Slide Lines in Lagrangian hydrocodes

Combined Force and Velocity Boundary Condition

R. Loubère[1]
M. Kuchařík, Richard Liska, Luboš Bednárik[2]

[1]Institut de Mathématique de Toulouse (IMT) and CNRS, Toulouse, France

[2]FNSPE, Czech Technical University in Prague, Czech Republik.

EULAG, June 2012

Plan

- Context
  - 2D Lagrangian Staggered Hydrodynamics
  - Motivation for slide-line
  - Design principles
- Slide line method
  - Contact force
  - Velocity correction (slide-line vertex)
  - Improvements
- Numerical tests
  - Sanity checks
  - More advanced tests (Sedov, Rayleigh-Taylor)
- Beyond 2D and two-materials
- Conclusions and perspective

Context

2D Staggered Lagrangian Hydrodynamics

2D Hydrodynamics in Lagrangian formulation

Euler hydrodynamics equations in an Inertial Confinement Fusion physics.
Lagrangian formulation to follow multiple materials : large aspect ratio, shock waves, expansion, high deformation.

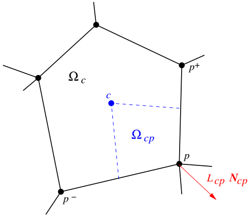
2D Compatible Staggered Lagrangian scheme

Thermodynamical variables at cell centers (ρ, ε), kinematics variables at vertexes ($\boldsymbol{X}$, $\boldsymbol{U}$).
Conservative scheme (mass, momentum, total energy) on polygonal grid.
Artificial viscosity to deal with shocks.

Subcell based discretisation (mass is conserved)

$$\rho_c = m_c / V_c, \quad m_p \frac{d\,\vec{v}_p}{d\,t} = \vec{F}_p = \sum_{c \in \mathcal{C}(p)} \vec{F}_{cp}$$

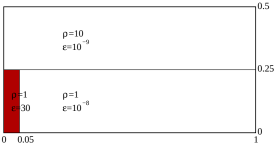
$$m_c \frac{d\,\varepsilon_c}{d\,t} = \sum_{p \in \mathcal{P}(c)} \vec{F}_{cp}.\vec{v}_p$$

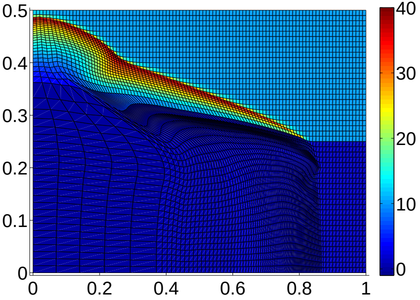


Context

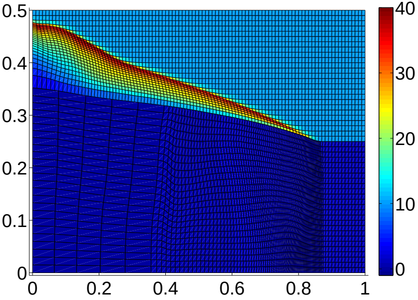
Motivation for slide line

Shear flows, vorticity are problematic for any Lagrangian methods.
Example taken from [Caramana (JCP, 2009)]





One mesh



Two meshes + slide line

Context

Design principles and methods

Design principles

- Across a slide line normal velocity/acceleration is preserved, pressure is constant, tangential velocity is free.
- Conservation is ensured.
- No friction, no impact, no void opening. (Our choice !)
- Sliding treatment should not generate any inter-penetration or void opening. Robustness.
- Independence by respect to the meshes.
- Ideal situations must be perfectly solved.

