Development and applications of implicit Immersed Boundary Methods for flows in complex media.

Andrzej Wyszogrodzki
Piotr Smolarkiewicz

National Center for Atmospheric Research

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Sopot, Poland

Model formulation

Nonhydrostatic, anelastic, Navier-Stokes equations.

$$\frac{d\,\mathrm{v}}{dt} = -\nabla \pi' - g\frac{\theta'}{\overline{\theta}} + D_m(\kappa_m, e, \mathrm{v}) - \beta\,\mathrm{v} - \alpha_m\,\mathrm{v}$$

$$\frac{d\theta'}{dt} = -\mathrm{v}\cdot\nabla\theta_e + D_h(\kappa_h, e, \theta) - \beta(\theta - \theta_B) - \alpha_h\theta'$$

$$\frac{dC}{dt} = -\mathrm{v}\cdot\nabla C + D_h(\kappa_h, e, C)$$

$$\frac{de}{dt} = S(e) - \beta e \qquad \nabla(\overline{\rho}\cdot\mathrm{v}) = 0$$

reference state: $\overline{\rho}$, $\overline{\theta}$

π' normalized pressure perturbation from the ambient state

$$\pi' = (p - p_0)/\overline{\rho}$$

- Continuous adaptive mesh refinement (horizontal & vertical)

- “terrain-following” grid transformation (upper, lower boundary)

- IMB methods to simulate urban flows

- Passive tracer for T&D modeling

EULAG Immersed Boundary scheme

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Continuous feedback forcing approach for rigid boundaries - fictitious body forces introduced in equations of motion to represent internal boundaries (Goldstein et al. 1993), zero-order boundary reconstruction (stepwise geometry).

$$\frac{d\psi}{dt} = -\gamma \int_0^t \psi d\tau - \beta\psi + A\sin(\omega t)$$

Forced damped harmonic oscillator

For consistency with model NFT assume Crank-Nicholson time discretization

$$\psi^{n+1} = \widehat{\psi} + 0.5\delta t R^{n+1} \qquad \widehat{\psi} \equiv \psi^n + 0.5\delta t R^n$$

$$R^n = -\gamma \mathcal{I}^n(\psi) - \beta\psi^n + A\sin(\omega t^n)$$

$$\mathcal{I}^n(\psi) \equiv \delta t \sum_{k=1}^{n} 0.5(\psi^{k-1} + \psi^k)$$

Compact closed form of trapezoidal integral

$$\psi^{n+1} = \widehat{\widehat{\psi}} \; / \; [1 + 0.5\delta t(\beta + 0.5\delta t\gamma)]$$

$$\widehat{\widehat{\psi}} = \widehat{\psi} + 0.5\delta t\,[A\sin(\omega t^{n+1}) - \gamma(\mathcal{I}^n(\psi) + 0.5\delta t\psi^n)]$$

Explicit part

EULAG Immersed Boundary scheme

Simplifications: $\gamma \equiv 0$. **(no sensitivity within tested problems range)**

$$\psi^{n+1} = \frac{\psi^n(1 - 0.5\delta t\beta) + \delta t A \cos(\omega\delta t/2)\sin(\omega t^{n+1/2})}{1 + 0.5\delta t\beta}$$

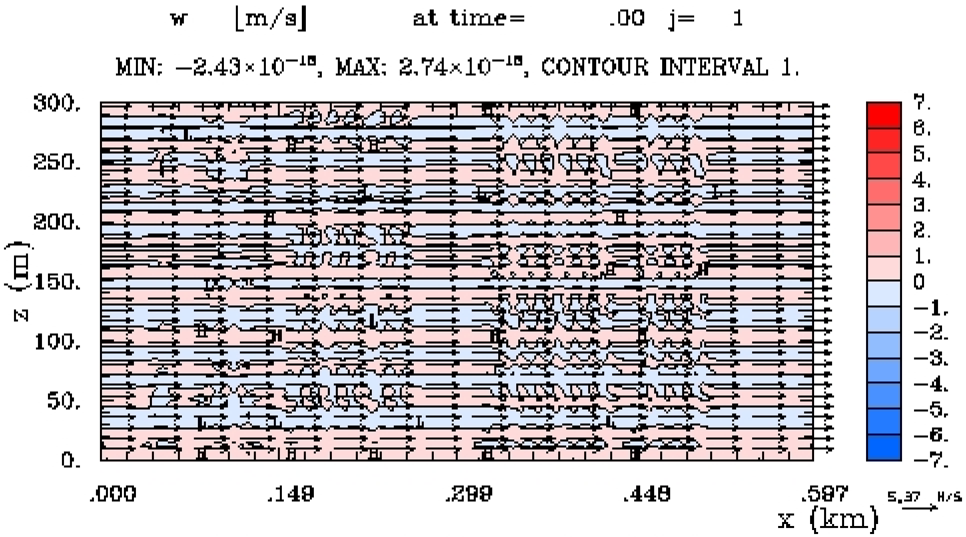
Forcing term A:

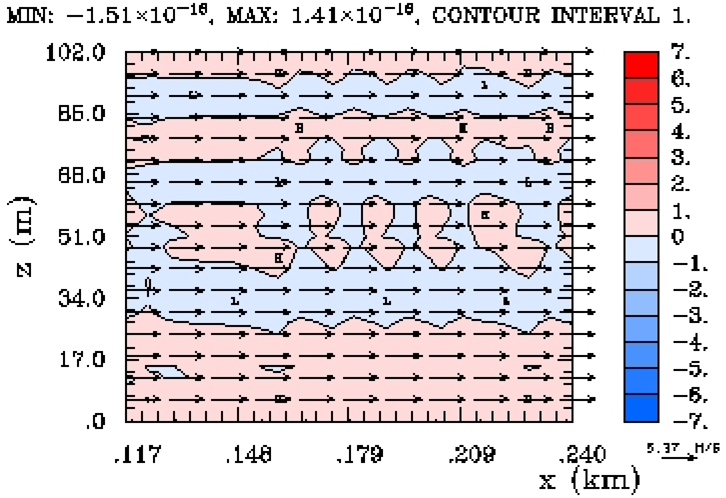
In general nontrivial external forcing A requires variable in time β

In problems at hand primary forcing is representing pressure gradient terms which responds instantaneously to the flow distortions

- **large scale component:** $A = \psi_o/T$ and $\omega = 2\pi/T$ with $T \gg \delta t$.
- **rapidly oscillating part:** $A \sim \mathcal{O}(\psi_o/\delta t)$ and $\omega \leq 2\delta t$
- **in both cases attenuation time** $\beta^{-1} = 0.5\delta t$ **is effective but allows for residual flow within building structures**
- **shorter time scale give stable but oscillatory solution**

EULAG Immersed Boundary scheme



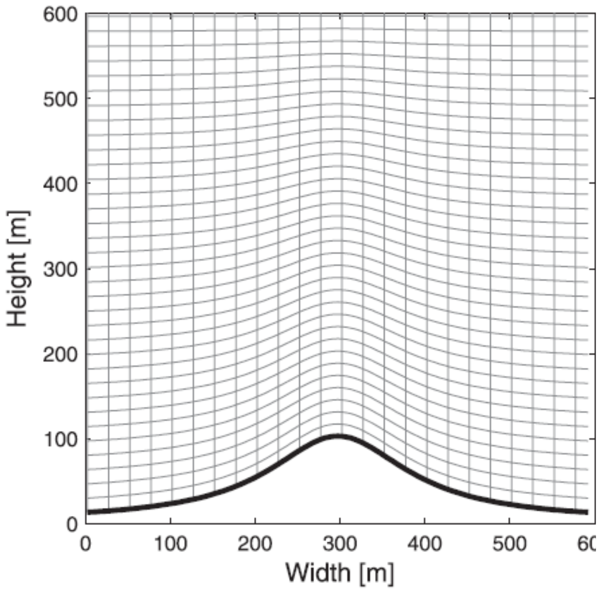


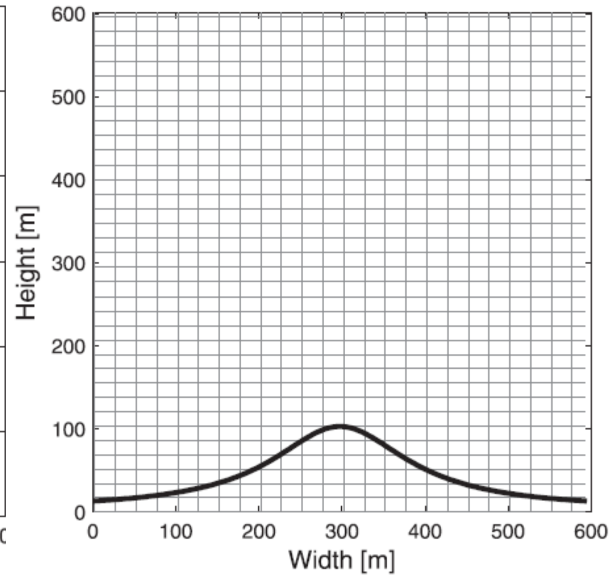
residual flow within object structures

$$A \equiv 0 \quad \beta^{-1} = 0.5\delta t$$

No external forcing: no pressure gradients within building - solution damps to zero within single time step.

IMB vs "terrain following" smooth topography





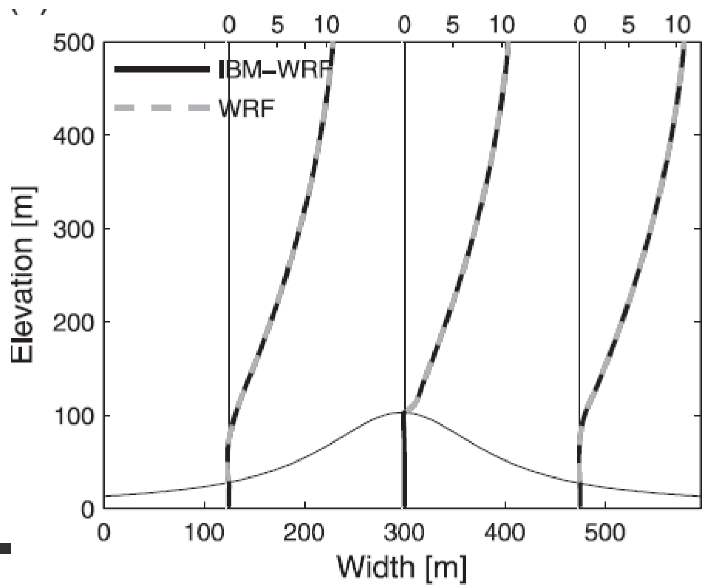
From Lundquist et al. 2010 IMB implementation in WRF

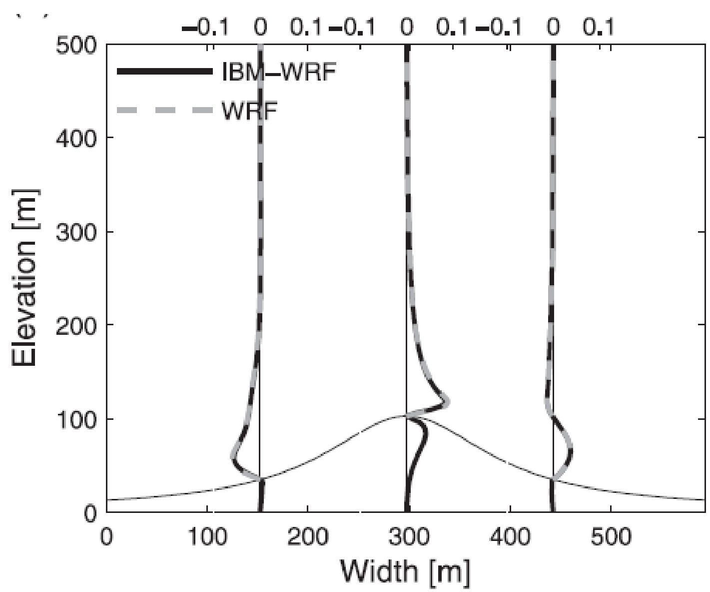
- direct forcing of Mohd-Yusof (1997)

- velocity or scalar value is modified at forcing points

Witch of Agnesi

$$h_t(x) = \frac{h_p}{1 + (x/a)^2}$$





IMB vs "terrain following" complex structures (e.g Pentagon)

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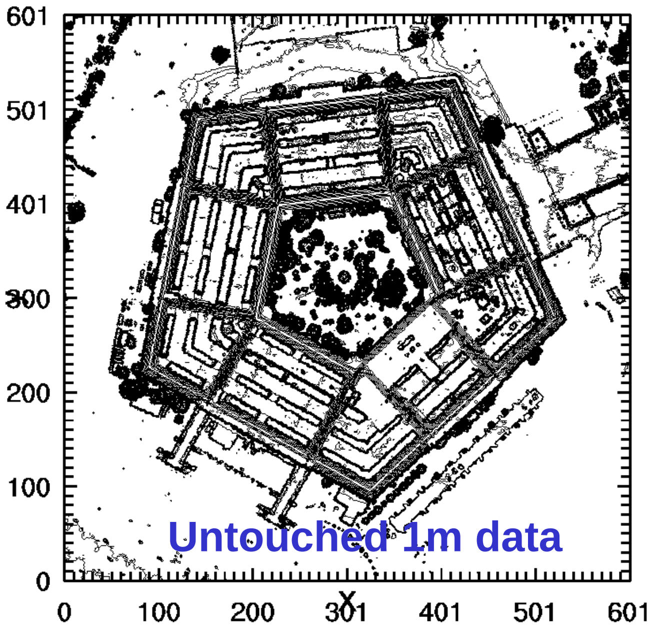
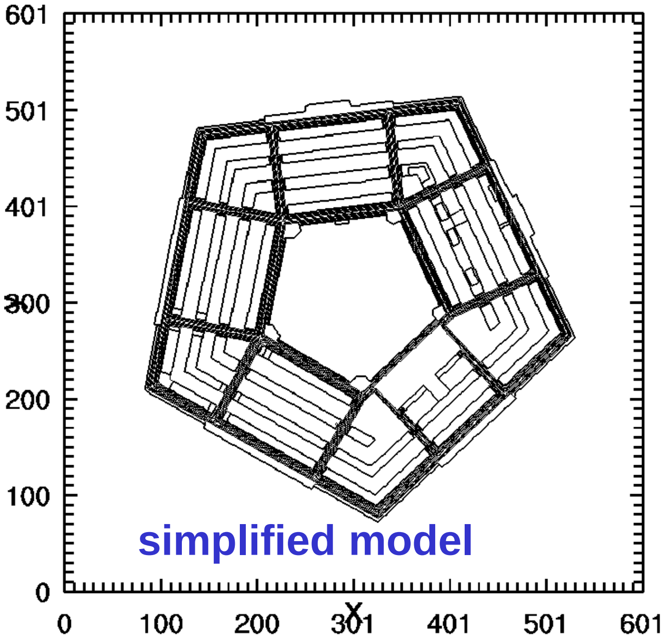
- **Building description derived from 1m resolution lidar dataset (April 2001)**
- **Domian: 600x600x31 @ $\Delta x=\Delta y=\Delta z=2m$**
- **7200 time steps @ $\Delta t=0.05$ s**
- **Rigid upper boundary**
- **Specified C_D on building and sfc**
- **Neutral stratification with prescribed velocity profile from standard LES simulations (Moeng and Sullivan)**

