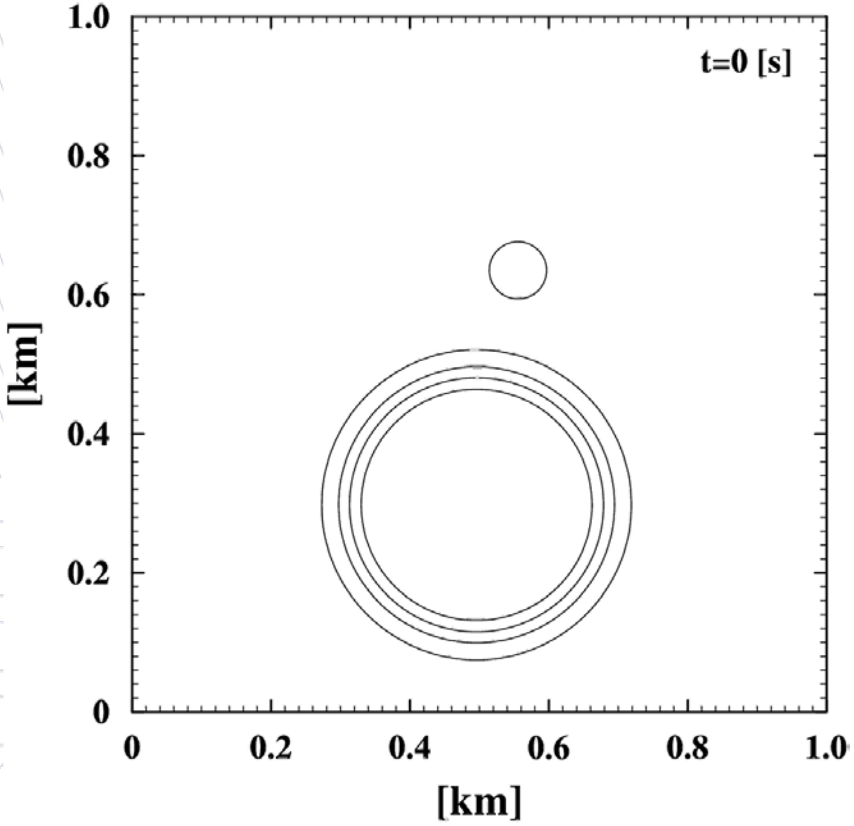
2. Interacting bubbles

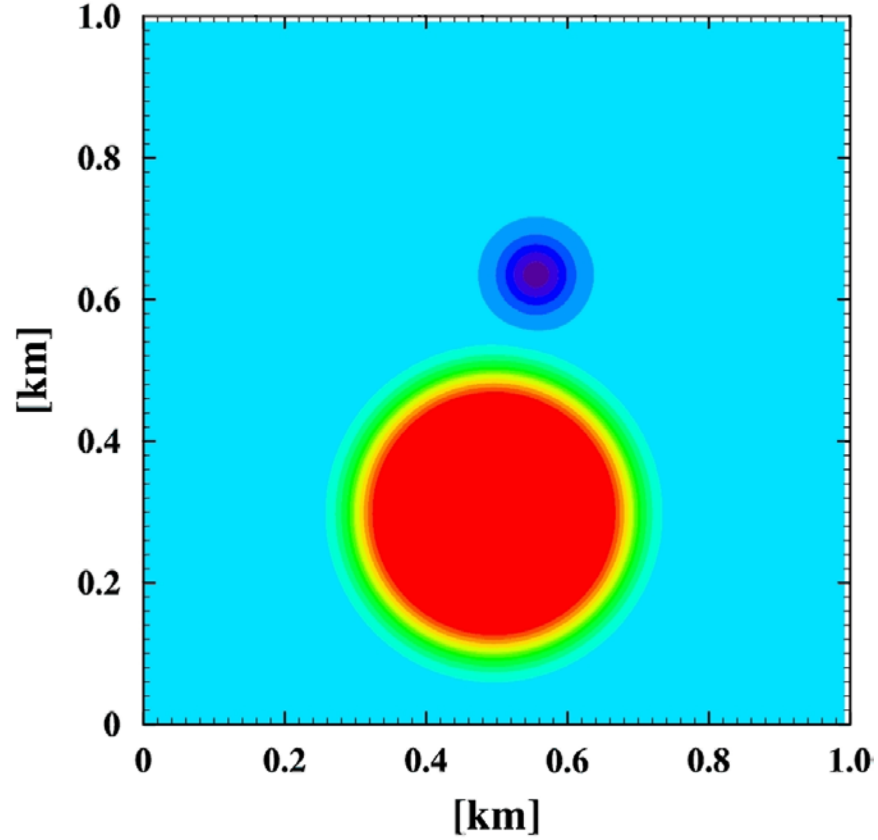


Experiment setup:

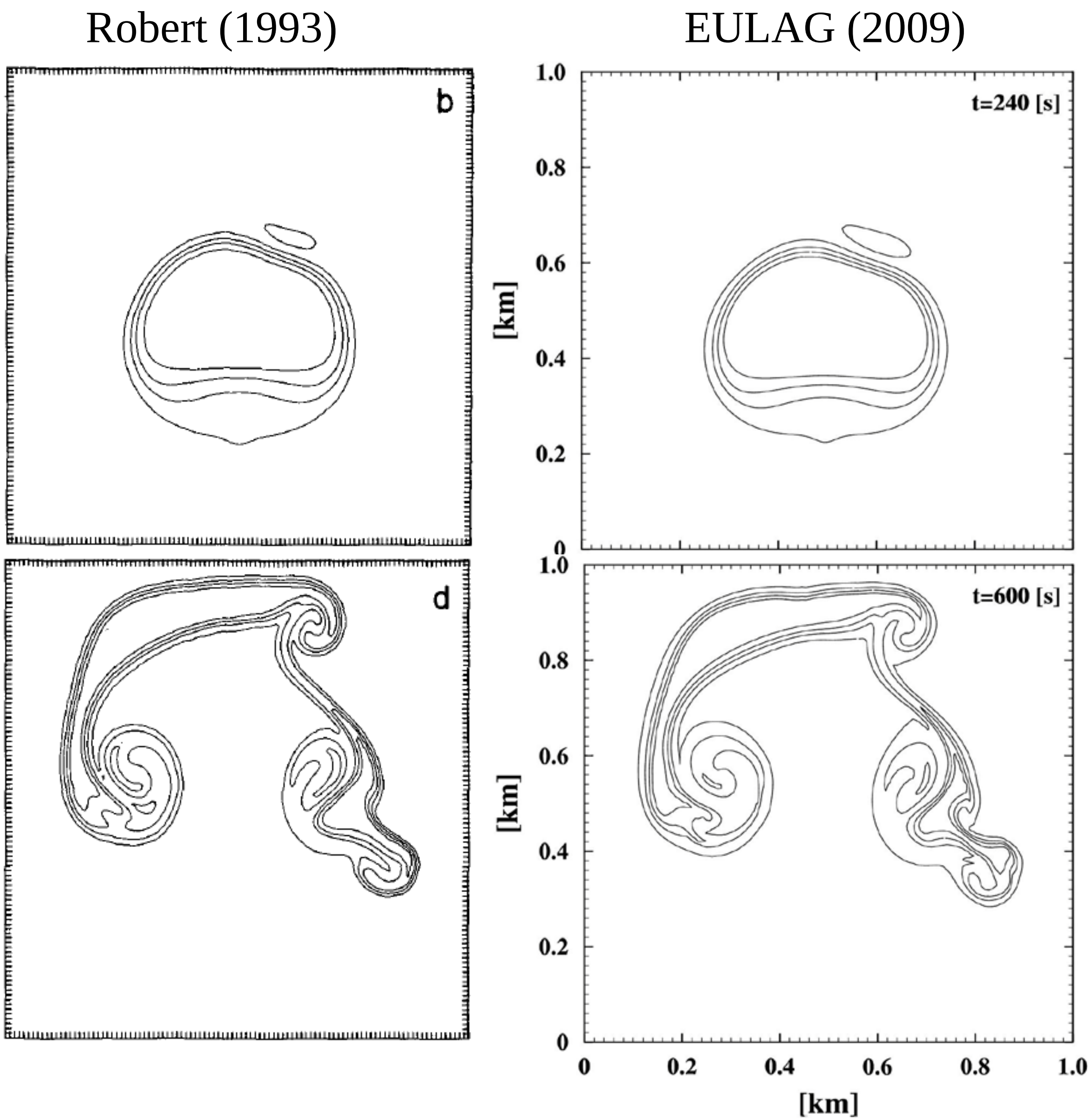
- domain size1km x 1 km
- resolution 16x16m,
  8x8m, 4x4m
- free-slip b.c.
- rigid boundaries
- no external flow
- no subgrid-scale model
- temperature perturbation:
  0.5K (rising thermal)
  -0.1K (descending thermal)

2. Interacting bubbles





2. Interacting bubbles



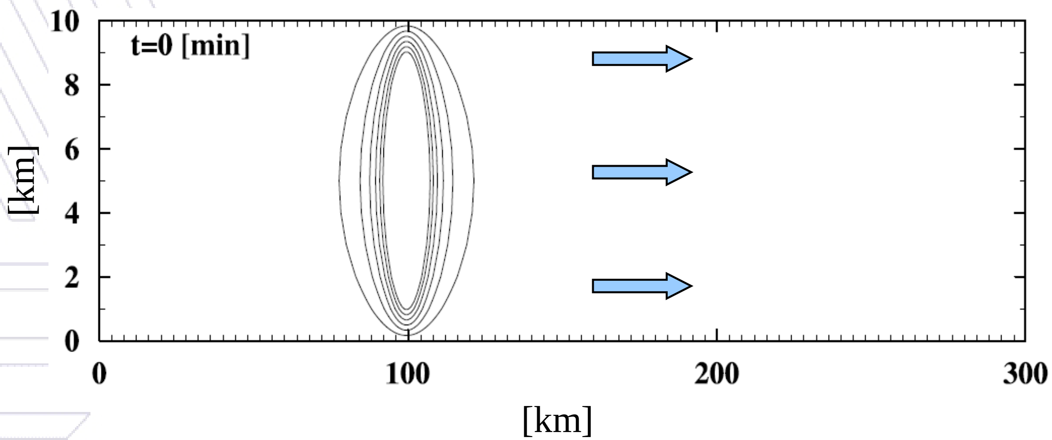
Inertia-gravity waves

reference:

Skamarock W. C. and Klemp J. B. Efficiency and accuracy of Klemp-Wilhelmson time-splitting technique. *Mon. Wea. Rev.* 122(11):2623-2630, **1994**

Constant flow within a short channel (300km)