Methods

Many different methods :
[Wilkins (Academic Press, 1964)] : Classical approach with master/slave sides.
[Benson (JCP, 1992)] : Review paper, followed by many contributors !
[Caramana (JCP, 2009)] : Approach compatible with our hydro scheme (based on forces, internal velocity BCs) $\longrightarrow$ our starting point

Slide Line method

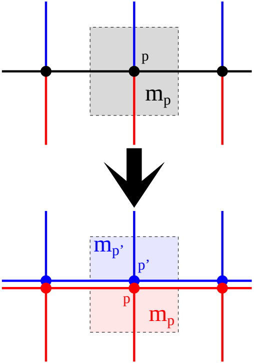
Contact force

- Momentum equation for standard point p, no sliding :

$$m_p \frac{d\,\vec{v}_p}{d\,t} = \vec{F}_p\,,$$

can be split for each separate half-point

$$m_p \frac{d\,\vec{v}_p}{d\,t} = \vec{F}_p + \vec{g}_p\,, \quad m_{p'} \frac{d\,\vec{v}_{p'}}{d\,t} = \vec{F}_{p'} + \vec{g}_{p'}\,.$$



- Here, $\vec{F}_p$, $\vec{F}_{p'}$ are any Lagrangian forces – pressure, anti-hourglass, viscosity, gravity, . . .
- To be computed : contact forces $\vec{g}_p$, $\vec{g}_{p'}$, representing opposite pressure force.
- Suppose p and p' still coincide $\Rightarrow$ same size, opposite orientation : $\vec{g}_{p'} = -\vec{g}_p$.
- Sum equations, same acceleration $\Rightarrow$

$$\vec{g}_p = -\vec{g}_{p'} = \frac{m_p\,\vec{F}_{p'} - m_{p'}\,\vec{F}_p}{m_p + m_{p'}}$$

Slide Line method

Contact force

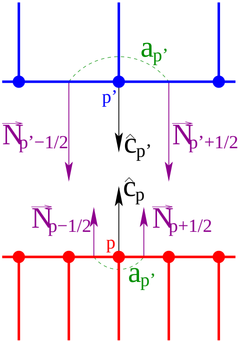
- Normal of each edge $\vec{N}_{p\pm\frac{1}{2}}$ used for construction of nodal normals $\vec{N}_p = (\vec{N}_{p-\frac{1}{2}} + \vec{N}_{p+\frac{1}{2}})/2$.
- Normalization : $a_p = \|\vec{N}_p\|$, $\hat{c}_p = \vec{N}_p / a_p$.
- All integrals (present in masses and forces) must be scaled to lengths of opposite edges.
- Lower node : quantities from upper side must be scaled by factor $a_p/a_{p'}$,

$$\vec{g}_p = \frac{m_p \frac{a_p}{a_{p'}} \vec{F}_{p'} - \frac{a_p}{a_{p'}} m_{p'} \vec{F}_p}{m_p + \frac{a_p}{a_{p'}} m_{p'}} = \frac{a_p \left(m_p \vec{F}_{p'} - m_{p'} \vec{F}_p\right)}{a_{p'} m_p + a_p m_{p'}}$$

- Upper node : scaling by $a_{p'}/a_p$,

$$\vec{g}_{p'} = -\frac{\frac{a_{p'}}{a_p} m_p \vec{F}_{p'} - m_{p'} \frac{a_{p'}}{a_p} \vec{F}_p}{\frac{a_{p'}}{a_p} m_p + m_{p'}} = \frac{a_{p'} \left(m_{p'} \vec{F}_p - m_p \vec{F}_{p'}\right)}{a_{p'} m_p + a_p m_{p'}}$$

- Both equations same up to prime sign.



Design : Simple situations are perfectly solved independently of the meshes

Slide Line method

Contact force

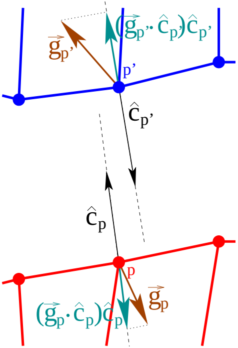
- Contact forces only introduce interaction between sides due to pressures in each side ⇒ direction in the slide line normal.
- Projection of the contact forces to the normal direction,

$$m_p \frac{d\,\vec{v}_p}{d\,t} = \vec{F}_p + (\vec{g}_p \cdot \hat{c}_p)\,\hat{c}_p,$$
$$m_{p'} \frac{d\,\vec{v}_{p'}}{d\,t} = \vec{F}_{p'} + (\vec{g}_{p'} \cdot \hat{c}_{p'})\,\hat{c}_{p'}\,.$$

