An unstructured mesh model for rotating stratified fluids

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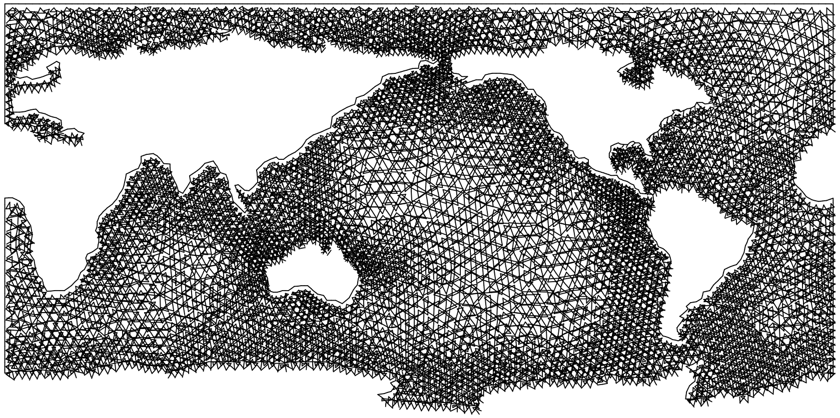

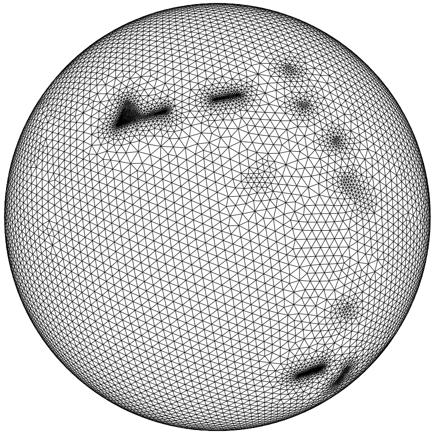

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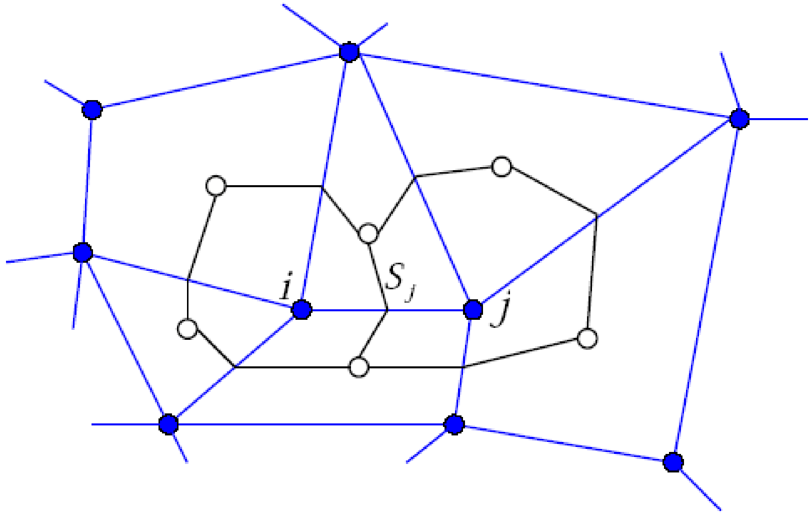
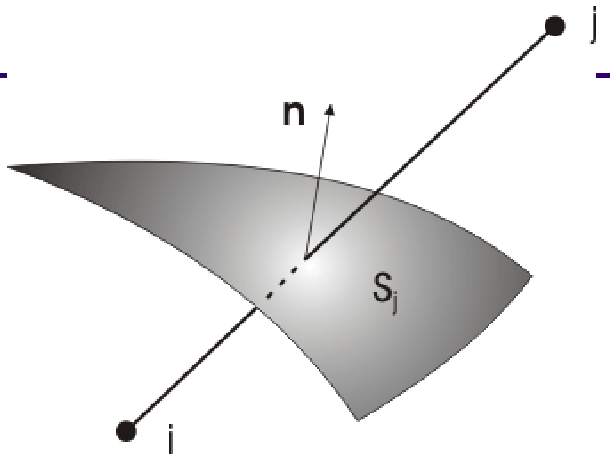
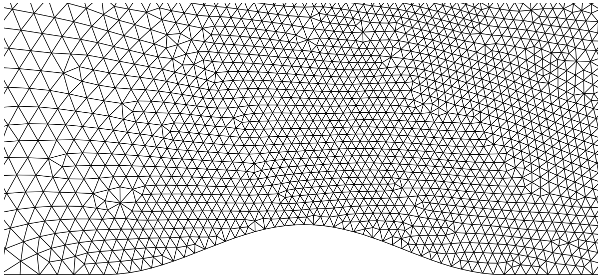
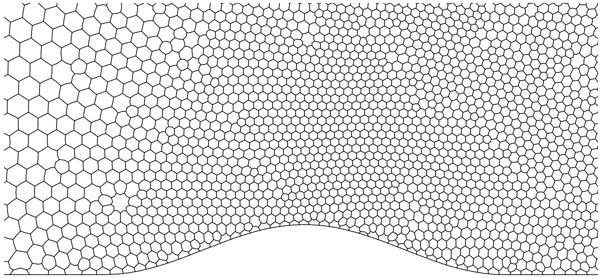
The edge-based discretisation





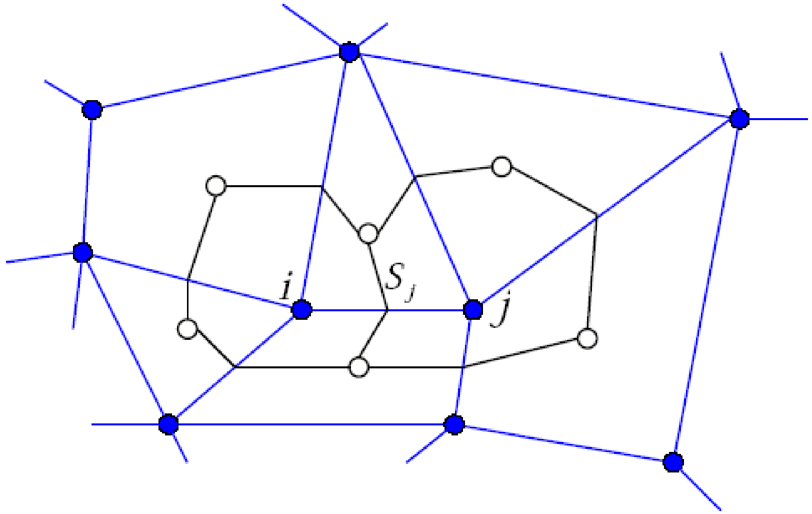




Edges

Median dual mesh --- Finite volumes

The edge-based discretisation



$$\Psi_i^{n+1} = \Psi_i^n - \frac{\delta t}{\mathcal{V}_i} \sum_{j=1}^{l(i)} F_j^{\perp} S_j$$

