

# Toward all-scale simulation of *moist* atmospheric flows with soundproof equations

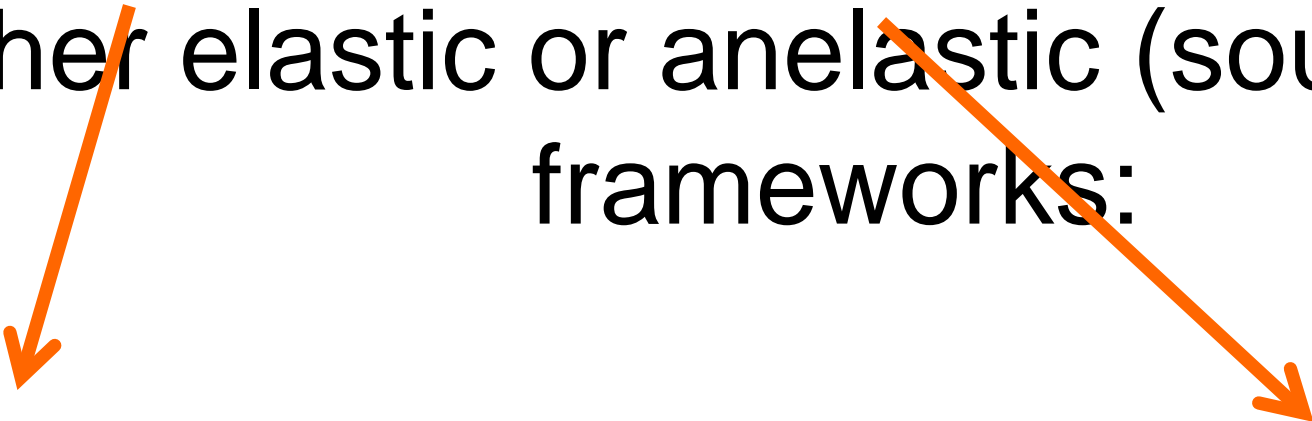
Wojciech W. Grabowski<sup>1</sup>, Marcin J. Kurowski<sup>2</sup>,  
Zbigniew P. Piotrowski<sup>2</sup>, and Piotr K. Smolarkiewicz<sup>1</sup>

*<sup>1</sup>National Center for Atmospheric Research  
Boulder, USA*

*<sup>2</sup>Institute of Meteorology and Water Management  
Warsaw, Poland*



# Traditional small-scale Boussinesq models are based on either elastic or anelastic (sound-proof) frameworks:



## **The Simulation of Three-Dimensional Convective Storm Dynamics**

JOSEPH B. KLEMP

*National Center for Atmospheric Research<sup>1</sup>, Boulder, Colo. 80307*

ROBERT B. WILHELMSON

*University of Illinois, Urbana 61801*

(Manuscript received 22 September 1977, in final form 21 February 1978)

### **ABSTRACT**

A new three-dimensional cloud model has been developed for investigating the dynamic character of convective storms. This model solves the compressible equations of motion using a splitting procedure which provides numerical efficiency by treating the sound wave modes separately. For the subgrid turbulence processes, a time-dependent turbulence energy equation is solved which depends on local buoyancy, shear and dissipation. First-order closure is applied to nearly conservative variables with eddy coefficients based on the computed turbulence energy. Open lateral boundaries are incorporated in the model that respond to internal forcing and permit gravity waves to propagate out of the integration domain with little apparent reflection. Microphysical processes are included in the model using a Kessler-type parameterization. Simulations conducted for an unsheared environment reveal that the updraft temperatures follow a moist adiabatic lapse rate and that the convection is dissipated by water loading of the updraft. The influence of a one-directional shear on the storm development is also investigated. A simulation with a veering and backing wind profile exhibits interesting features which include a double vortex circulation, cell splitting and secondary cell formation.

**Klemp and Wilhelmson JAS 1978**

Split-explicit integration: acoustic modes integrated with time sub-stepping

## **A Scale Analysis of Deep Moist Convection<sup>1</sup> and Some Related Numerical Calculations**

FRANK B. LIPPS AND RICHARD S. HEMLER

*Geophysical Fluid Dynamics Laboratory/NOAA, Princeton University, Princeton, NJ 08540*

(Manuscript received 6 December 1981, in final form 27 May 1982)

### **ABSTRACT**

A scale analysis valid for deep moist convection is carried out. The approximate equations of motion are anelastic with the time scale set by the Brunt-Väisälä frequency. A new assumption is that the base state potential temperature is a slowly varying function of the vertical coordinate. It is this assumption that eliminates the energetic inconsistency discussed by Wilhelmson and Ogura (1972) for a non-isentropic base state. Another key result is that the dynamic pressure is an order of magnitude smaller than the first-order temperature and potential temperature. In agreement with observations, the kinetic energy is found to be an order of magnitude smaller than the first-order thermodynamic energy.

A set of six numerical simulations representing moderately deep moist convection is carried out. The base state is an idealized maritime tropical sounding with no vertical wind shear. The first calculation (Run A) shows the growth and dissipation of a typical shower cloud. The remaining calculations have small changes in either initial conditions or model equations from Run A. These calculations indicate the sensitivity of the present model to different approximations and give additional evidence for the validity of the scale analysis.

**Lipps and Hemler JAS 1982**

Explicit integration: acoustic modes eliminated in the anelastic system

Moist thermodynamics involve two aspects:

- ***formulation of saturation conditions;***  
this determines if phase transition is  
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Moist thermodynamics involve two aspects:

- ***formulation of saturation conditions;***  
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*there are some issues that need to be resolved...*

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**Lipps and Hemler JAS 1982**

Explicit integration: acoustic modes  
eliminated in the anelastic system

$$q_{vs}(p, T) \approx 0.622 \frac{e_s(T)}{p}$$

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Since typically potential temperature  $\Theta$  is predicted, one needs pressure for converting  $\Theta$  into  $T$  in addition to using it in  $q_{vs}(p, T)$ .  $\theta = T(p_{oo}/p)^{R/c_p}$



$$q_{vs}(p, T) \approx 0.622 \frac{e_s(T)}{p}$$

$$\Delta q_{vs} = \frac{\partial q_{vs}}{\partial e_s} \frac{de_s}{dT} \Delta T + \frac{\partial q_{vs}}{\partial p} \Delta p$$

$$\Delta T = \frac{\partial T}{\partial \theta} \Delta \theta + \frac{\partial T}{\partial p} \Delta p$$

$$\frac{\Delta q_{vs}}{q_{vs}} = \beta_L \frac{\Delta \theta}{\theta} - \left( 1 + \frac{R}{c_p} \beta_L \right) \frac{\Delta p}{p}$$

$$\beta_L = L/R_v T \quad 15-20 \text{ for tropospheric temperature range}$$

$$\frac{R}{c_p} \sim 0.3$$

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$$\frac{|\Delta q_{vs}|}{q_{vs}} \sim 10 \frac{|\Delta \theta|}{\theta} + 10 \frac{|\Delta p|}{p}$$

$$\frac{|\Delta \theta|}{|\Delta p|} \quad ?$$

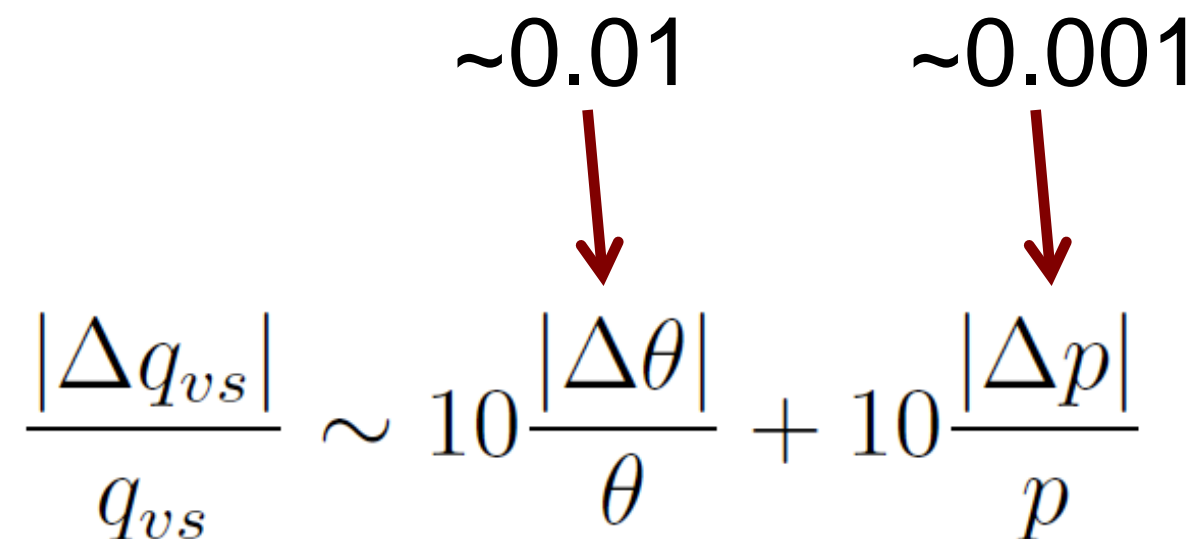
# Small-scale dynamics: LES/CRM simulation

$$|\Delta\theta| \sim 5 \text{ K}$$

$$|\Delta p| \sim 100 \text{ Pa (1hPa; } \sim \rho u^2)$$

$$\frac{|\Delta q_{vs}|}{q_{vs}} \sim 10 \frac{|\Delta\theta|}{\theta} + 10 \frac{|\Delta p|}{p}$$

$\sim 0.01$        $\sim 0.001$



Pressure perturbations are typically neglected in anelastic small-scale models. Environmental pressure profile is used to convert  $\Theta$  into  $T$  (e.g., Lipps and Hemler 1982).

“Since dynamics only care about pressure *gradient*, dynamic pressure is only known to a constant and thus should not be used in thermodynamics...”  
(T. Clark, personal communication)



However, such a statement cannot be universally valid. For instance, in a funnel cloud or a tornado, condensation happens because pressure is significantly lower than in the unperturbed environment...