- Final projected contact forces :

$$(\vec{g}_p \cdot \hat{c}_p)\ \hat{c}_p = -\frac{m_p\left(\vec{F}_{p'}\cdot\hat{c}_{p'}\right)+m_{p'}\left(\vec{F}_p\cdot\hat{c}_p\right)}{a_{p'}\,m_p + a_p\,m_{p'}}\ a_p\,\hat{c}_p\,,$$
$$(\vec{g}_{p'} \cdot \hat{c}_{p'})\ \hat{c}_{p'} = -\frac{m_{p'}\left(\vec{F}_p\cdot\hat{c}_p\right)+m_p\left(\vec{F}_{p'}\cdot\hat{c}_{p'}\right)}{a_{p'}\,m_p + a_p\,m_{p'}}\ a_{p'}\,\hat{c}_{p'}\,.$$

- Again same up to prime symbol.



Design : Normal component of acceleration is the same on both sides

Slide line method

Preventing Inter-Penetration by Velocity Correction

- Contact forces represent pressure (and other) forces from opposite side, no mechanism to prevent inter-penetration.
- From now, we need to start distinguishing between master and slave side, slave point (p, defined in initialization) must follow its master point :
  $$\vec{v}_p^{n+1} = \vec{v}_p^{n+1,\dagger} + \left[\left(\vec{v}_{p'}^{n+1,\dagger} + \vec{v}_{p'}^{n}\right) \cdot \hat{c}_{p'}\right] \hat{c}_{p'} - \left[\left(\vec{v}_{p}^{n+1,\dagger} + \vec{v}_{p}^{n}\right) \cdot \hat{c}_{p'}\right] \hat{c}_{p'} \, .$$
- Here, corrected $\vec{v}_p^{n+1}$ is used for nodal motion, $\vec{v}_p^{n+1,\dagger}$ includes contact forces.
- Fix removes excessive velocity in the direction of inter-penetration (slide line normal) from the final velocity.
- Prevents slave node to move in this direction more than master does.
- Some sort of “internal velocity boundary condition”.

Design : Inter-penetration or void opening are reduced, but conservation is lost.

Goodness criteria for the slide line method

Measure of Energy Discrepancy

Total energy is lost !

- Change of internal energy from energy equation : $m_c\,(\varepsilon_c^{n+1} - \varepsilon_c^n) = -\sum_{p\in P(c)} \vec{F}_p^c \cdot \Delta\vec{r}_p$.
- Change of kinetic energy : $\frac{1}{2}\,m_p\,\left((\vec{v}_p^{n+1})^2 - (\vec{v}_p^n)^2\right)$.
- Compatible discretization $\Rightarrow$ both values must be identical. However, due to velocity correction, energy discrepancy occurs.

Measure of Energy Discrepancy

- Discrepancy in point p during one timestep :
  $\Delta W_p^{n\rightarrow n+1} = \left[\frac{1}{2}\,m_p\,\left((\vec{v}_p^{n+1})^2 - (\vec{v}_p^n)^2\right)\right] - \left[\sum_{c\in C(p)} \vec{F}_p^c \cdot \Delta\vec{r}_p\right]$.
- Integration over space and time :
  $\Delta W^{n\rightarrow n+1} = \sum_{\forall p} \Delta W_p^{n\rightarrow n+1}$, $\Delta W = \sum_{n=1}^{N} \Delta W_{n\rightarrow n+1}$.
- Can be also generalized for non-compatible discretizations.

Check : Measure the impact of the slide line treatment.