$$F_j^{\perp} = [v_j^{\perp}]^+ \Psi_i^n + [v_j^{\perp}]^- \Psi_j^n$$

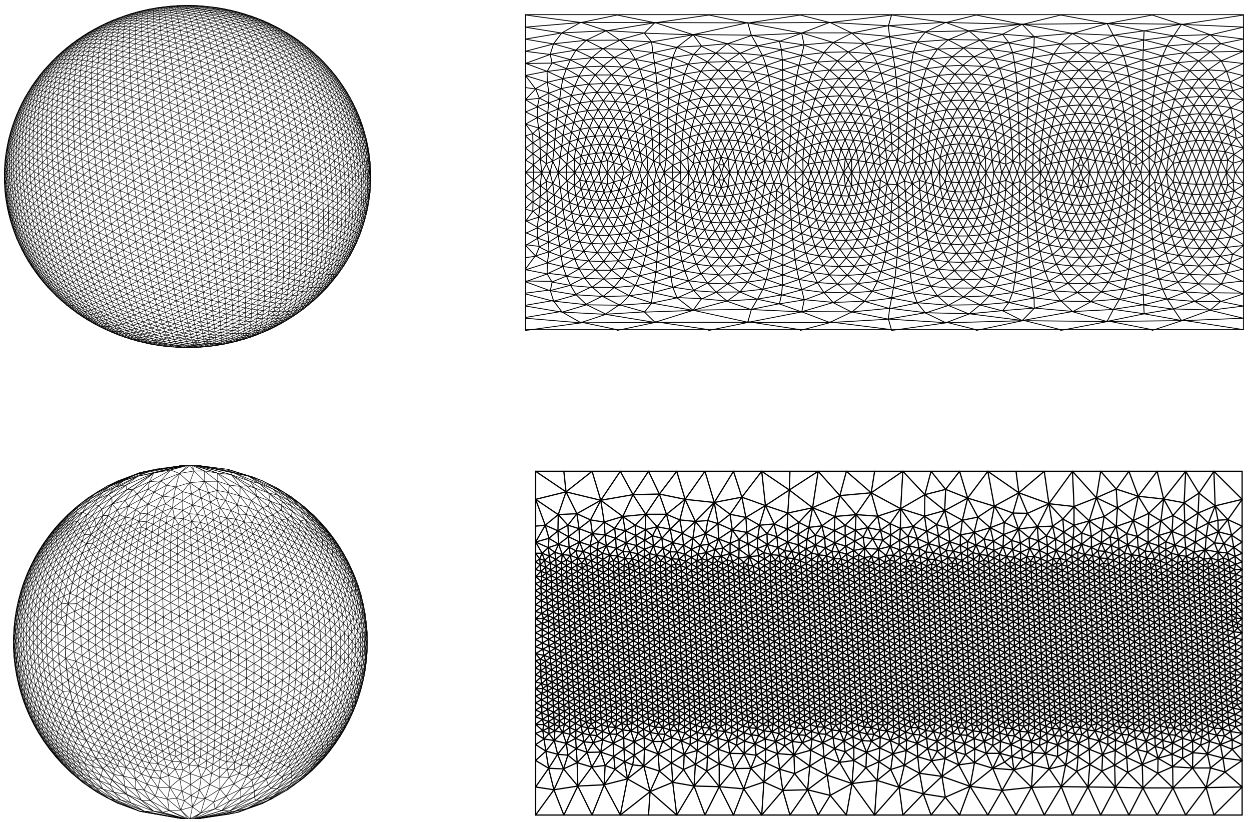
$$[v]^+ := 0.5(v + |v|) \quad , \quad [v]^- := 0.5(v - |v|)$$

$$\left(\frac{\partial \Phi}{\partial x^I}\right)_j = \frac{1}{\overline{\mathcal{V}}_j} \left(\sum_{m=1}^{l(i)} \overline{\Phi}^{i,m} S_m^I + \sum_{m'=1}^{l(j)} \overline{\Phi}^{j,m'} S_{m'}^I \right), \quad \overline{\mathcal{V}}_j \equiv \mathcal{V}_i + \mathcal{V}_j,$$

(MPDATA Smolarkiewicz & Szmelter , J. Comput. Phys. 2005)

$$\frac{\partial G\mathbf{\Phi}}{\partial t} + \nabla \cdot (\mathbf{V}\mathbf{\Phi}) = G\mathbf{R}$$

Geospherical framework



(Szmelter & Smolarkiewicz, J. Comput. Phys. 2010)

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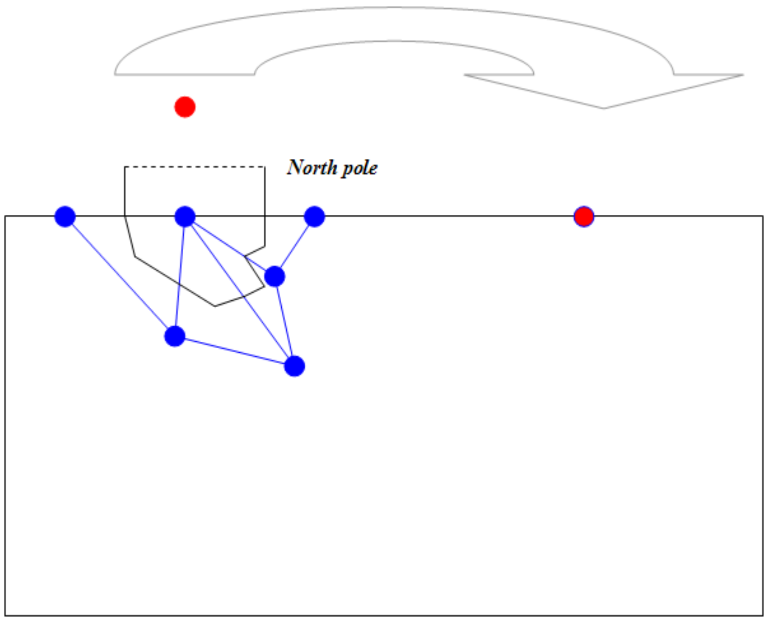
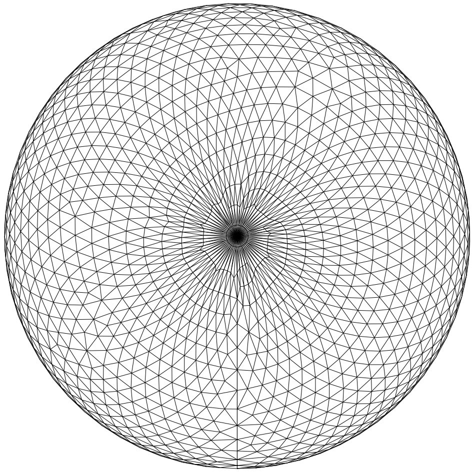
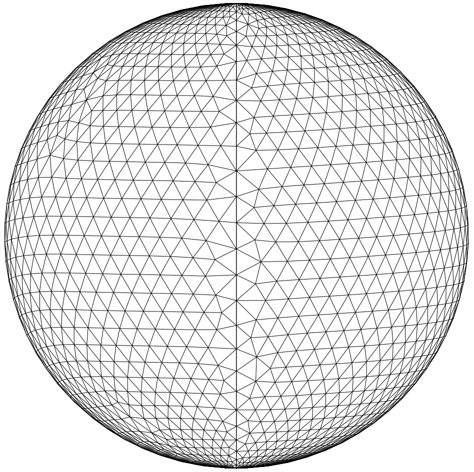
Pole treatment





Periodic boundaries



A general NFT MPDATA unstructured mesh framework

$$\frac{\partial G\mathbf{\Phi}}{\partial t} + \nabla \cdot (\mathrm{V}\mathbf{\Phi}) = G\mathrm{R}$$

$$\forall_{i,n} \quad \mathbf{\Phi}_i^{n+1} = \mathcal{A}_i(\mathbf{\Phi}^n + 0.5\delta t\mathrm{R}^n,\ \mathrm{V}^{n+1/2}, G) + 0.5\delta t\mathrm{R}_i^{n+1}$$