Initial potential temperature perturbation



Testing model for subtle phenomenon

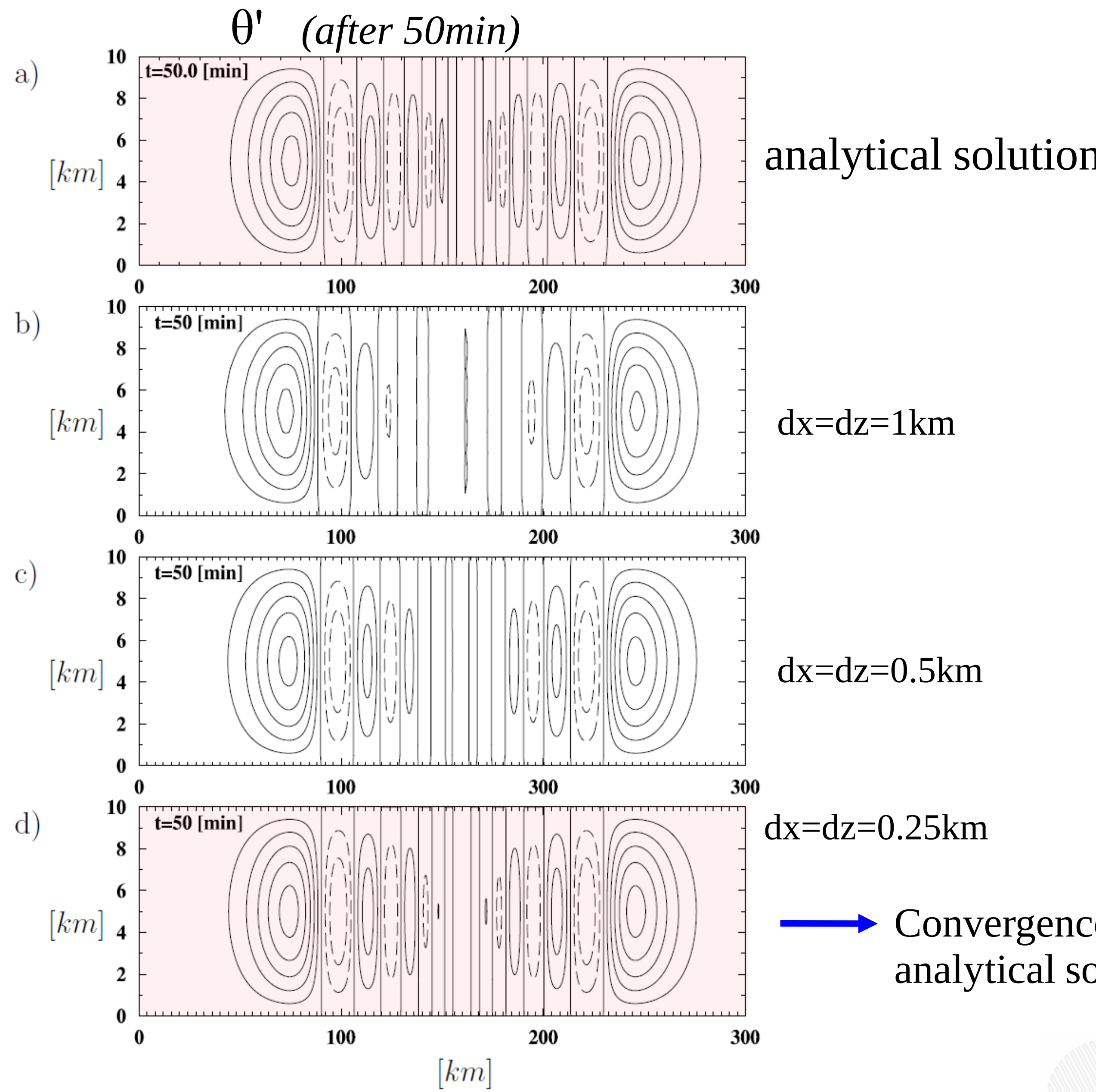
Setup overview:

- domain size 300x10km
- resolution 1x1km, 0.5x0.5km, 0.25x0.25km
- rigid free-slip b.c.
- periodic lateral boundaries
- constant horizontal flow 20m/s
- no subgrid mixing
- hydrostatic balance
- stable stratification $N=0.01\ s^{-1}$
- max. temperature perturbation 0.01K

Convergence study for resolution

Convergence study for resolution

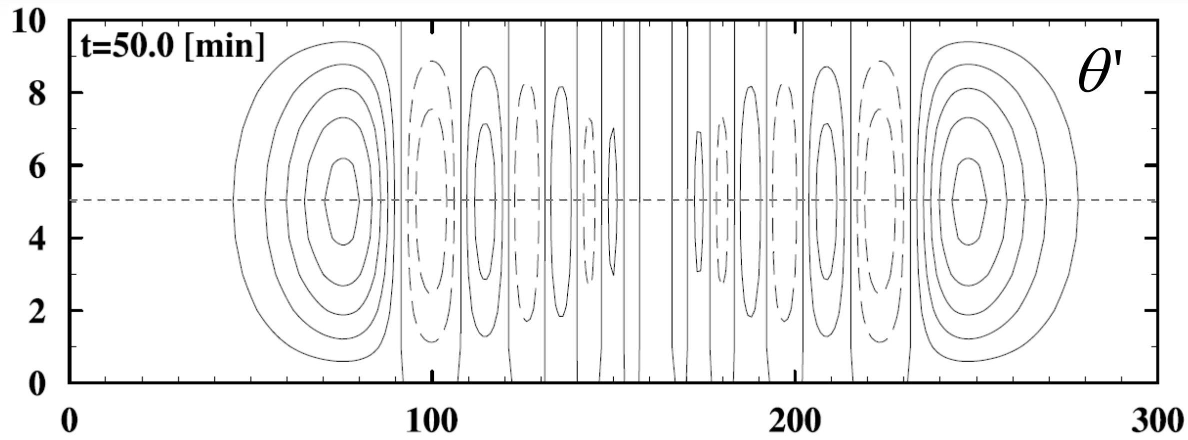
EULAG incompressible Boussinesq



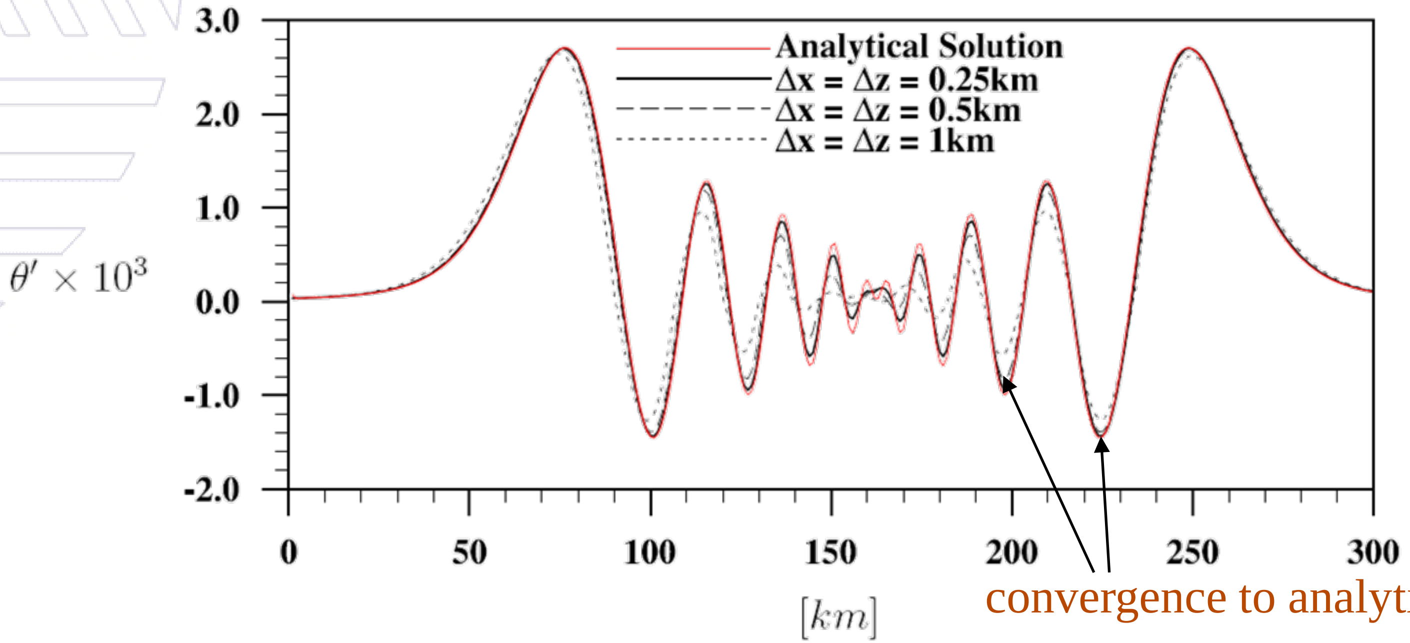
→ Convergence to analytical solution

3. Profiles of potential temperature along 5000m height

Convergence study for resolution

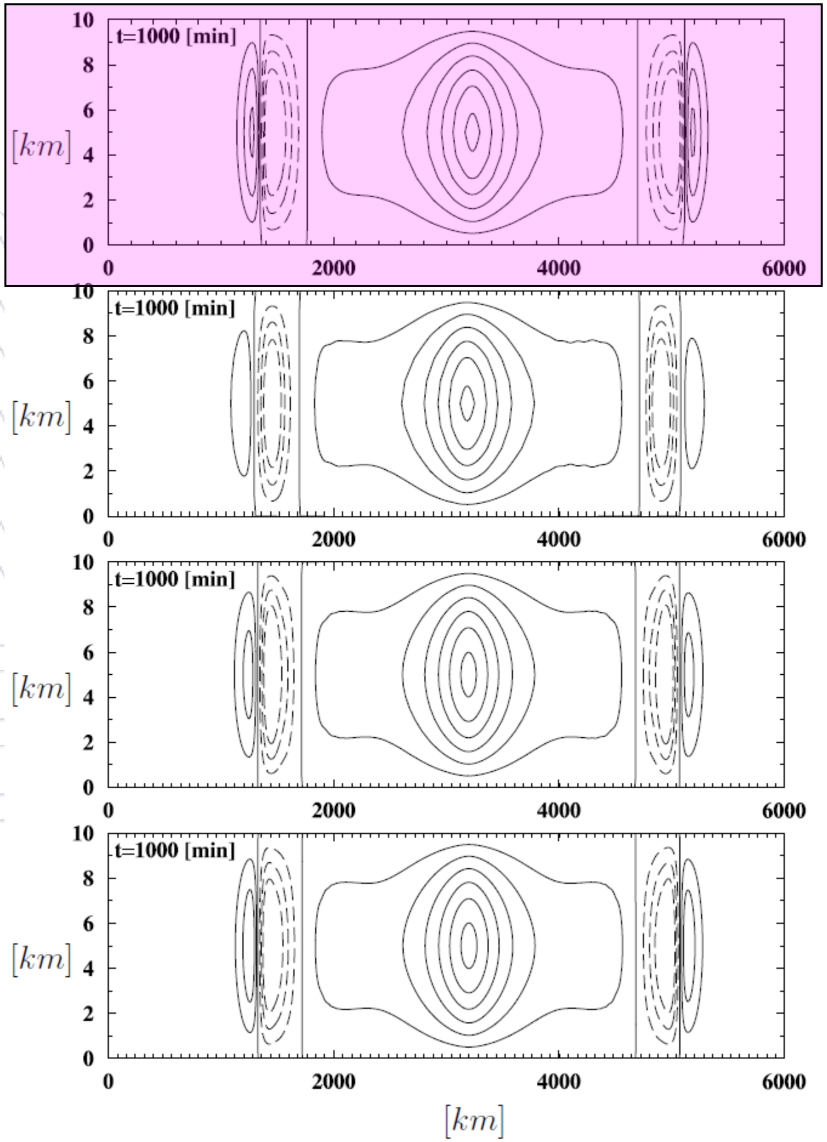


Profiles of potential temperature along 5000m height



convergence to analytical solution

3. Time evolution of potential temperature in long channel (6000 km)



Analytical solution based on linear approximation (Skamarock and Klemp 1994)