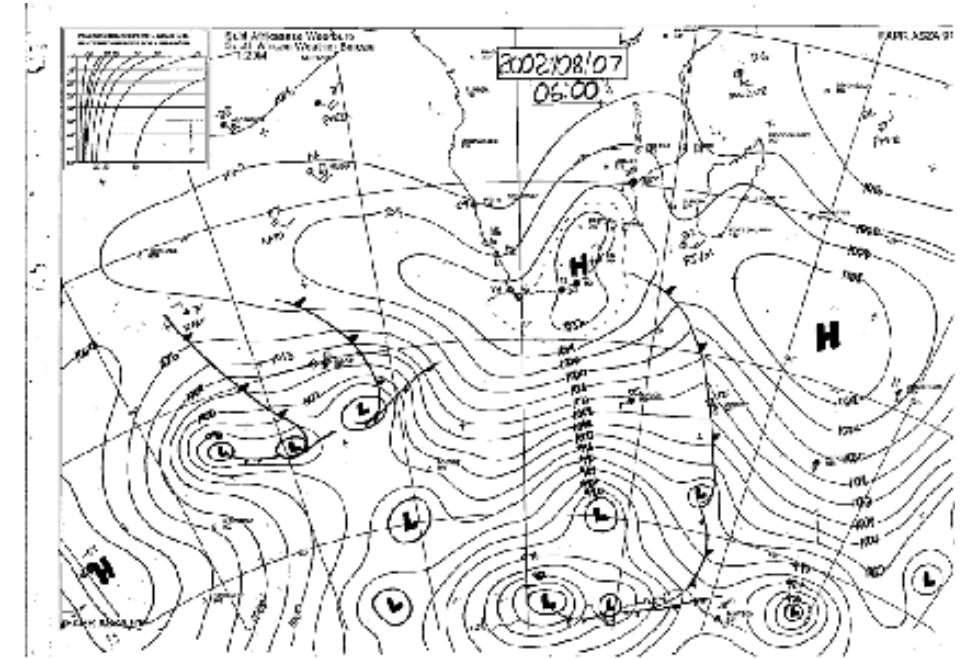
$$u = 100 \text{ m/s}, \quad \rho u^2 \sim 100 \text{ hPa}$$



# Synoptic-scale dynamics: weather simulation

$$|\Delta\theta| \sim 10 \text{ K}$$

$$|\Delta p| \sim 10 \text{ hPa}$$



$$\frac{|\Delta q_{vs}|}{q_{vs}} \sim 10 \frac{|\Delta\theta|}{\theta} + 10 \frac{|\Delta p|}{p}$$

~0.01                      ~0.01

↓                                      ↓

Hydrostatic pressure variations associated with mesoscale and larger-scale weather systems need to be included in moist thermodynamics when converting  $\Theta$  into  $T$ ...

Can one use pressure from the elliptic pressure solver in the sound-proof system in the same way as the pressure in the compressible system?

If the answer is yes, then our problems are solved!

We compare numerical solutions of relatively simple dry dynamics problems from anelastic and compressible EULAG model versions.

**Anelastic EULAG:** standard (including moist processes).

**Compressible EULAG:** dry gas dynamics equations (Smolarkiewicz and Szmelter *JCP* 2009).

# 2D RISING BUBBLE SIMULATIONS

Neutrally stratified environment  $\Theta = 300$  K

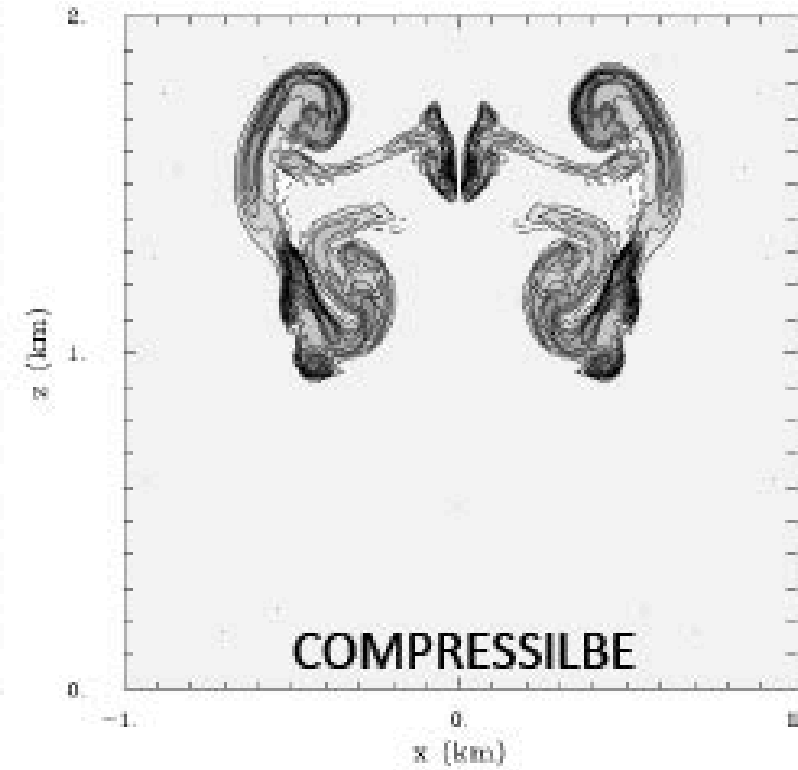
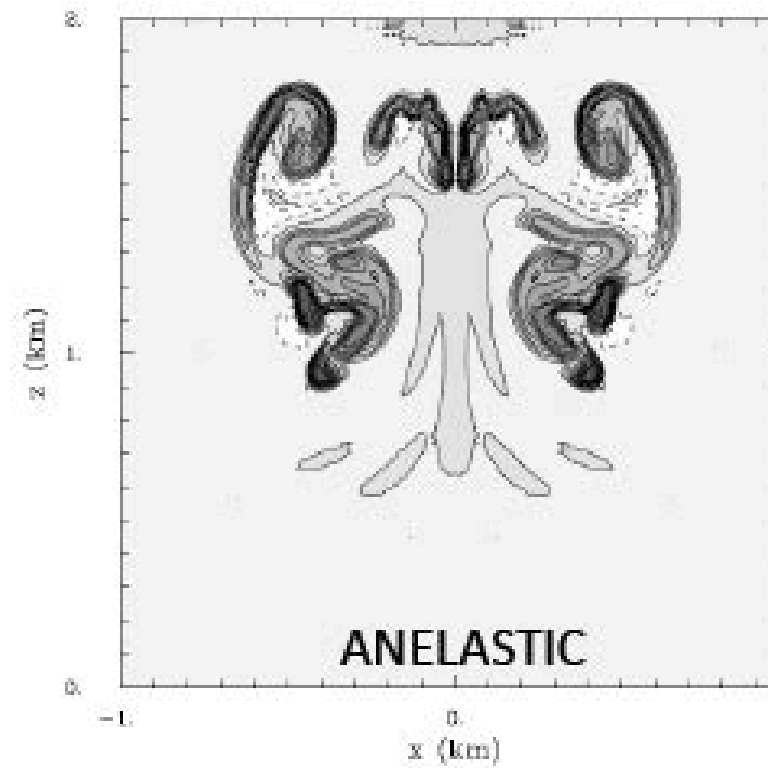
Spherical 250-m initial perturbation  $\Delta\Theta = 0.5$  K and 5 K

Domain: 2 x 2 km<sup>2</sup>, gridlength: 12.5 m

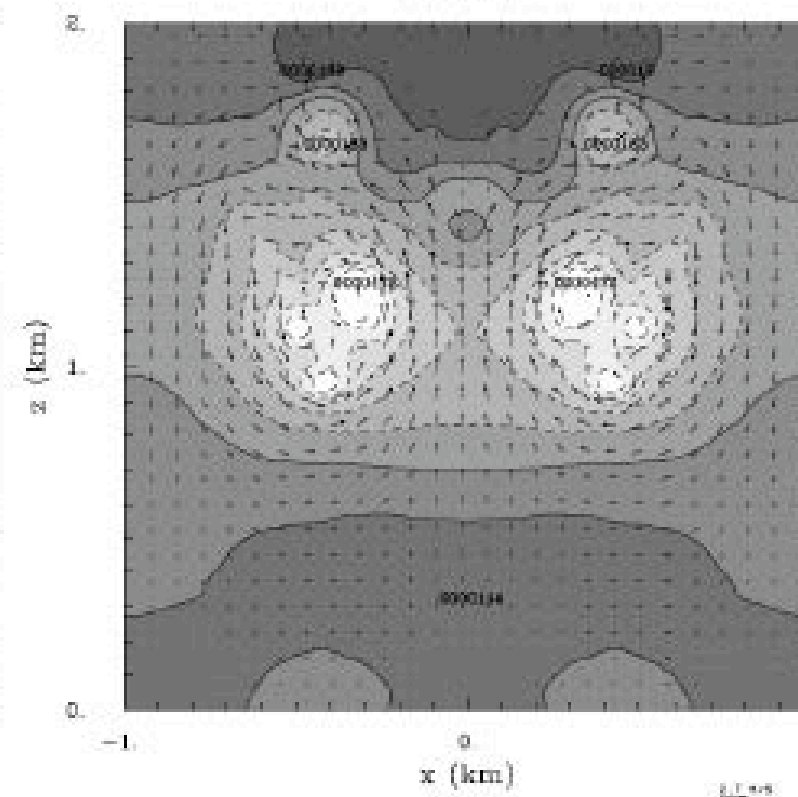
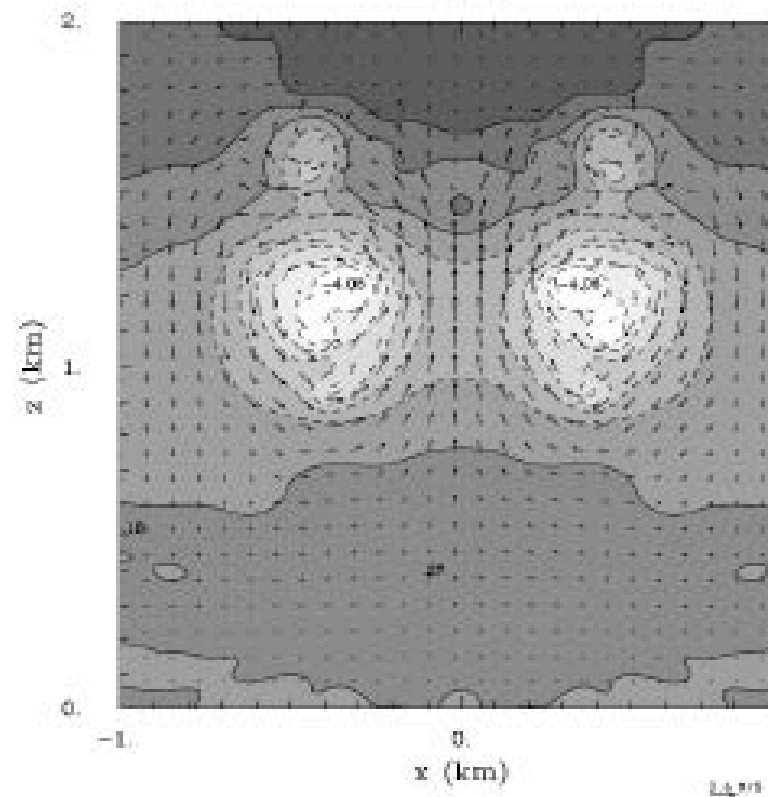
Timestep: 1/0.01 s (anel/compr) for  $\Delta\Theta = 0.5$  K  
0.2/0.003 s (anel/compr) for  $\Delta\Theta = 5$  K  
0.04/0.003 s (anel/compr) for  $\Delta\Theta = 50$  K  
0.01/0.003 s (anel/compr) for  $\Delta\Theta = 500$  K