(Smolarkiewicz 91, Smolarkiewicz & Margolin 93; Mon. Weather Rev.)

$$R^{n+1} = R(t + \delta t) + \mathcal{O}(\delta t^3)$$

Linear implicit integration

preconditioned non-symmetric

Krylov-subspace elliptic solver

Nonlinear implicit integration

$$\forall_i \quad \mathbf{\Phi}_i^{n+1,\ \mu} = \mathbf{\Phi}_i^* + 0.5\delta t\mathrm{R}_i^{n+1,\ \mu-1} \quad \mu = 1, .., m$$

(Smolarkiewicz & Szmelter, J. Comput. Phys. 2009)

A shallow water model

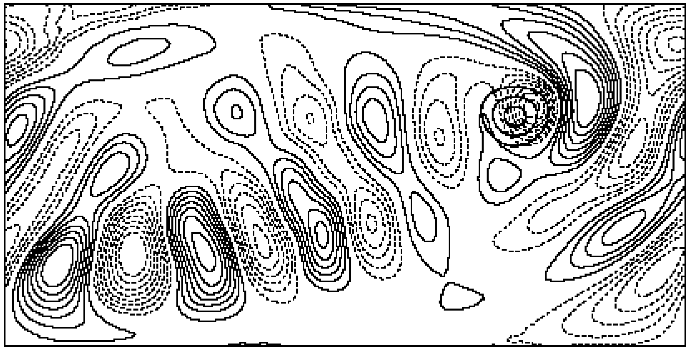
$$\frac{\partial G\Phi}{\partial t} + \nabla \cdot (\mathbf{V}\Phi) = GR \qquad \Phi_i^{n+1} = \mathcal{A}_i(\Phi^n + 0.5\delta t R^n,\ \mathbf{V}^{n+1/2}, G) + 0.5\delta t R_i^{n+1}$$

$$\frac{\partial G\mathcal{D}}{\partial t} + \nabla \cdot (G\mathbf{v}^*\mathcal{D}) = 0 \ ,$$

$$\frac{\partial GQ_x}{\partial t} + \nabla \cdot (G\mathbf{v}^*Q_x) = G\left(-\frac{g}{h_x}\mathcal{D}\frac{\partial H}{\partial x} + fQ_y - \frac{1}{G\mathcal{D}}\frac{\partial h_x}{\partial y}Q_xQ_y\right) \ ,$$

$$\frac{\partial GQ_y}{\partial t} + \nabla \cdot (G\mathbf{v}^*Q_y) = G\left(-\frac{g}{h_y}\mathcal{D}\frac{\partial H}{\partial y} - fQ_x + \frac{1}{G\mathcal{D}}\frac{\partial h_x}{\partial y}Q_x^2\right) \ ,$$

Zonal orographic flow (*Grose&Hoskins; Williamson)*



meridional velocity component

$\delta\xi_k/\delta\xi_{k+1}$	$2^{1/2}/2^0$	$2^0/2^{-1/2}$	$2^{-1/2}/2^{-2/2}$	$2^{-2/2}/2^{-3/2}$
$L_1(H'_k)/L_1(H'_{k+1})$	1.75	1.87	1.88	2.00
$L_1(v'_k)/L_1(v'_{k+1})$	1.76	2.04	1.89	2.03
$L_2(H'_k)/L_2(H'_{k+1})$	1.93	1.92	1.90	1.98
$L_2(v'_k)/L_2(v'_{k+1})$	1.77	2.01	1.93	2.00

The second-order asymptotic convergence rate

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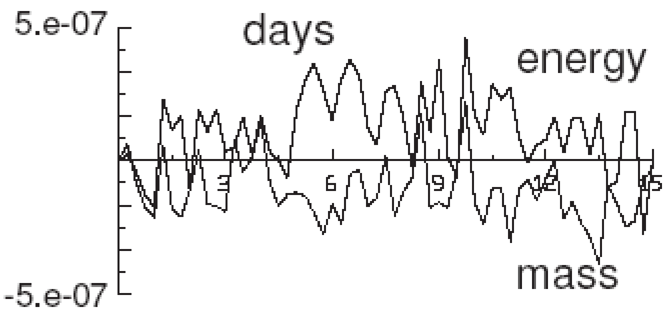
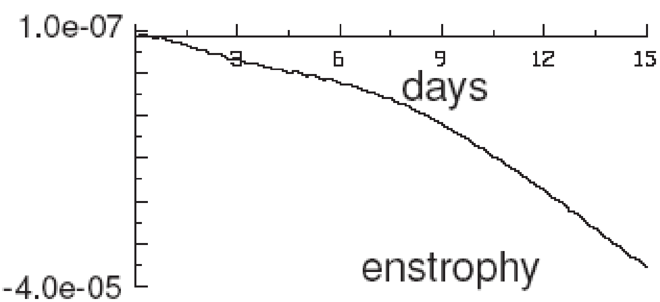
Zonal orographic flow; mass, energy and potential enstrophy conservation errors after 15 simulated days.

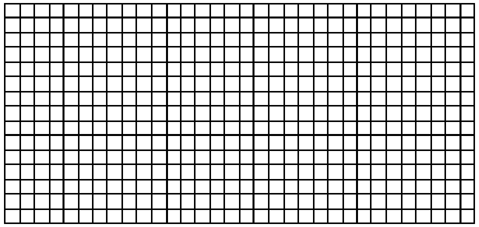
	mass	energy	potential enstrophy
single precision	$-2.8 \cdot 10^{-8}$	$-6.1 \cdot 10^{-9}$	$-3.7 \cdot 10^{-5}$
double precision	$+3.9 \cdot 10^{-15}$	$+2.3 \cdot 10^{-8}$	$-3.7 \cdot 10^{-5}$



