Simulation of breaking inertia-gravity waves using DNS and LES

Mark Fruman   Ulrich Achatz
IAU, Goethe Universität-Frankfurt

Sebastian Remmler   Stefan Hickel   Felix Rieper
Technische Universität München   Deutscher Wetterdienst

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- Three-dimensionalization of IGW breaking
- Three-dimensional DNS of a breaking IGW

Inertia-gravity waves (motivation)

- Atmospheric gravity waves carry momentum from the location of generation and deposit it where they “break”
- play an important role in driving circulation (cf. QBO, cold summer mesopause)
- Spatial scales from below model grid and observational instrument resolution (tens of metres) to synoptic scales (hundreds of kilometres)

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⇒ effect on mean flow in climate and forecast models must be parameterized:

  - Relate dynamics of breaking (time scales, saturation amplitude, . . .)
  - to properties of the background (stratification, shear, . . .)
  - and wave properties (amplitude, wavelength, direction, . . .)

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- Gravity wave breaking is a physically relevant test case for LES of stratified turbulence

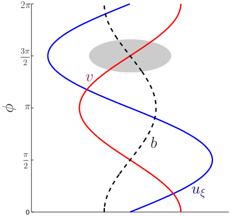
Inertia-gravity waves

- Consider the inviscid Boussinesq Equations on an f plane with constant background stratification N:

$$\frac{Du}{Dt} - fv = -\frac{\partial P}{\partial x}, \qquad \frac{Db}{Dt} + N^2 w = 0,$$

$$\frac{Dv}{Dt} + fu = -\frac{\partial P}{\partial y}, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

$$\frac{Dw}{Dt} - b = -\frac{\partial P}{\partial z},$$



- An inertia-gravity wave is a solution of the form

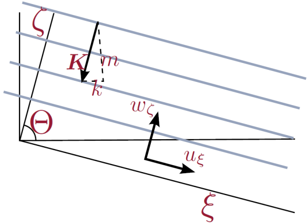
$$[\hat{u}, \hat{v}, \hat{w}, \hat{b}] \exp\left[i(kx + mz - \omega t)\right]$$

where:

$$\boxed{\omega^2 = \left(\frac{k^2}{k^2+m^2}\right) N^2 + \left(\frac{m^2}{k^2+m^2}\right) f^2} \quad \text{and} \quad \boxed{[\hat{u}, \hat{v}, \hat{w}, \hat{b}] = a\left[\frac{\omega}{k}, -i\frac{f}{k}, -\frac{\omega}{m}, i\frac{N^2}{m}\right]}$$

IGW stability problem (Achatz Phys. Fluids 2005, JAS 2007)

- Stability problem best solved in a reference frame rotated such that vertical is parallel to the wavevector of wave and moving with the phase speed of the wave. Let ξ be the rotated x-coordinate, ζ the rotated vertical coordinate, and (u_ξ, w_ζ) the corresponding velocity components.



- Boussinesq equations in the rotated frame:

$$\frac{Du_\xi}{Dt} - f \sin\Theta \; v + \frac{\partial p}{\partial \xi} + \cos\Theta \; b = \nu \nabla^2 u_\xi, \qquad \frac{Db}{Dt} + N^2(-\cos\Theta \; u_\xi + \sin\Theta \; w_\zeta) = \mu \nabla^2 b,$$

$$\frac{Dv}{Dt} + f(\sin\Theta \; u_\xi + \cos\Theta \; w_\zeta) + \frac{\partial p}{\partial y} = \nu \nabla^2 v, \qquad \frac{\partial u_\xi}{\partial \xi} + \frac{\partial v}{\partial y} + \frac{\partial w_\zeta}{\partial \zeta} = 0,$$

$$\frac{Dw_\zeta}{Dt} - f \cos\Theta \; v + \frac{\partial p}{\partial \zeta} - \sin\Theta \; b = \nu \nabla^2 w_\zeta .$$

- Equations are discretized in space and time and solved numerically.