$$\Delta\Theta = 0.5 \text{ K}; \quad t = 15 \text{ min}$$

$\Theta'$

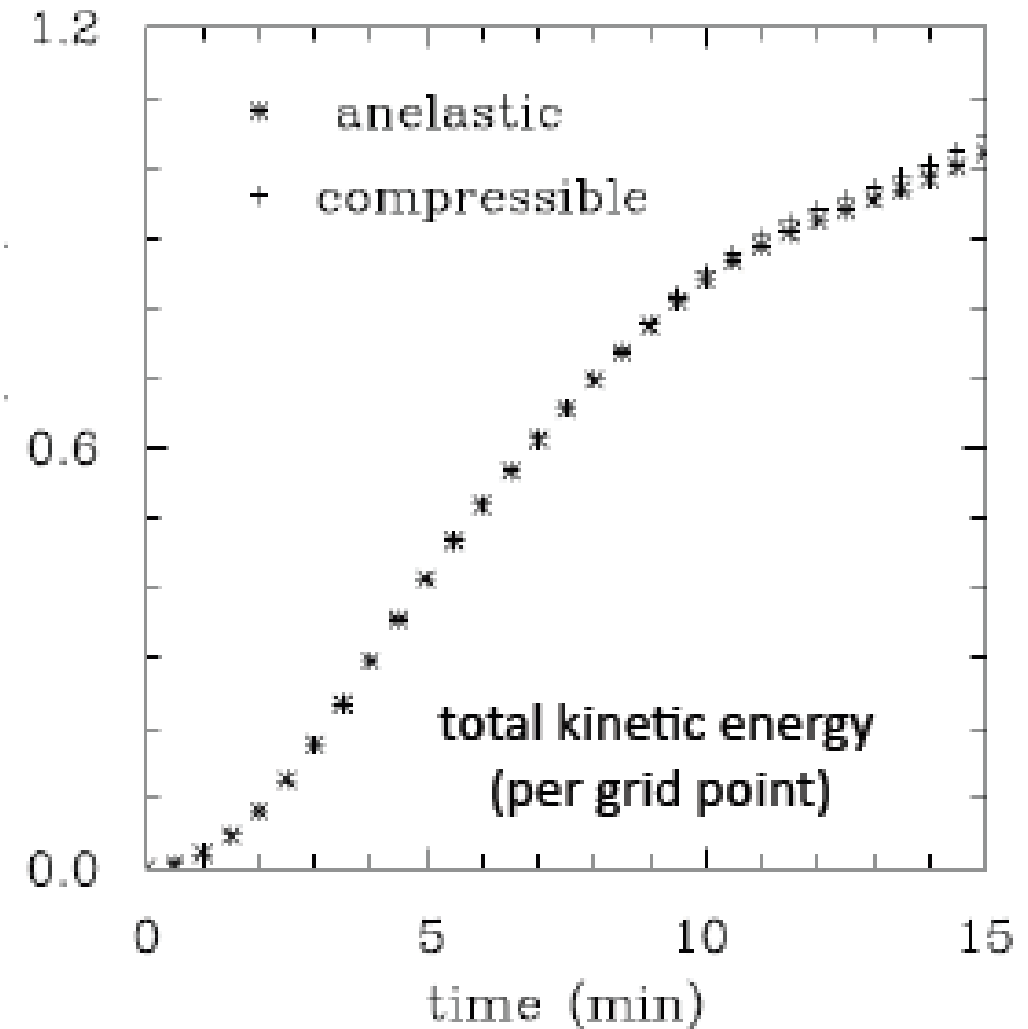
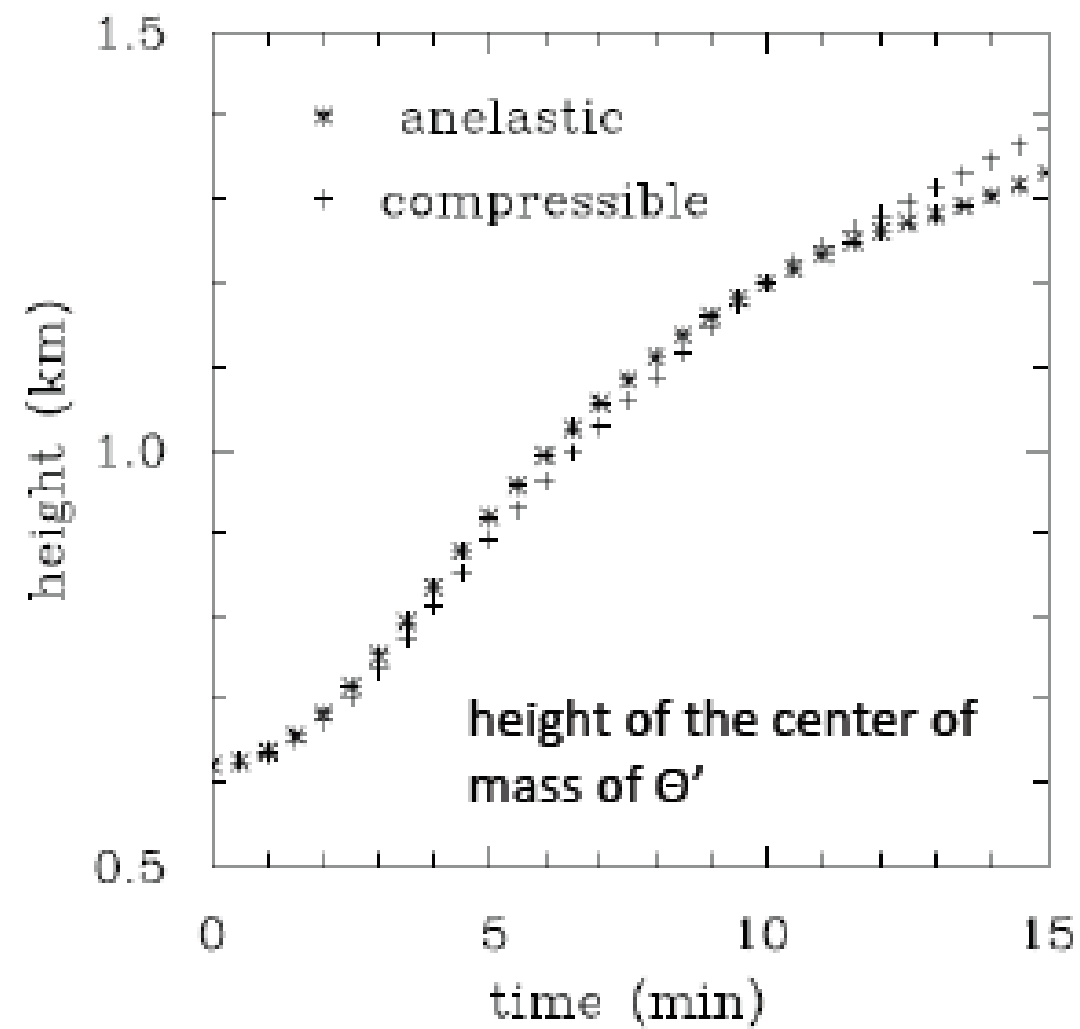