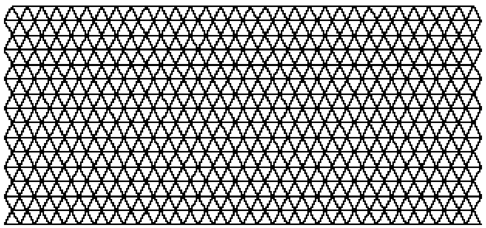


single precision

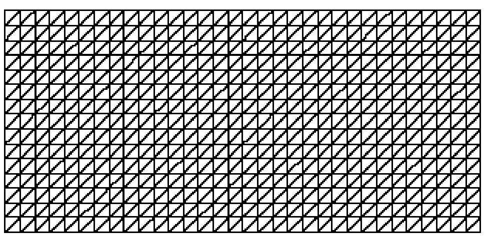
Rossby-Haurwitz Wave



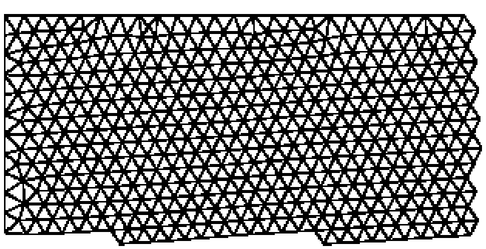
Cartesian



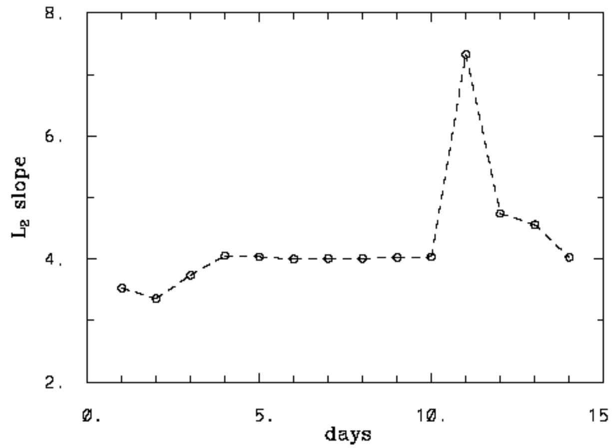
equilateral triangular

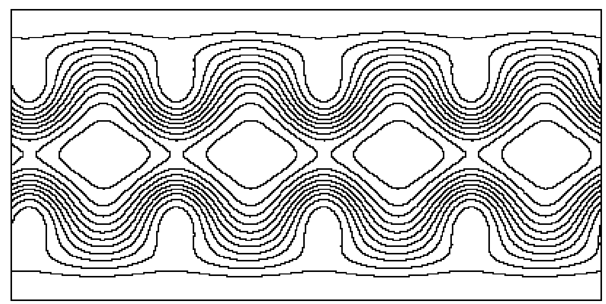


triangularized Cartesian

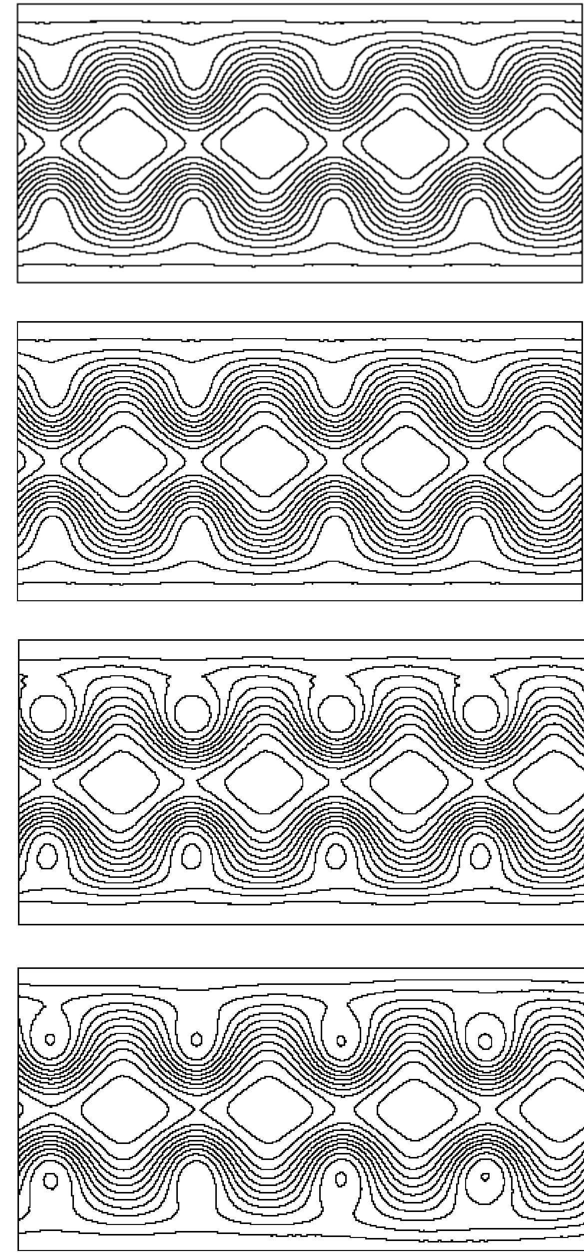


irregular triangular





5 days



14 days

A global hydrostatic model

isentropic, isosteric/ isopycnic

$$\frac{\partial G\Phi}{\partial t} + \nabla \cdot (\mathbf{V}\Phi) = GR$$

$$\Phi_i^{n+1} = \mathcal{A}_i(\Phi^n + 0.5\delta t R^n,\ \mathbf{V}^{n+1/2}, G) + 0.5\delta t R_i^{n+1}$$

$$\frac{\partial G\mathcal{D}}{\partial t} + \nabla \cdot (G\mathbf{v}^*\mathcal{D}) = 0\ ,$$

$$\frac{\partial GQ_x}{\partial t} + \nabla \cdot (G\mathbf{v}^*Q_x) = G\left(-\frac{1}{h_x}\mathcal{D}\frac{\partial M}{\partial x} + fQ_y - \frac{1}{G\mathcal{D}}\frac{\partial h_x}{\partial y}Q_xQ_y\right)\ ,$$

$$\frac{\partial GQ_y}{\partial t} + \nabla \cdot (G\mathbf{v}^*Q_y) = G\left(-\frac{1}{h_y}\mathcal{D}\frac{\partial M}{\partial y} - fQ_x + \frac{1}{G\mathcal{D}}\frac{\partial h_x}{\partial y}Q_x^2\right)\ ,$$

$$\frac{\partial M}{\partial \zeta} = \Pi\ .$$

$$\mathcal{D}^{n+1} = \partial p^{n+1}/\partial\zeta \quad \downarrow$$

$$\partial M^{n+1}/\partial\zeta = \Pi^{n+1} \quad \uparrow$$

Rotating stratified fluid
An Eulerian-Lagrangian form

Isentropic model

$$\zeta = \theta$$

$$\Pi = c_p(p/p_o)^{R_d/c_p}$$

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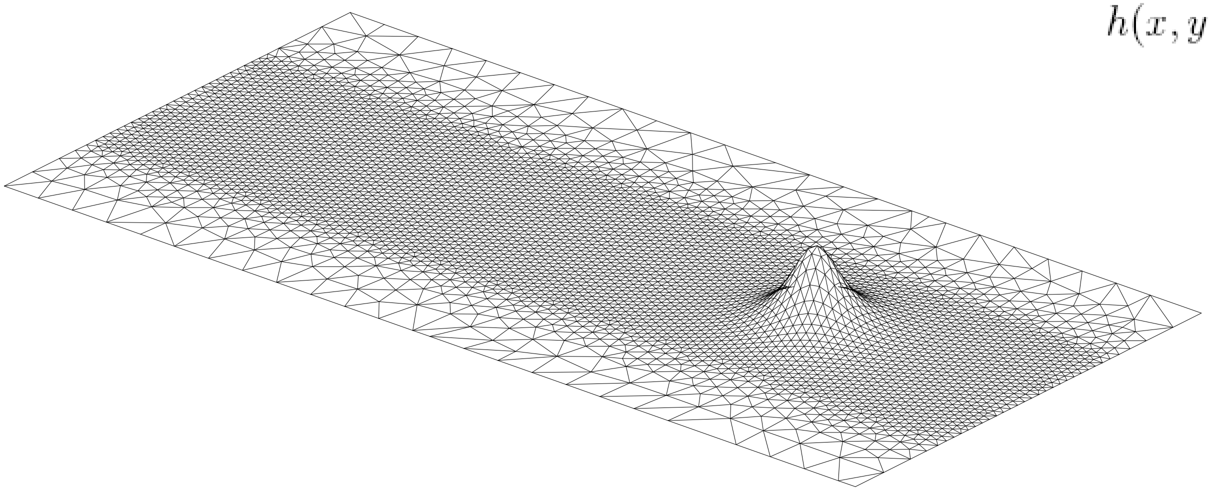
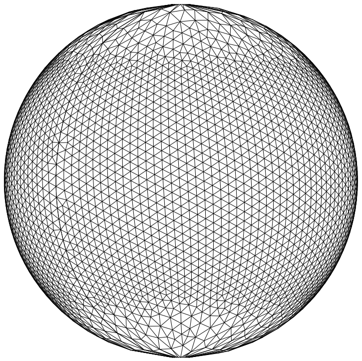
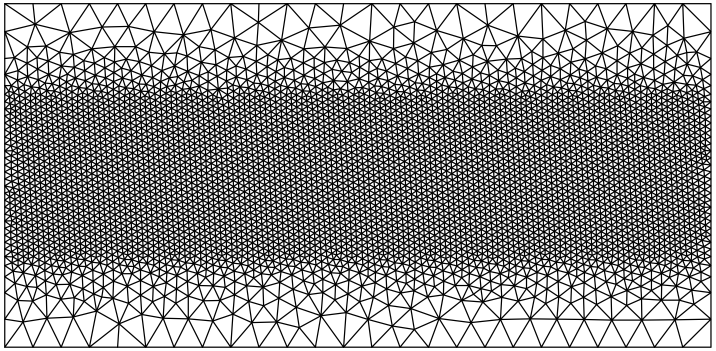
A stratified 3D mesoscale flow past an isolated hill