IGW stability problem (Achatz Phys. Fluids 2005, JAS 2007)

- Inertia gravity wave (IGW) case study:
  - wavelength 6 km; propagation angle 89.5°
  - nondim. amplitude 1.2 $\Rightarrow$ statically unstable
  - Very long period ($\sim$ 8 hours)

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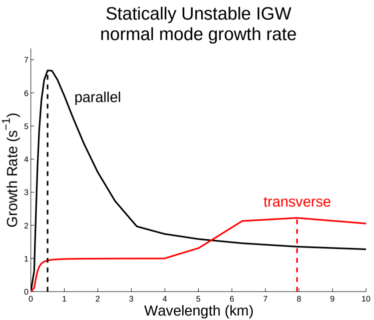
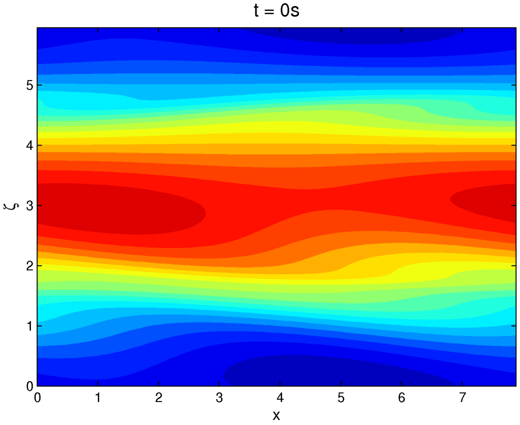
- Linearize system about IGW solution, seek normal modes:



Growth rates of leading parallel and transverse normal modes

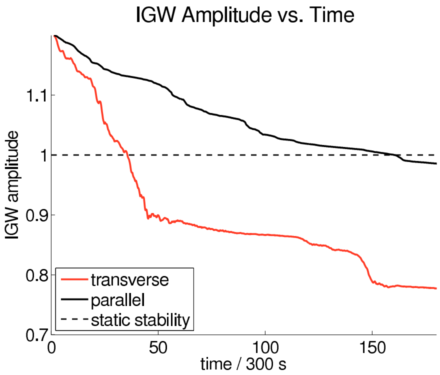
IGW stability problem (Achatz Phys. Fluids 2005, JAS 2007)

- Linear instability modes used to initialize "2.5 dimensional" nonlinear simulations
- e.g. buoyancy perturbation from IGW pertubed by leading transverse mode (20 minute integration):



IGW stability problem (Achatz Phys. Fluids 2005, JAS 2007)

- 2.5D nonlinear simulations initialized with leading transverse and parallel normal modes, projected onto IGW mode:



- Nonlinear DNS shows transverse normal mode leads to more dissipation of the wave.

The DNS take ~~weeks~~ days . . . can an LES method get same result?

Comparison of 2.5D LES of breaking IGW

1. Smagorinsky scheme
   - Filtered Reynolds' average stress and heat flux modeled as an eddy viscosity and diffusion, with eddy viscosity proportional to local rate of strain

$$\overline{v_i' v_j'} \approx -\nu_{sgs} \left(\frac{\partial \overline{v}_i}{\partial x_j} + \frac{\partial \overline{v}_j}{\partial x_i} \right), \quad \nu_{sgs} = (C_s \Delta)^2 \sqrt{\frac{1}{2} \sum_{i,j=1}^{3} \left(\frac{\partial \overline{v}_i}{\partial x_j} + \frac{\partial \overline{v}_j}{\partial x_i} \right)^2}$$

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2. MUSCL
   - Monotone Upwind-centred Schemes for Conservation Laws
   - Finite-volume scheme using flux limiting: high-order central-difference discretization everywhere except near local extrema where low-order upwind scheme used.
   - Due to numerical dissipation of upwind scheme, behave somewhat like an implicit LES.

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3. Implicit LES scheme ALDM . . .

ALDM (Hickel et al. JCP 2006, Hickel & Adams Phys. Fluids 2007)

- Generic 1-D conservation law:

$$\boxed{\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left[F(u)\right] = 0}$$

Filtering: LES model:

$$\frac{\partial u_N}{\partial t} + \frac{\partial}{\partial x} F_N(u_N) = \mathcal{G}_{SGS}$$

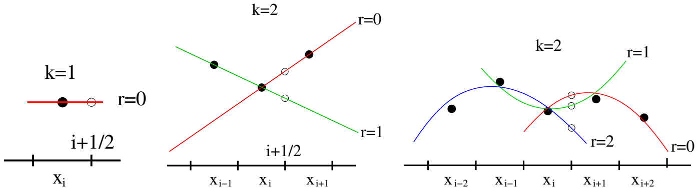
Discretization: Finite Volume Method:

$$\frac{\partial u_N}{\partial t} + \frac{\partial}{\partial x} F_N(u_N) = \mathcal{G}_{num}$$

- In practice, $\mathcal{G}_{num}$ is comparable in size to $\mathcal{G}_{SGS}$, which motivates … Implicit LES:
  - Discretize F_N and u_N such that the truncation error $\mathcal{G}_{num}$ acts as an implicit sub-gridscale model $\mathcal{G}_{SGS}$.
  - Principle behind Adaptive Local Deconvolution Method (ALDM)