Improvement : interpolation of entity

- Up to now – [Caramana (JCP, 2009)], interaction with closest point (in both contact force and velocity correction formulas).
- Produce disturbances of the slide line, especially in case of different aspect ratios, many points interact with only one
- All quantities on the opposite side must be interpolated :

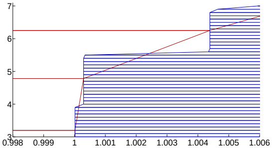
$$(\vec{g}_p \cdot \hat{c}_p)\ \hat{c}_p = -\frac{m_p\left(\vec{F}_{p'}\cdot\hat{c}_{p'}\right)+m_{p'}\left(\vec{F}_p\cdot\hat{c}_p\right)}{a_{p'}\ m_p + a_p\ m_{p'}}\ a_p\ \hat{c}_p\,,$$

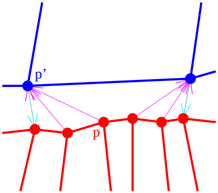
$$(\vec{g}_{p'} \cdot \hat{c}_{p'})\ \hat{c}_{p'} = -\frac{m_{p'}\left(\vec{F}_p\cdot\hat{c}_p\right)+m_p\left(\vec{F}_{p'}\cdot\hat{c}_{p'}\right)}{a_{p'}\ m_p + a_p\ m_{p'}}\ a_{p'}\ \hat{c}_{p'}\,,$$

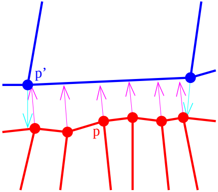
$$\vec{v}_p^{n+1} = \vec{v}_p^{n+1,\dagger} + \left[\left(\vec{v}_{p'}^{n+1,\dagger} + \vec{v}_{p'}^{n}\right)\cdot\hat{c}_{p'}\right]\hat{c}_{p'}$$

$$-\left[\left(\vec{v}_p^{n+1,\dagger} + \vec{v}_p^{n}\right)\cdot\hat{c}_{p'}\right]\hat{c}_{p'}\,,$$

i.e. normal $\hat{c}$, characteristic length a, mass m, force $\vec{F}$, and velocity $\vec{v}$.





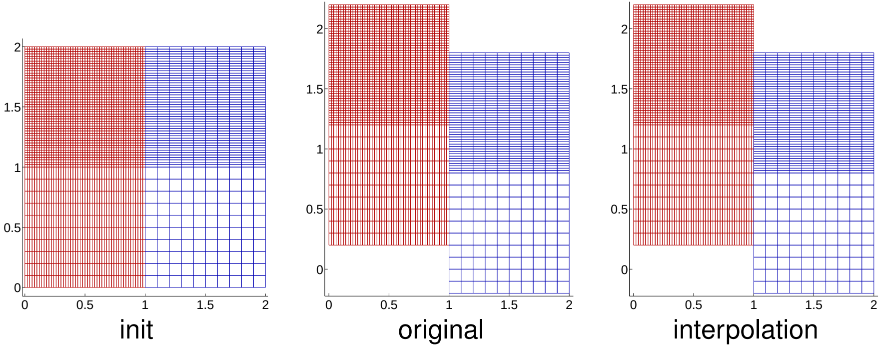


Numerical Examples

Sanity checks : Pure sliding

Vertically sliding blocks

- Non-uniform meshes, straight slide line, opposite velocity.
- Same pressures $\Rightarrow$ contact forces must cancel exactly with pressure forces.
- Test of scaling.
- $\Delta W = 0$ for all methods, slide line perfect.

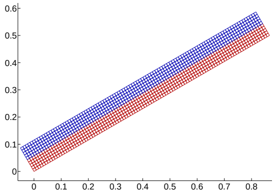


Numerical Examples

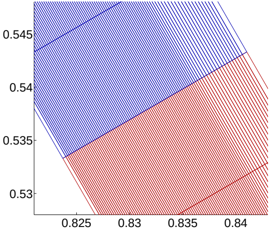
Sanity checks : Salzmann-Like Piston

Oblique piston (from left to right)