$p'$

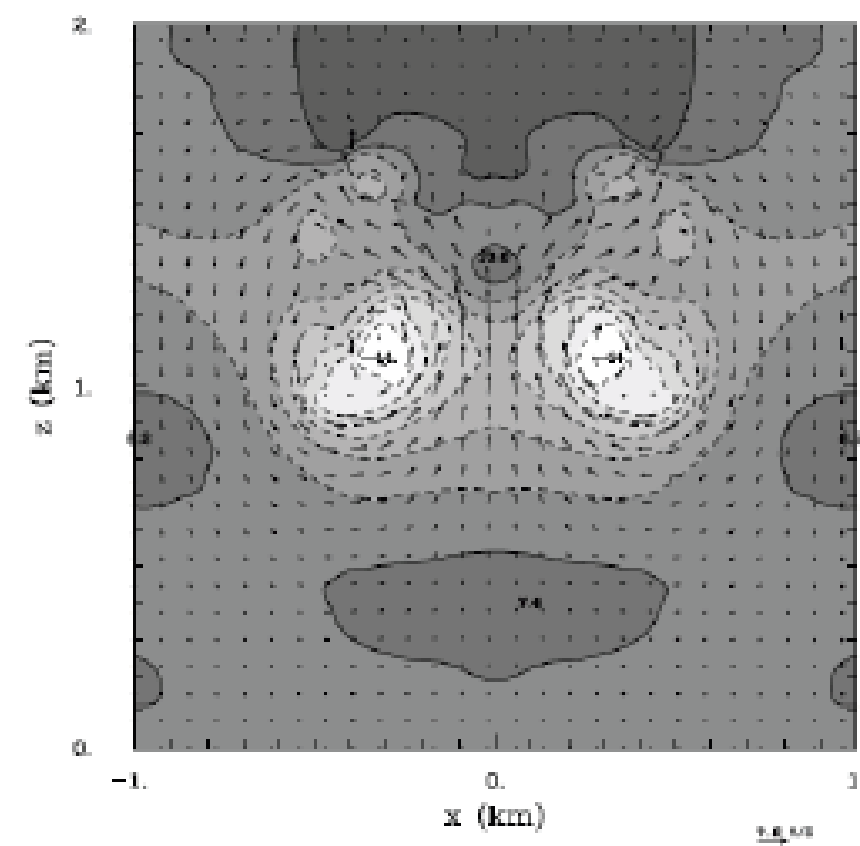
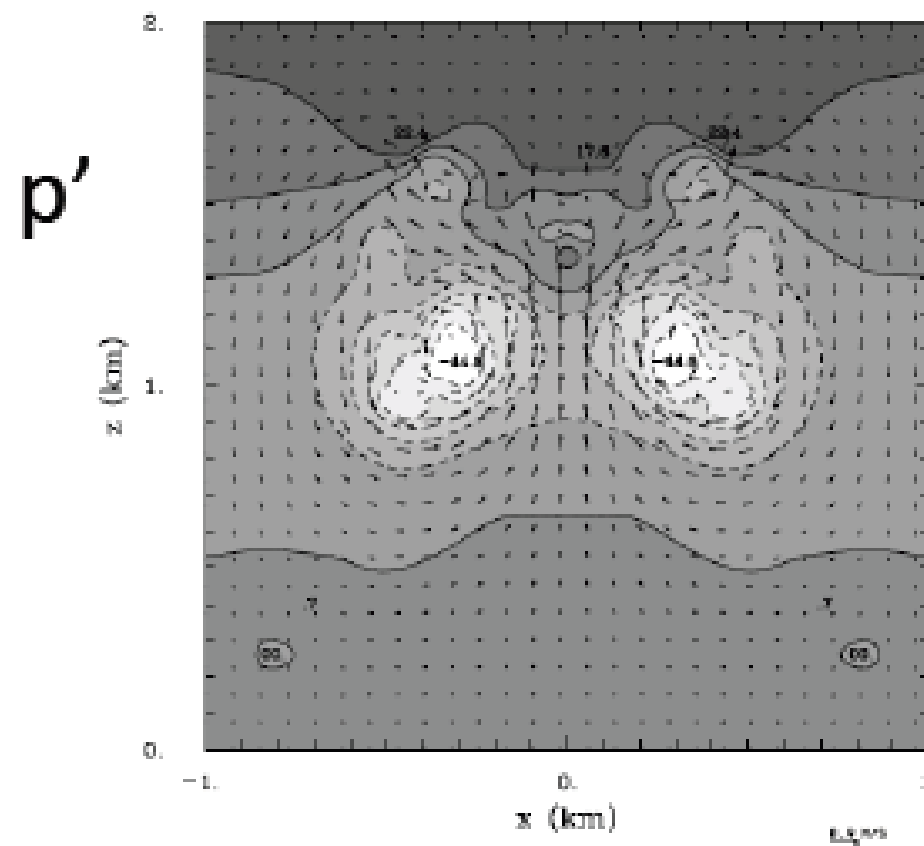
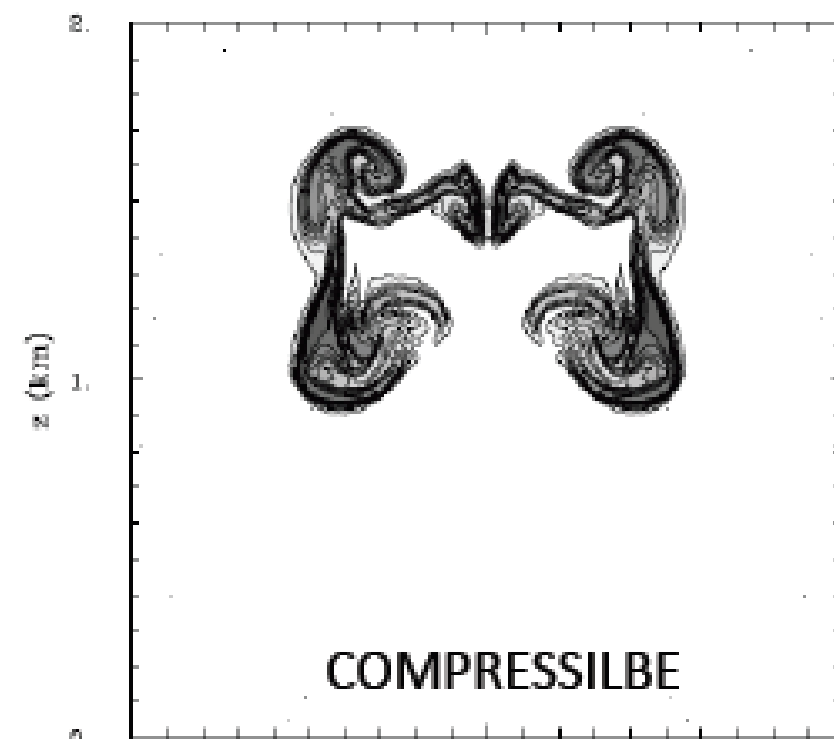
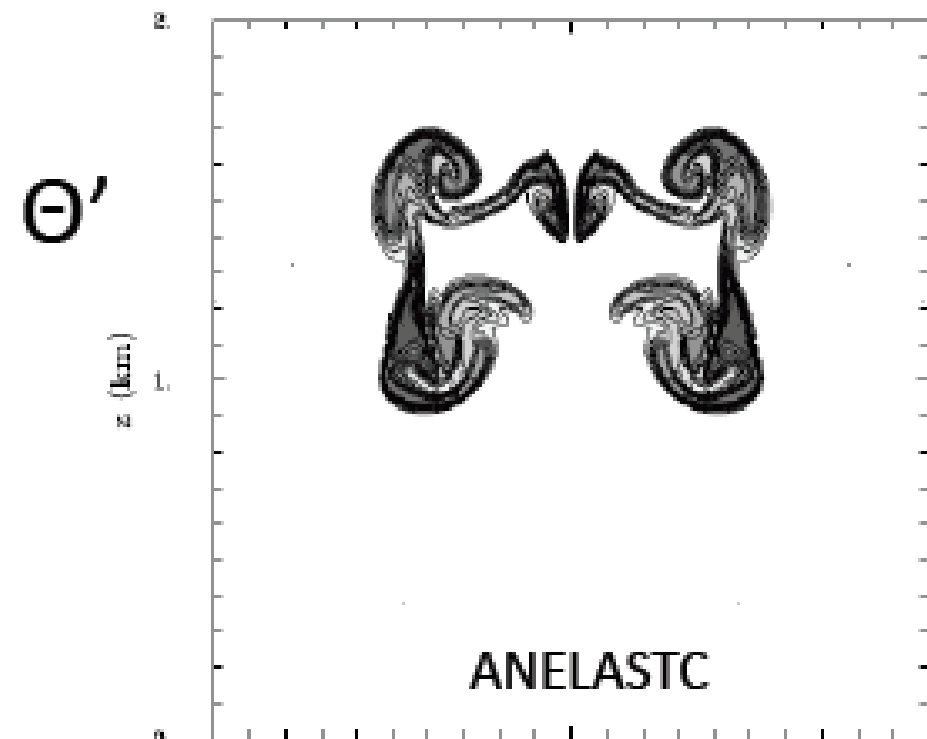




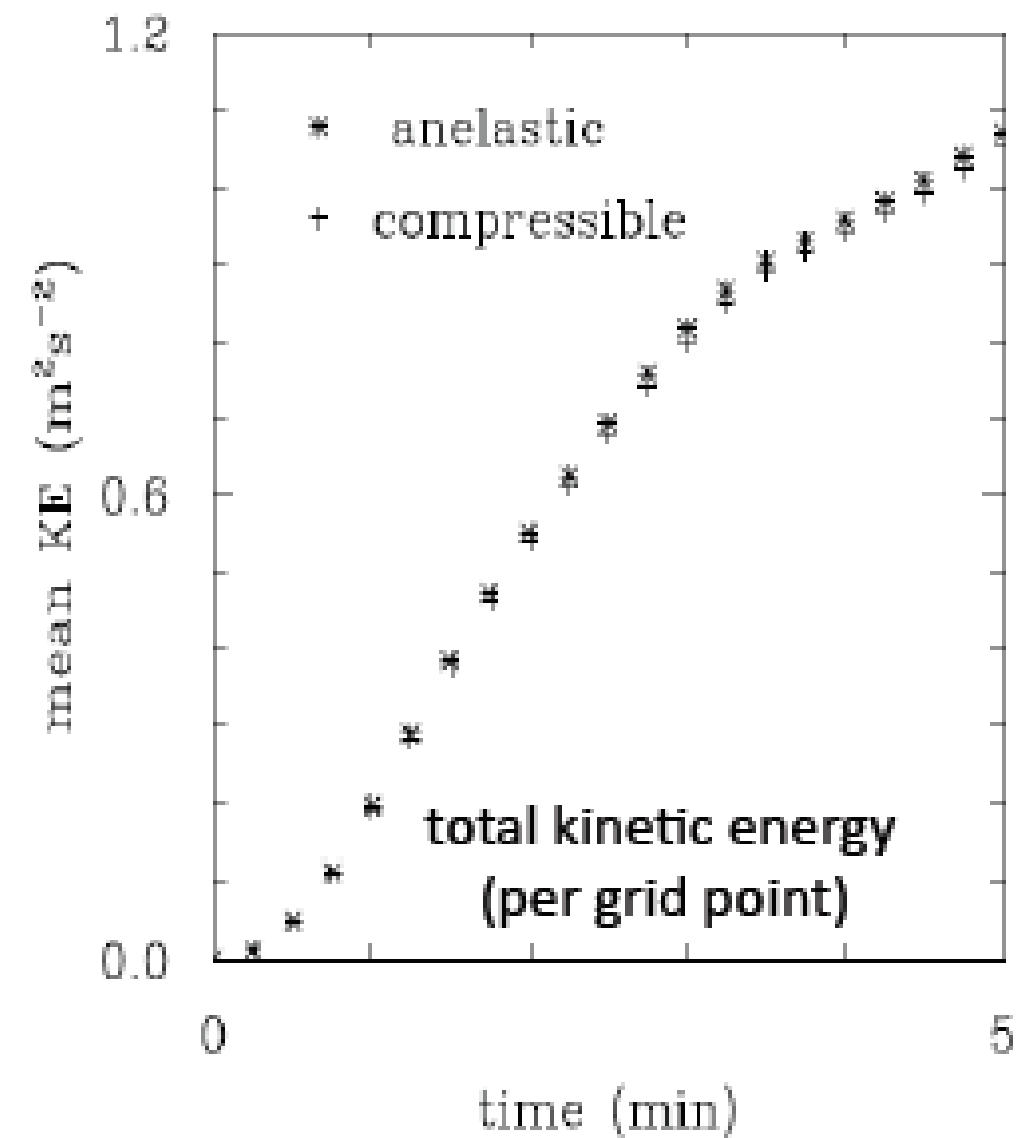
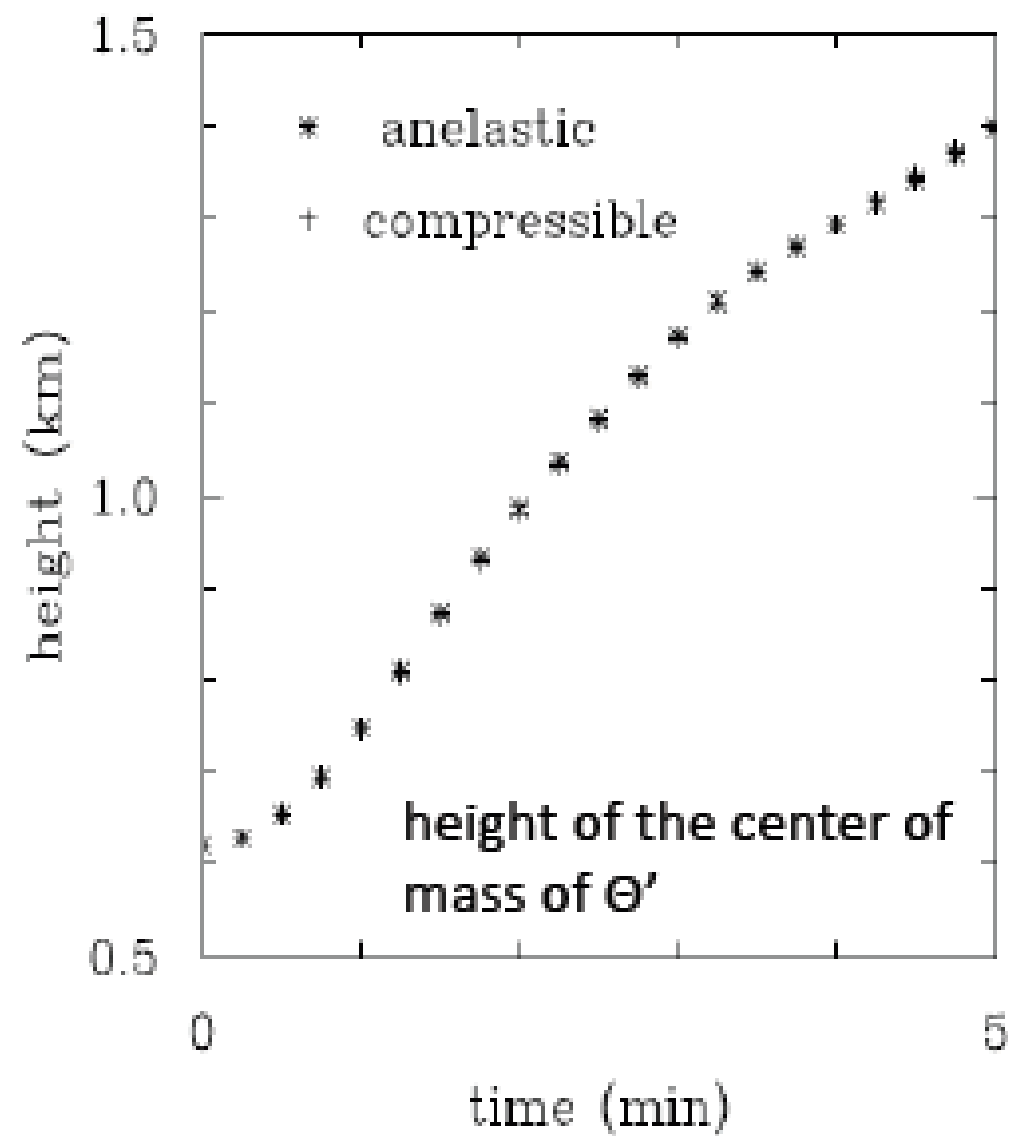
$$\Delta\Theta = 0.5 \text{ K}$$