dx = 20 km
dz = 1km

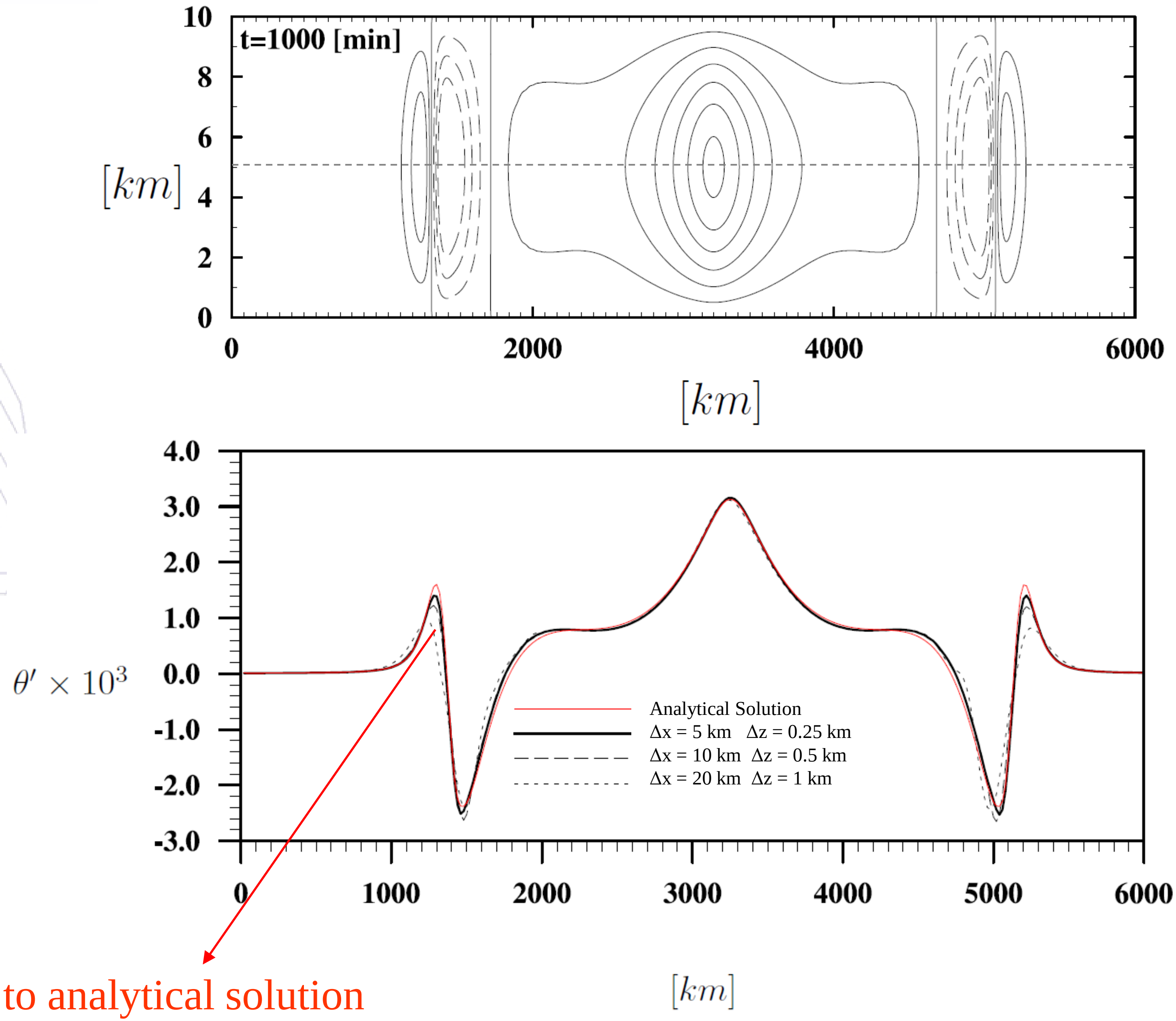
dx = 10 km
dz = 0.5 km

dx = 5km
dz = 250 m

Numerical solution from EULAG (inocompressible Boussinesq approach)

$$\frac{dx}{dz} = 20$$

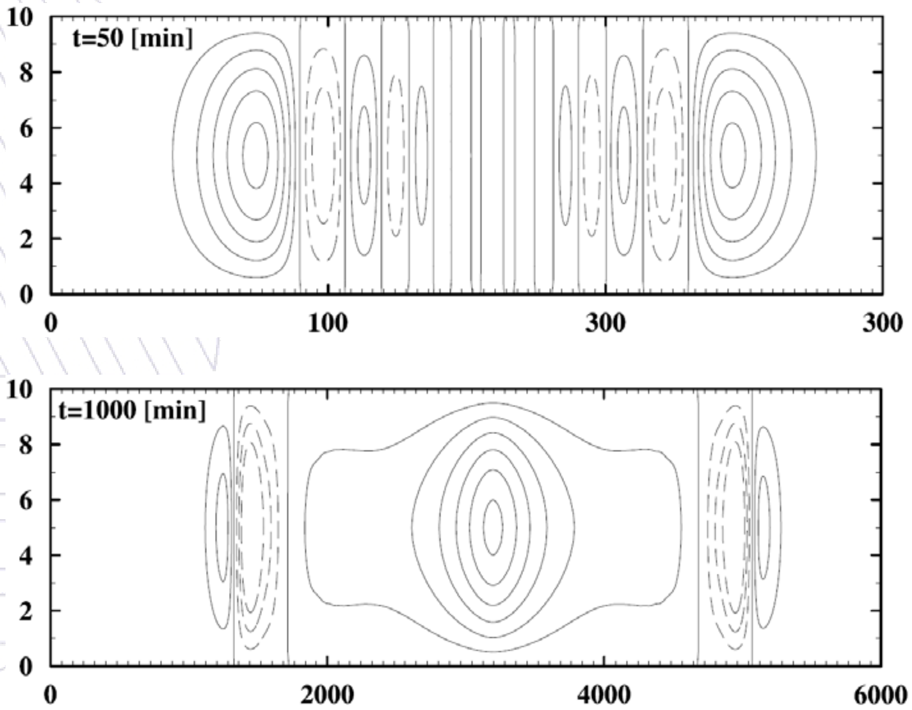
3. Profiles of potential temperature along 5000m height



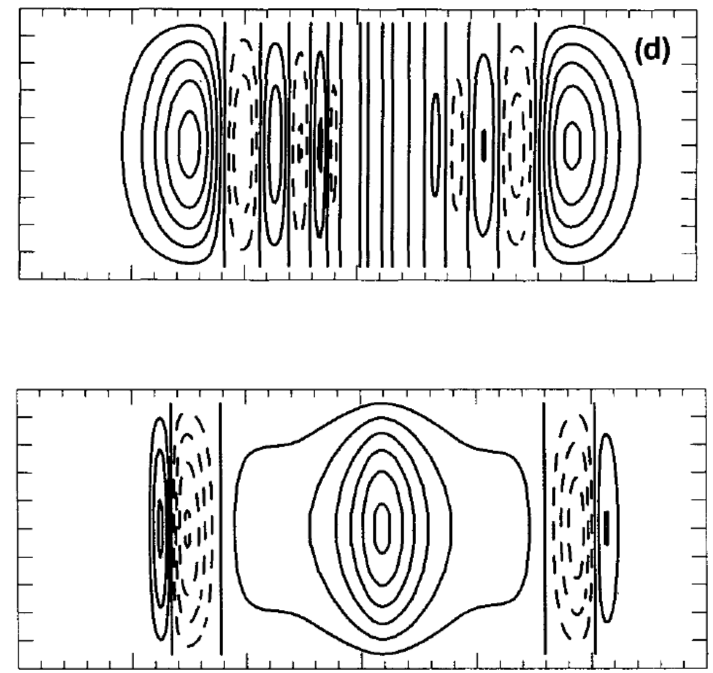
Convergence to analytical solution

3. Comparison with compressible model

EULAG (*Incompressible Boussinesq*)



Klemp and Wilhelmson (JAS, 1978)
(*Compressible*)



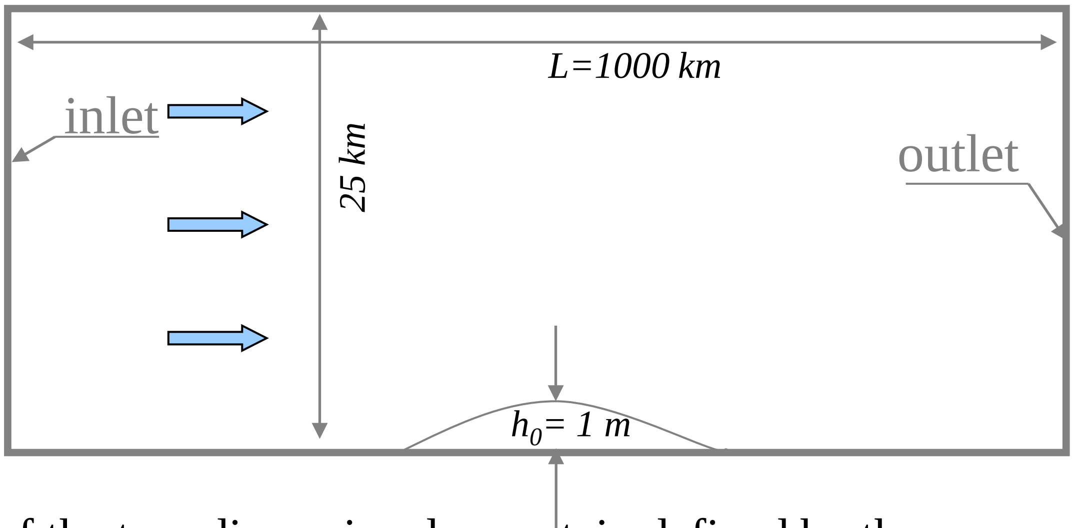
Mountain waves

references:

Bonaventura, L., A semi-implicit semi-Lagrangian scheme using the height coordinate for a nonhydrostatic and fully elastic model of atmospheric flows. J. Comput. Phys. 158(2):186-213, **2000**

J. P. Pinty, R. Benoit, E. Richard, and R. Laprise. Simple tests of a semi-implicit semi-lagrangian model on 2d mountain wave problems. Monthly Weather Review, 123(10):3042–3058, 1995

2D simulation of linear hydrostatic waves over a mountain.