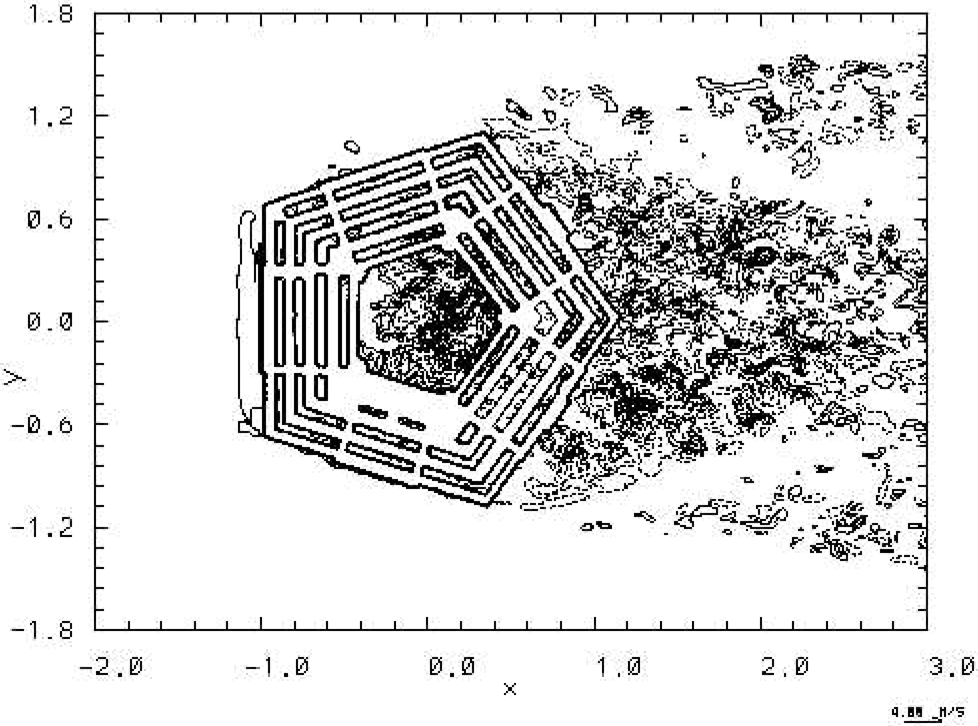




IMB vs "terrain following" complex structures

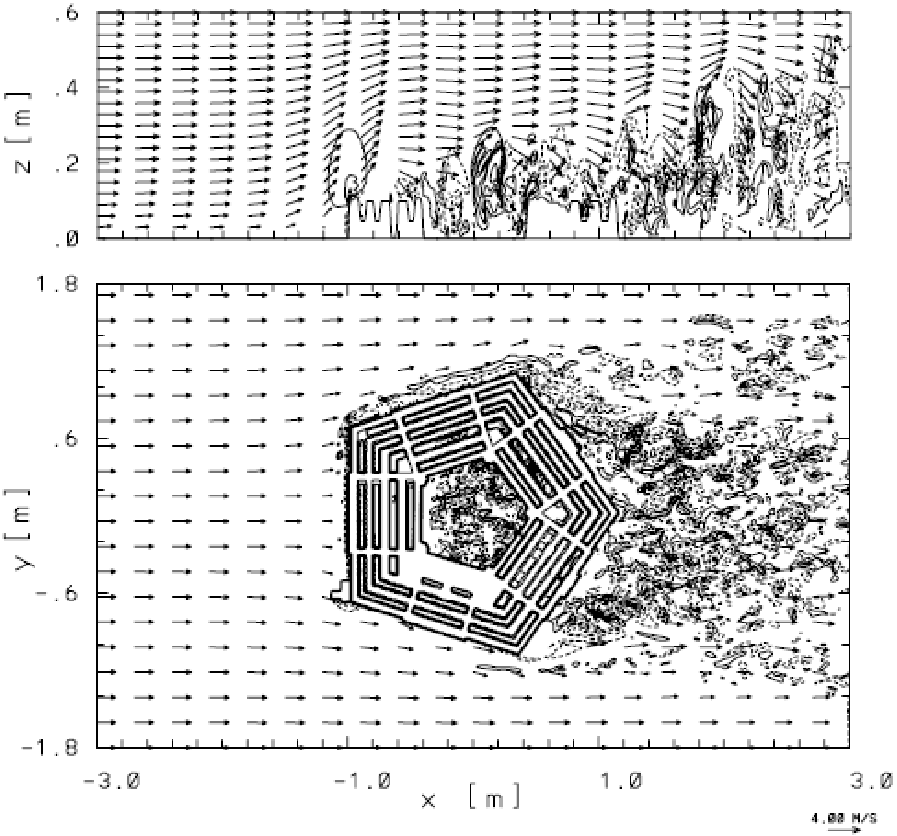
US EPA Meteorological Wind Tunnel at Fluid Modeling facility

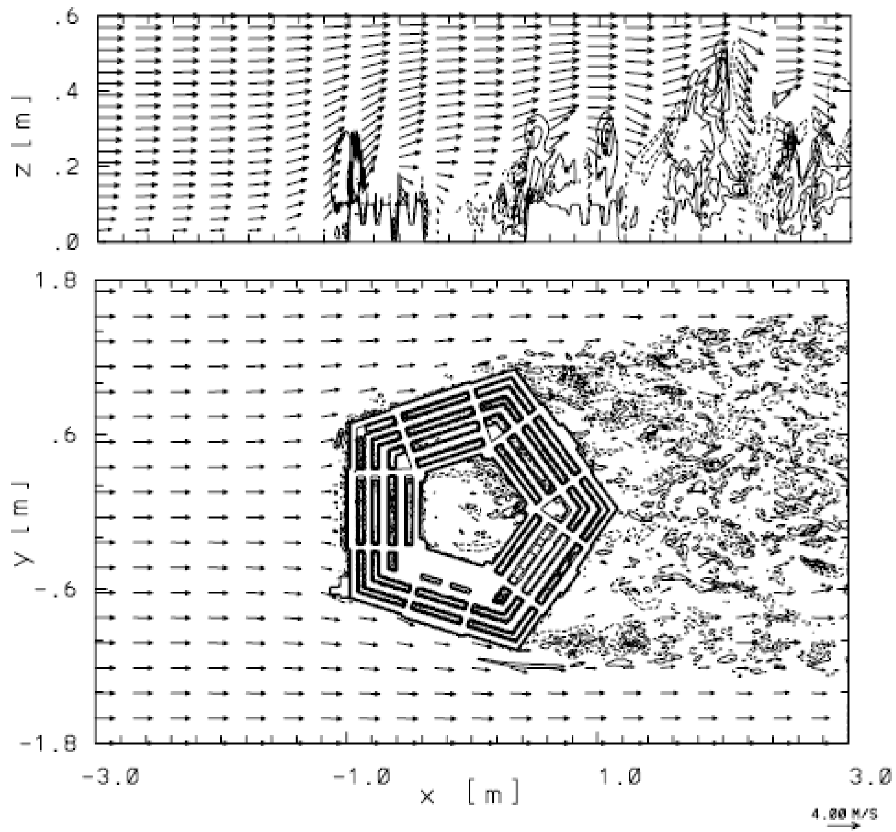
EULAG instantaneous vertical velocity field



IMB vs "terrain following" complex structures

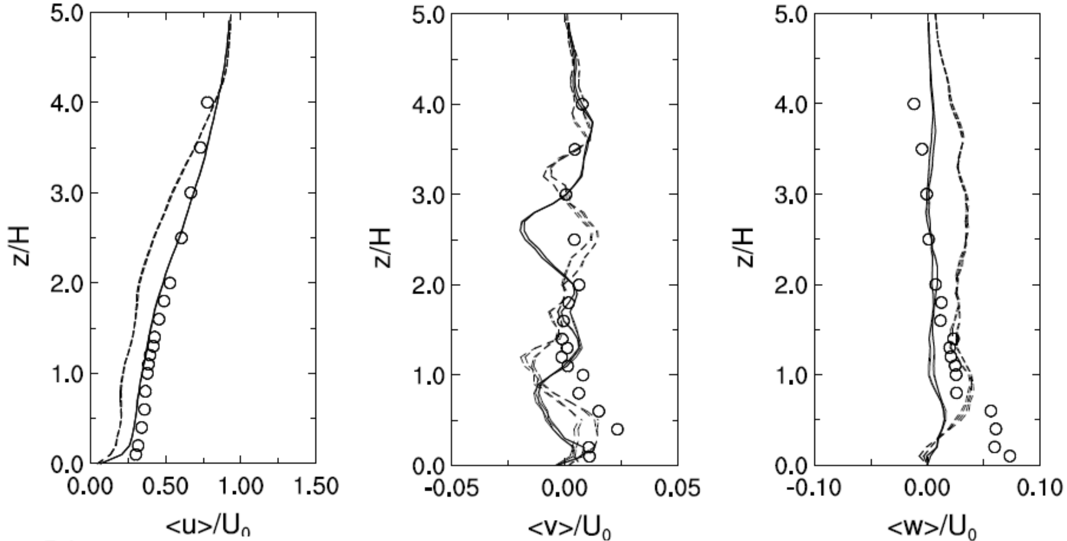
EULAG instantaneous vertical velocity fields



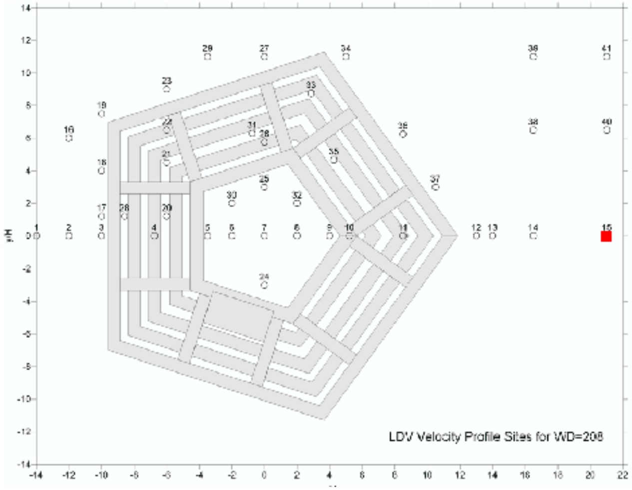


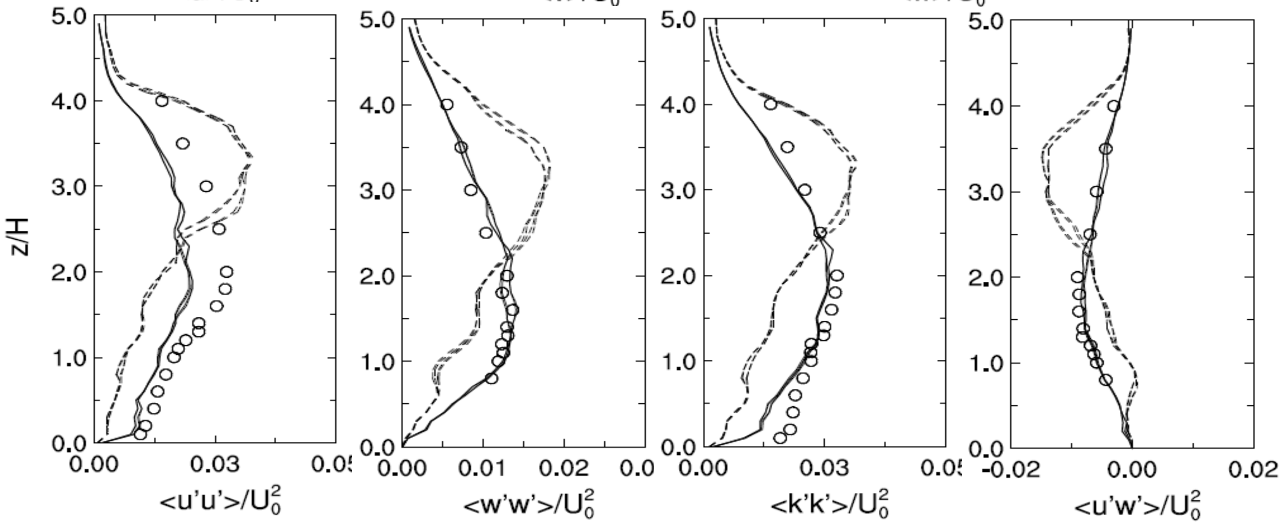
IMB vs “terrain following” complex structures

Velicity profiles



Vertical profiles in the building wake





solid – IMB
dashed – TF
circles -observation

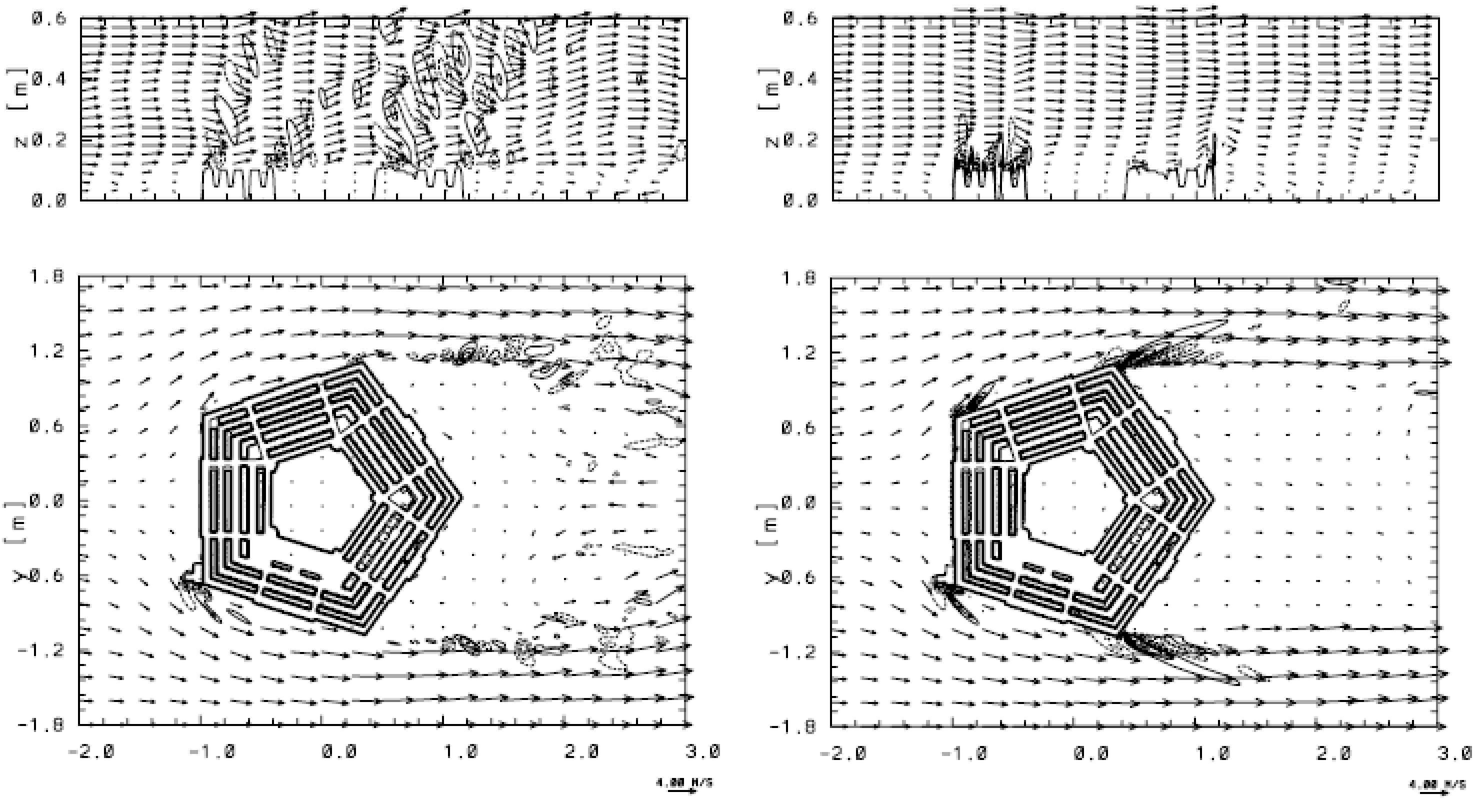
Fluctuation fields

IMB vs “terrain following” complex structures

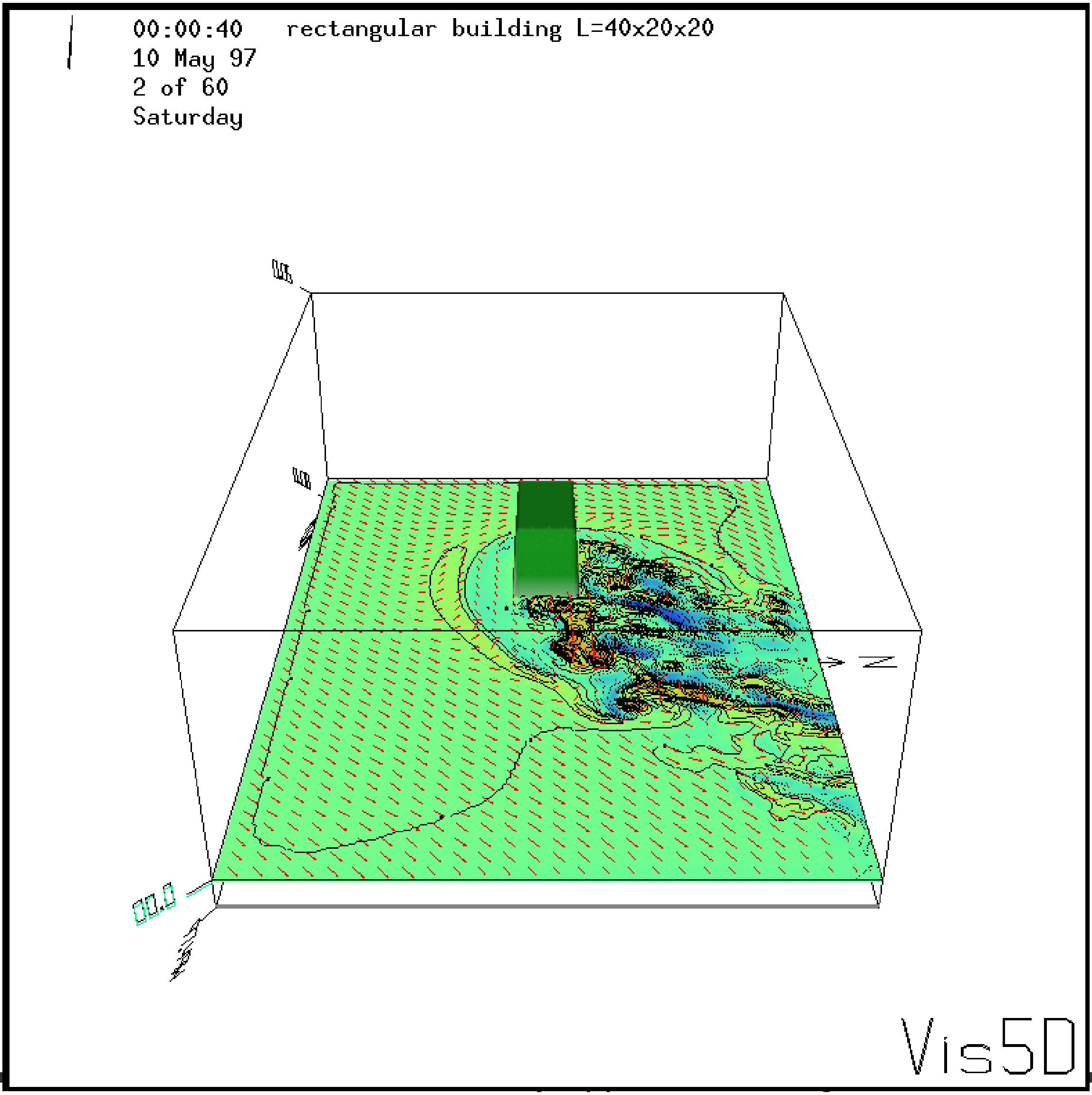
low Froude number $Fr \equiv U_0/Nh \lesssim 0.5$

or strongly stratified flow $N = \sqrt{g\theta_e^{-1} \mathrm{d}\theta_e/\mathrm{d}z} = 80\ \mathrm{s}^{-1}$