$$h(x, y) = h_0[1. + (l/\mathcal{L})^2]^{-3/2},$$

(4532 points)





Reduced planets (Wedi & Smolarkiewicz, QJR 2009)

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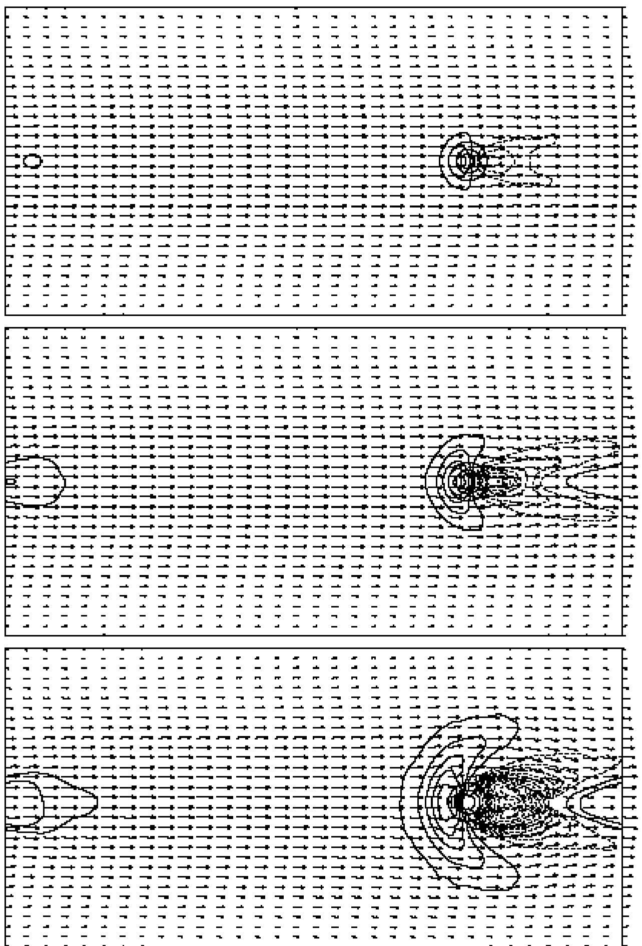
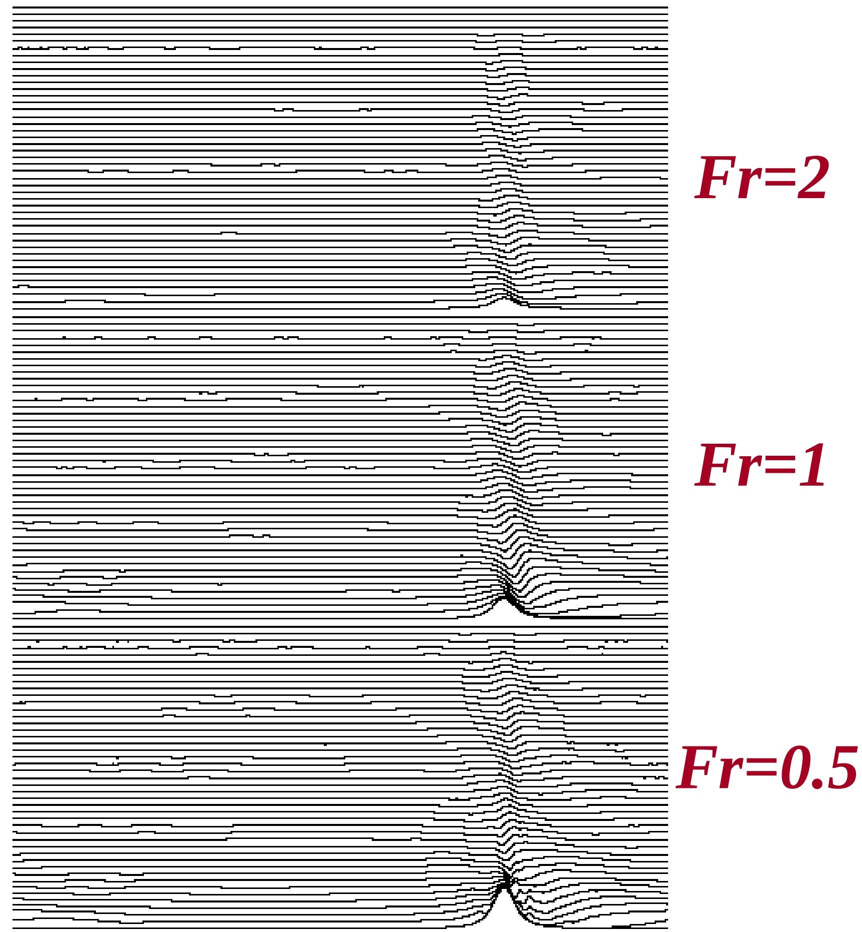
Global stratified flow past
An isolated hill

(Smolarkiewicz & Rotunno, J. Atmos. Sci. 1989)
(Hunt & Snyder J. Fluid Mech. 1980)