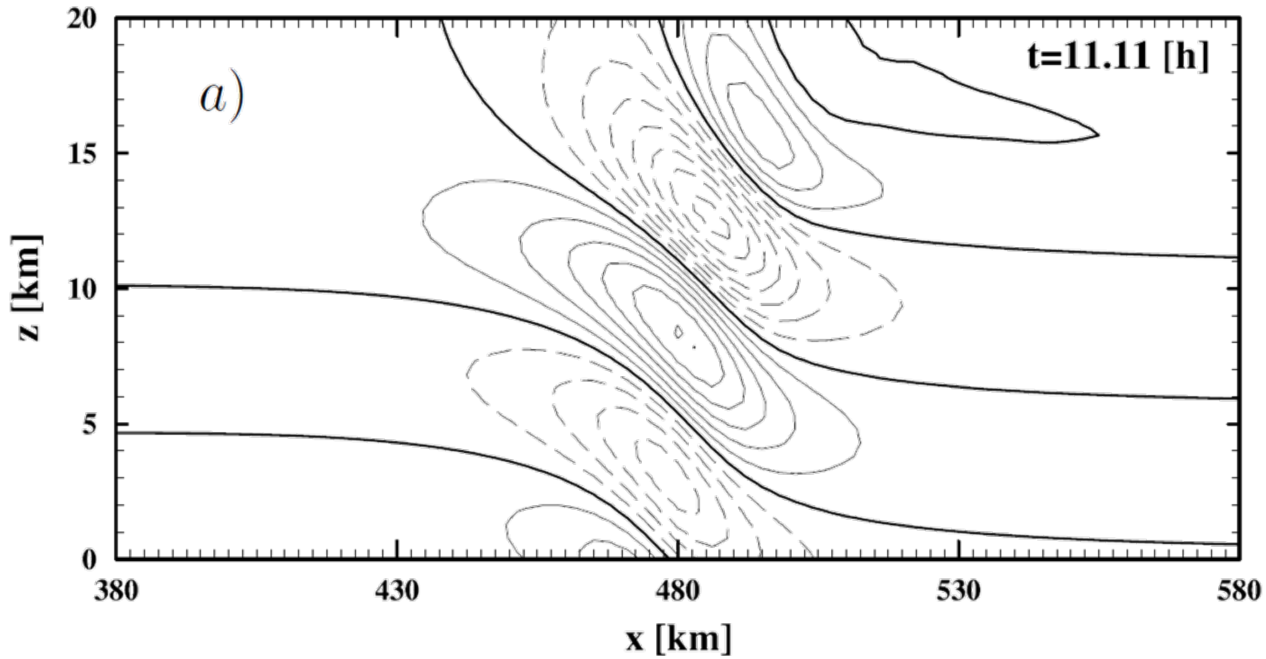


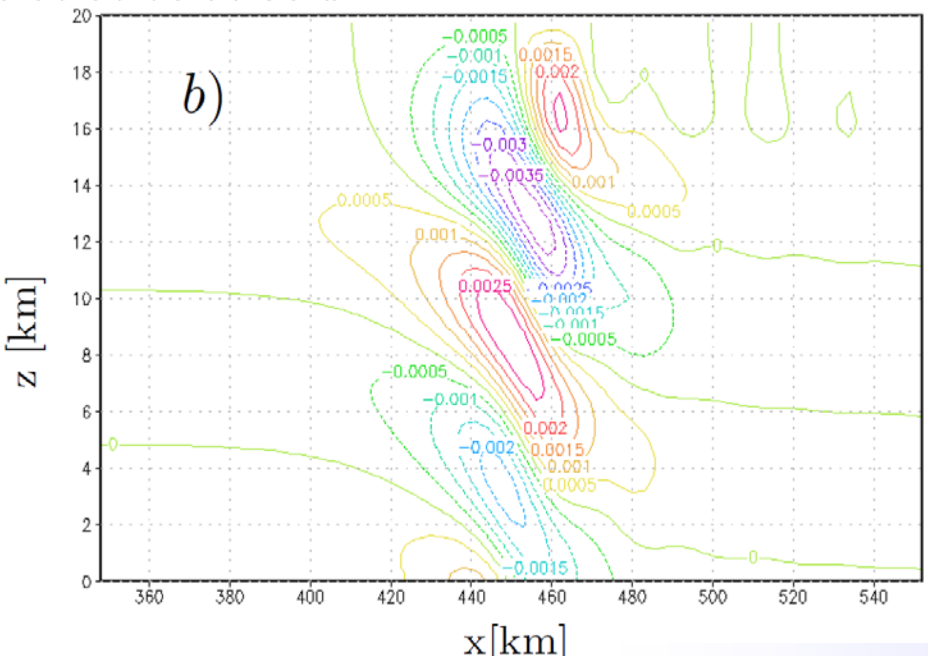
• Profile of the two-dimensional mountain defined by the symmetric Agnesi formula:

$$h(x) = \frac{h_0}{1+\left(\left(x-x_0\right)/a\right)^2}, \quad 0 \le x \le L \qquad a = 16\,km \qquad N = 0.0187 s^{-1} \qquad aN/U << 1$$

- Initial horizontal velocity U = 32 m/s
- Grid resolution Δx = 3km, Δz = 250 m
- Time step size Δt = 40 s
- Terrain following coordinates have been used
- Profiles of vertical and horizontal sponge zones from Pinty et al. (MWR 1995)

4. Linear hydrostatic regime – Comparison COSMO vs. EULAG





Vertical velocity field at $t = Ut/s = 80$

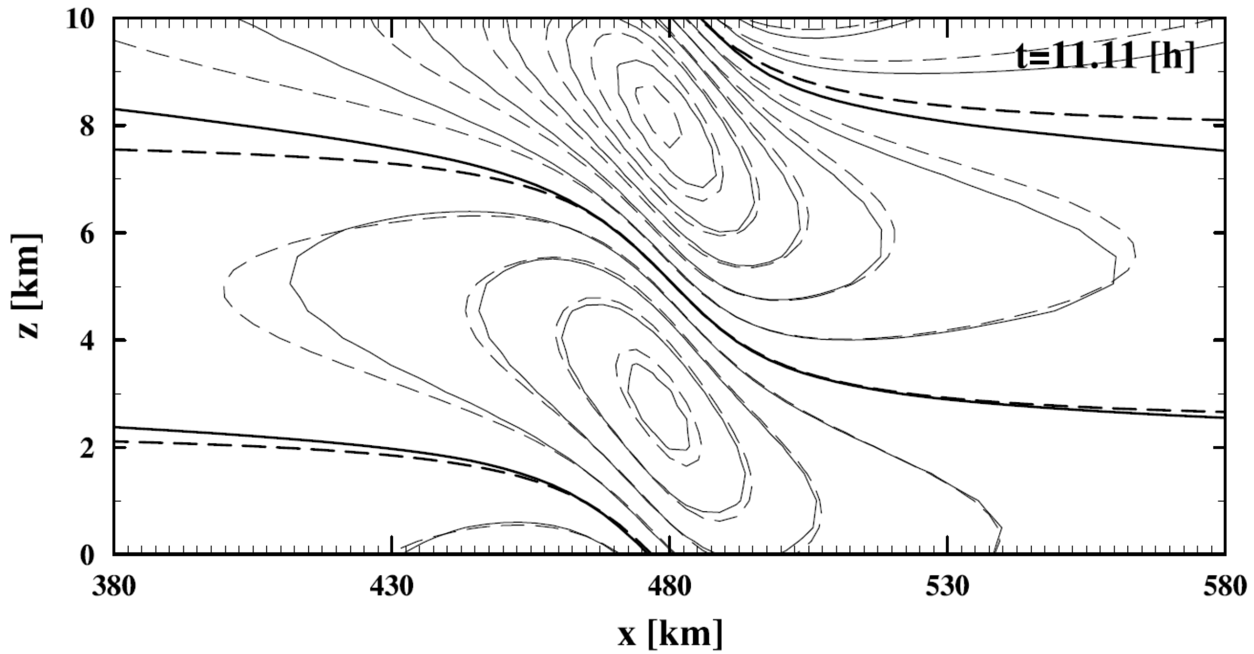
Vertical wind perturbation with contour interval of $0.05 cms^{-1}$ obtained from:
***a*)** anelastic **EULAG** model
***b*)** fully compressible model - current **COSMO** dynamical core.

The dashed line indicate negative values. Attenuation time scale of the absorber applied to top boundary is $\tau = 300s$.

4. Linear hydrostatic regime – comparison with analitycal solution

Horizontal component of velocity

EULAG (solid line) versus analytical solution (dashed line) after 11.11 hours.



Analytical solution developed by Klemp and Lilly (JAS. 1978):

$$u(x,\theta) = N h_0 \gamma e^{C_p \theta / 2R} \frac{\{\gamma x - (1 - C_p/2R)\}\cos\gamma\theta + \{\gamma + (1 - C_p/2R)x\}\sin\gamma\theta}{\{\gamma^2 + (1 - C_p/2R)^2\}x^2}$$

where $\gamma = \dfrac{\mathbf{g}}{N\overline{u}}, \quad \theta = \ln(\vartheta / \vartheta_0)$

ϑ_0 is surface level potential temperature

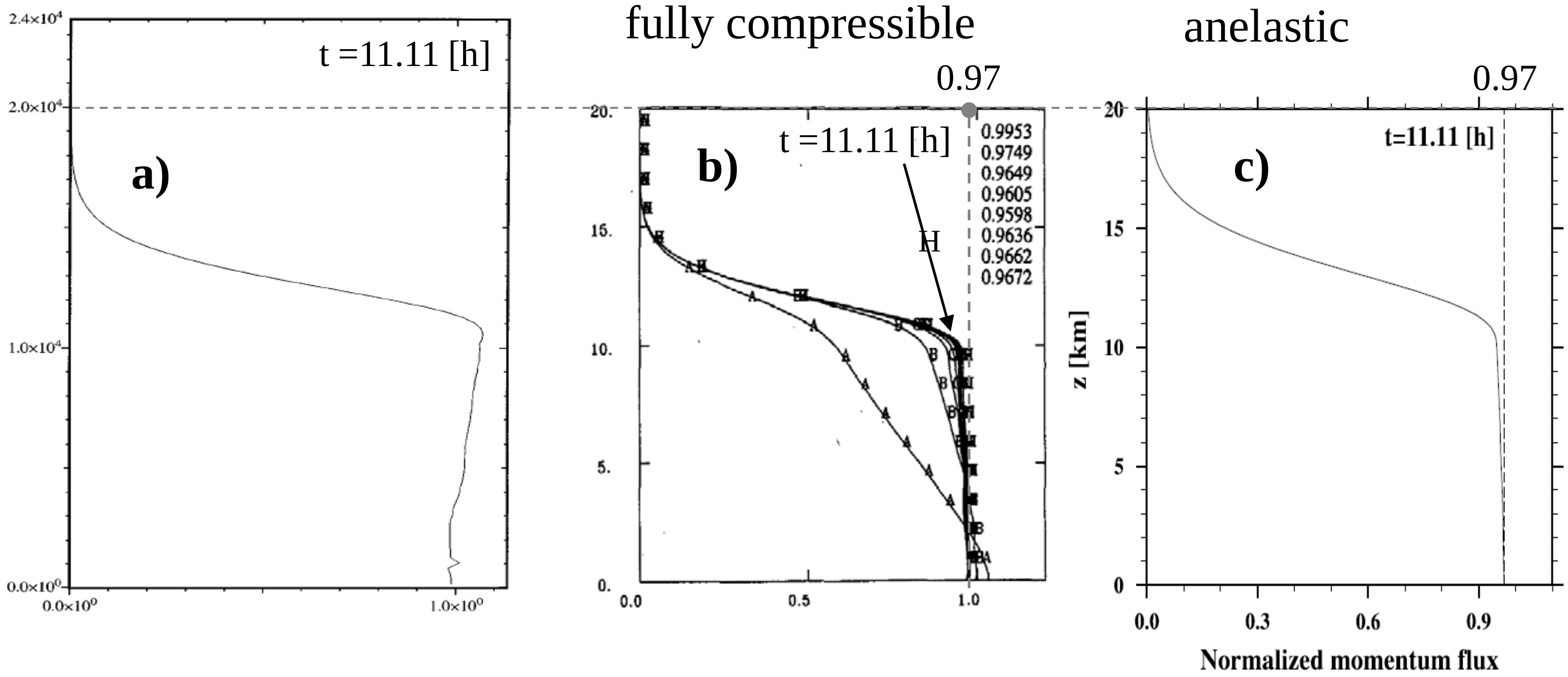
4. Linear hydrostatic regime – normalized vertical momentum flux