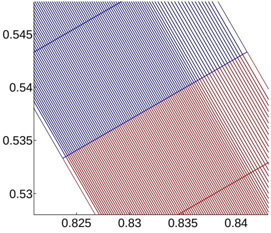
- Sanity test : Salzmann piston with uniform mesh, straight slide line, rotated by $\pi/6$.
- Pressure gradient parallel with interface $\Rightarrow$ contact forces cancel pressure forces.
- Test of scaling and projection.
- $\Delta W = 0$ for all methods, slide line perfect.



init

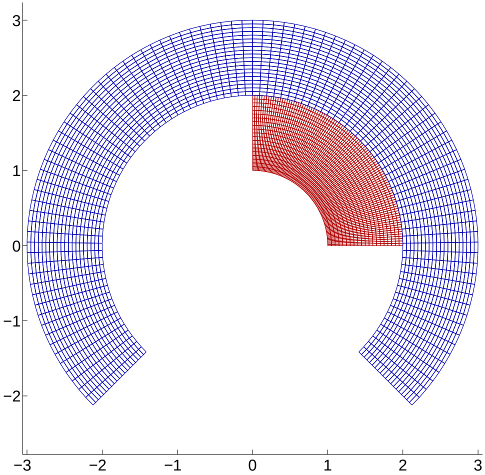


original

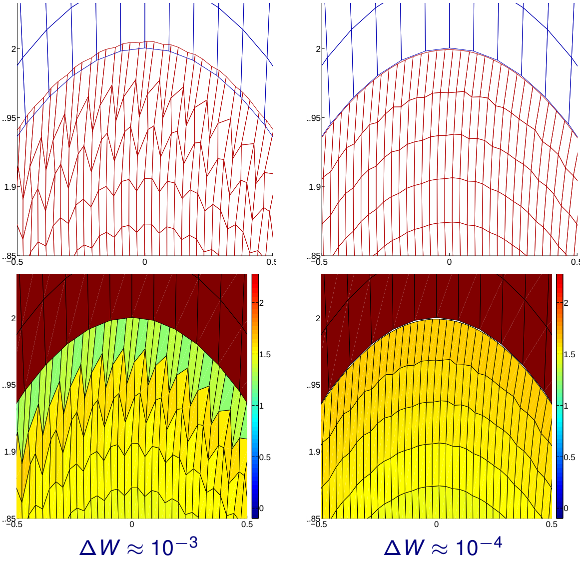


interpolation

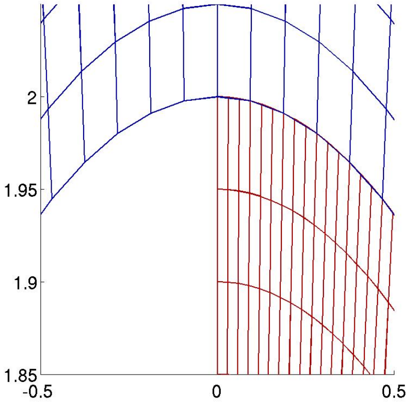
- 2 meshes of 100×20 cells : static heavy outer master mesh ($\rho = 10^4$) and moving light inner slave mesh ($\rho = 1$).
- Same pressure $p = 1$, $T = 0.65$ (original method fails soon after).
- Original method $\Rightarrow$ serious distortions (driven by staircase problem), interpolation helps.