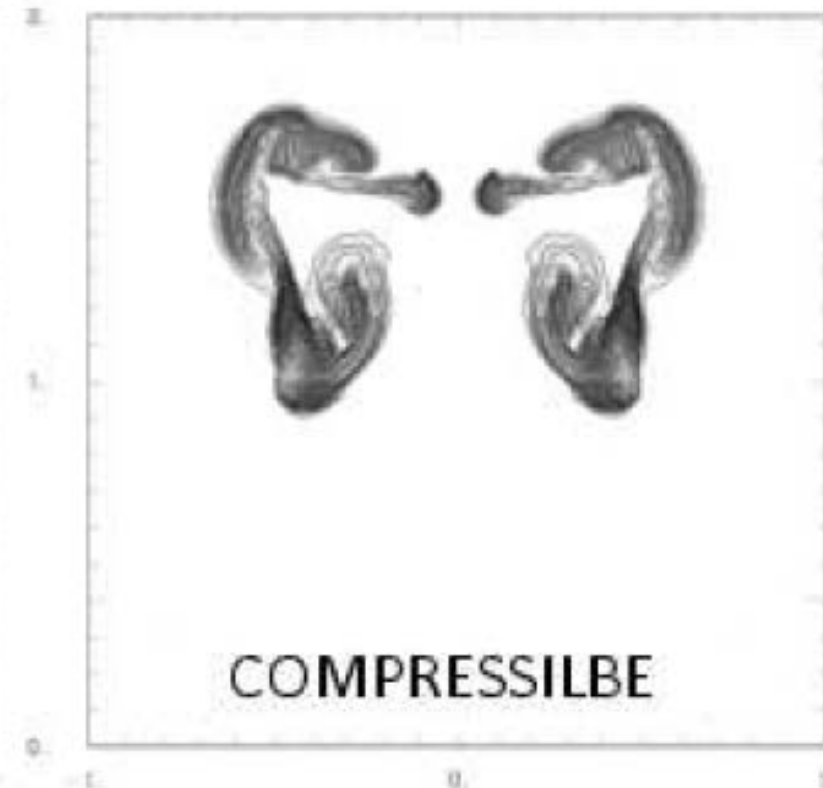
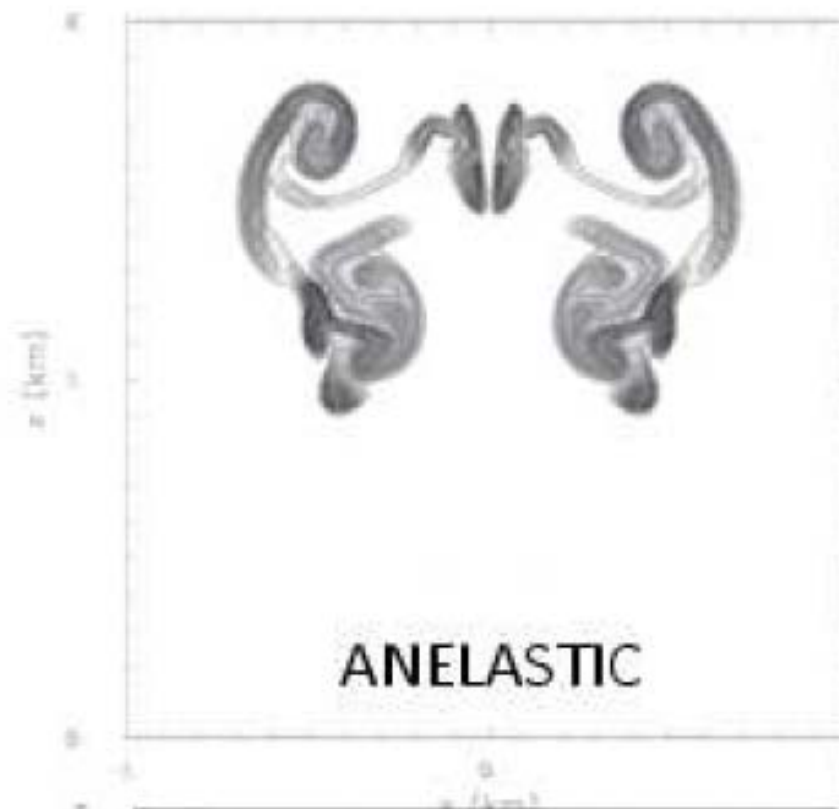
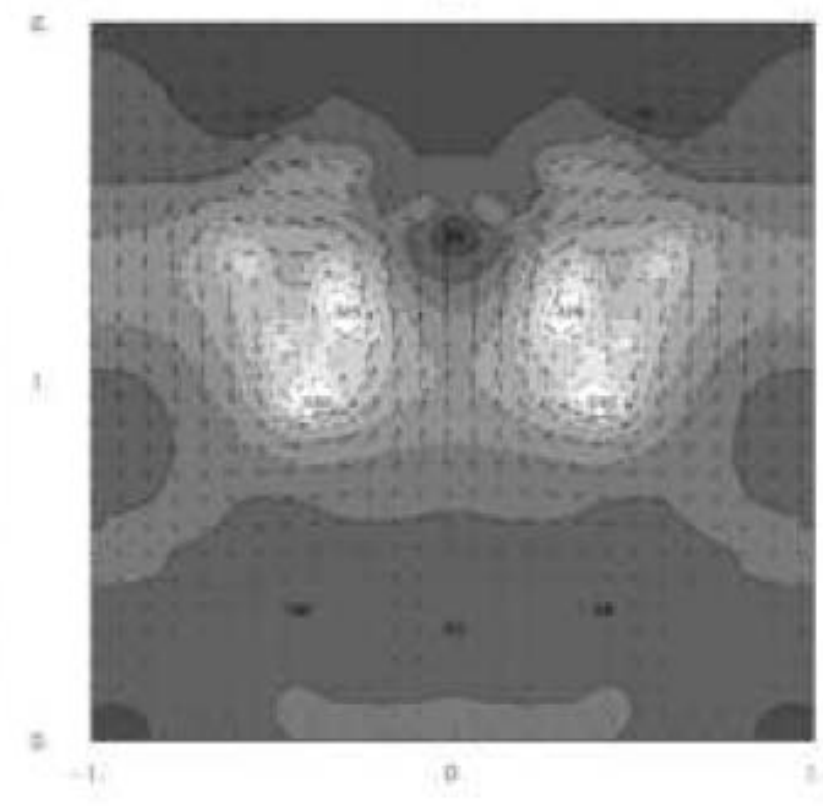
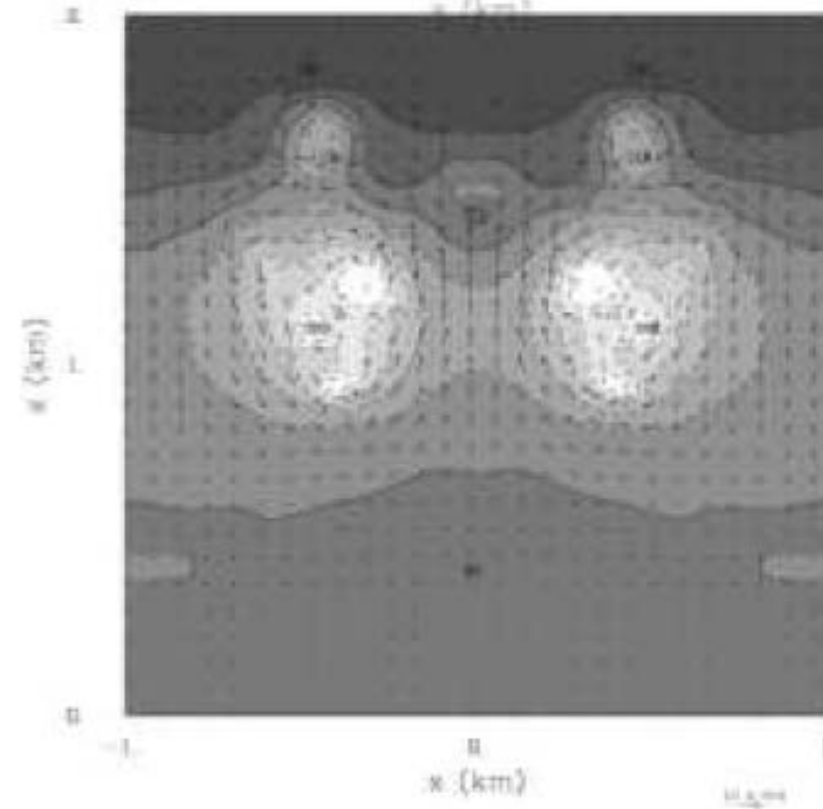
$$\Delta\Theta = 5 \text{ K}; \quad t = 5 \text{ min}$$