Fully compressible models (a) and (b) versus EULAG (c)

Bonaventura (JCP. 2000)

Pinty et al. (MWR. 1995) fully compressible

EULAG (2009) anelastic

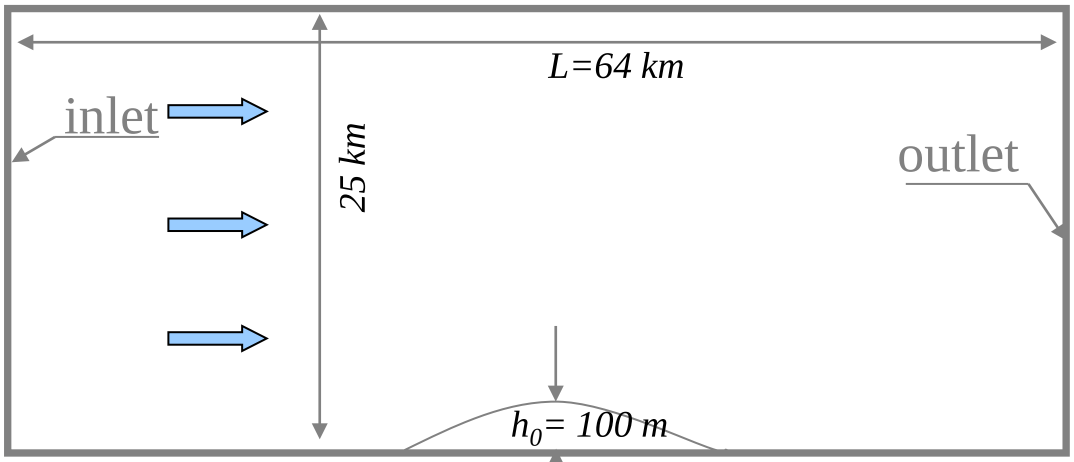


The flux normalized by linear analitic solution (Klemp and Lilly JAS. 1978):

$$M_{analitic} = (\pi/4)\rho_0 h_0^2 \frac{g\gamma}{\gamma^2 + \left(1 - \frac{C_p}{2R}\right)^2} \approx (\pi/4)\rho_0 N\bar{u} h_0^2$$

4. Linear non-hydrostatic regime

2D simulation of linear non-hydrostatic waves over a mountain.



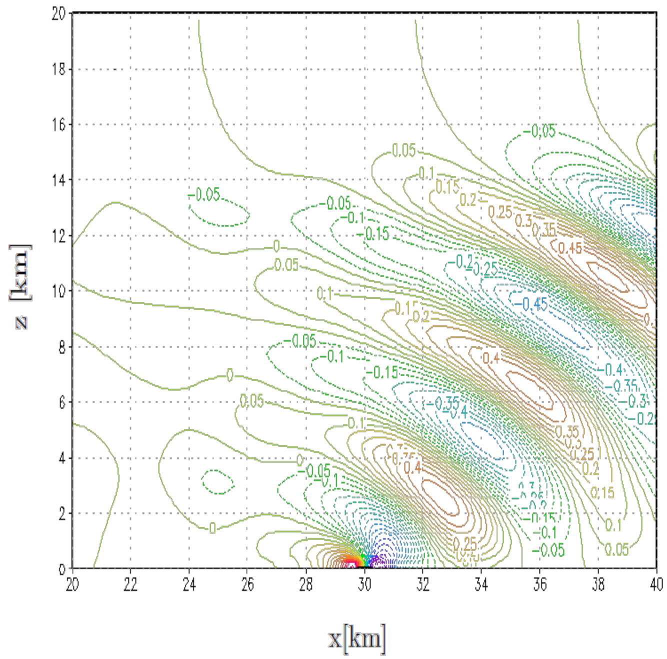
- Profile of the two-dimensional mountain defined by the symmetric Agnesi formula:

$$h(x) = \frac{h_0}{1 + ((x - x_0)/a)^2}, \quad 0 \le x \le L \qquad a = 500m \qquad N = 0.0187 s^{-1} \qquad aN/U \sim 1$$

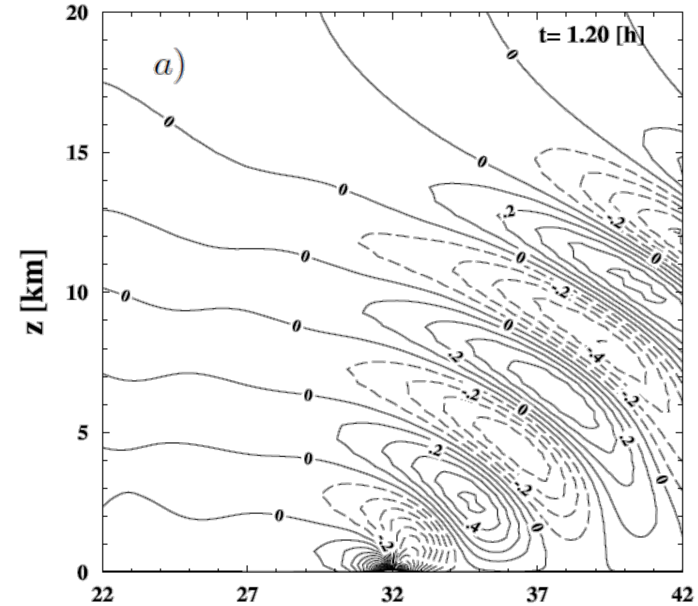
- Initial horizontal velocity U = 14 m/s
- Grid resolution Δx = 100m, Δz = 250 m
- Time step size Δt = 4 s
- Terrain following coordinates have been used
- Profiles of vertical and horizontal sponge zones from Pinty et al. (MWR 1995)

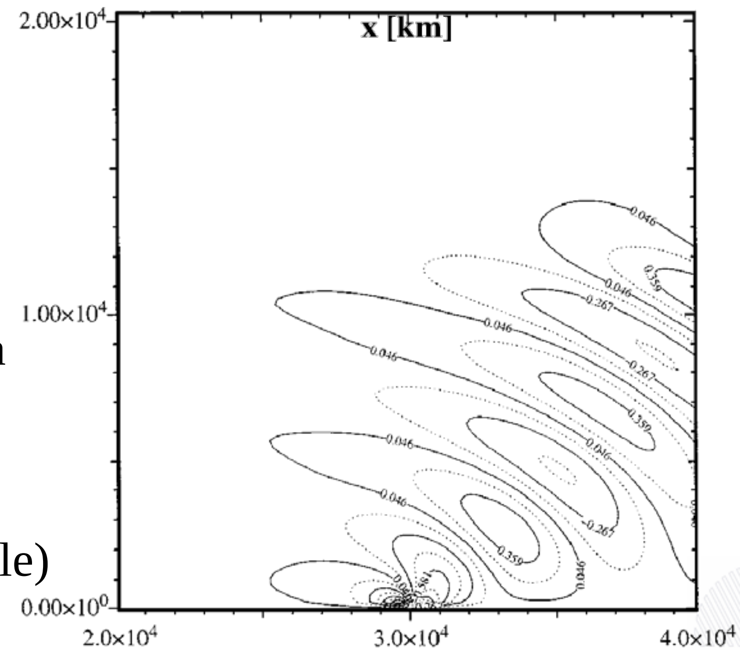
4. Linear non-hydrostatic regime – comparison with other models

COSMO (compressible)



EULAG (anelastic approximation)





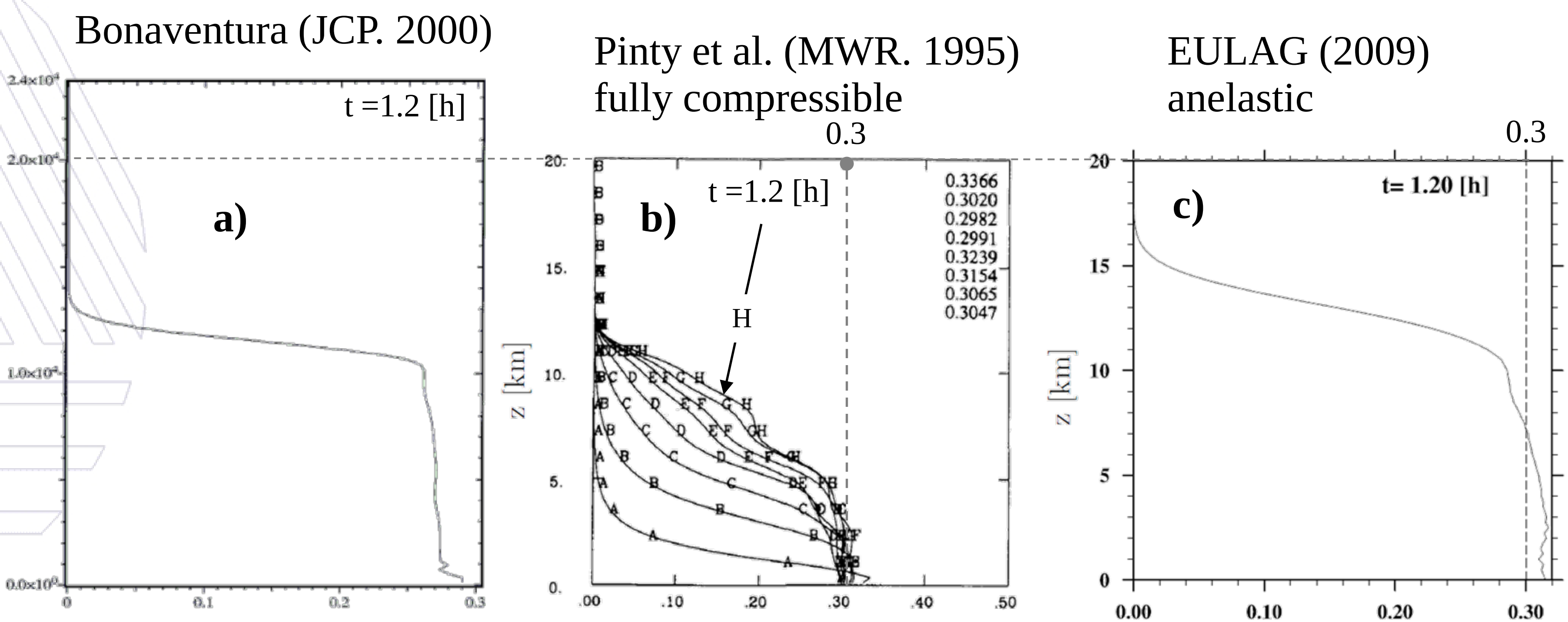
Horizontal component of velocity

EULAG versus COSMO and Bonaventura after 1.2 hours of simulation

Bonaventura (JCP. 2000, compressible)