ALDM (Hickel et al. JCP 2006, Hickel & Adams Phys. Fluids 2007)

- Reconstructed $\tilde{u}$ at each cell face is a function of 6 interpolating polynomials $p_{k,r}(x)$ (where $k = 1, \ldots, 3$ and $r = 0, \ldots, k-1$):

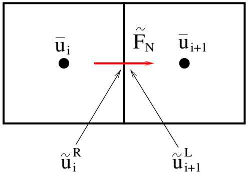


$$\tilde{u} = \sum_{k=1}^{3} \sum_{r=0}^{k-1} \omega_{k,r} p_{k,r}(x), \quad \text{where} \quad \omega_{k,r} \sim \frac{1}{3} \gamma_{k,r} \beta_{k,r}(u)$$

- $\beta_{k,r}$ – smoothness of stencil (k, r), $\gamma_{k,r}$ – tunable parameters

ALDM (Hickel et al. JCP 2006, Hickel & Adams Phys. Fluids 2007)

- Add nonlinear numerical viscosity:



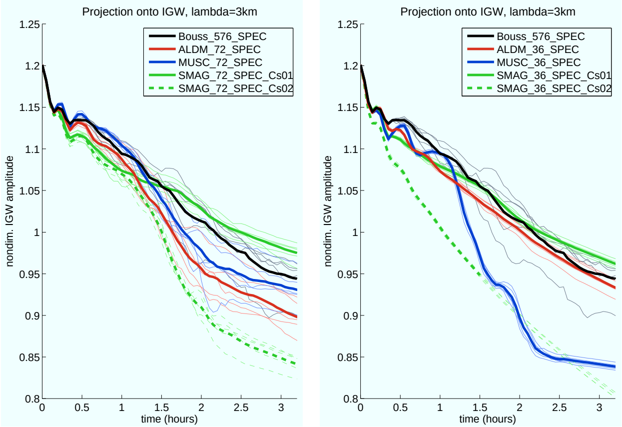
$$\tilde{F}_N(x_{i+1/2}) = F\left(\frac{\tilde{u}^R_{i+1/2} + \tilde{u}^L_{i+1/2}}{2}\right) - \sigma_{i+1/2}(\tilde{u}^R_{i+1/2} - \tilde{u}^L_{i+1/2})$$

where $\sigma_{i+1/2} = \sigma^{\rho u} \left|\overline{u}_{i+1} - \overline{u}_i\right|$ ($\sigma^{\rho u}$ is another tunable parameter)

- 5 independent parameters σ, γ^0_{31}, γ^+_{31}, γ^+_{32}, γ^+_{21} tuned once and for all so that spectral energy flux matches DNS for 3D, homogeneous, isotropic turbulence.

Comparison of 2.5D LES of breaking IGW

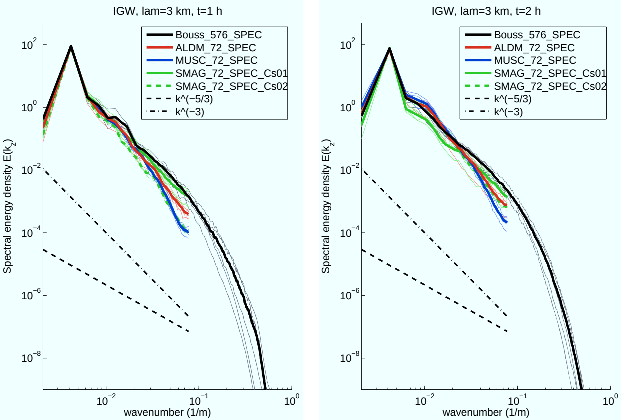
- Projection of nonlinear solution onto free-mode corresponding to unstable 3 km IGW as a function of time.



- Quantity of interest for gravity wave drag parameterization is final amplitude to which a wave decays after it breaks
- MUSCL and Smagorinsky w/ $C_s = 0.2$ too dissipative at low resolution.

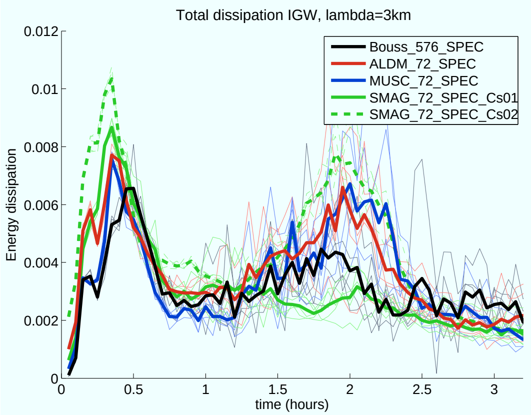
Comparison of 2.5D LES of breaking IGW

- ξ-averaged energy spectra after one and two hours model time.