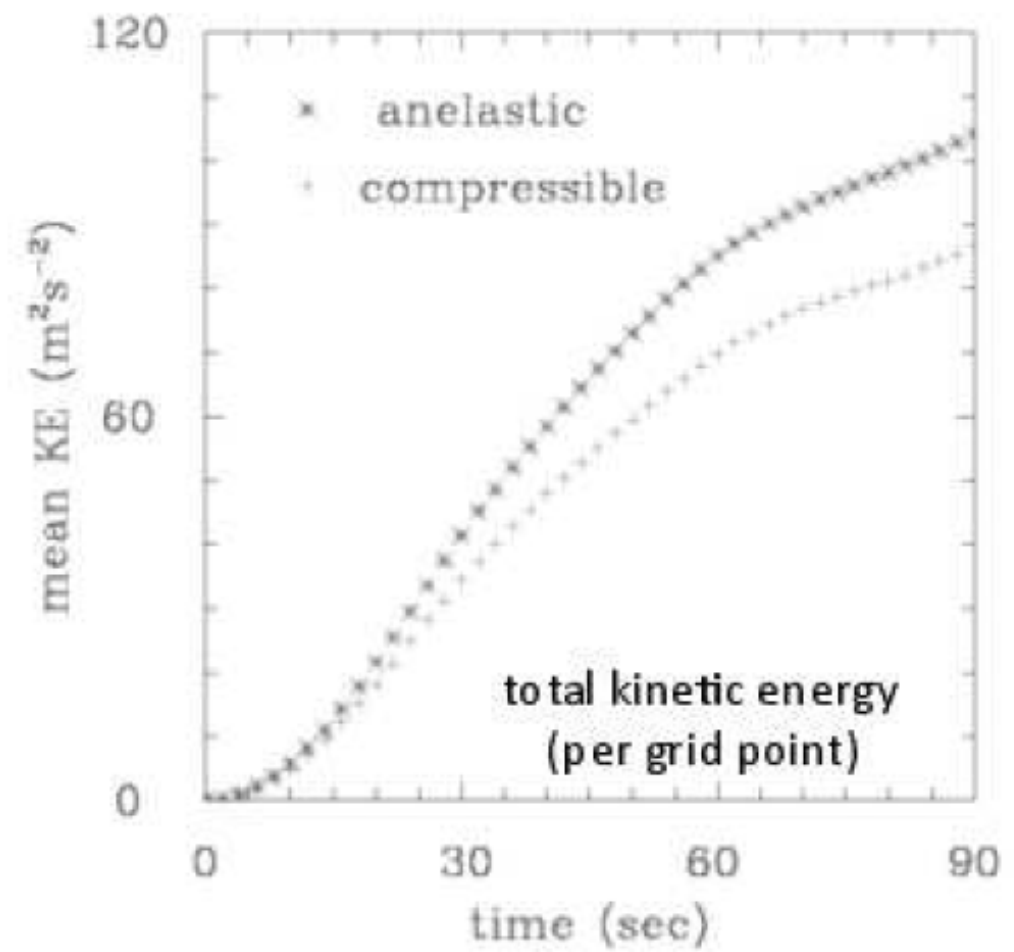
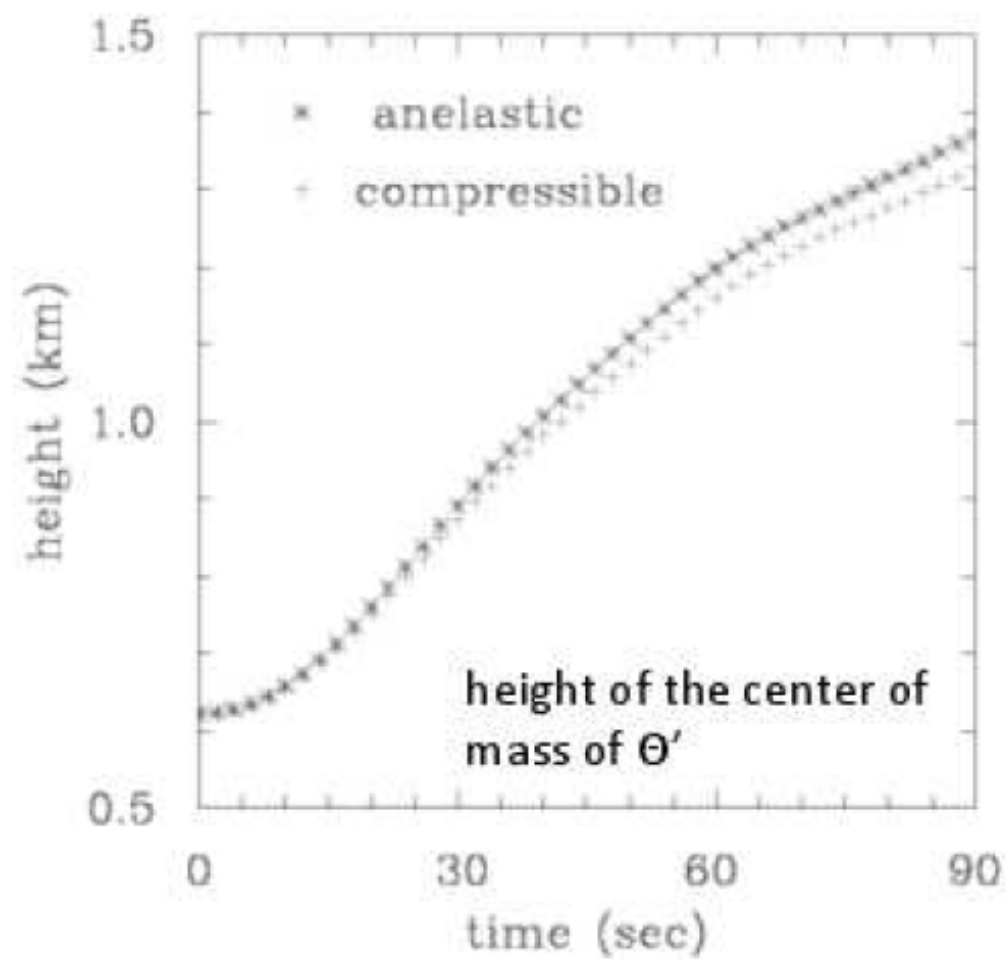
$$\Delta\Theta = 5 \text{ K}$$



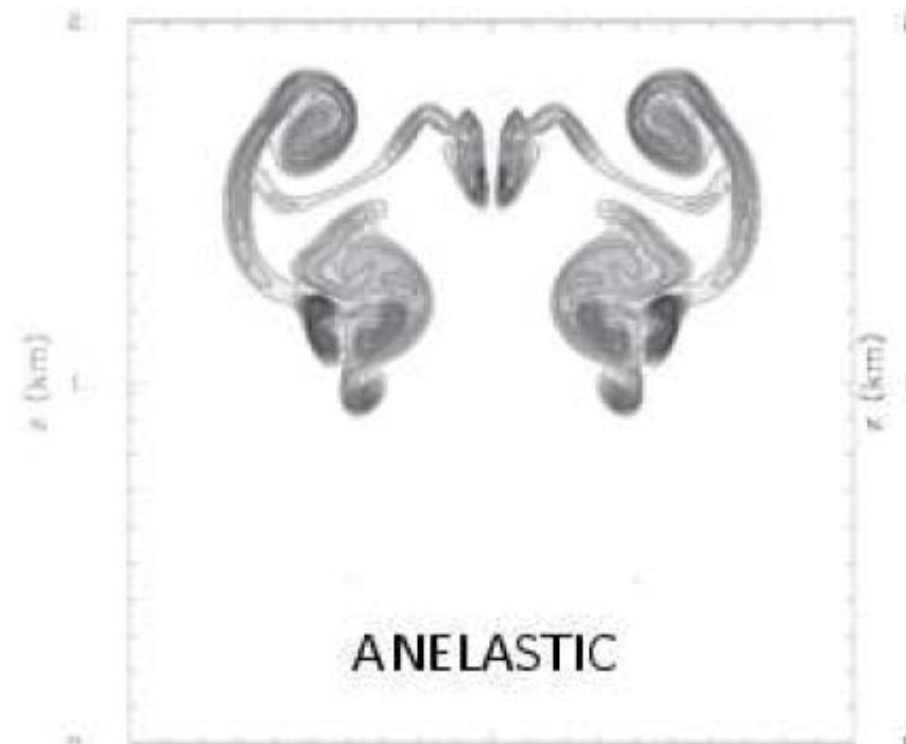
$$\Delta\Theta = 50 \text{ K}; \quad t = 1.5 \text{ min}$$