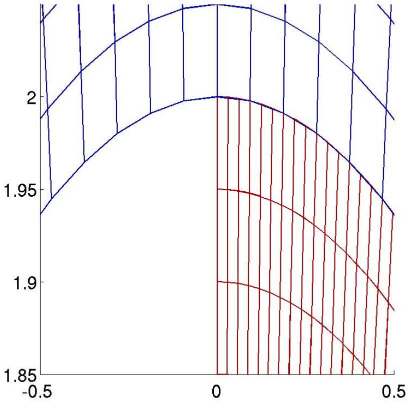


Numerical Examples



Numerical Examples

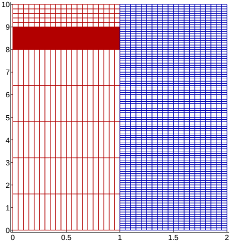


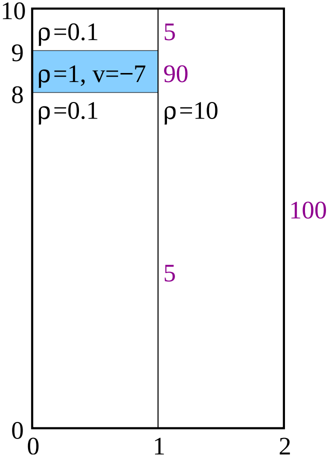


Numerical Examples

Bullet in Channel

- Inspired by real experiments from PALS laser facility [Badziak et al. (APL, 2010)].
- Left mesh : heavy "bullet" ($\rho = 1$) in "air" ($\rho = 0.1$). Right mesh : master "solid" ($\rho = 10$). $p = 1$, static except bullet.
- Both meshes have 20×100 cells, walls everywhere.
- Strong differences in aspect ratios.
- $T = 1.17$ (original fails soon).
- Sliding bullet makes high pressure below and low pressure above $\Rightarrow$ deformation of slide line.



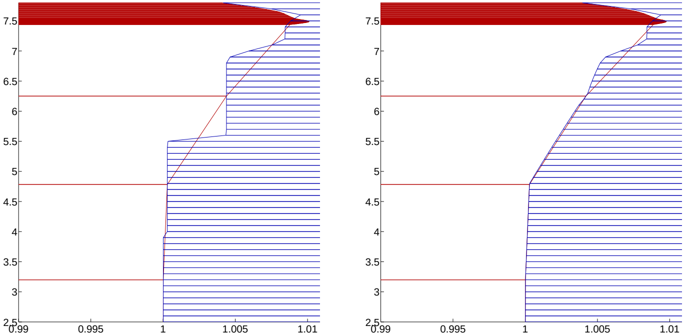


Numerical Examples

Bullet in Channel

Early time $T = 0.1$

- Slide line starts to be in motion $\Rightarrow$ staircase problem for original method due to 1-to-1 interaction.



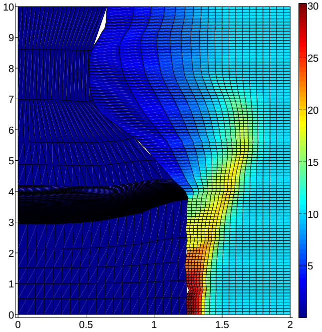
original meshes **interpolation meshes**

Numerical Examples

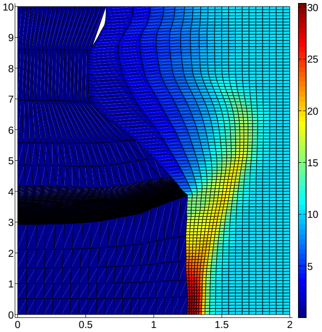
Bullet in Channel

Later time $T = 1.17$

- Bullet and interface deformed.
- Original method soon to fail – biggest inter-penetration and voids !



original mesh+density



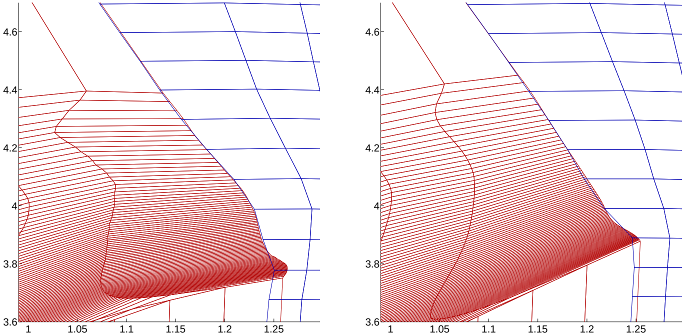
interpolation mesh+density

Numerical Examples

Bullet in Channel

Later time $T = 1.17$

- Zoom on slide line at bullet.
- Bigger interpenetration and void for original method.