EULAG instantaneous vertical velocity fields

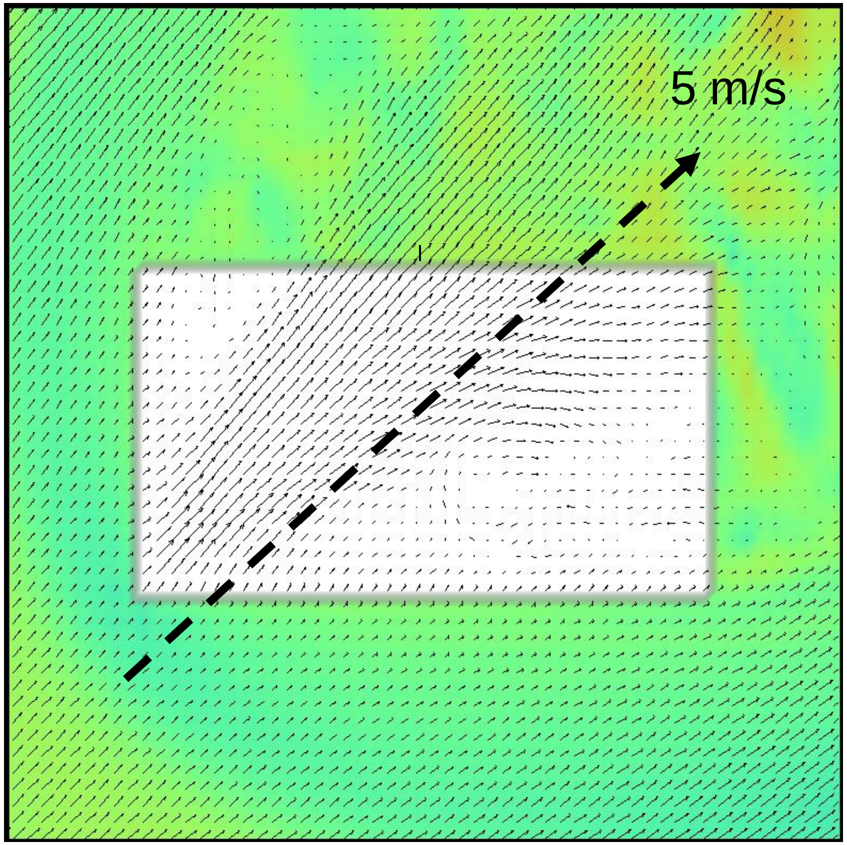


Application to flow around simple building structures

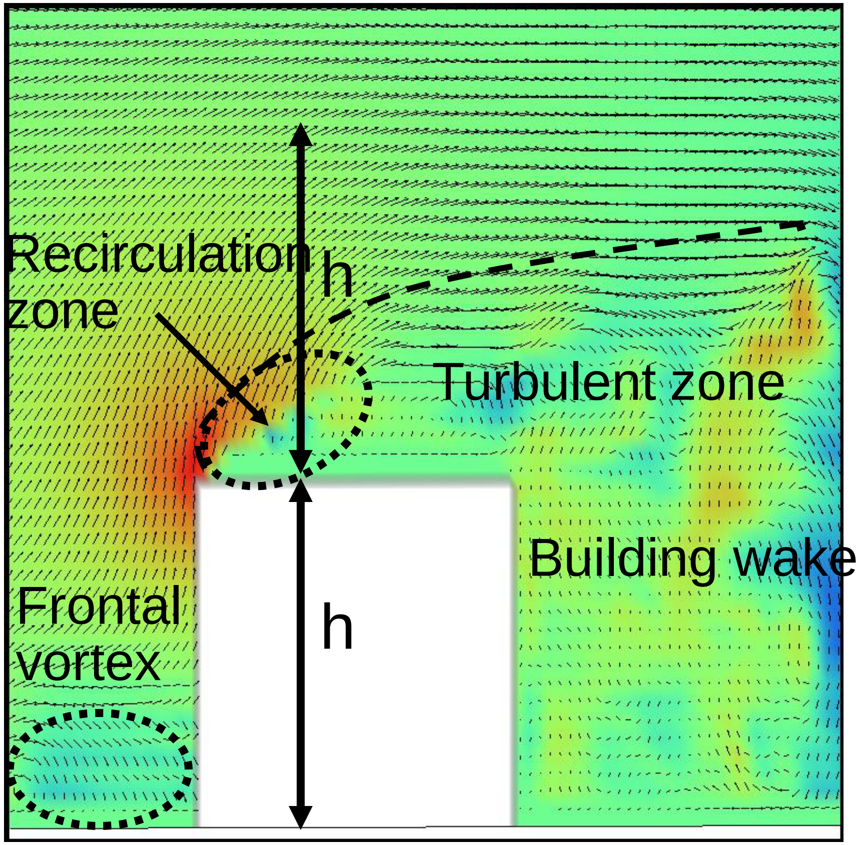


w field
neutral
U=5 m/s

Flow around simple building structures

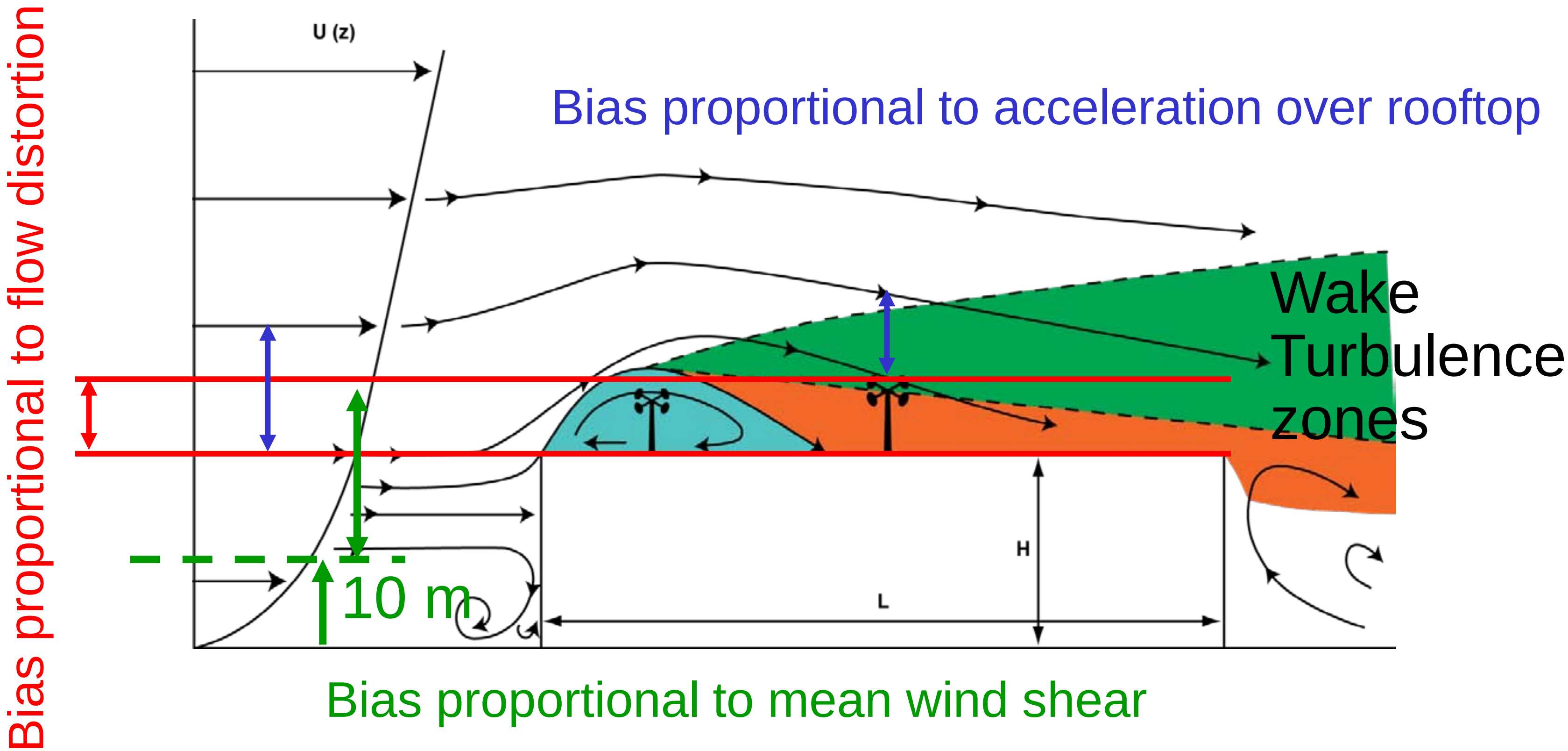


z=2m above rooftop



Vertical plane along dashed line

Building effects typically extend at least one building height above rooftop



Redrawn from Wilson 1979

Assessment of building-induced errors from rooftop anemometer observations for mesoscale NWP & T&D applications

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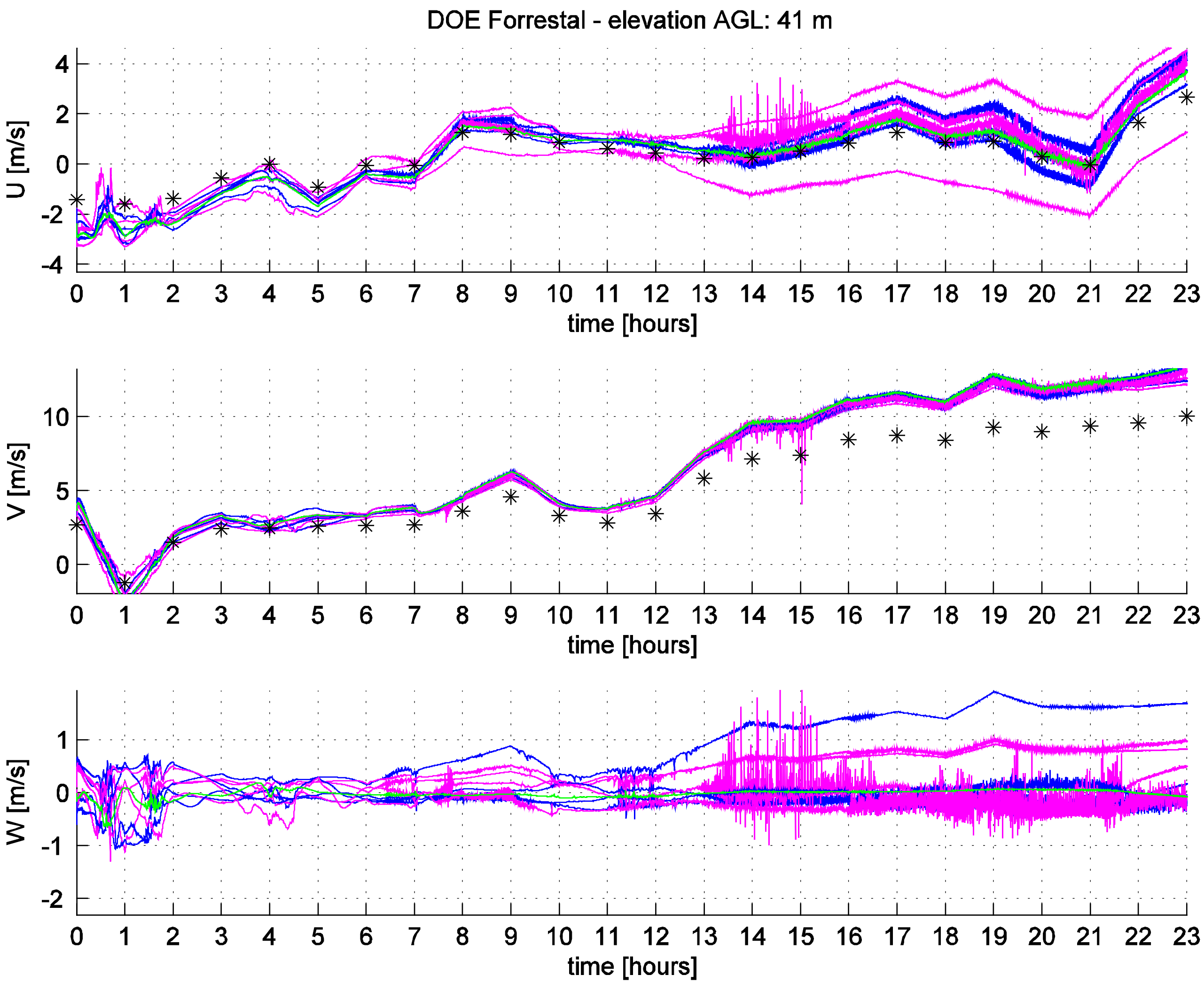
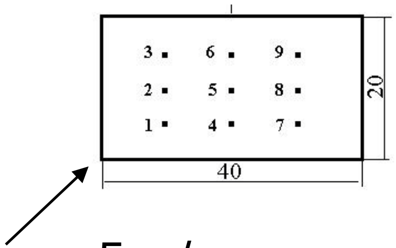


Eulag anemometer winds

- Anbient flow

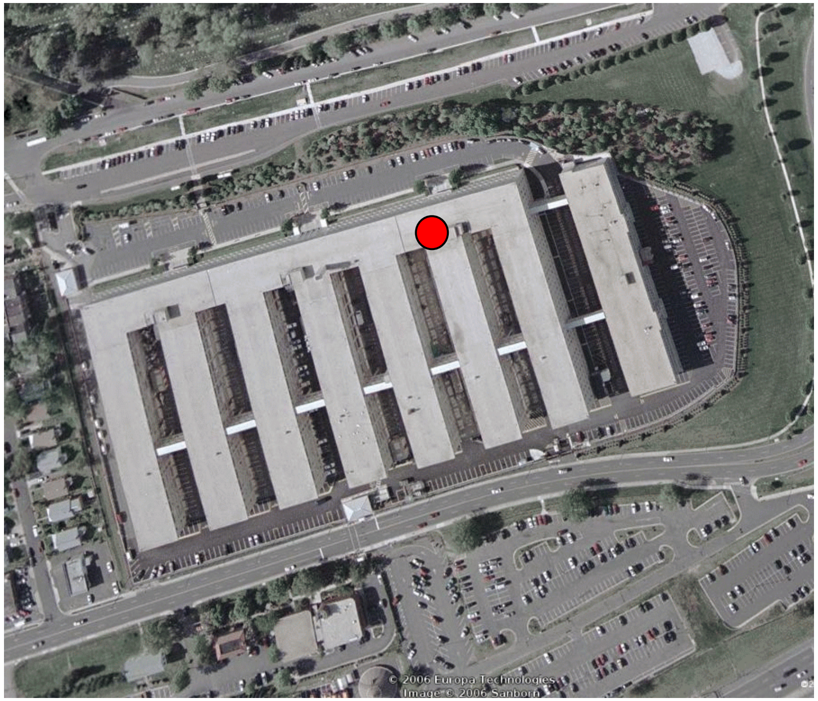
- Eulag rooftop winds at 9 locations



5 m/s

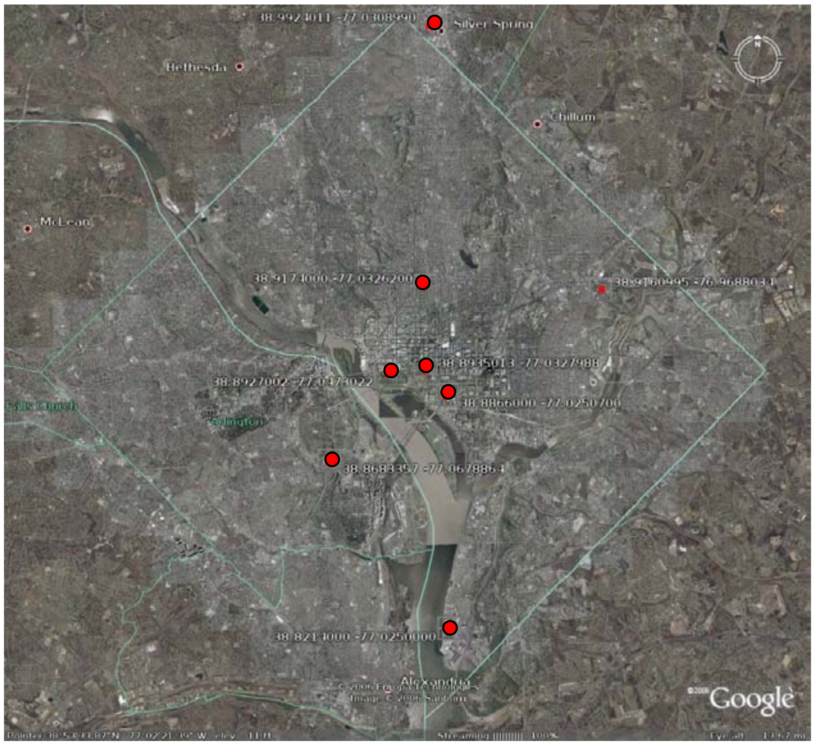
Application - DCNet

- Actual urban environment and building shapes – 1m resolution
- Time-averaged readings from anemometer winds 10 m above rooftop