 $\Theta'$ 

 $p'$ 


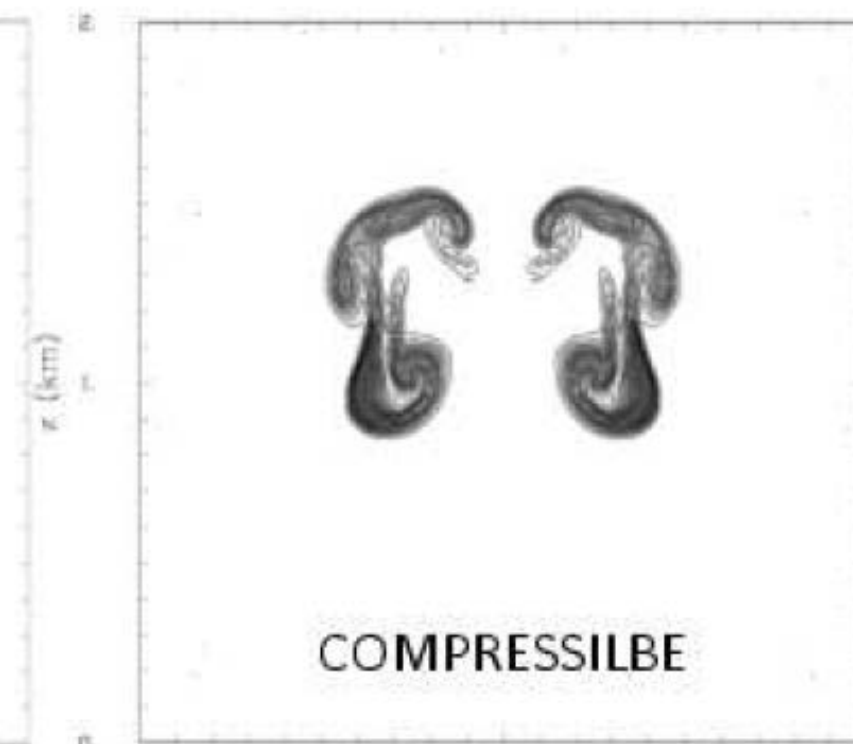
$$\Delta\theta = 50 \text{ K}$$



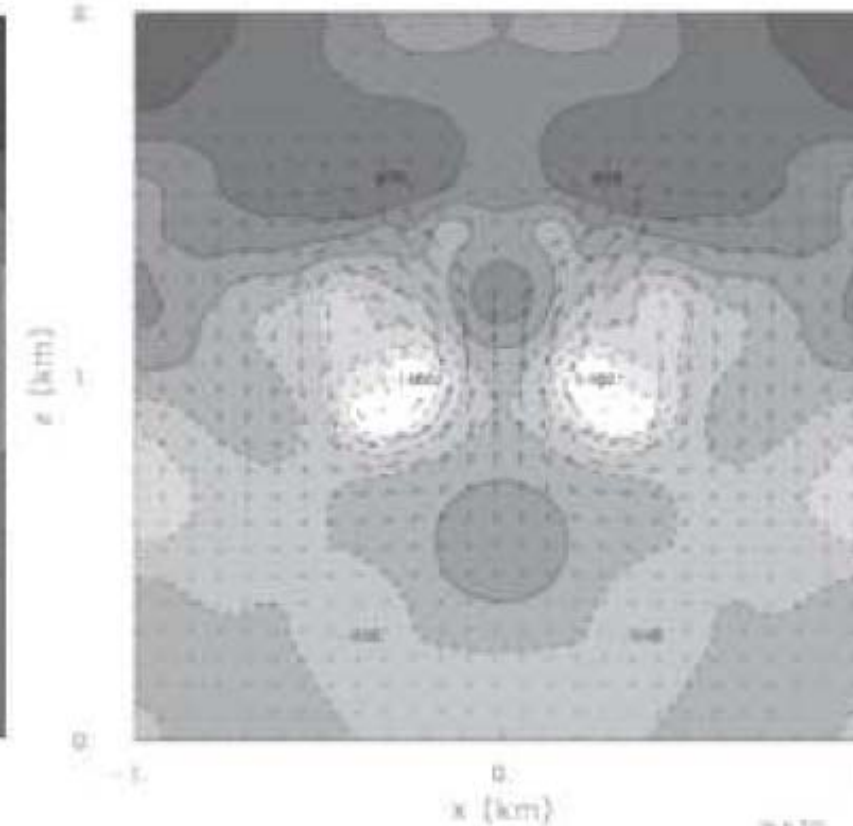
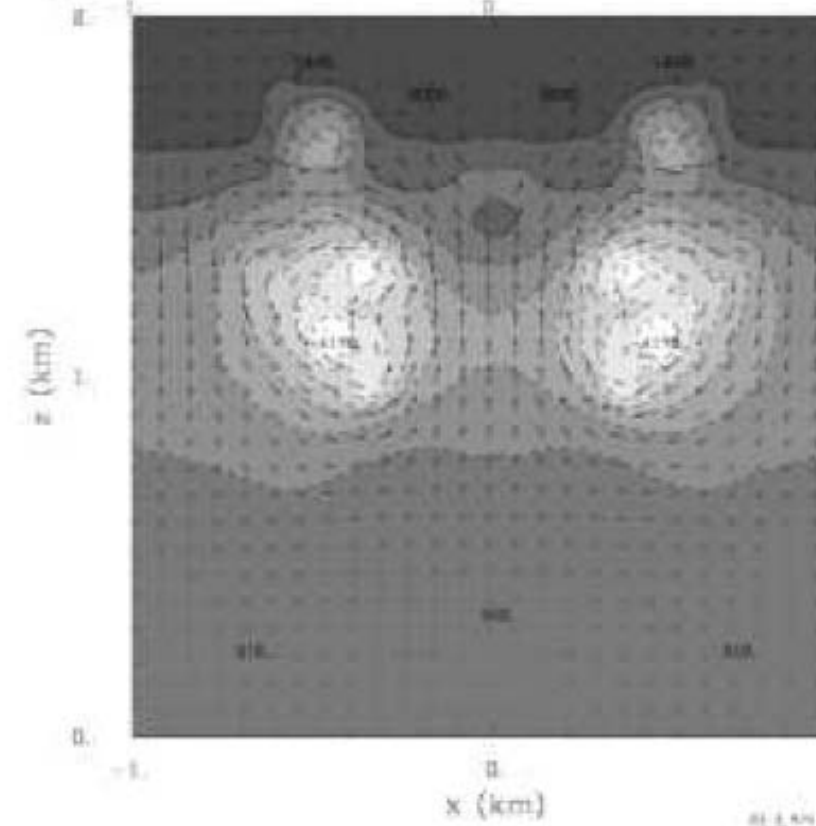
$$\Delta\Theta = 500 \text{ K}; \quad t = 0.5 \text{ min}$$

 $\Theta'$ 


ANELASTIC

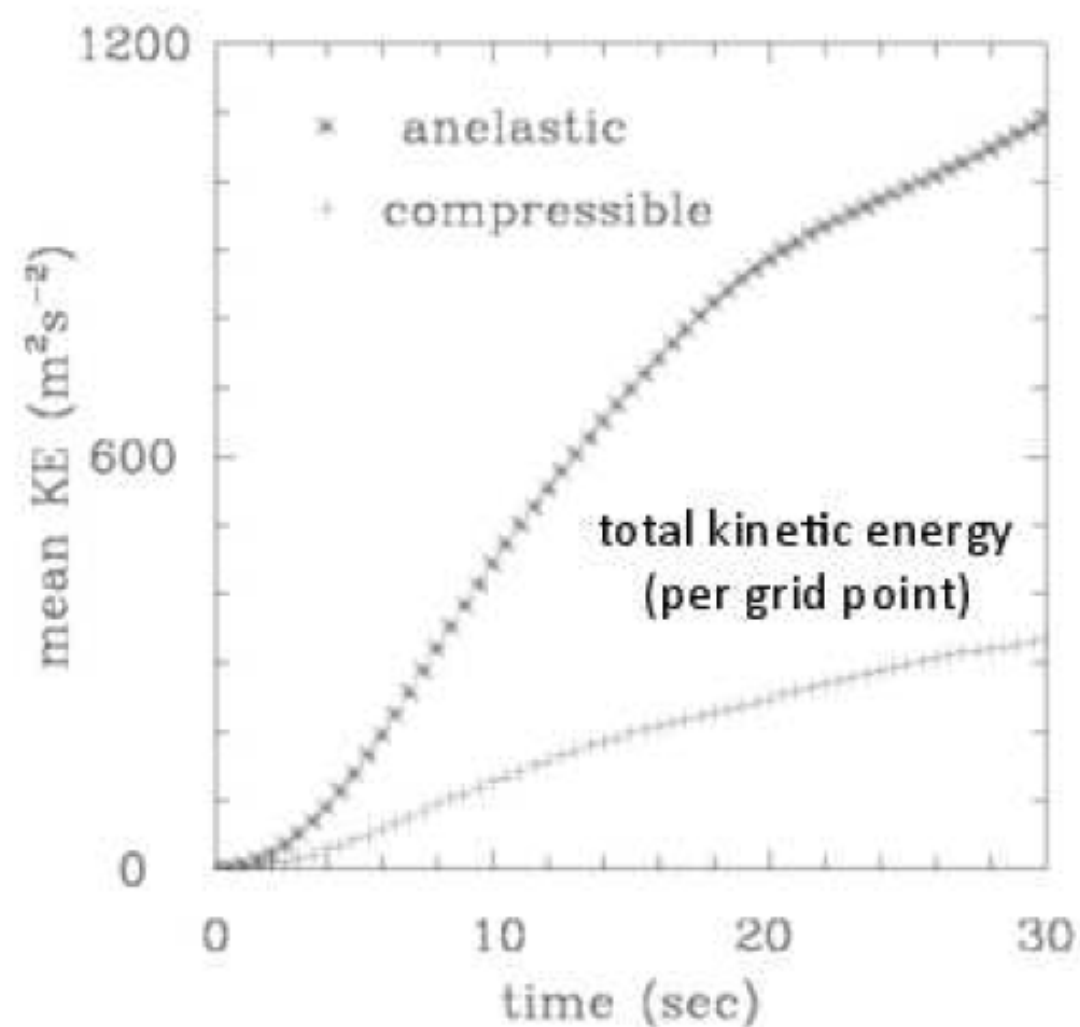
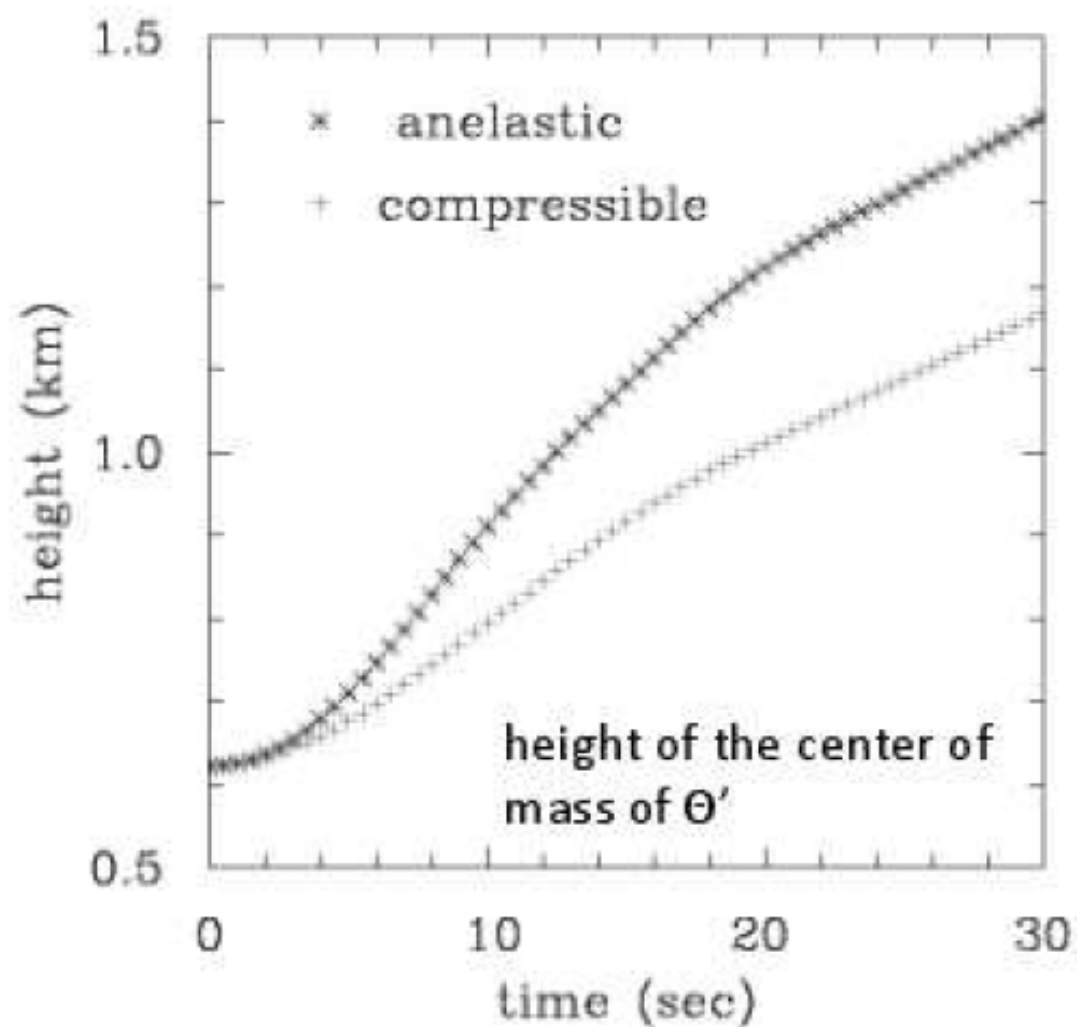