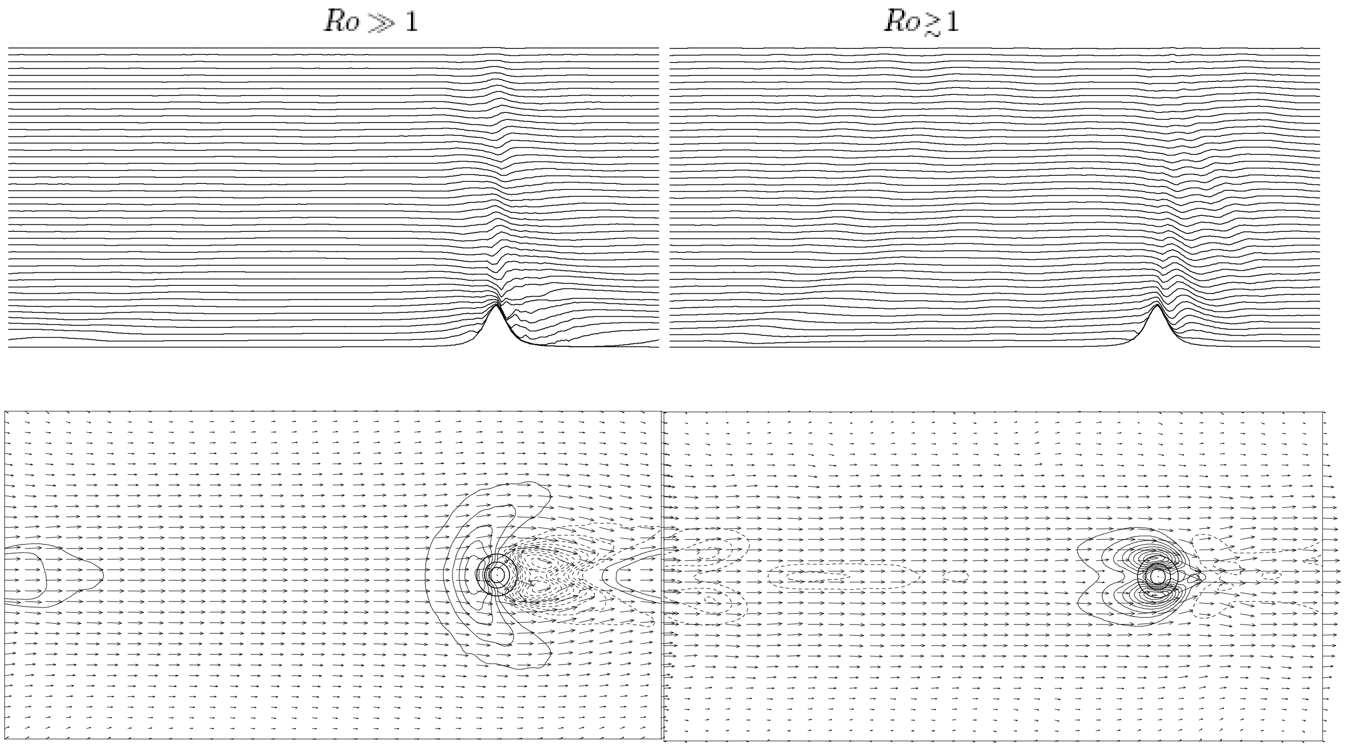
4 hours

$$Fr = U_0/Nh$$





Fr=0.5



isentropic surface
height ≈ 0.125wavelength
of the mountain wave.

(Smith, Advances in Geophys 1979)
(Hunt, Olafsson & Bougeault, QJR 2001)

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A local area non-hydrostatic model

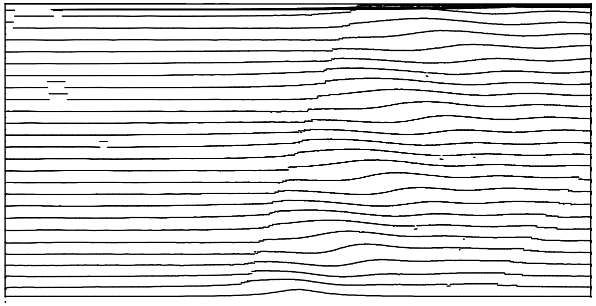
$$\frac{\partial \phi}{\partial t} + \nabla \bullet (\mathbf{V}\phi) = R$$

$$\phi_i^{n+1} = \mathcal{A}_i(\phi^n + 0.5\delta t R,\ \mathbf{V}^{n+1/2}) + 0.5\delta t R^{n+1}$$

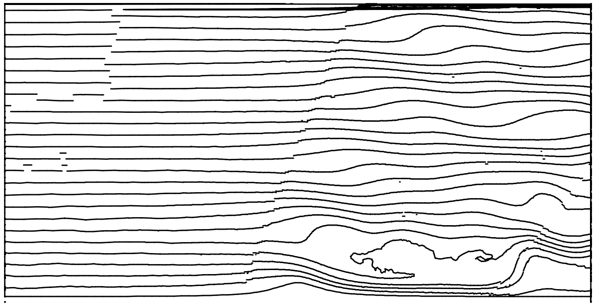
$$\nabla \bullet (\mathbf{V}\rho_o) = 0\ ,$$

$$\frac{\partial \rho_o V^I}{\partial t} + \nabla \bullet (\mathbf{V}\rho_o V^I) = -\rho_o \frac{\partial \tilde{p}}{\partial x^I} + g\rho_o \frac{\theta'}{\theta_o}\delta_{I2}$$

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \bullet (\mathbf{V}\rho_o \theta) = 0\ .$$



$Fr \lesssim 2$



$Fr \lesssim 1.$

3% in wavelength
8% in propagation angle
Wave amplitude loss 7% over 7 waveneghts