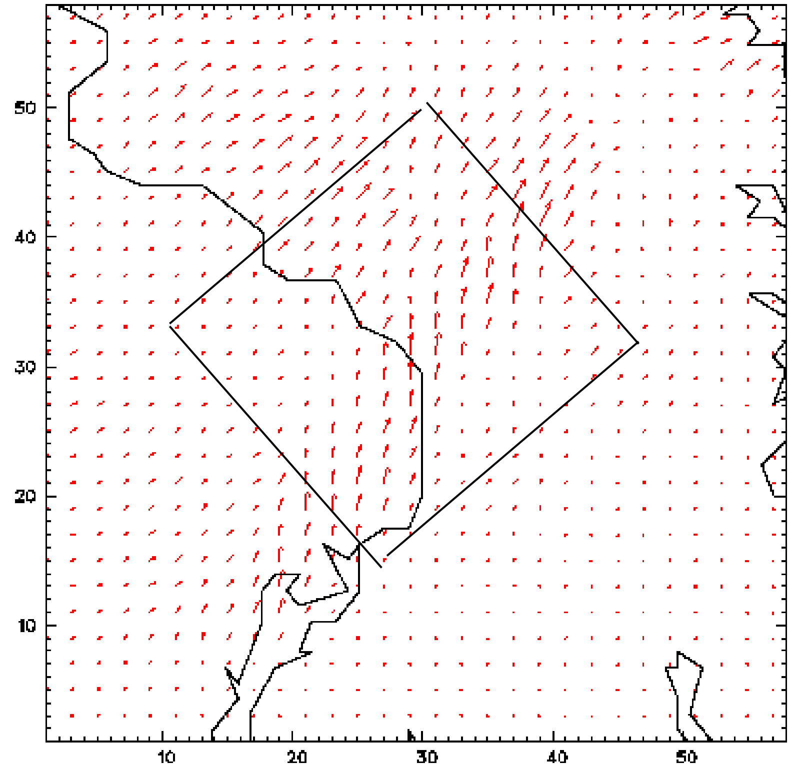


Application - DCNet

7 buildings with rooftop anemometers, all at 10m above rooftop



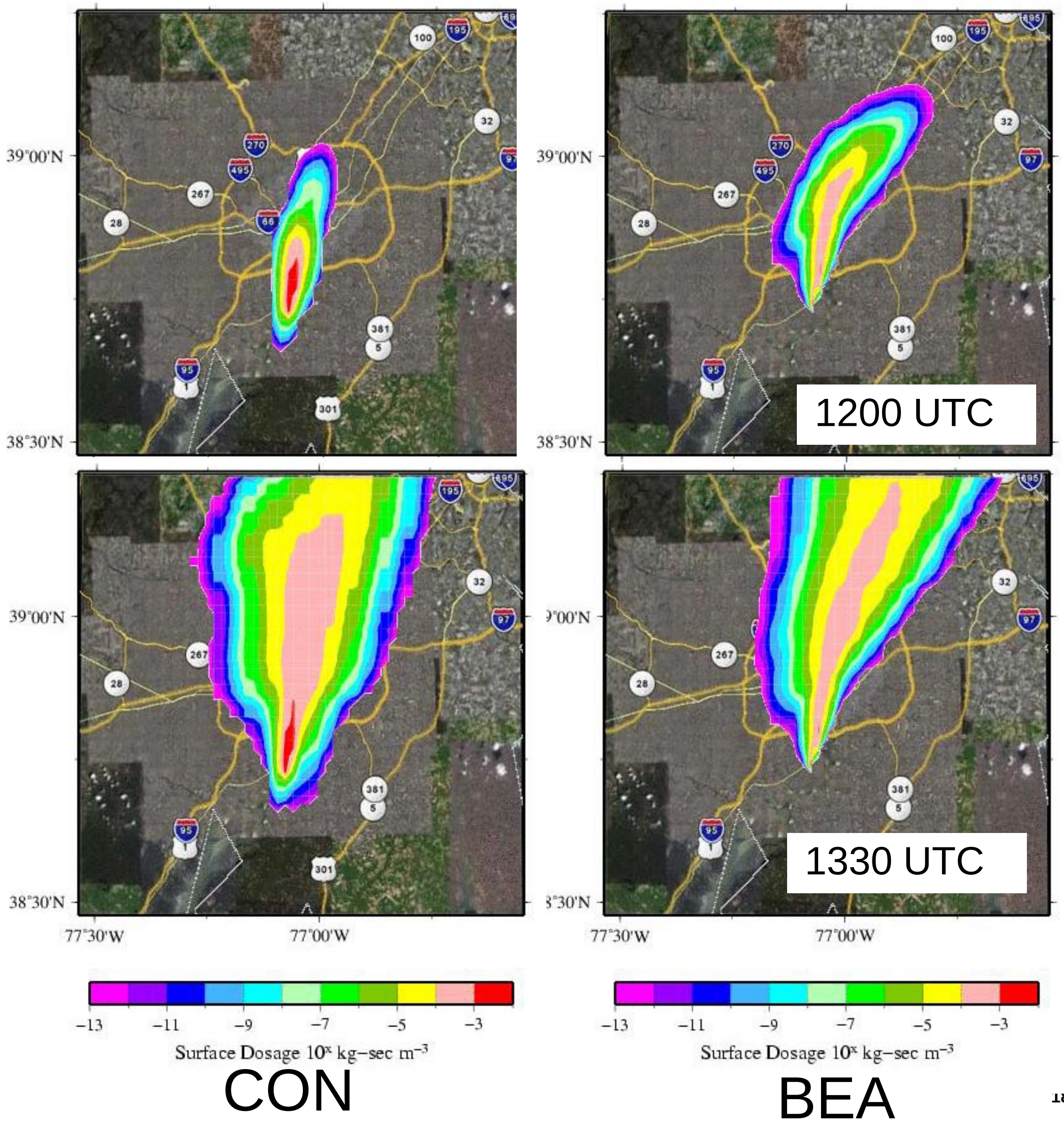
● DCNet rooftop anemometer locations



CON-BEA vector wind difference after RTFDDA simulation time= 12 hrs. at 40 m AGL (max ~ 5 m/s)

SCIPUFF derived dosages for SF6 release at 1100 UTC

- **MM5/RTFDDA 4 nests**
  - **40.5, 13.5, 4.5, 1.5 km**
  - **Start: 0Z 13 Jan 2005**
  - **End: 0Z 14 Jan 2005**
- **3 runs**
  - **NAT: No assimilation of wind observations**
  - **CON: Assimilation of nature (NAT) run**
  - **BEA: Assimilation of EuLag-generated building winds**
  - **Assimilations over first 11 h, free forecast over last 13 hours**



Multiscale urban flows

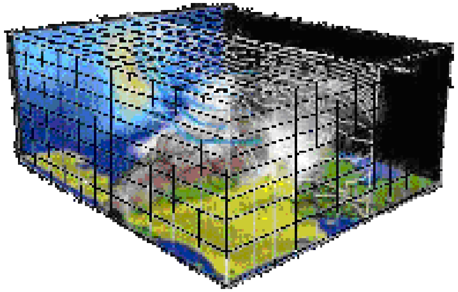
Two way coupling between WRF/EULAG

WRF provides initial and lateral boundary conditions for EULAG in two modes
- Isolated sounding data mode – short term, quasi steady conditions, small scale urban domain
- Unsteady (temporal-based coupling) mode – linear interpolation of the WRF data in time and space

⟶ Building geometry flow features resolved explicitly with immersed boundary (IB) approach

Downscale data transfer

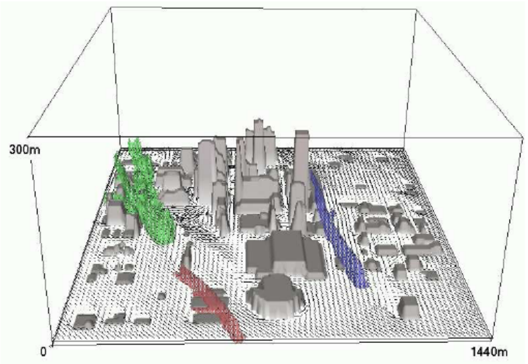
Mesoscale modeling system:
WRF-Noah/UCM forecast model



Coupler:

MCEL Library

Urban T&D modeling system:
EULAG LES/CFD model



Upscale data transfer:

Turbulence and wind fields explicitly resolved by EULAG are feedback to WRF-urban
- EULAG fields are volumetrically averaged to (coarser) WRF mesh
- WRF urban framework introduce source terms in the momentum and turbulence equations

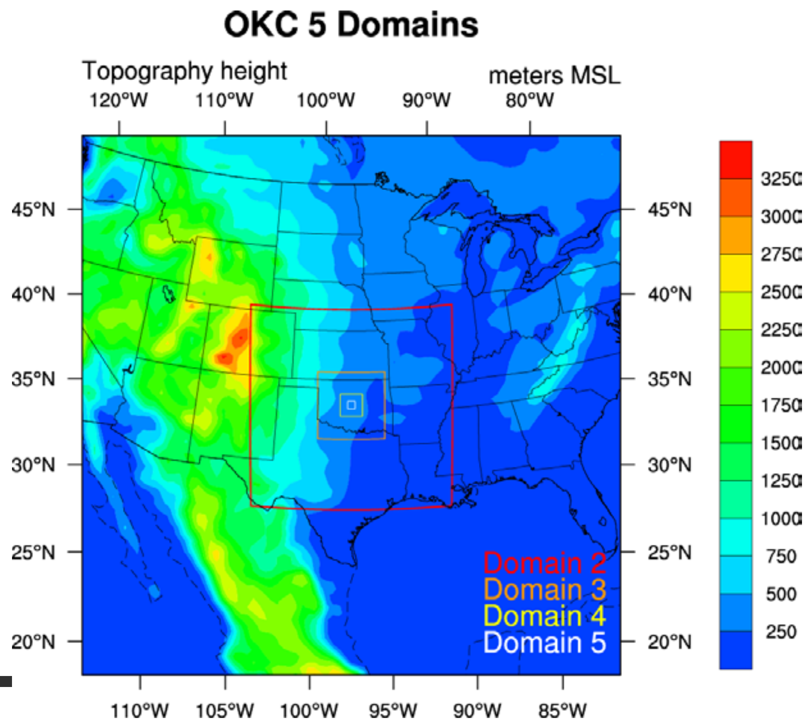
⟶ The coupling impact urban and downstream weather forecast

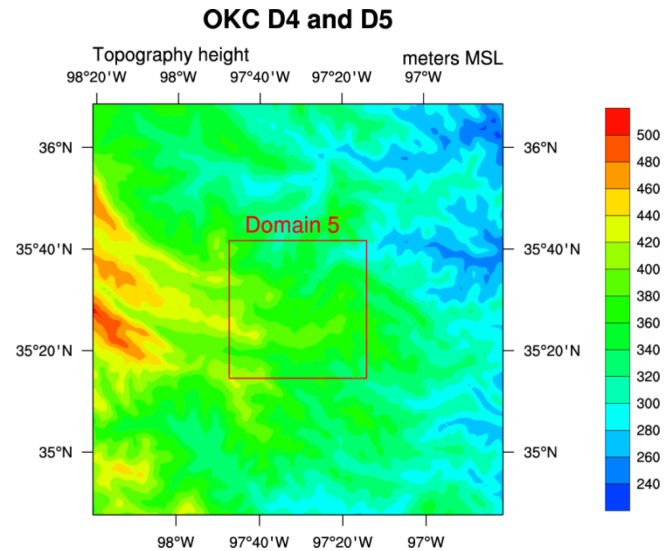
Multiscale urban flows
setting up mesoscale model

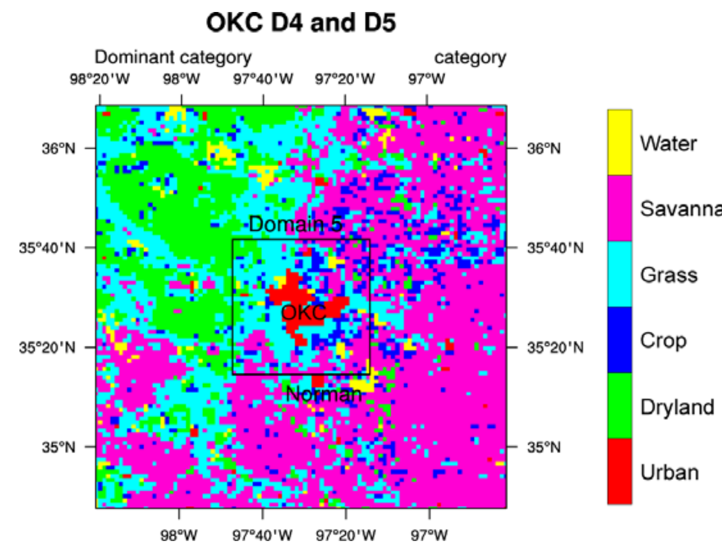
WRFv2.2 modeling domains - terrain heights (m) and land-use types

5 two-way nested domains, grid spacing and grid sizes:

- D1: 40.5km (90* 90*38)
- D2: 13.5km (100*100*38)
- D3: 4.5km (100*100*38)
- D4: 1.5km (100*100*38)
- D5: 0.5km (100*100*38)