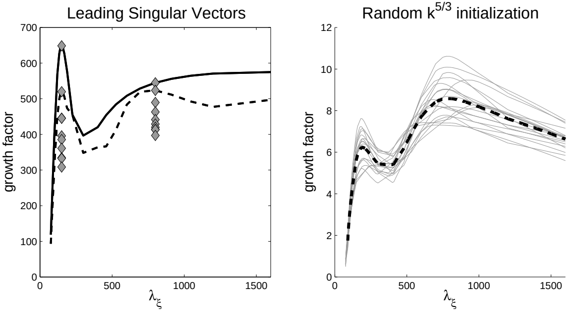
Comparison of 2.5D LES of breaking IGW

- Total energy dissipation vs. time



Three-Dimensionalization (Fruman & Achatz, JAS 2012)

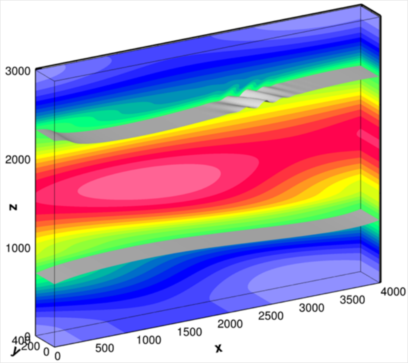
- Linearize Boussinesq equations about time dependent 2.5D nonlinear integration
- Seek singular vectors: initial perturbations whose energy grows by largest factor in given optimization time



- Identify length scale of fastest growing perturbations in third dimension

3D DNS of breaking IGW

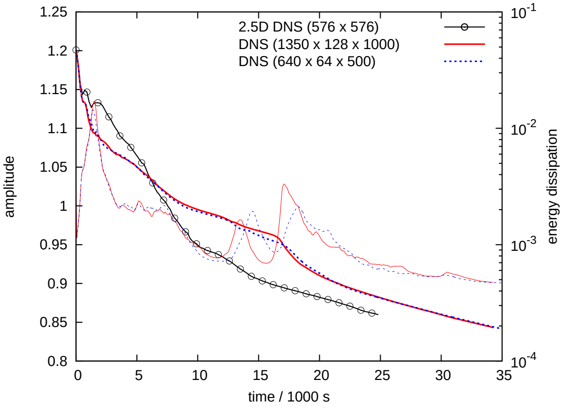
- Initial condition of buoyancy field for 3D DNS:
  - Unstable IGW + transverse NM + secondary SV
  - Domain 3 km × 4 km × 400 m.



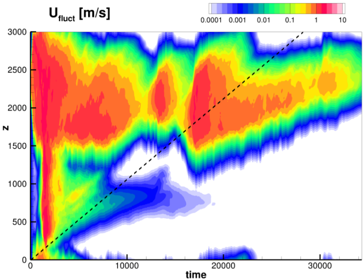
- Movie of Q criterion

3D DNS of breaking IGW

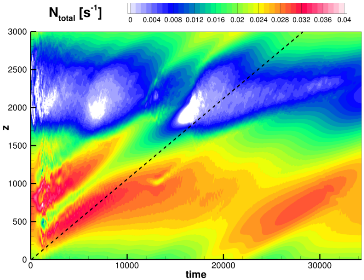
- Projection of solution on IGW (thicker line)
- Total energy dissipation (thinner line)



3D DNS of breaking IGW



Energy fluctuations (total energy less mean in ξ direction)



Total buoyancy frequency N

(dashed line represents point fixed in space)

Summary

- Inertia-gravity wave breaking is important to the circulation in the atmosphere but, due to the wide range of scales involved, is difficult to study in detail.
- Linear stability analysis in a reference frame moving with the wave gives the dominant length scales of the breaking in the plane perpendicular to the wavevector.
- Gravity-wave breaking a physically relevant test case for LES of stratified turbulence.

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- Gravity-wave breaking a physically relevant test case for LES of stratified turbulence.
- Comparison of LES schemes in 2.5D:
  - Classic Smagorinsky scheme with coefficient $C_s = 0.1$ agrees best with DNS in terms of the amplitude decay of the wave and the spectral energy density, but results very sensitive to choice of C_s.
  - Implicit LES scheme ALDM somewhat too dissipative but shows consistent performance under change of resolution.
  - At sufficiently high resolution, numerical scheme MUSCL might be an effective LES.

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  - 3D LES (underway)
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THANK YOU FOR YOUR ATTENTION!