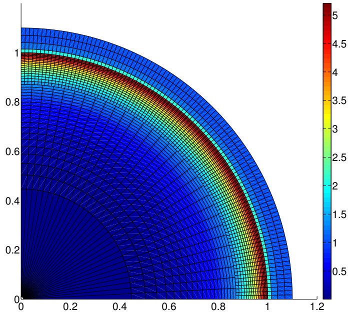
original mesh **interpolation mesh**

Numerical Examples

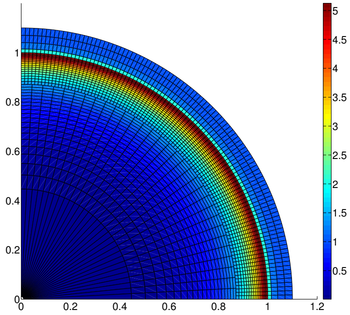
Sedov problem with interface (slide-line)

Test symmetry violation due to slide-line machinery

- Sedov point explosion on polar mesh (unit density, zero pressure) at $t = 1$.
- Radius $r \in \langle 1/100, 1.1\rangle$ and angle $\theta \in \langle 0, \pi/2\rangle$
- Slide-line at $r = 0.5$, 20 cells in r direction, $100/31$ outer/inner cells in angular direction. Meshes do not coincide



(a) original



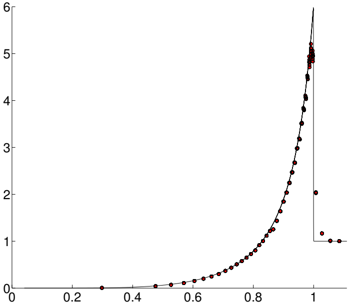
(b) interpolated interact.

Numerical Examples

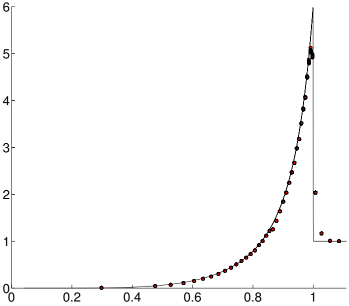
Sedov problem with interface (slide-line)

Symmetry violation due to slide-line machinery

- Sedov point explosion on polar mesh (unit density, zero pressure) at $t = 1$.
- Radius $r \in \langle 1/100, 1.1 \rangle$ and angle $\theta \in \langle 0, \pi/2 \rangle$
- Slide-line at $r = 0.5$, 20 cells in r direction, $100/31$ outer/inner cells in angular direction. Meshes do not coincide



(a) original

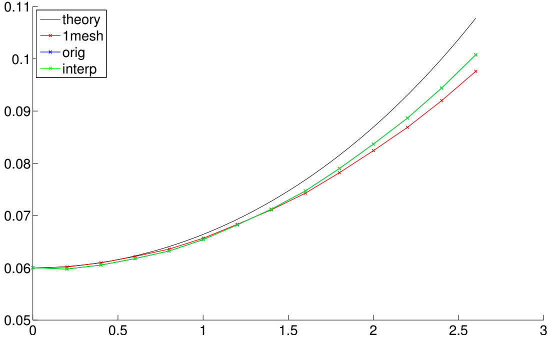


(b) interpolated interact.

Numerical results

Rayleigh-Taylor Instability with Sliding

- Goal : analyse the influence of the slide line treatment on the growth rate of the Rayleigh-Taylor instability problem against the theoretical rate.
- Two simulations : 1 single 100×600 mesh (no sliding) and two 100×300 meshes (separated by a slide line)
- Upper side is the master side.