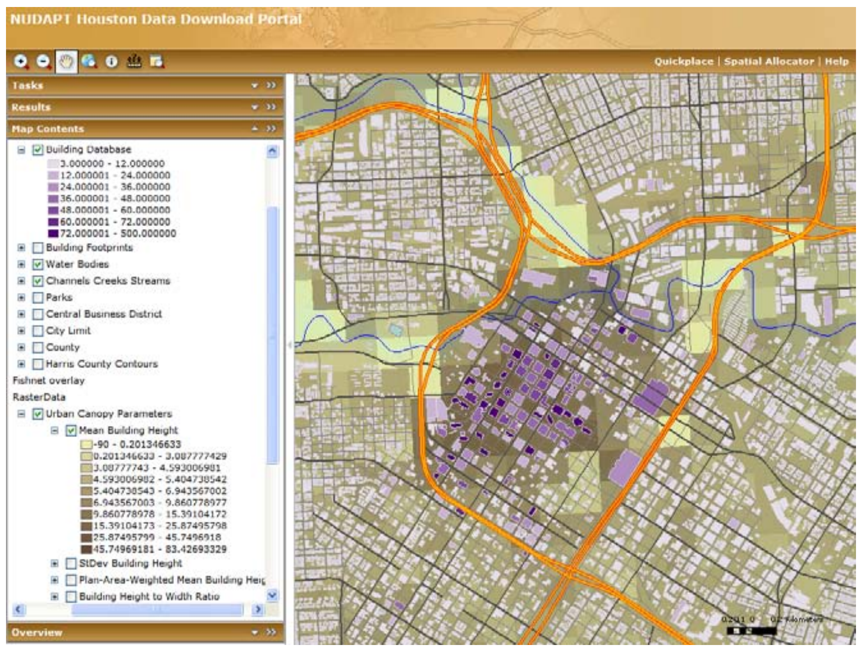


Multiscale urban flows buildings database

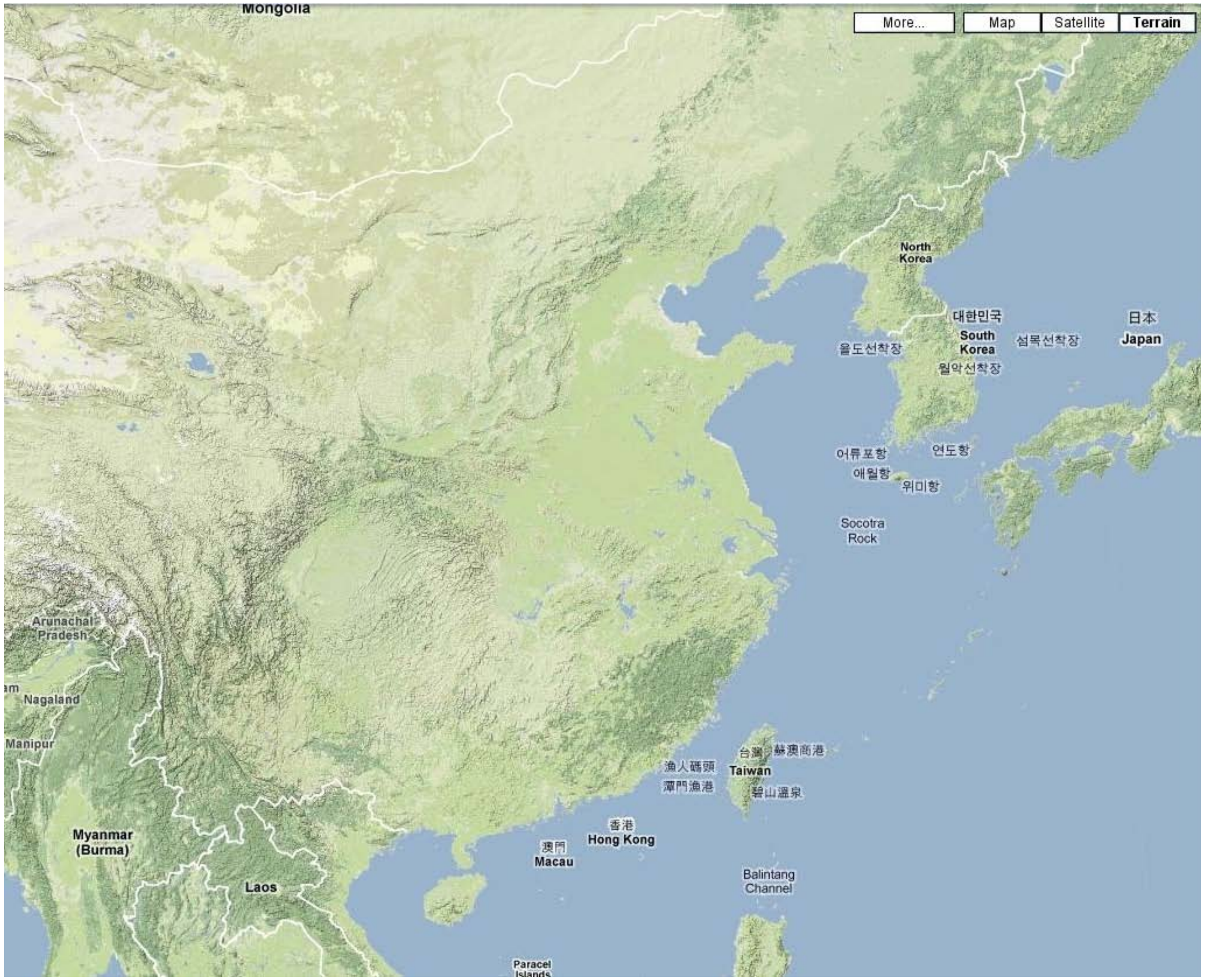
National Urban Database and Access Portal Tools (NUDAPT)

Jason Ching (NERL/USEPA)

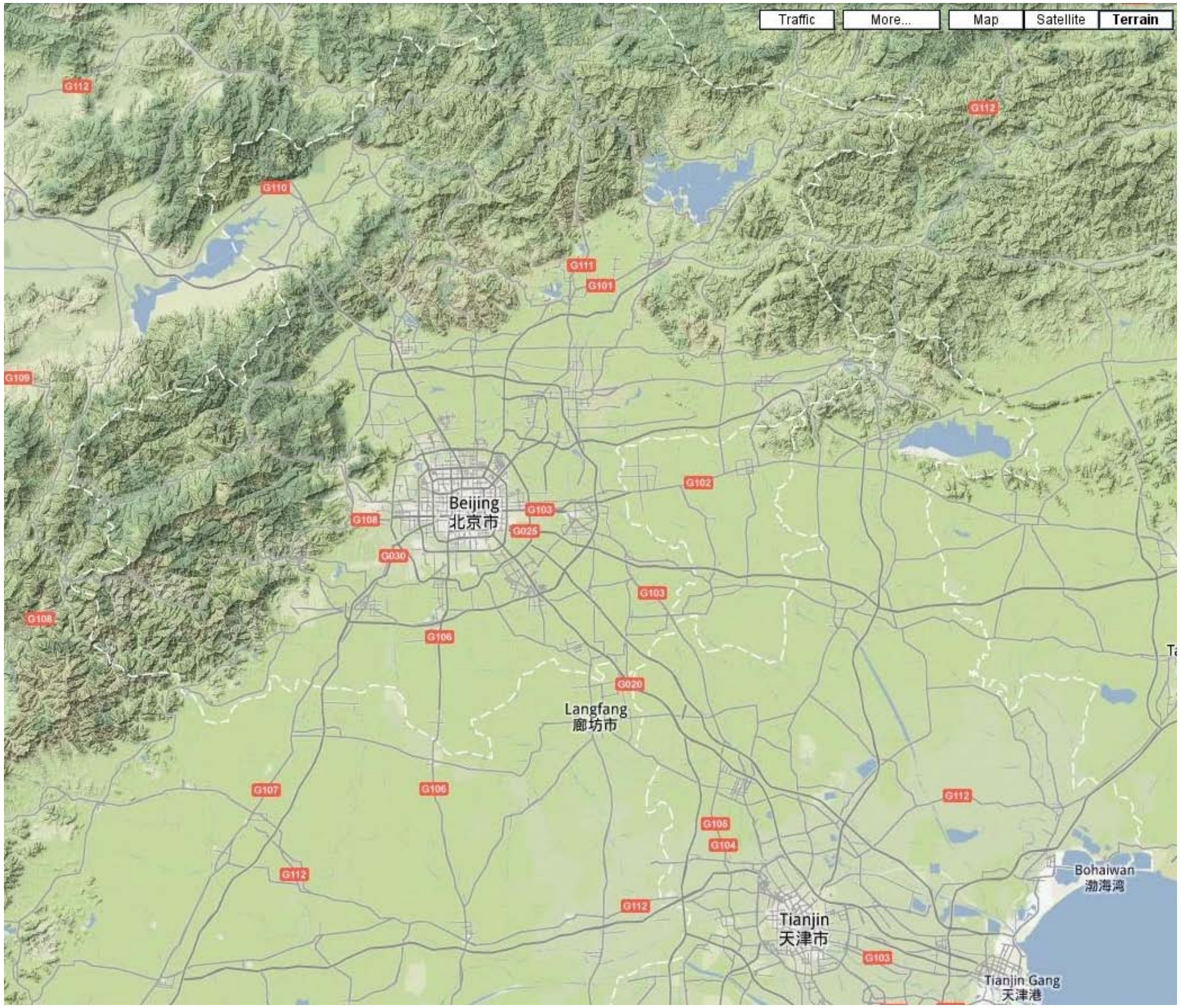
- Web-based system data download portal
- Source data is stored with daughter Urban Canopy Parameters (UCPs) in a centralized repository for easy comparison and download