4. Linear non-hydrostatic regime – normalized vertical momentum flux

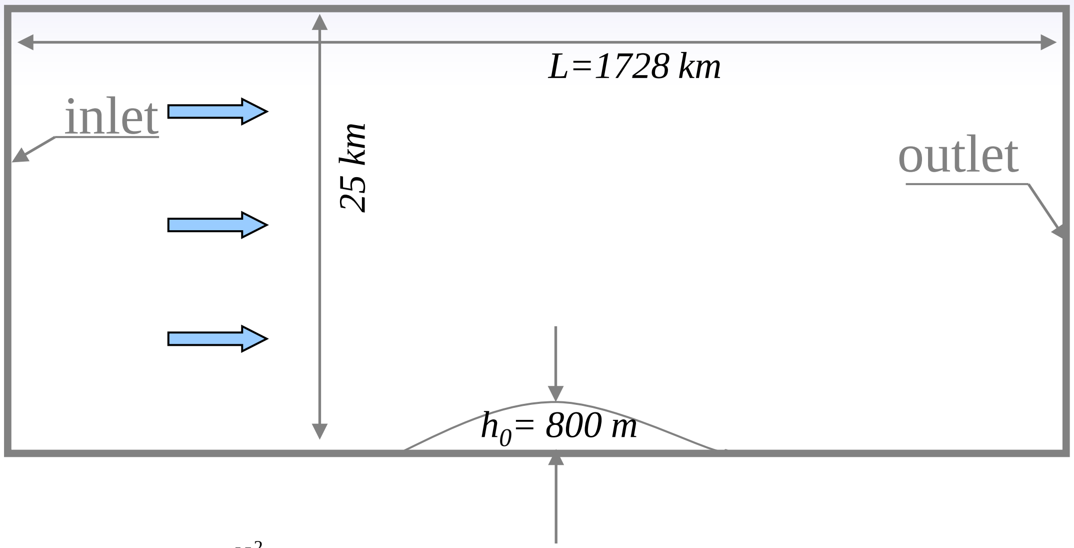
Fully compressible models (a) and (b) versus EULAG (c)



The flux normalized by linear analitic solution from (Klemp and Lilly JAS. 1978)

4. Setup of the 2D simulation of mountain waves in non-linear hydrostatic regime.

- Profile of the two-dimensional mountain defines the symmetrical Agnesi formula.



$$h(x) = \frac{h_0}{1+\left(\left(x-x_0\right)/a\right)^2}, \quad 0 \le x \le L$$

$$a = 16\,km$$

$$N = 0.02 s^{-1}$$

$$\theta(z) = \theta_0 e^{\frac{N^2}{g}z}$$ Clark-Farley formula

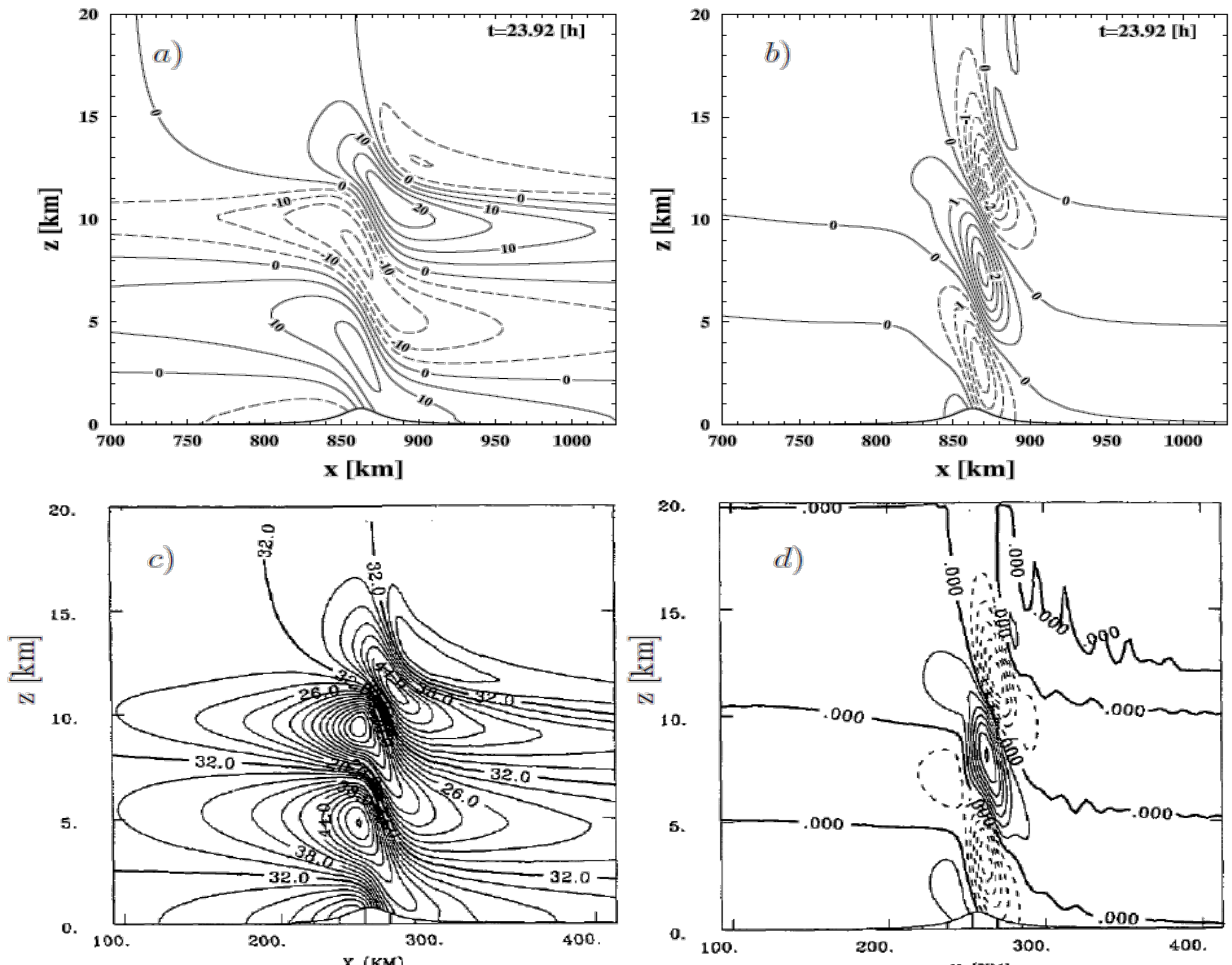
$$\rho(z) = \rho_0 e^{\frac{N^2}{g}z}\left(1-\frac{g^2}{C_p\theta_0 N^2}\right)^{\frac{Cv}{Rg}}$$

where $C_p = 1004\,Jkg^{-1}K^{-1}$, $C_v = 717\,Jkg^{-1}K^{-1}$
and $Rg = 287\,Jkg^{-1}K^{-1}$

- Initial horizontal velocity U = 32 m/s
- Grid resolution Δx = 2.8 km, Δz = 200 m
- Time step size Δt = 30 s
- Terrain following coordinates have been used
- Problem belongs to non-linear hydrostatic regime
- Profiles of vertical and horizontal sponge zones from Pinty et al. (MWR 1995)

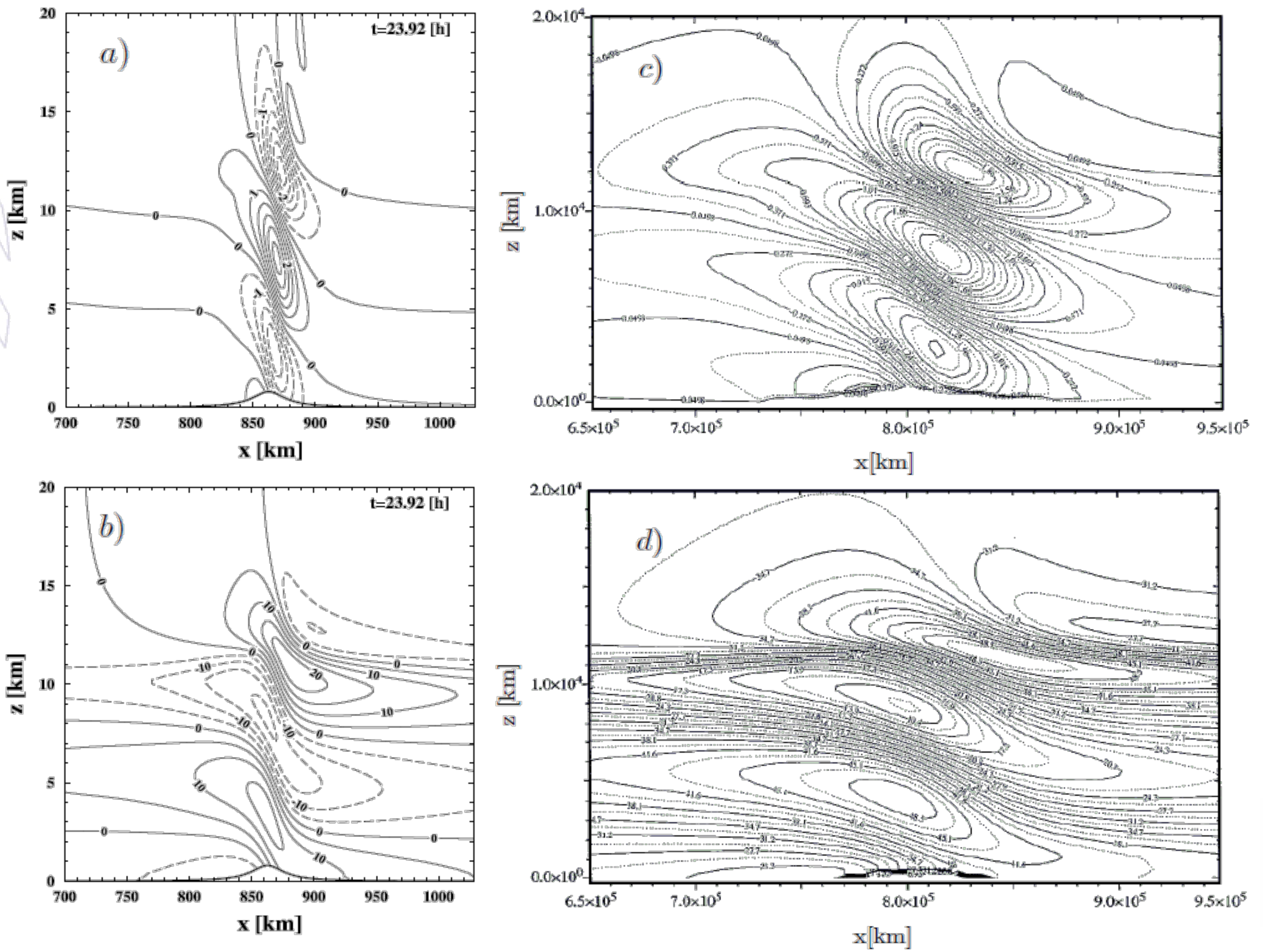
4. Non-linear hydrostatic regime. Comparison of two numerical results from two different approaches i.e. anelastic (EULAG) and compressible (MC2)

Velocity flow field in hydrostatic nonlinear regime *a*) EULAG - horizontal wind perturbation, *b*) EULAG - vertical velocity, *c*) compressible model tested by Pinty et al. (1995) - horizontal wind perturbation and *d*) vertical velocity. The dashed line indicate negative values.