Growth rate as a function of time (linear phase of RT instability)

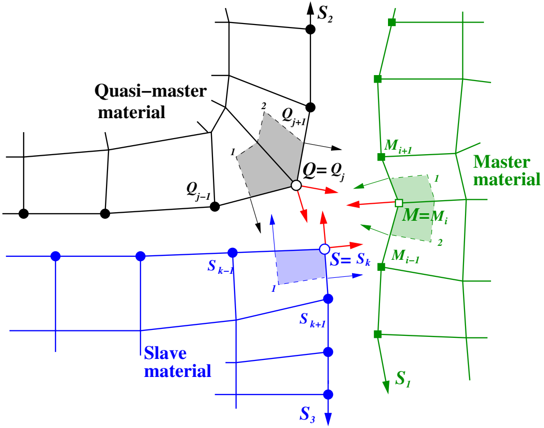
1 mesh = no slide line
orig/interp = with slide line
theory = analytical growth rate

Better growth rate when the slide line machinery is used.

Beyond 2D and two-materials

T junction

Notation



- Master (green), Quasi-master (black) and Slave (blue)
- Generic points : M, Q, S
- Apart from triple point slide line works as usual
- Contact force $\boldsymbol{G}_{MS}$ applies from Master onto Slave, $\boldsymbol{G}_{QM}$ applies from Quasi-master onto Master, etc.

Pair of contact forces

Momentum equations for triple point P in matrix form

$$\begin{pmatrix} m_M \\ m_Q \\ m_S \end{pmatrix} \frac{d}{dt}\boldsymbol{U}_P = \begin{pmatrix} \boldsymbol{F}_M \\ \boldsymbol{F}_Q \\ \boldsymbol{F}_S \end{pmatrix} + \begin{pmatrix} \boldsymbol{0} & \boldsymbol{G}_{MQ} & \boldsymbol{G}_{MS} \\ \boldsymbol{G}_{QM} & \boldsymbol{0} & \boldsymbol{G}_{QS} \\ \boldsymbol{G}_{SM} & \boldsymbol{G}_{SQ} & \boldsymbol{0} \end{pmatrix}.$$

Final form of pair of forces

$$\begin{aligned} \boldsymbol{G}_{MQ} + \boldsymbol{G}_{MS} &= \frac{m_M(\boldsymbol{F}_Q + \boldsymbol{F}_S) - (m_Q + m_S)\boldsymbol{F}_M}{(m_Q + m_S) + m_M}, \\ -\boldsymbol{G}_{MQ} + \boldsymbol{G}_{QS} &= \frac{m_Q(\boldsymbol{F}_M + \boldsymbol{F}_S) - (m_M + m_S)\boldsymbol{F}_Q}{(m_M + m_S) + m_Q}, \\ -\boldsymbol{G}_{MS} - \boldsymbol{G}_{QS} &= \frac{m_S(\boldsymbol{F}_M + \boldsymbol{F}_Q) - (m_M + m_Q)\boldsymbol{F}_S}{(m_M + m_Q) + m_S}. \end{aligned}$$

Need rescaling and velocity correction (Q+S constrained by M then S constrained by Q)

Beyond 2D and two-materials

Three dimensions

Planes of contact

- Point-to-point slide line treatment almost applies directly in 3D
- Compatible slide line treatment does not depend on dimensions
- Choice of contact normals is important
- Computer machinery is important

Conclusion and Perspectives

Conclusions

- Slide lines from [Caramana (JCP, 2009)] implemented in 2D staggered compatible Lagrangian code. Apply to our scheme almost ideally.
- However some improvements : interpolation, numerical surface tension
- Numerical tests on gases show potentiality of the method
- Beyond 2-D, 2-mats $\rightarrow$ (upcoming results)

Perspectives

- Material strength and impact of materials. Both are mandatory to validate such an approach.
- Full ALE with appropriate BCs
- Test in 3D

Acknowledgments

THANK YOU !

This research was supported in parts by the Czech Ministry of Education grants MSM 6840770022, MSM 6840770010, MEB021020, and LC528, and the Czech Science Foundation project P201/10/P086. The authors warmly thank the support of the French embassy in Prague, Czech Repulic under the Vltava program which has provided the conditions for this work to reach maturity. The authors thank E.J. Caramana for clarifications of many details and discussion of many issues related to this topic.