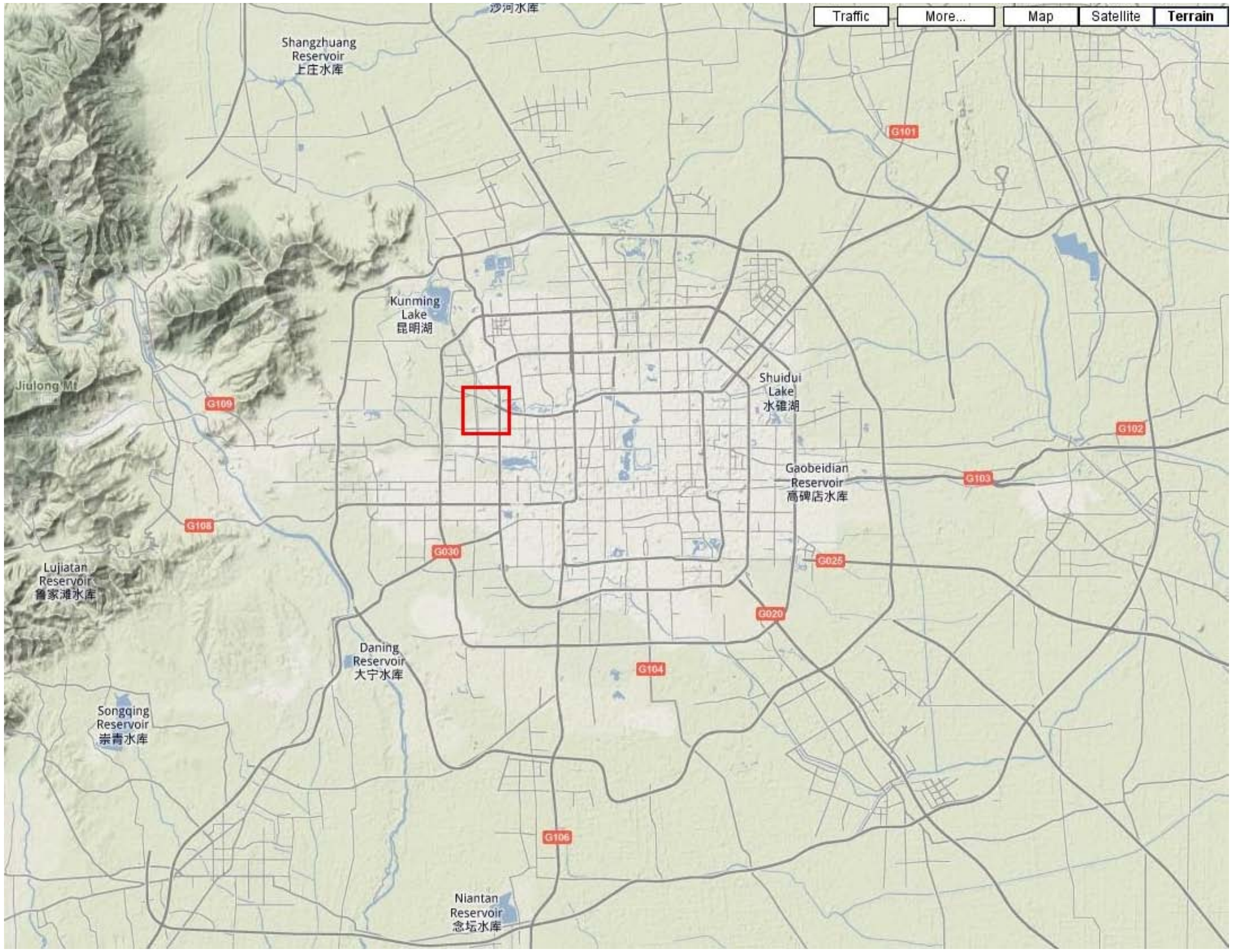
Multiscale urban flows



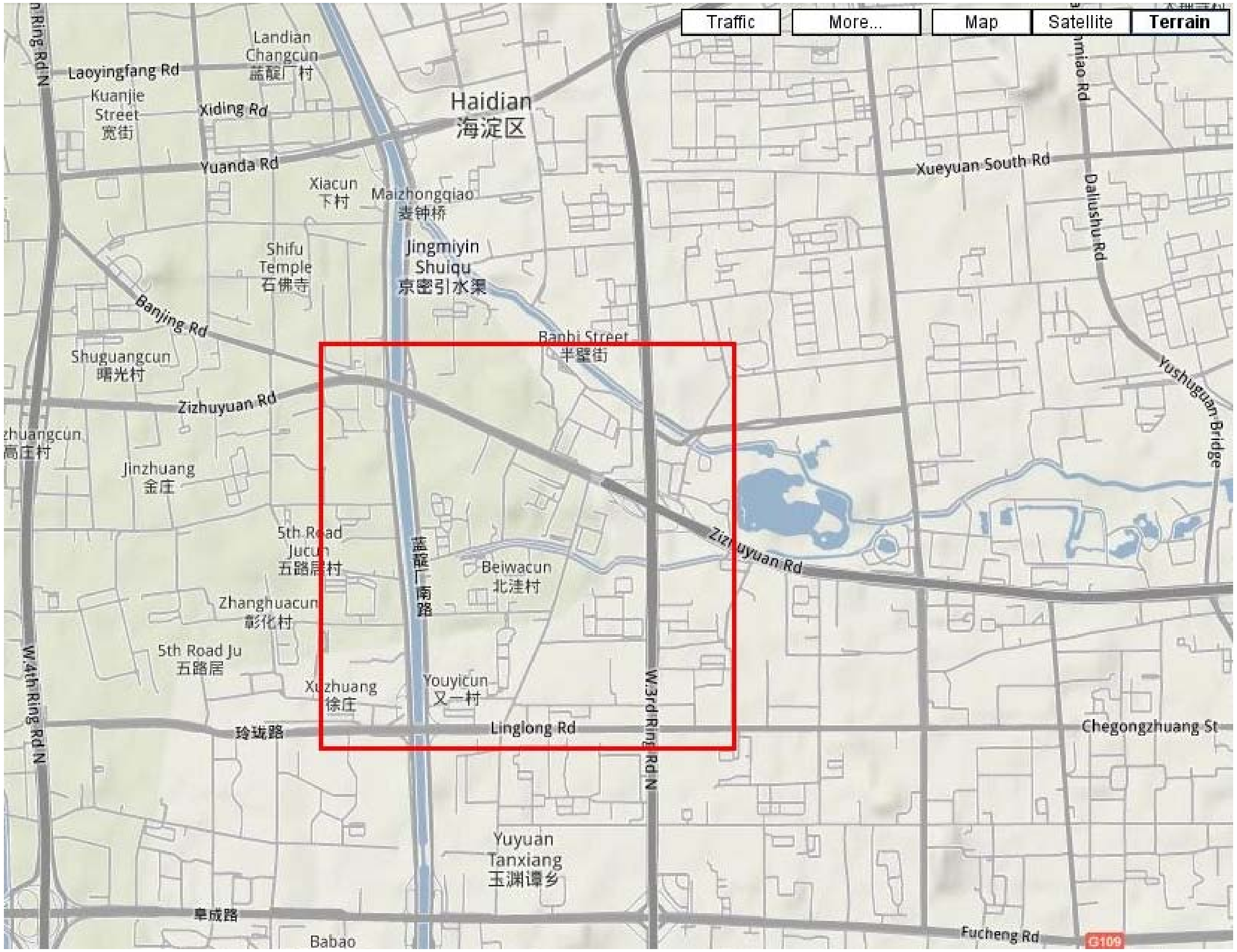
Multiscale urban flows



Multiscale urban flows

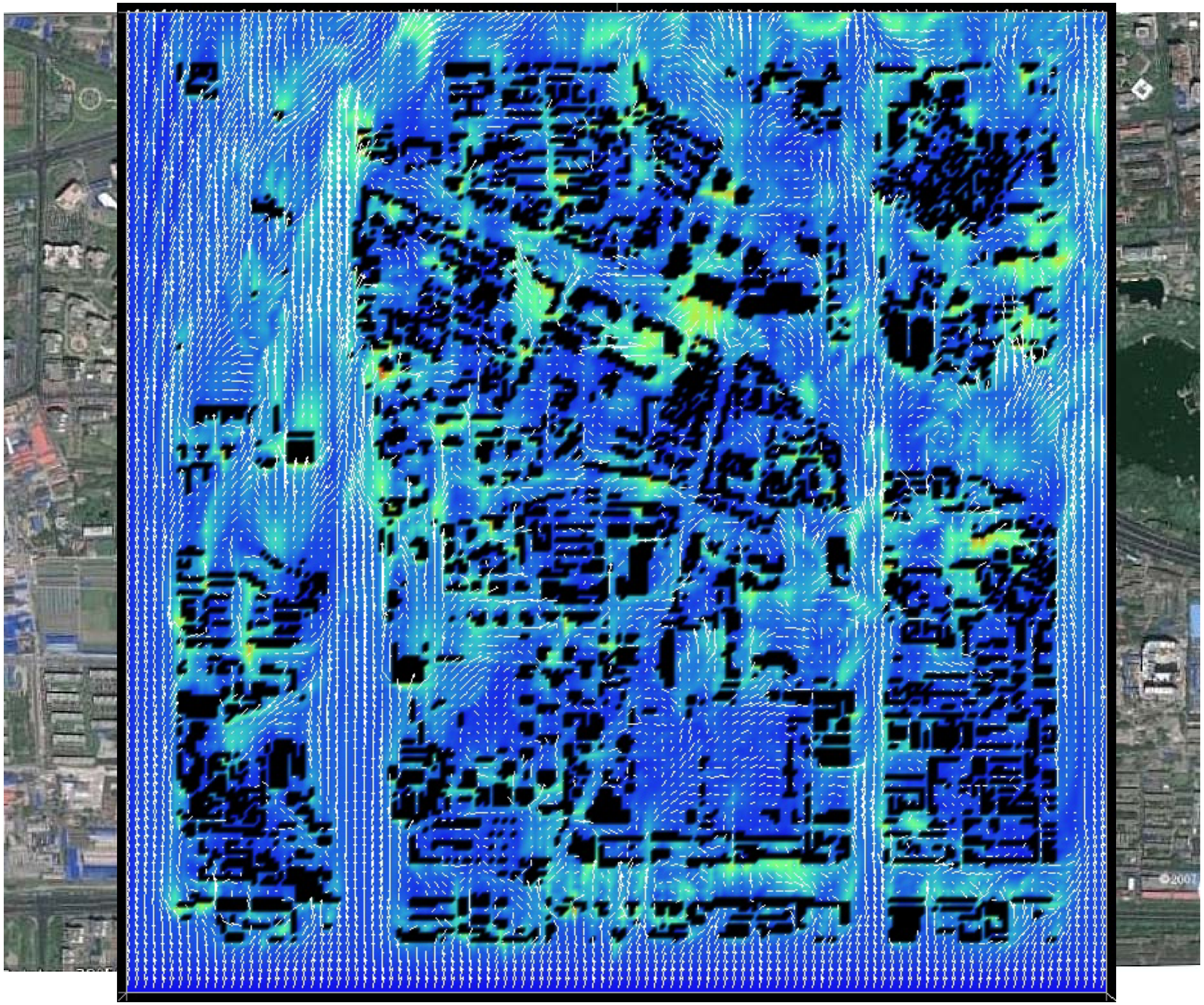


Multiscale urban flows



Multiscale urban flows

Multiscale urban flows



Multiscale urban flows
Joint Urban 2003 experiment, IOP6 daytime case

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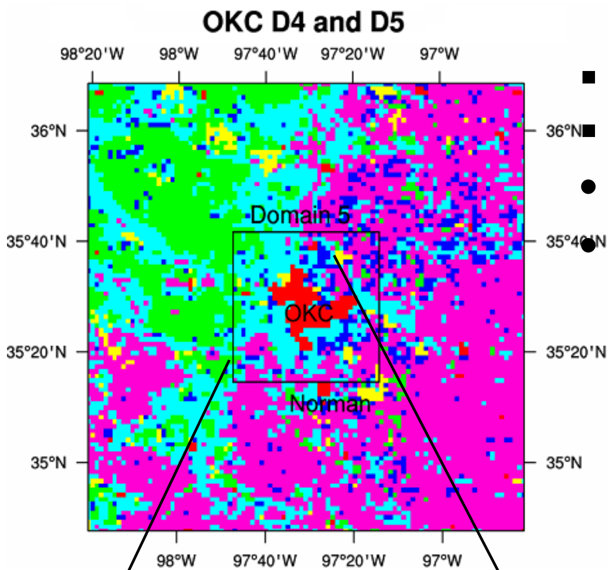
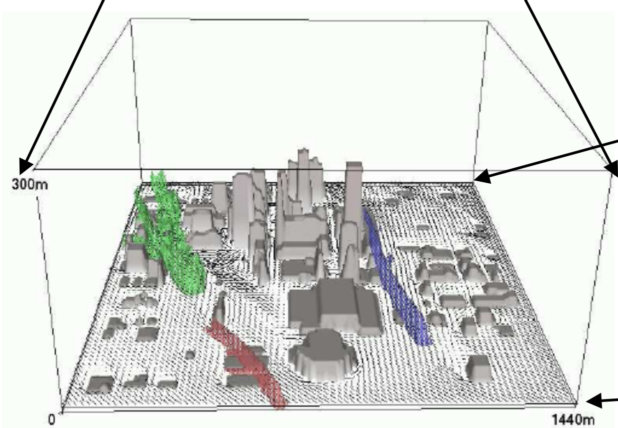
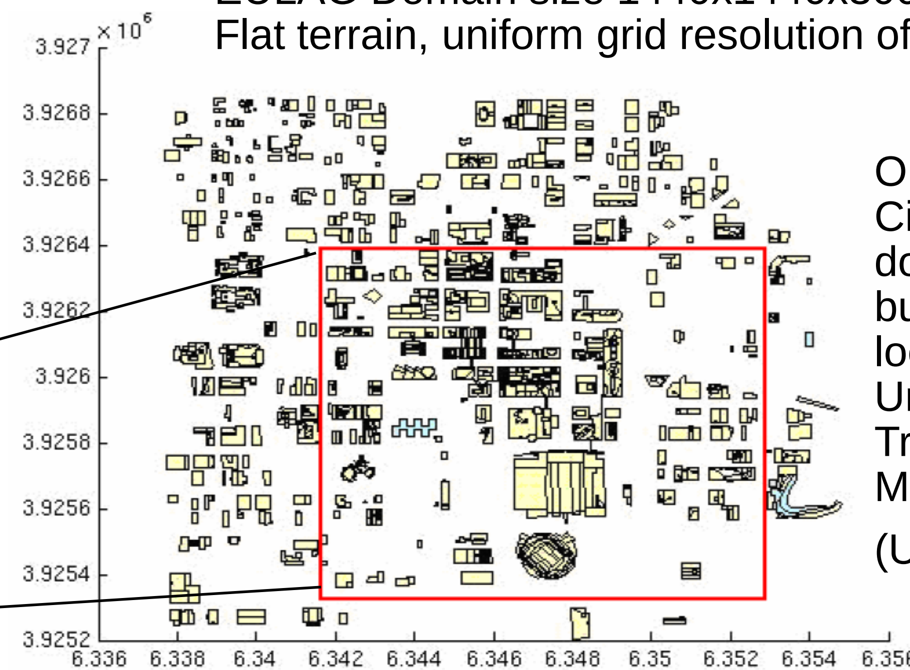


- WRF Initial and BC: NCEP EDAS data at 40 km resolution
- 12-hour forecast
- Mesoscale 500 m grid dumped at 5 minute intervals
- Data set velocity, temperature, and turbulence fields

EULAG Domain size 1440x1440x300m,
Flat terrain, uniform grid resolution of ~5m





Oklahoma City downtown, building locations at Universal Transverse Mercator

(UTM) grid

Multiscale urban flows
Joint Urban 2003 experiment, IOP6 daytime case

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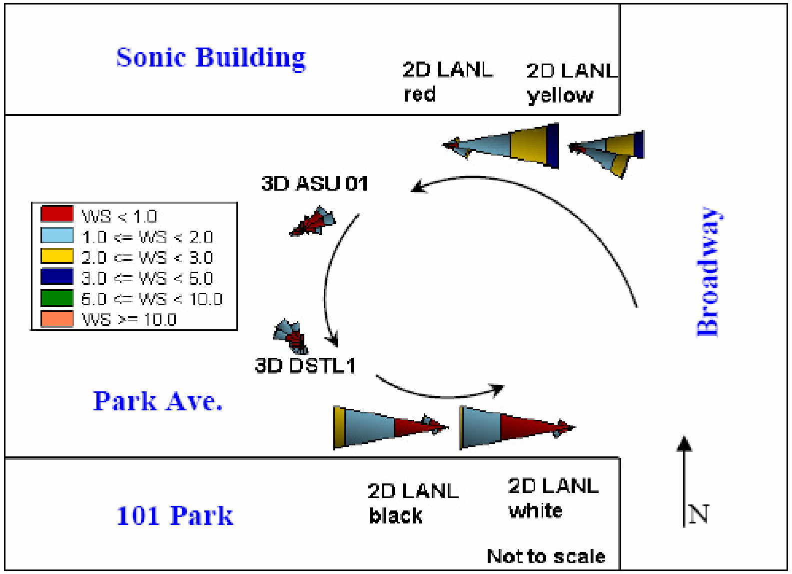
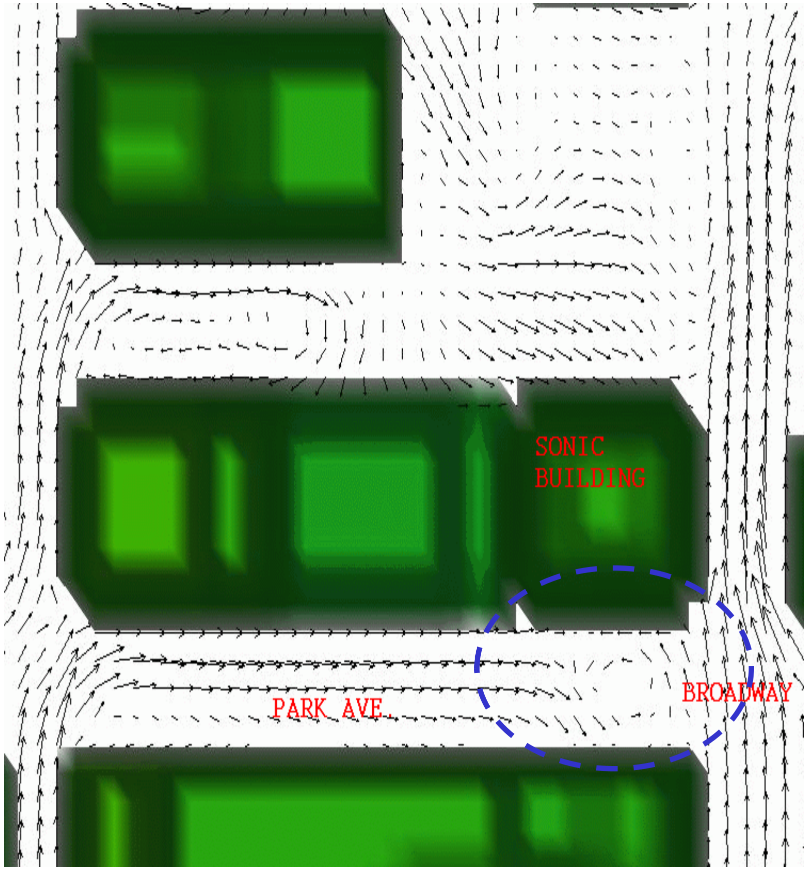
EULAG - Validation street level canyon winds

PWIDS winds



Wind rose plots indicating the presence of an end vortex at one of the street ends for a half hour period on IOP 06 (July 16, 2003, 09:00-09:30 hrs. CDT). After Pol and Brown 2004.



EULAG wind filed near Park Ave and Broadway at time 9:15 am CDT

EULAG - Validation street level canyon winds

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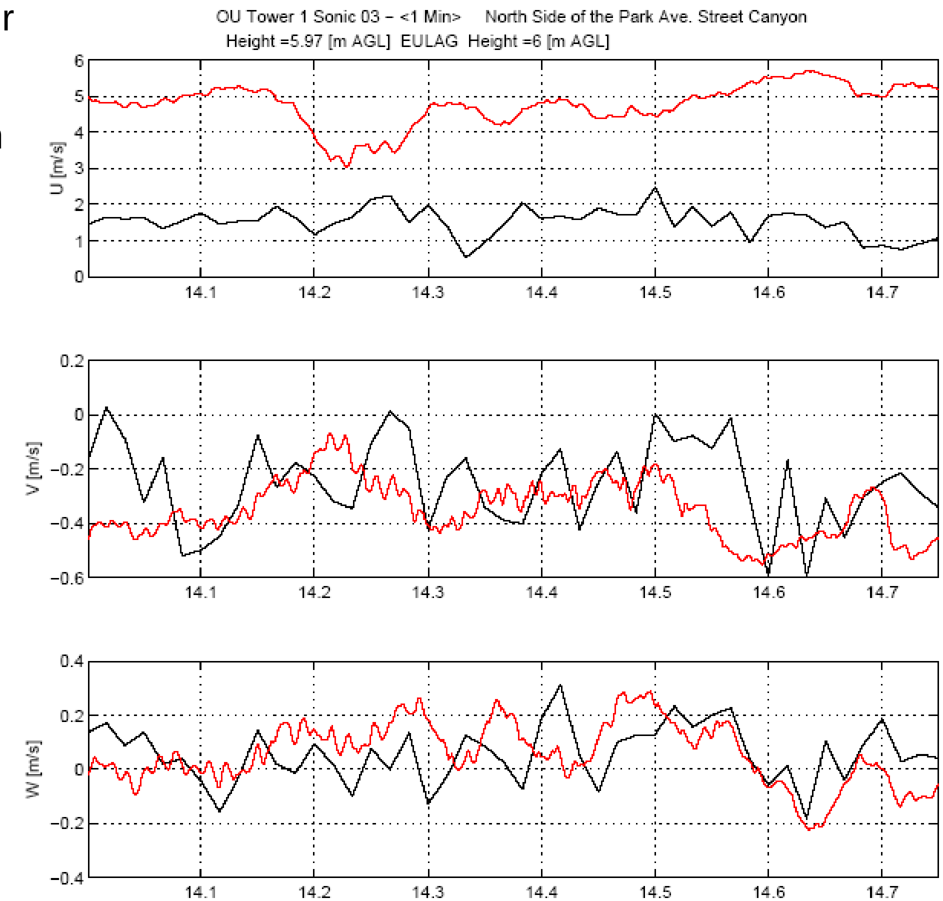
Comparison of sonic measured wind with LES EULAG simulations.

Time series of velocity fluctuations over a first hour of IOP6 cycle on July 16.

Two Oklahoma University 3D sonic towers are on the south and north side of the street canyon.

- Vertical flow structure
- Turbulence statistics
- Proper time averaging
- Grid resolution



Multiscale urban flows
Joint Urban 2003 experiment, IOP6 daytime case

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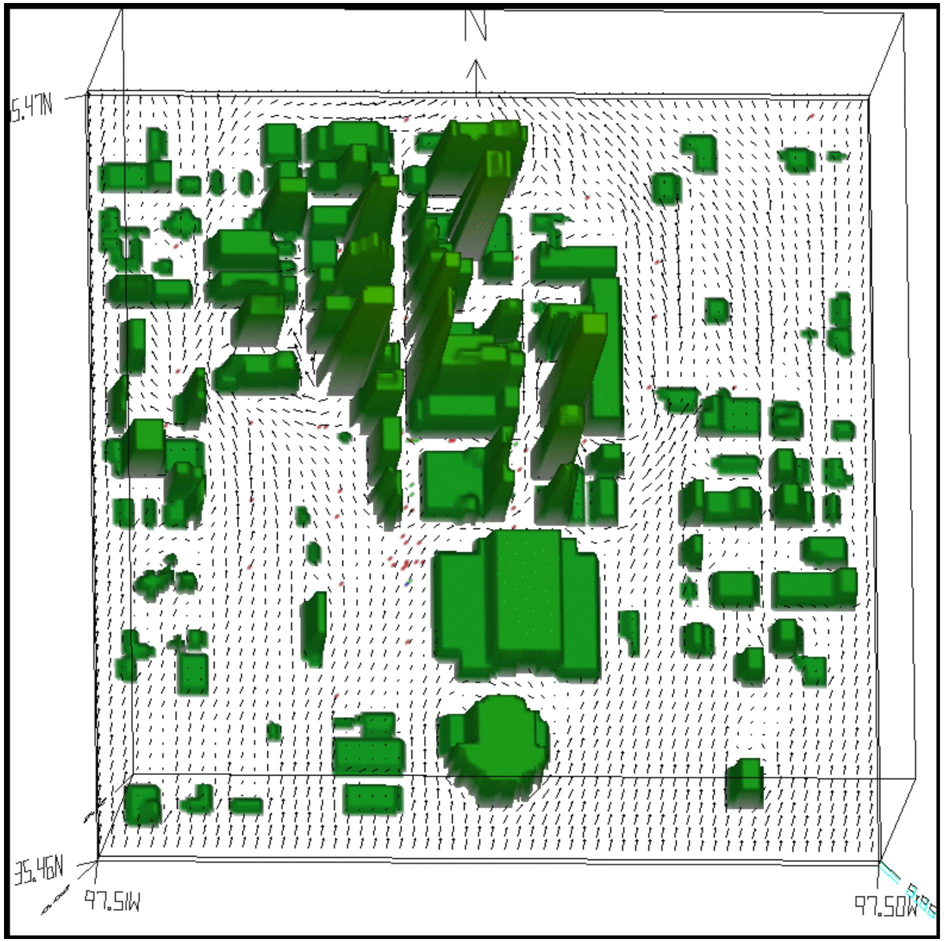
EULAG - T&D SF6 gas tracer concentrations