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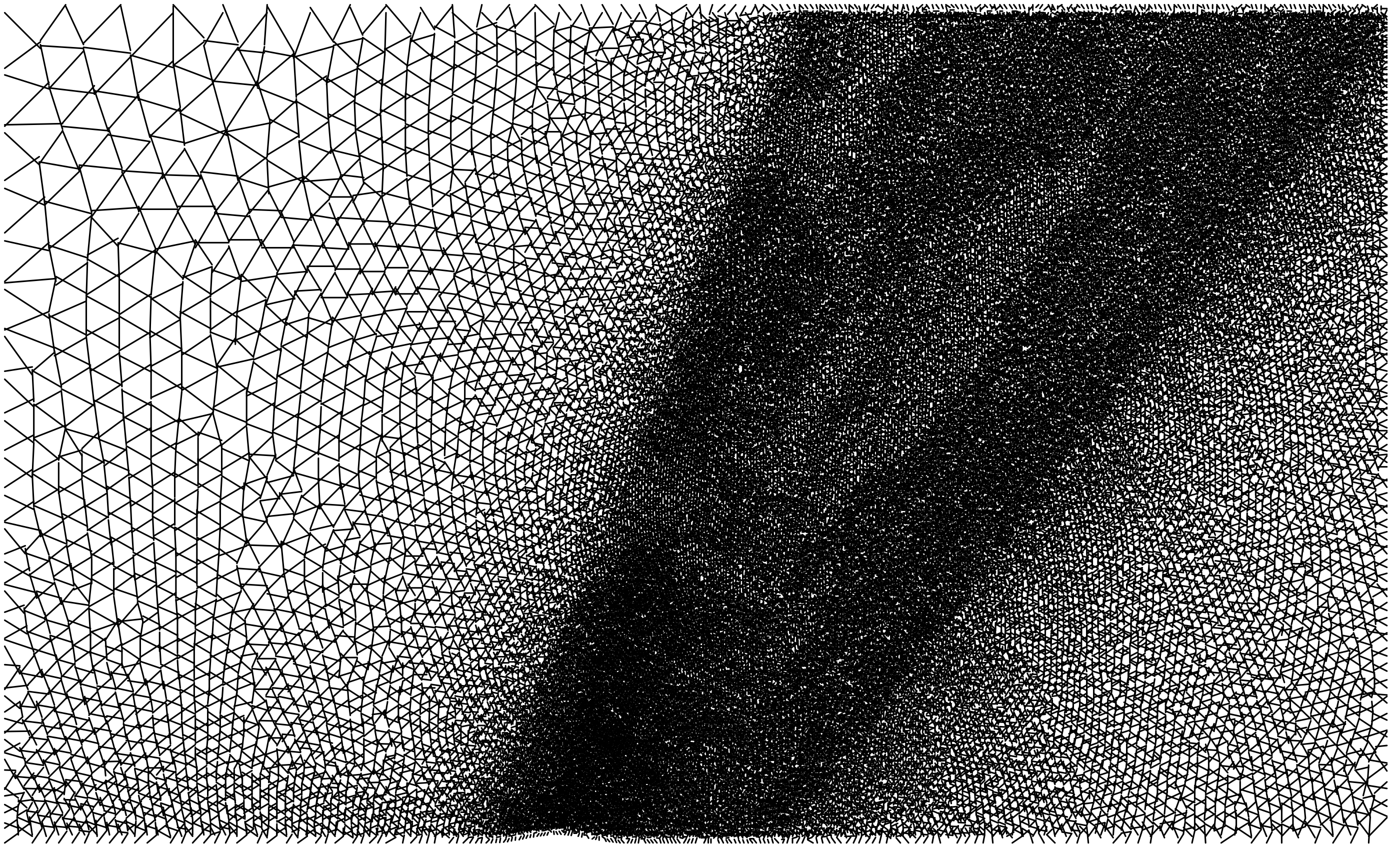
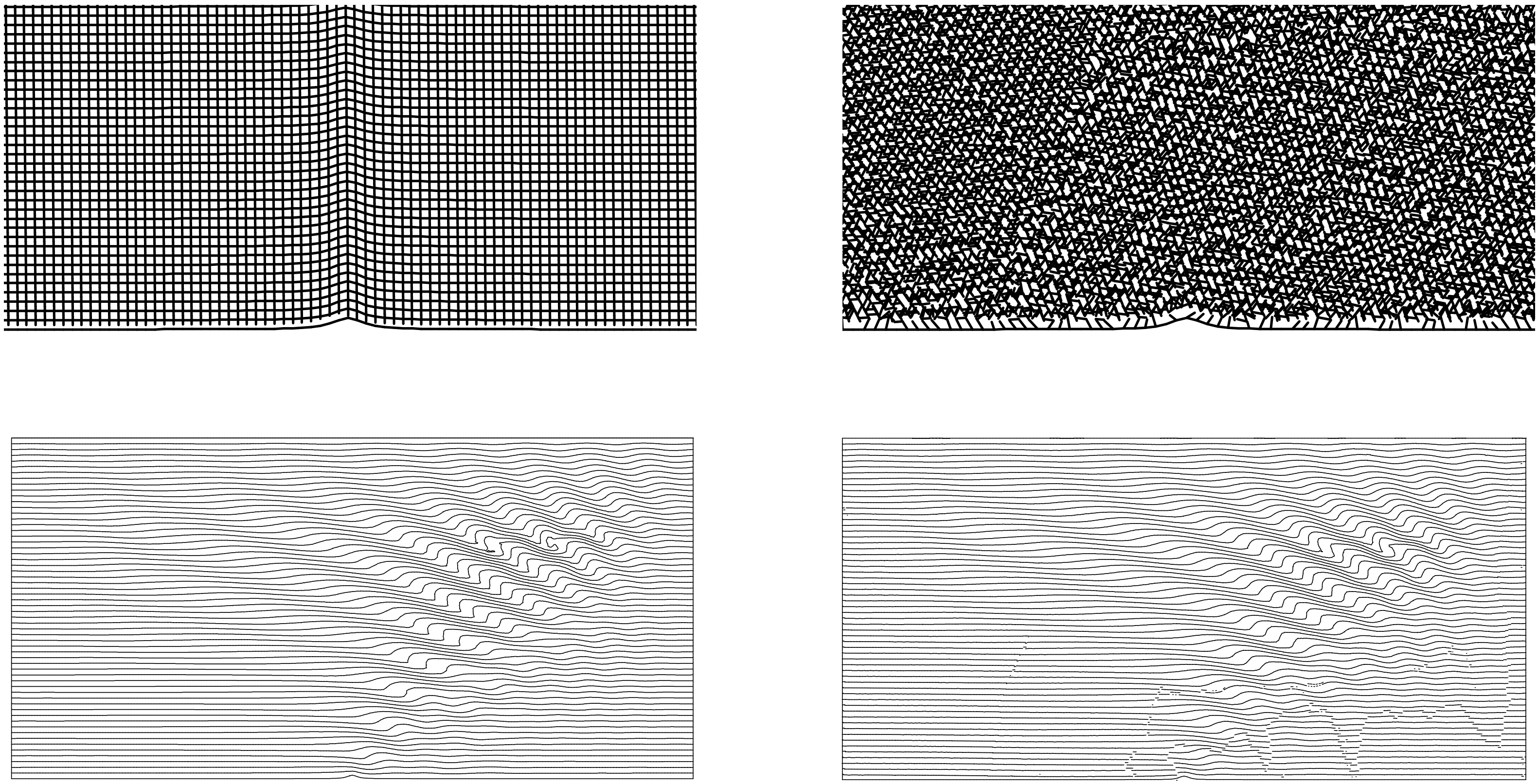
Computational mesh



Breakdown of a vertically propagating gravity wave



(Smolarkiewicz & Margolin, Atmos. Ocean 1997)

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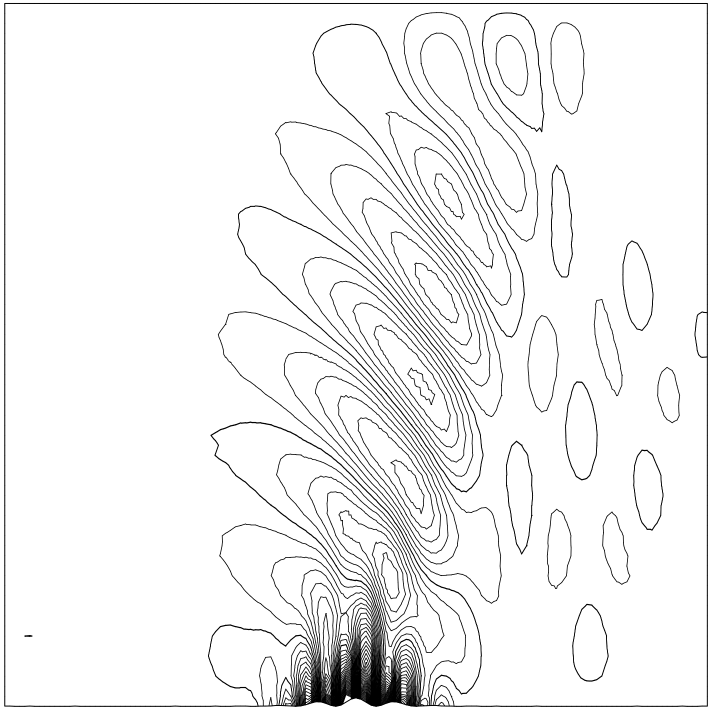
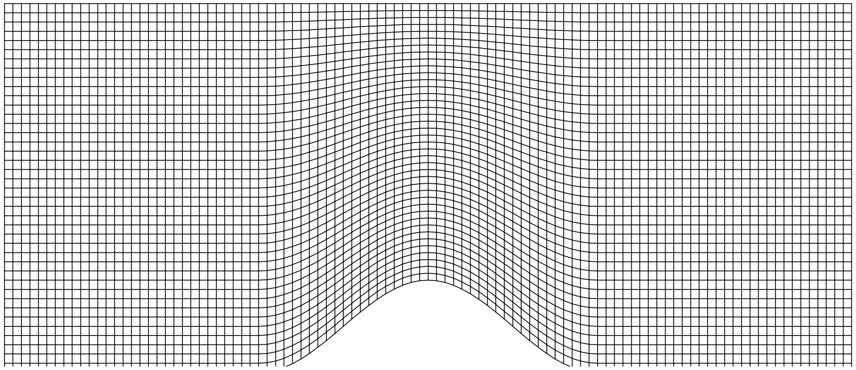
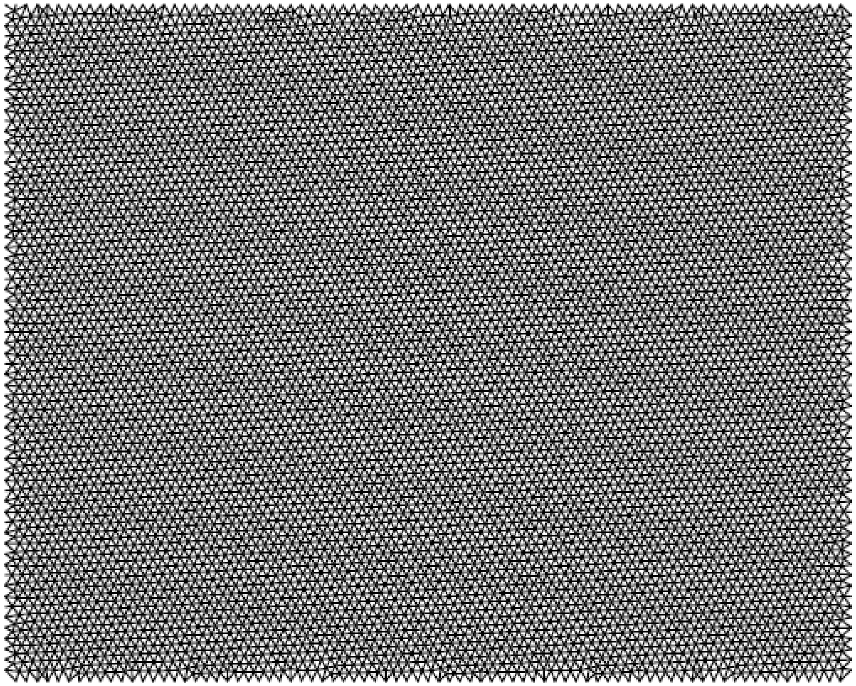
Schar Mon. Wea. Rev. 2002