COMPRESSIBLE

 $p'$ 




$$\Delta\theta = 500 \text{ K}$$



## 2D HYDROSTATIC GRAVITY WAVE

Stably stratified environment with a typical tropospheric stability

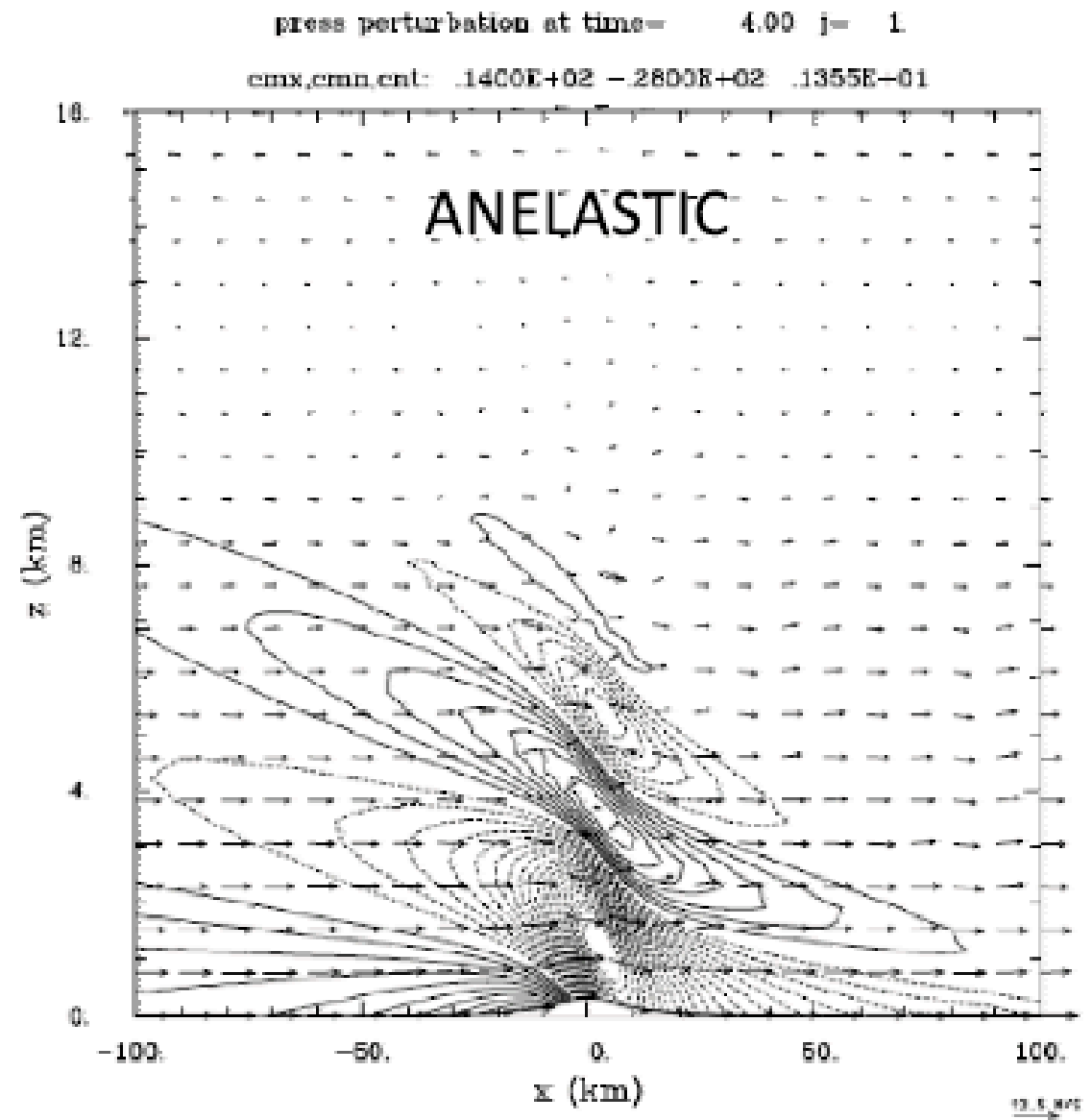
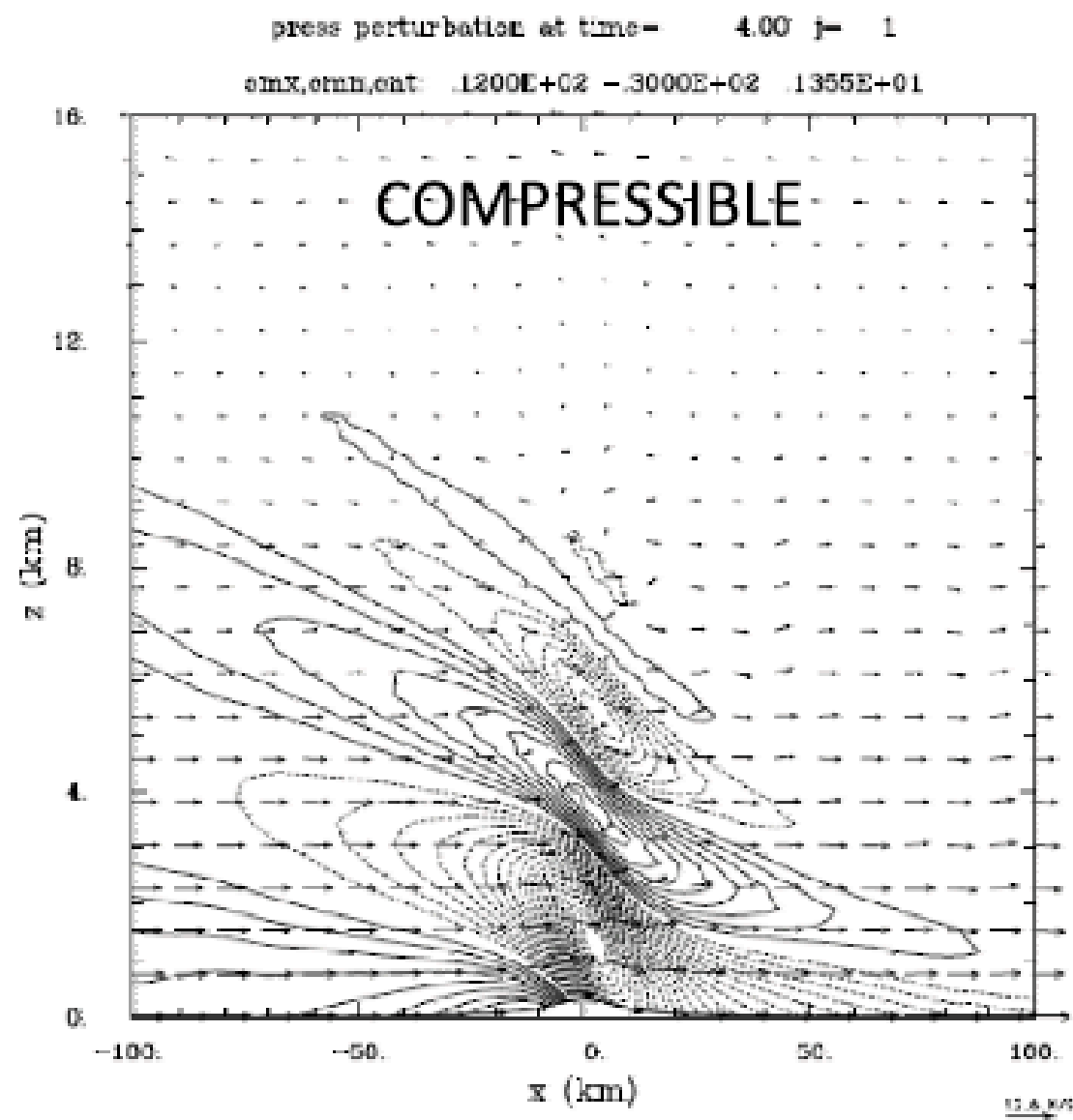
Bell-shaped mountain, 10 km half-width, 314 m height

Impinging flow: 10 m/s, decreasing with height (reverses at 12 km)

Domain: 240 x 20 km<sup>2</sup>, gridlength: 500/375 m

Timestep: 10/0.1 s (anelastic/compressible)

# Pressure perturbations at $t = 4$ hrs:



Anelastic pressure solutions agree relatively well with the pressure field from the compressible system.

It thus follows that one can use the elliptic pressure solution in the moist thermodynamics!

# SUMMARY

All-scale sound-proof simulation of atmospheric moist dynamics requires incorporation of the pressure perturbations into moist thermodynamics.

With a proper design of the pressure solver, elliptic pressure solution agrees well with the solution from the compressible system. The compressible system is inefficient, but it does provide benchmark solutions to which anelastic solutions can be compared.

Development of moist solvers that use the anelastic pressure solution and moist compressible solvers will follow, with specific model tests as outlined above.