IOP6 release scenarios

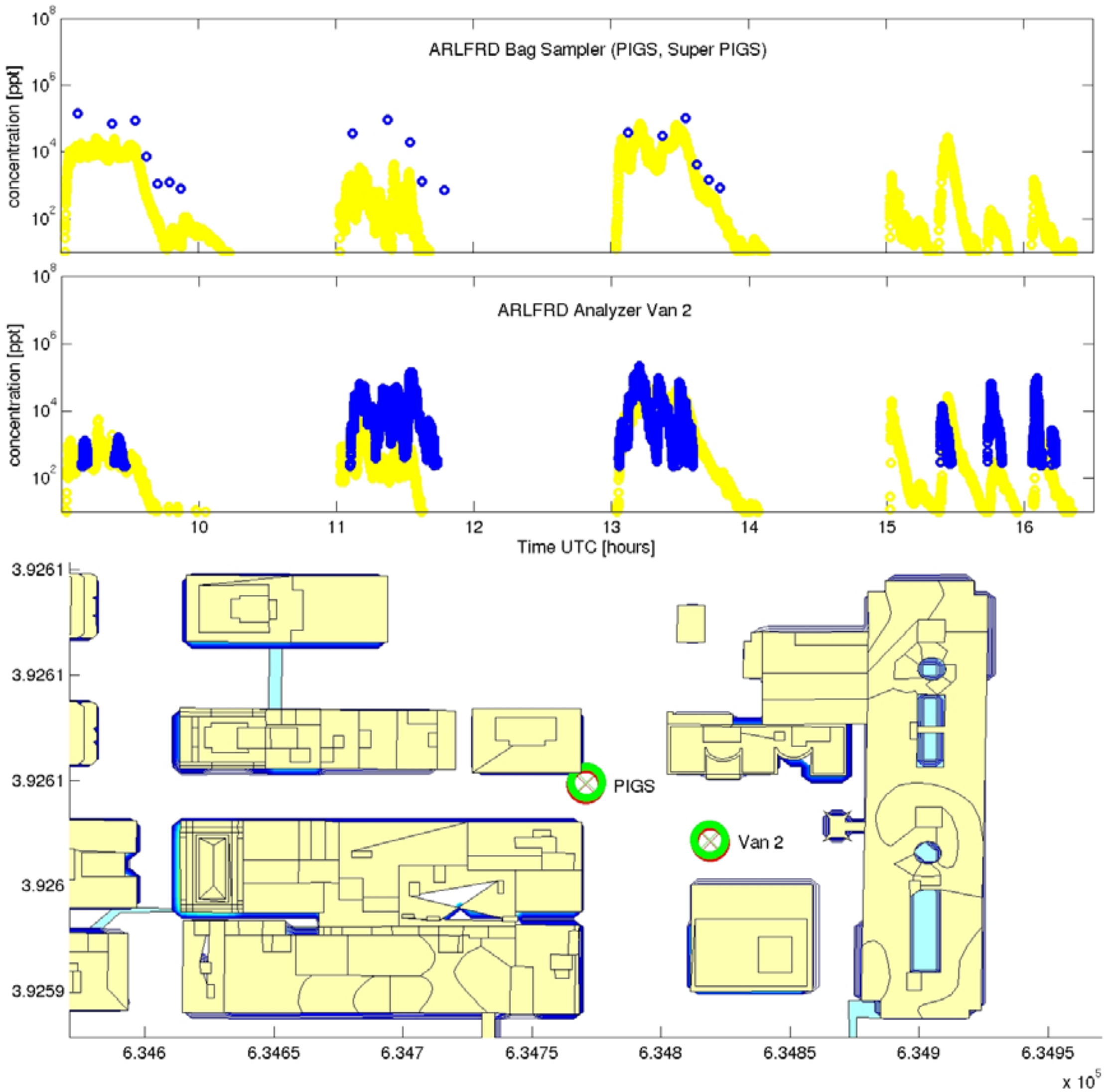
IOP	Date	Release Type	Start (CDT)	Release Amount
6	7/16/03	30-Min	0900	3.0
		30-Min	1100	3.2
		30-Min	1300	3.0
		Puff	1500	498
		Puff	1520	499
		Puff	1540	510
		Puff	1600	500

source located at Botanical Gardens (near Sheridan & Robinson).

Dispersion footprint for IOP6 9:00 am



Multiscale urban flows
Joint Urban 2003 experiment, IOP6 daytime case

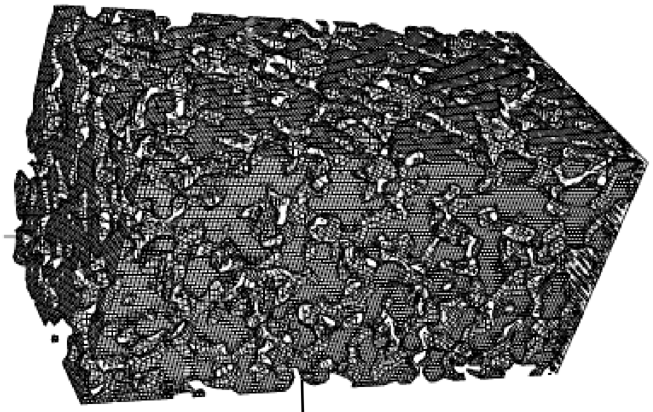


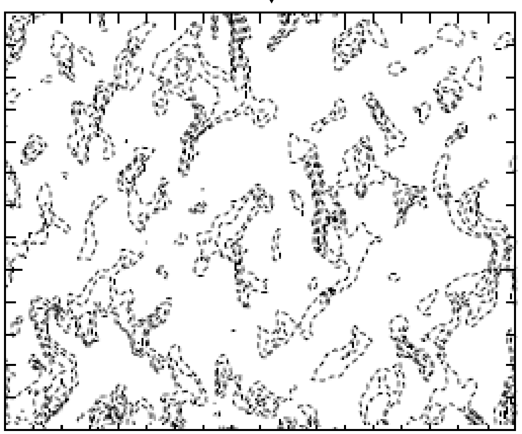
Model time series (yellow) of SF6 tracer sampled during IOP6 vs. observations (blue circles)

Darcy scale flows in porous media

Smolarkiewicz & Winter JCP (2010)

Porous medium





Microscopic flow from DNS

Stokes' regime - Stokes' drag is an effective resistive force of porous media on fluid parcels

Microscopic Reynolds number

$$\mathrm{Re} = ru / \nu \ll 1$$

r - characteristic scale of the pores

u - characteristic velocity scale

ν - kinematic viscosity of fluid

Violation of the creeping motion assumption

$$\mathrm{Re} \geq 1$$

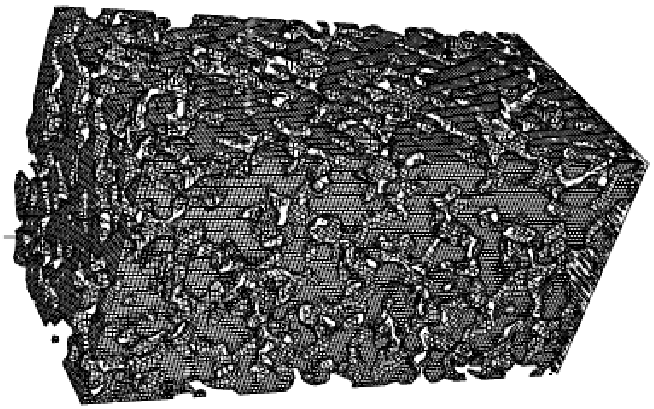
Transitional range where viscous effects are relatively small and the inertial forces dominate

$$\mathrm{Re} > 1$$

Motivation to use the Navier-Stokes equations (DNS) to problems with a broad range of hydraulic conductivities depending upon the media permeability and viscosity of the fluid

Urban flows analogy / diversity

Random construction of the pore space in unit volume



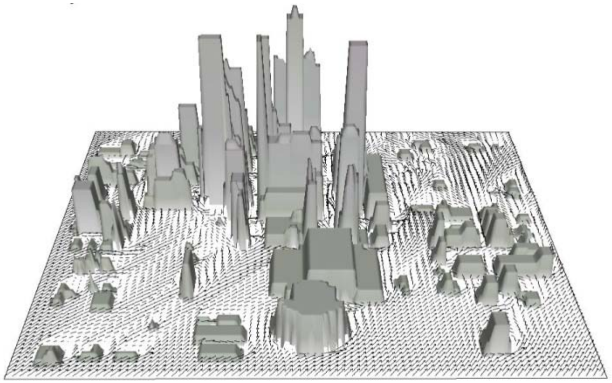
Smolarkiewicz & Winter JCP (2010)

$$\mathrm{Re} = ru / \nu \ll 1$$

$$\mathrm{Re} \geq 1$$

Statistical representation of the flow field properties in the unit volume of porous media

Construction of the pore space horizontally inhomogeneous and changes with height



Stokes properties violated in LES of the atmospheric urban flows

$$r \sim 10^1 \ldots 10^2 m$$

$$\nu \sim 1.5 \cdot 10^{-5} m^2 / s$$

$$u \sim 10^0 \ldots 10^1 m / s$$

$$\mathrm{Re} = ru / \nu \sim 10^5 \cdots 10^7$$

$$\mathrm{Re} \gg 1$$

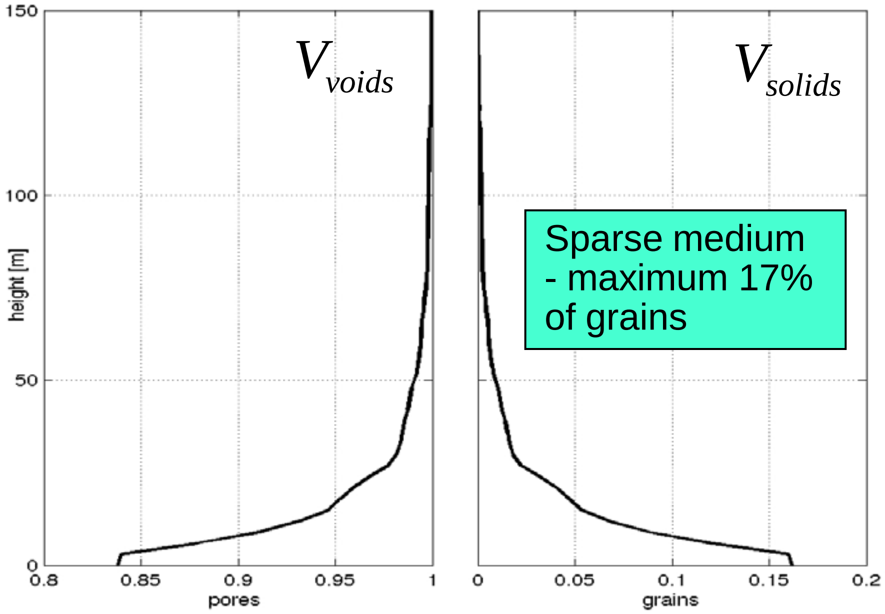
Flow field dominated by large coherent structures and local turbulent fluctuations
– horizontal inhomogeneity
– properties changes in vertical

Characteristics of urban ‘porous’ media

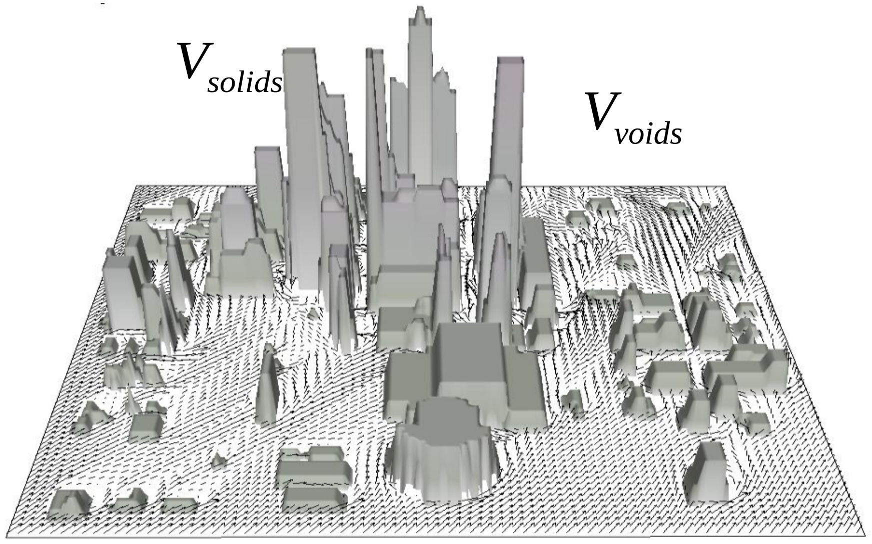
Porosity factor - fraction of the media available for transport, and may include both open and closed volumes