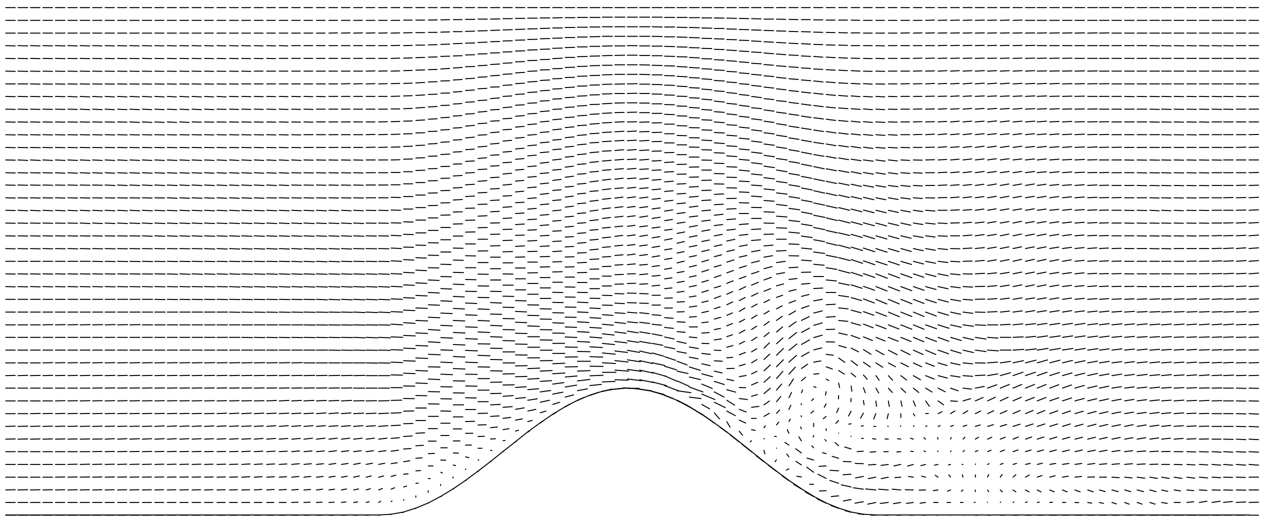
(Wedi & Smolarkiewicz JCP 2003)

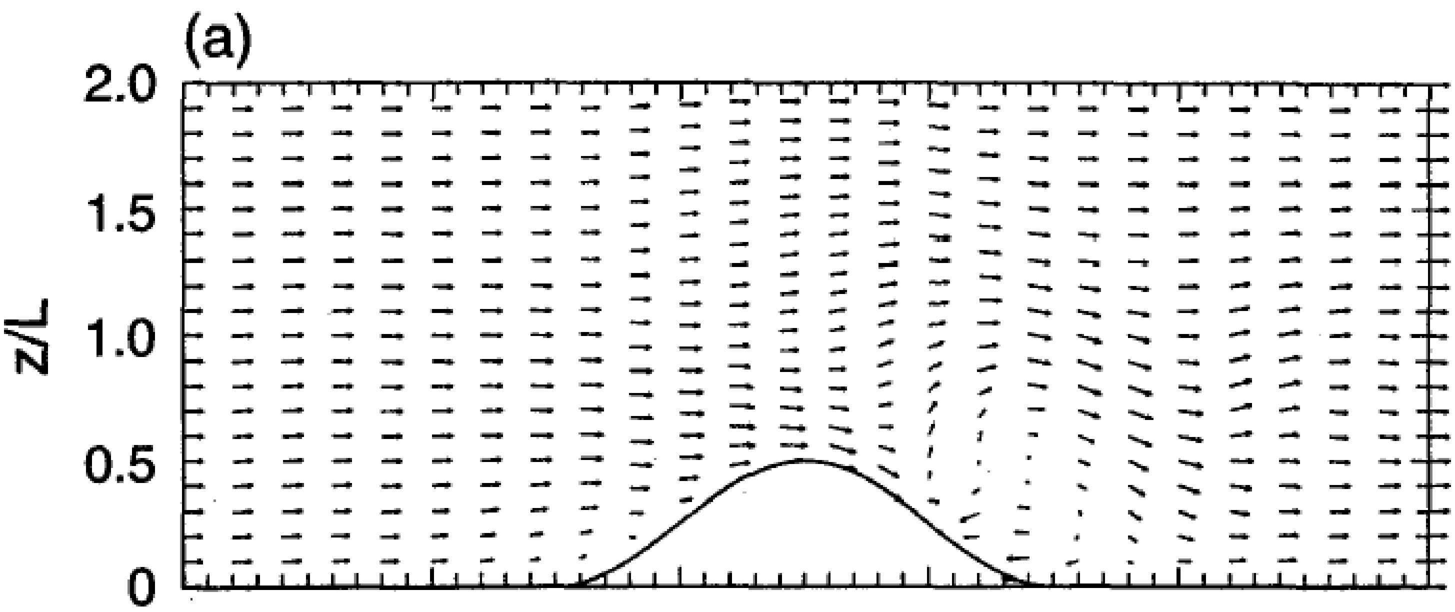
Low Froude Flow Past a Steep Three-Dimensional Hill





(Smolarkiewicz & Margolin, Atmos. Ocean 1997)




(a)
2.0
1.5
1.0
0.5
0
z/L

REMARKS

Presented work provides means for study and developments of unstructured mesh based schemes for atmospheric flows.

Future work: Generalisation of non-hydrostatic edge-based limited area models to global models