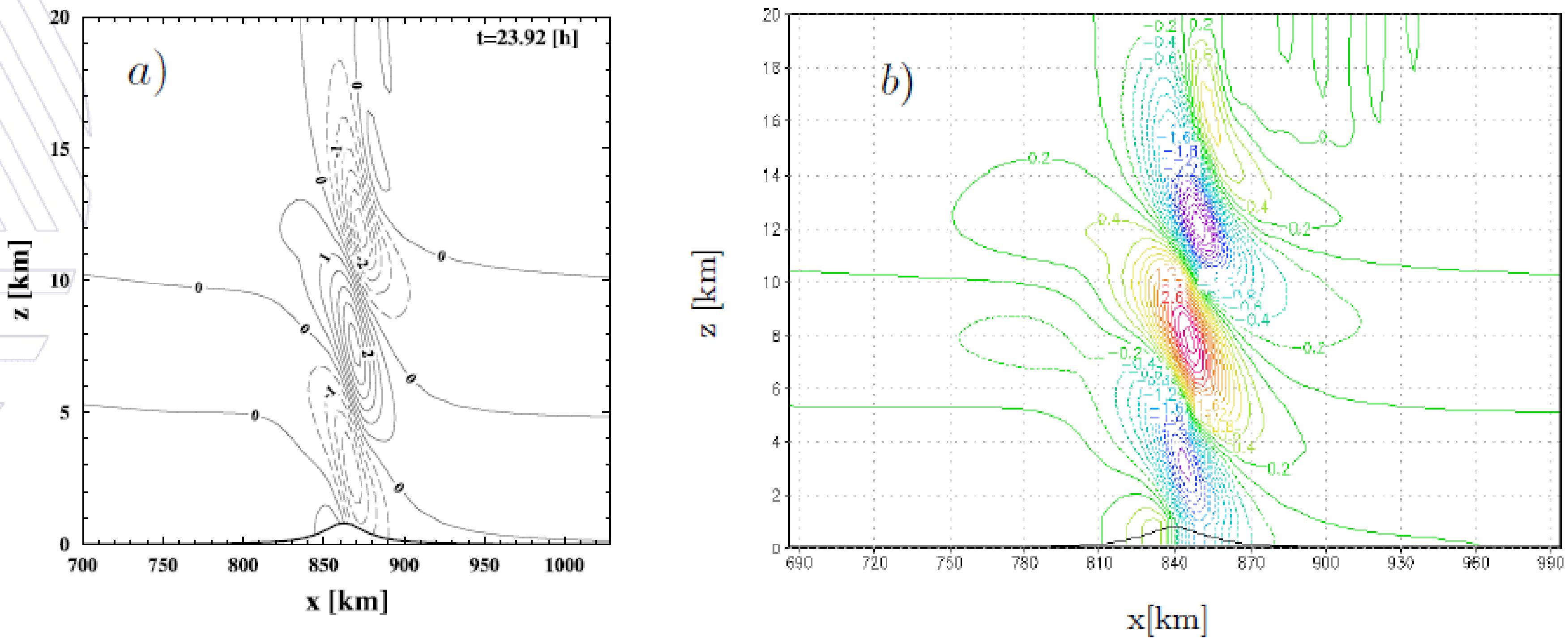
4. Non-linear hydrostatic regime. Comparison of two numerical results from two different approaches i.e. anelastic (EULAG) and compressible (Bonaventura 2000)

Comparison of velocity flow field in hydrostatic non-linear regime *a*) EULAG - vertical wind perturbation *b*) EULAG – horizontal velocity, *c*) Bonaventura's compressible model – vertical wind perturbation and *d*) horizontal velocity. The dashed line indicate negative values.



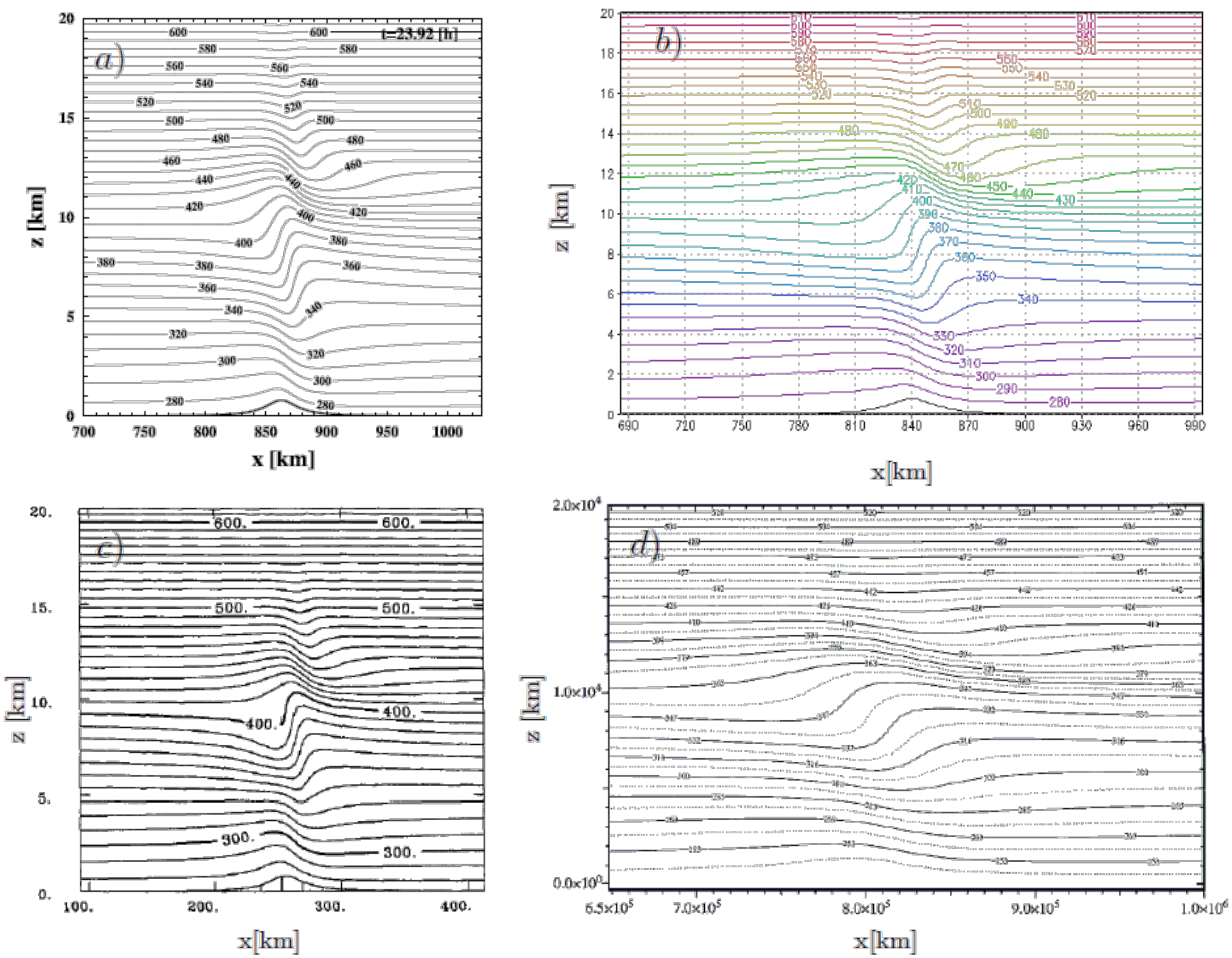
4. Non-linear hydrostatic regime. Comparison of two numerical results from two different approaches i.e. anelastic (EULAG) and compressible (COSMO)

Comparison of velocity flow field in hydrostatic non-linear regime
a) EULAG vertical wind perturbation
b) COSMO compressible model

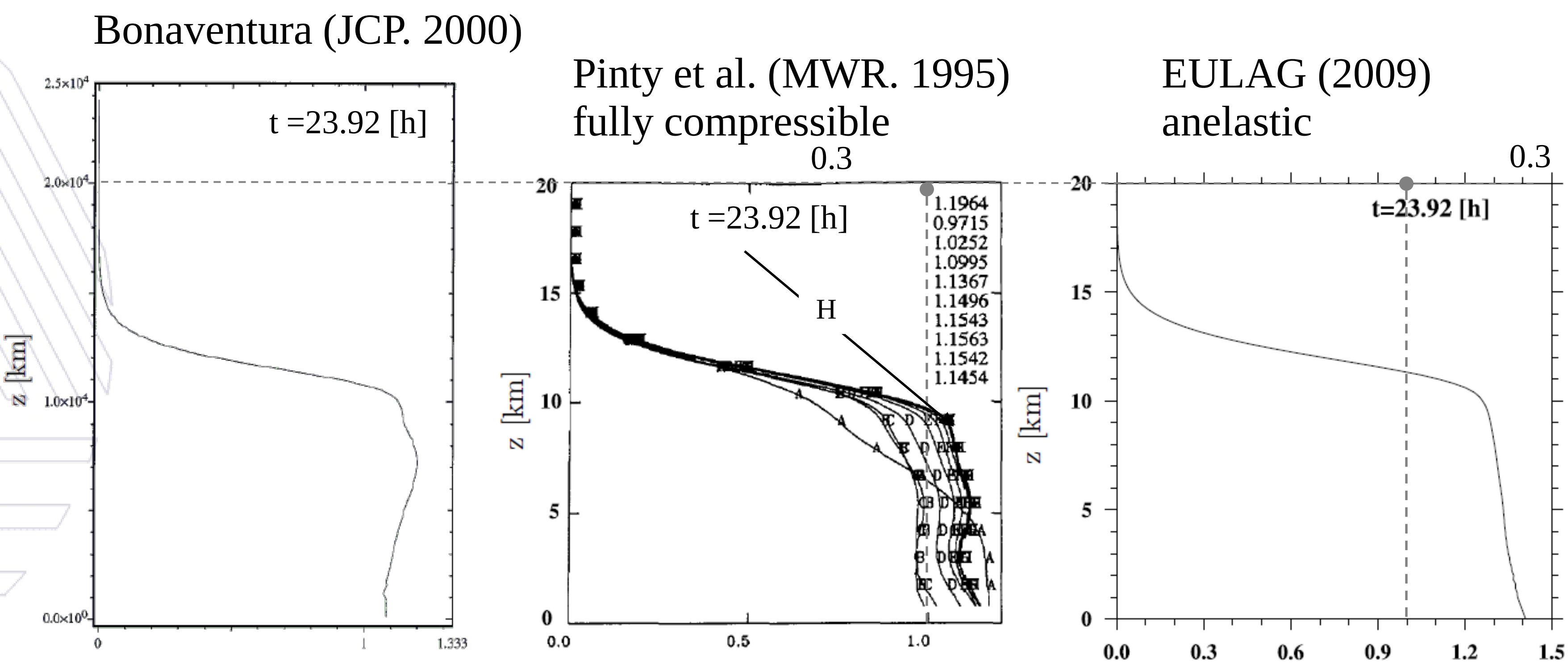


4. Nonlinear hydrostatic regime. Comparison of four numerical results from different approaches (anelastic and compressible) and four different codes (COSMO, EULAG, MC2, Bonaventura's model)

Potential temperature field in the hydrostatic nonlinear regime from all four different models. *a*) EULAG - anelastic non-hydrostatic semi-implicit Eulerian model, *b*) COSMO - fully compressible model, *c*) The fully compressible 2D non-hydrostatic semi-implicit semi-Lagrangian MC2 model. Results from Pinty et al. (1995), *d*) compressible Bonaventura's model.