$$\phi = \frac{V_{voids}}{V_{voids} + V_{solids}}$$

Normalized vertical distribution of pores & grains



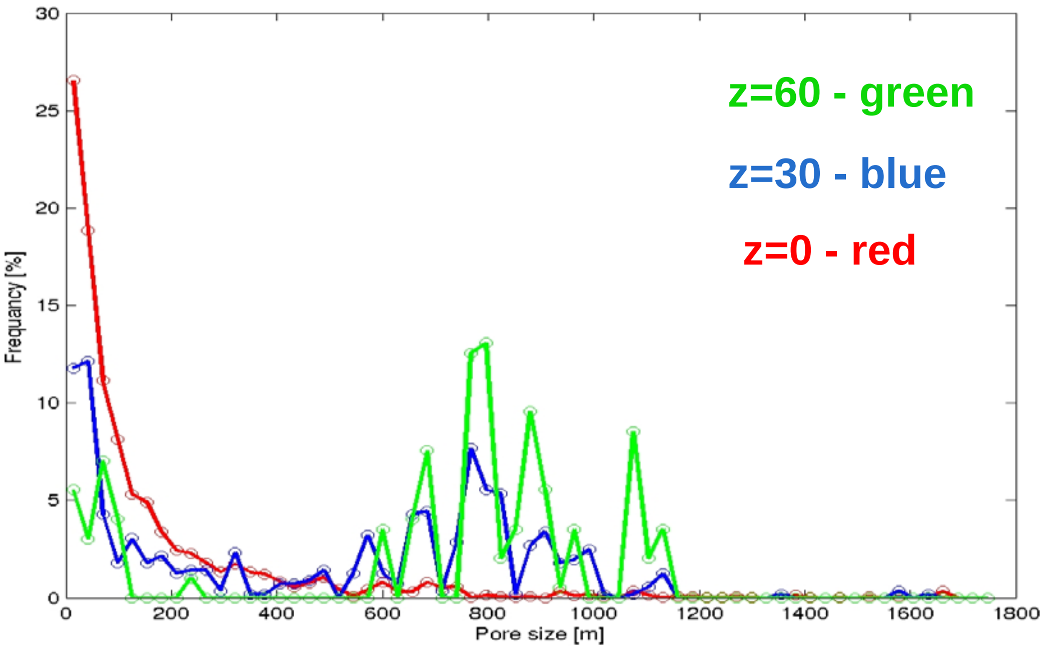
3D building structure



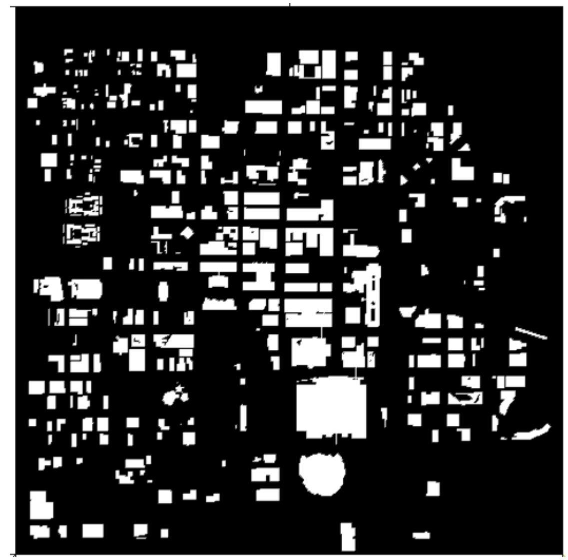
dx=dy=4m dz=3m dt=0.25s
N=M=448 L=51

Characteristics of urban ‘porous’ media

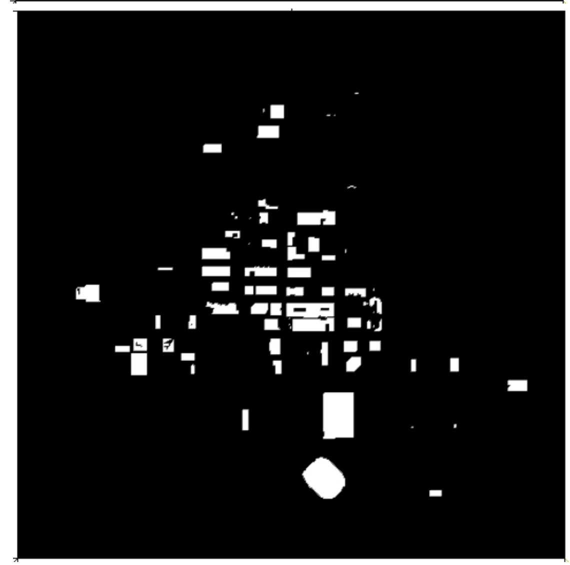
Pore size frequency distributions



z=0



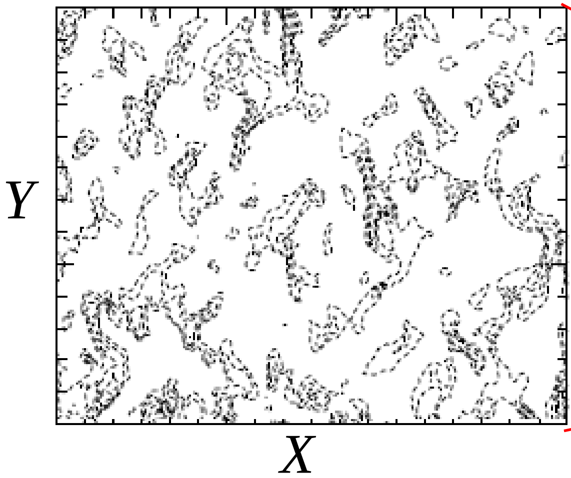
z=60

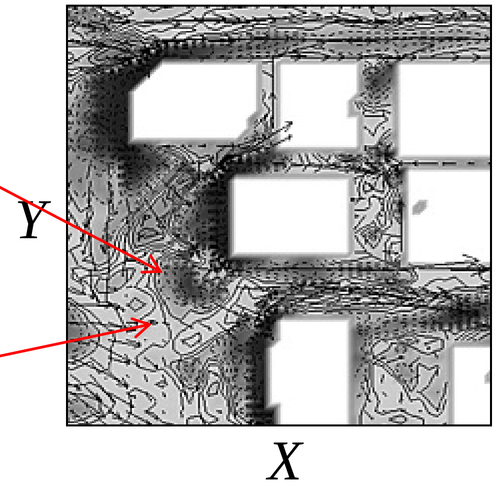


Urban flows analogy / diversity

Random construction of the pore space in unit volume

Construction of the pore space horizontally inhomogeneous and changes with height





Darcy scale flows

$$u = -(\kappa / \mu)\nabla p$$

$$\kappa \sim -\frac{\nu < u >_{XY}}{< (\rho^{-1}\partial p / \partial x) >_{XY}}$$

μ - dynamic viscosity

ν - kinematic viscosity

κ – permeability factor (measure of the ability to transmit fluids)

LES scale flows

$$< \ldots >_{XY} \sim 1/N \sum_{i=1}^{N} < \ldots >_{\Delta x \Delta y}$$

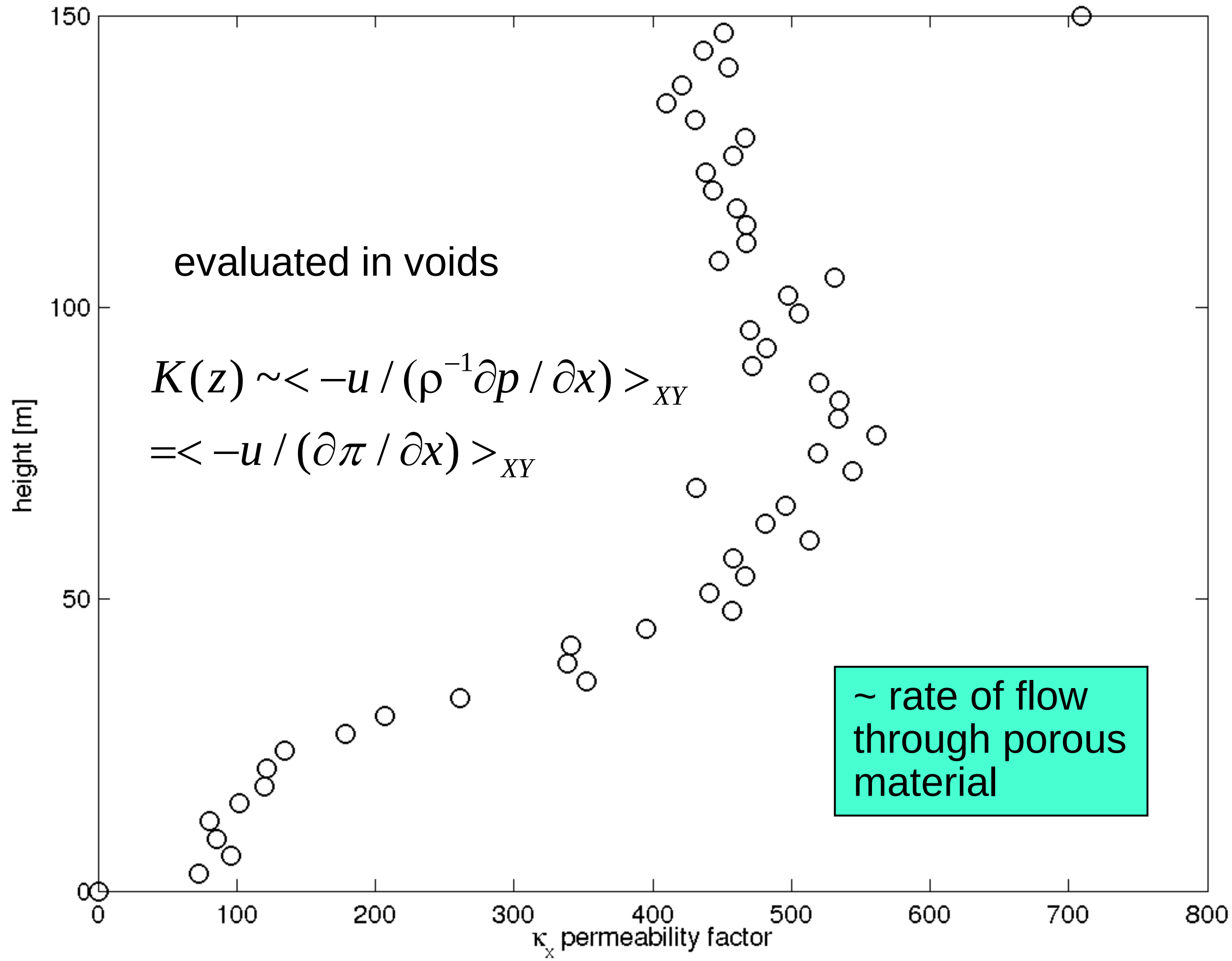
$$< \kappa > \sim - < \frac{u}{\rho^{-1}\partial p / \partial x} >_{XY}$$

Averaging true within LES model grid volume and in areas with small Re number

Evaluating urban ‘porous’ media analogy

NCAR

Effective conductivity in function of the model height



Characteristics of urban media flow

Tortuosity – characteristic nondimensional flow parameter within the porous structures

$$T(z) = \frac{\int_A v(l)\lambda(l)dl \,/ \int_A v(l)dl}{L} = \frac{<\lambda>}{L}$$

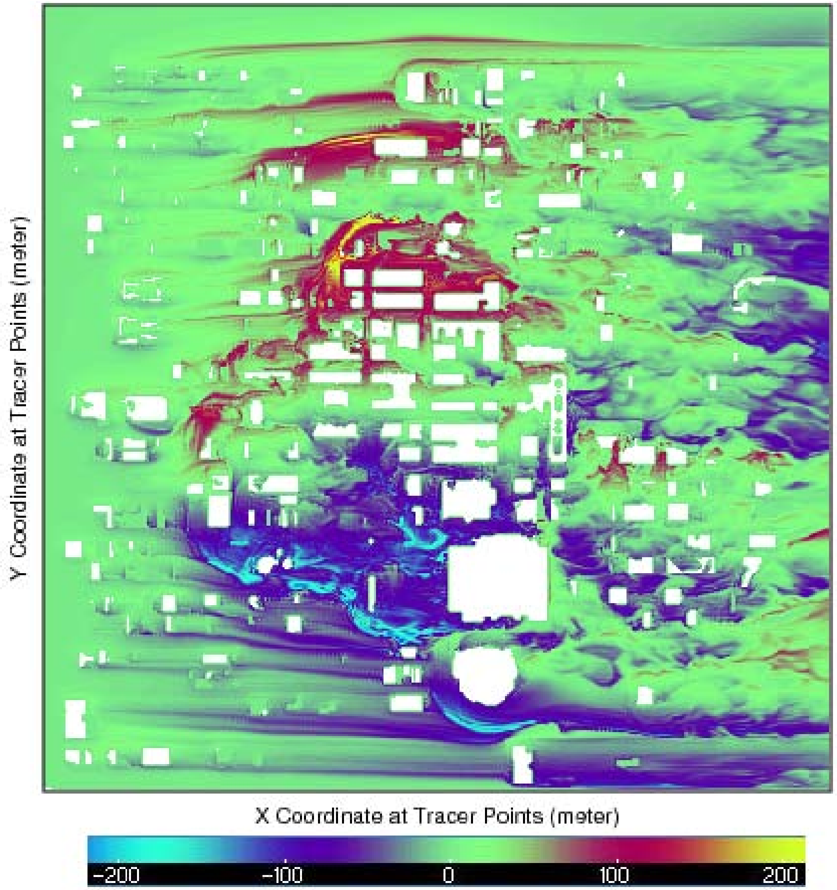
λ - length of flow trajectories passing through an arbitrary crosssection *A* in a time unit
L - length of the media sample
v – velocity component in *l* direction

- quantifies an effective length of a path that fluid must travel in order to navigate through the media.
- not easily computable with mesh-based methods
- employ Lagrangian displacements δ - tortuosity measure which consists of statistics of the Eulerain field

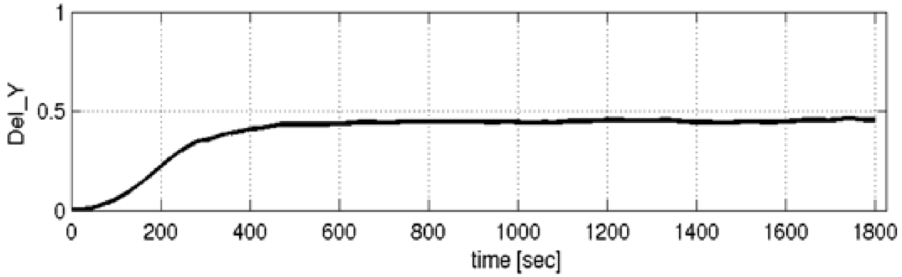
$$\frac{d\delta(\overline{\mathbf{x}},t)}{dt} = \mathrm{v}(\overline{\mathbf{x}},t) \qquad \delta(\overline{\mathbf{x}},t) = \overline{\mathbf{x}} - \overline{\mathbf{x}}_0$$

Structure of the cross wind component of the Lagrangian displacements

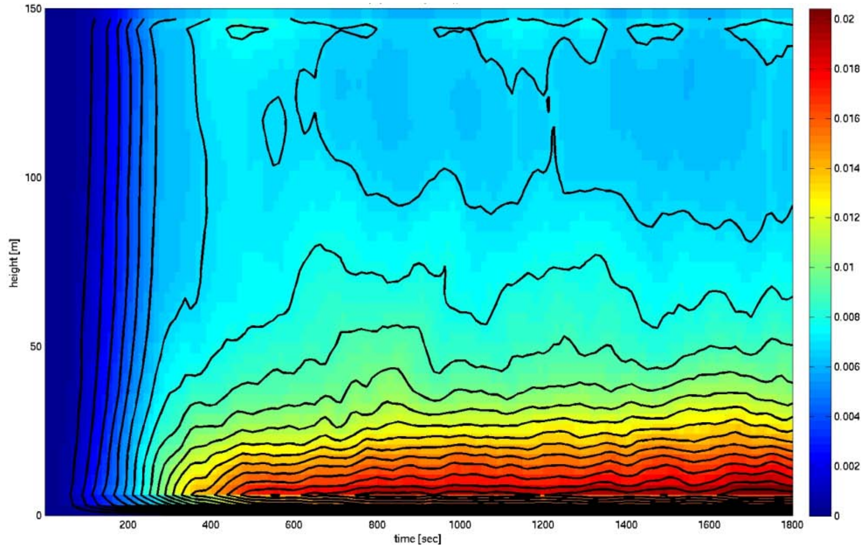
Horizontal cross-section at the height of 10 m above the ground



Domain averaged *L2* norms history

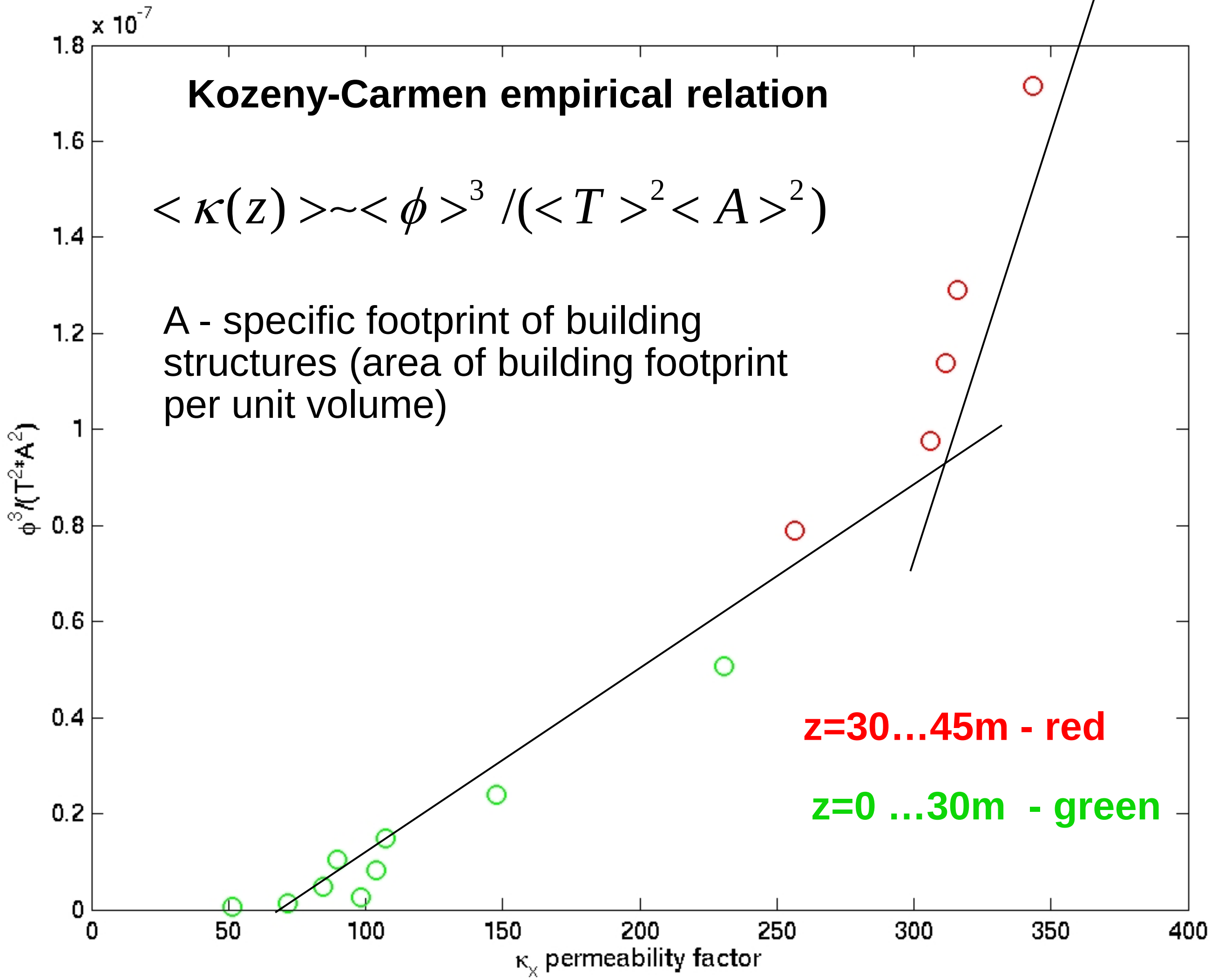


Horizontally averaged time history in the function of the model height.



Evaluating urban 'porous' media analogy

Relation between conductivity, porosity and tortuosity



Future work

- More validation against wind tunnel experiments
- Higher order boundary reconstruction methods
- Near wall flow model
- Sources (fluxes) for heat and scalars at building walls and rooftops