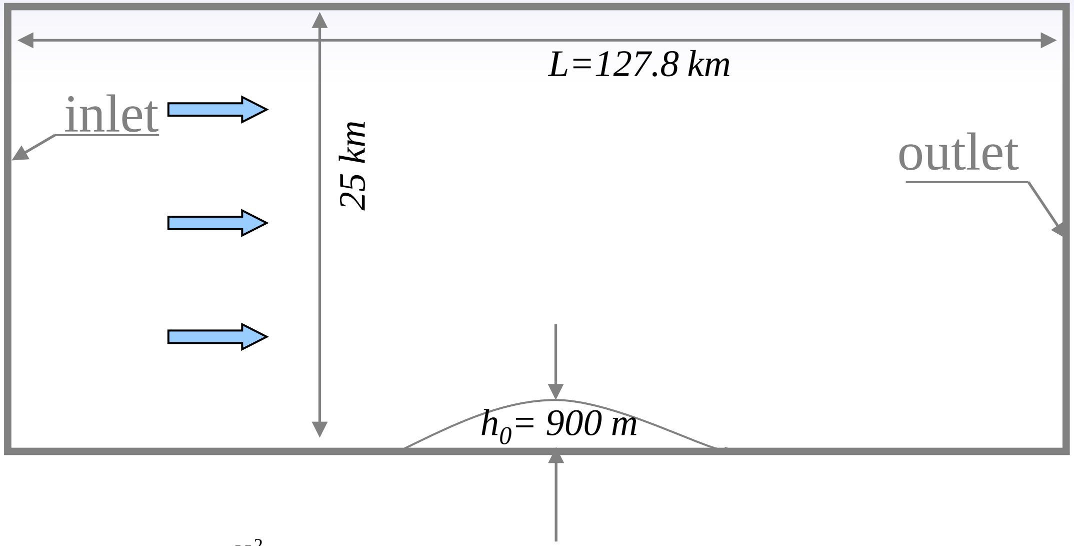
Normalized vertical flux for the hydrostatic nonlinear case from two different models. *a*) The fully compressible 2D nonhydrostatic semi-implicit semi-Lagrangian MC2 model. Results from Pinty et al. (1995), *b*) EULAG - anelastic non-hydrostatic semi-implicit Eulerian model, c) compressible Bonaventura's model.



The flux normalized by linear analitic solution from (Klemp and Lilly JAS. 1978)

4. Setup of the 2D simulation of mountain waves in non-linear non-hydrostatic regime.

- Profile of the two-dimensional mountain defines the symmetrical Agnesi formula.



$$h(x) = \frac{h_0}{1 + \left((x - x_0)/a\right)^2}, \quad 0 \le x \le L$$

$$a = 1km$$

$$N = 0.02 s^{-1}$$

$$\theta(z) = \theta_0 e^{\frac{N^2}{g} z}$$ Clark-Farley formula

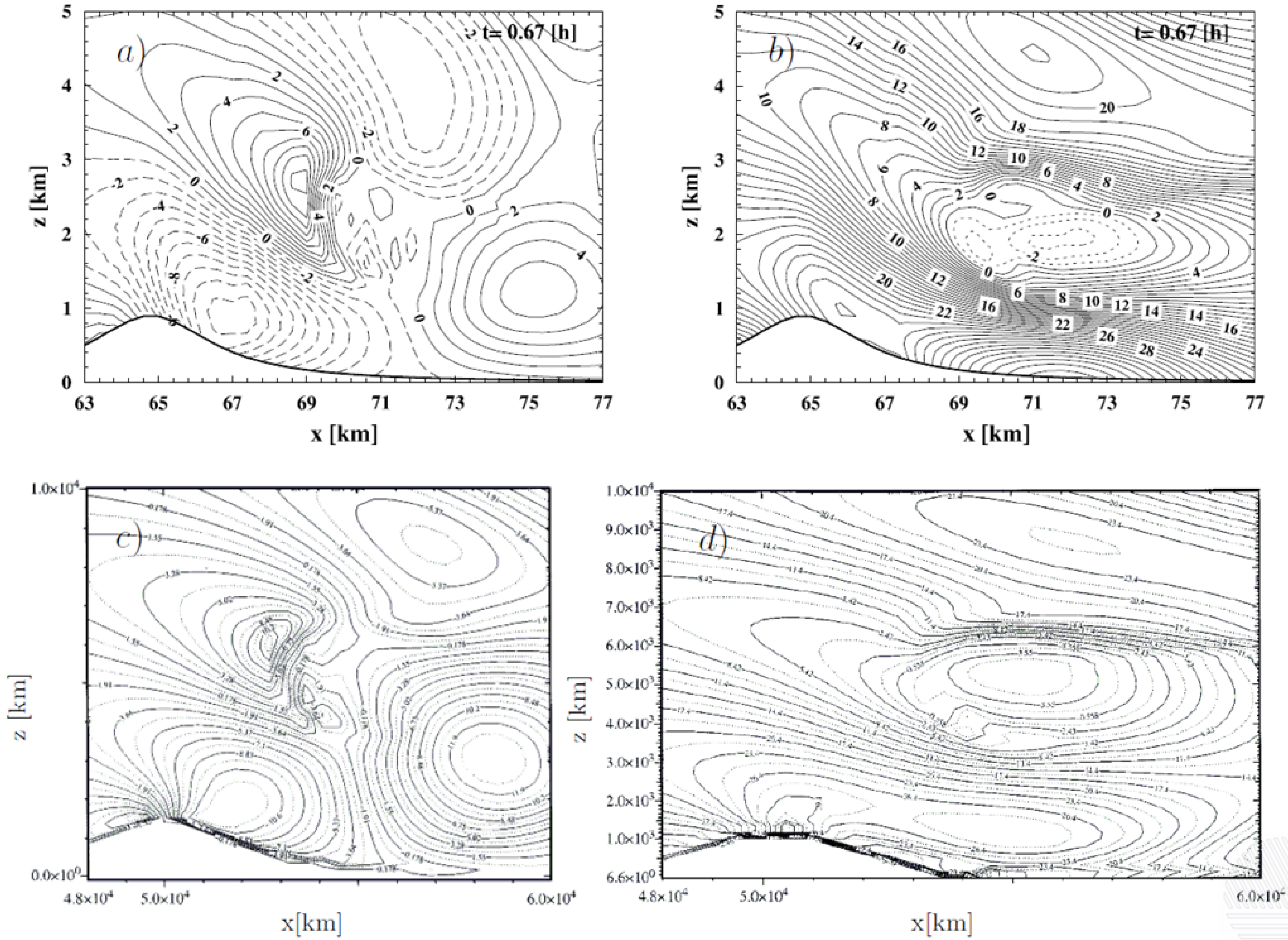
$$\rho(z) = \rho_0 e^{\frac{N^2}{g} z} \left(1 - \frac{g^2}{C_p \theta_0 N^2}\right)^{\frac{Cv}{Rg}}$$

where $C_p = 1004 Jkg^{-1}K^{-1}$, $C_v = 717 Jkg^{-1}K^{-1}$
and $Rg = 287 Jkg^{-1}K^{-1}$

- Initial horizontal velocity U = 13.28 m/s
- Grid resolution Δx = 200 m, Δz = 100 m
- Time step size Δt = 4 s
- Terrain following coordinates have been used
- Problem belongs to nonlinear nonhydrostatic regime

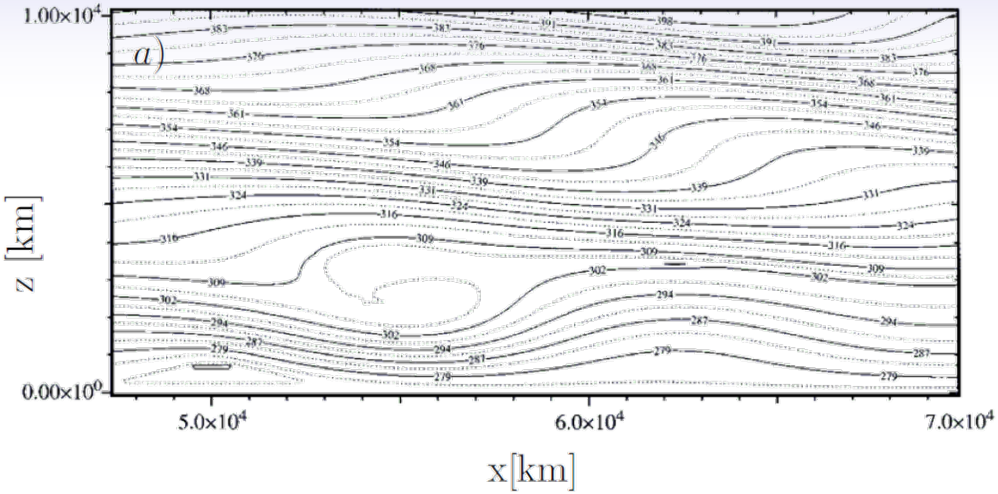
4. Non-linear non-hydrostatic regime.

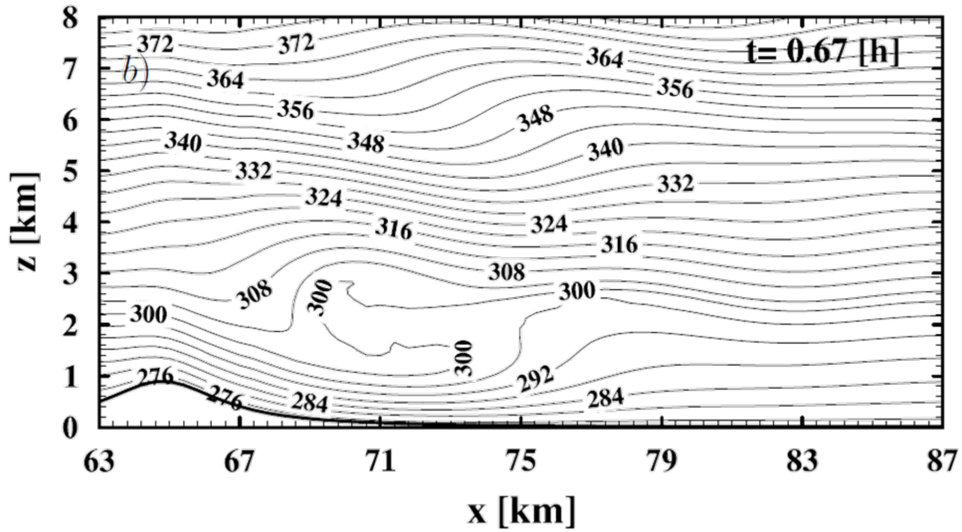
Comparison of velocity flow field in non-hydrostatic non-linear regime
a) EULAG - horizontal wind perturbation with contour interval $1ms^{-1}$,
b) EULAG - vertical velocity with contour interval of $1cms^{-1}$. Bonaventura's compressible model *c*) horizontal wind perturbation and *d)* vertical velocity.



4. Non-linear non-hydrostatic regime.

Potential temperature in the non-hydrostatic non-linear regime at $t = 0.67h$ computed using *a*) the fully compressible Bonaventura's model, *b*) EULAG anelastic model.





CONCLUSIONS

Results computed using Eulag code converge to analytical solutions when grid resolutions increase.

In considered problems we showed that anelastic approximation gives both qualitative and quantitative agreement with fully compressible models.

EULAG gives correct results even if computational grids have significant anisotropy.

Near future:

implementation EULAG dynamical core to COSMO model

Thank you …

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