Conservative reconstruction methods for the semi-Lagrangian advection scheme in the IFS

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Outline

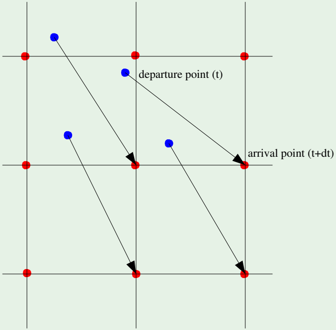
1. Introduction
2. “Non-interpolating” Methods
3. Implementation and Results
   - Method 1
   - Method 2
   - Method 3
4. Conclusions

IFS Dynamics

Spectral, semi-implicit, semi-Lagrangian on a (reduced) Gaussian grid

Semi-Lagrangian

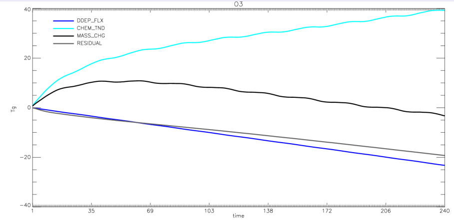
Re-mapping on the model grid at each time step (backward trajectories)



$$\psi_{arrival}^{t+dt} = \psi_{departure}^{t} (+\text{Sources})$$

Budget of chemical species in the IFS (MACC)

O_3



(J. Flemming)

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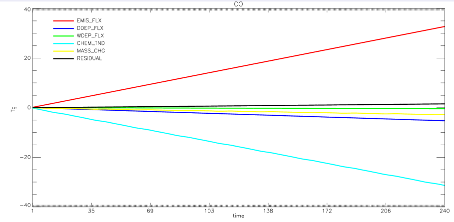


(J. Flemming)

Current S.L. interpolation in the IFS

Variables, equations

Parameters per unit of total mass: (u,v), T, q_k and the hydrostatic surface pressure π_s (mass/m^2 in vertical columns)

Equation for tracers

Mixing ratio at a grid point, no cells associated with the Gaussian grid, no control of the "continuity" of the discretized fluid.

$$\frac{dq_k}{dt} = 0$$

Interpolations

Pure grid point interpolation (no cell control), linear (4+4) or quasi-cubic (4+12+12+4) with or without monotonic (non-oscillatory) filter (the result of each cubic interpolation remains between the values at the 2 nearest grid points)

Cubic interpolations with the non-oscillatory filter are particularly non-conservative.

Outline

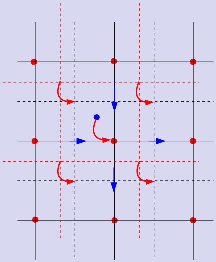
Eulerian pseudo-transport - method 1

From Smolarkiewicz and Pudykiewicz, 1992

- At departure time, computation of the grid point value at departure point thanks to an Eulerian pseudo-transport of the fluid with a constant pseudo-velocity $\vec{U}$ such as the departure point reaches its nearest grid point at the end of the pseudo-transport.
- In the transport equation, the wind is supposed to be constant (Smolarkiewicz and Pudykiewicz, 1992), even if different for each grid point. The deformation of the flow is not taken into account in the pseudo-transport:

$$\frac{\partial \psi}{\partial t} = -\vec{\nabla}.(\vec{U}\psi) = -\vec{U}.\vec{\nabla}\psi$$

Method 1

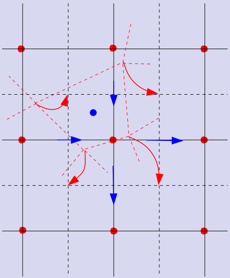


Eulerian pseudo-transport - method 2

More like Hill and Szmelter, 2010

- At departure time, move departure cells into the nearest grid box using an Eulerian scheme $\Longrightarrow$ get the amount of variable which will be inside the arrival cell at the next time step.
- The Eulerian pseudo-transport is done with a pseudo-wind field which knows aboud the real flow deformation: conservative, non-oscillatory remapping.
- Method 2 "includes" the mass budget (flux form equation). It is valid only for variables per unit volume (density type).

SL scheme with Eul. interpolation



Eulerian pseudo-transport - method 3

- Mix of method 1 and method 2: averaging the pseudo-winds computed with method 1 to built a field of pseudo-wind on a staggered grid (pseudo-wind at the edges of cells).
- The pseudo-wind field "knows" aboud the real flow deformation. Conservative, non-oscillatory method.
- Method 3 "includes" the mass budget (flux form equation). It is valid only for variables per unit volume (density type).

Outline

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Method 1 in the IFS

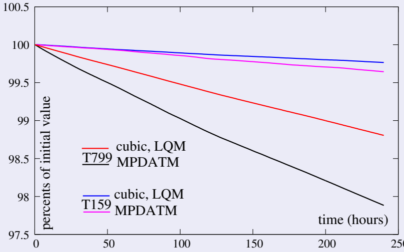
Method 1 was easy to implement in the IFS with a donor scheme and with different flavour of MPDATA

- use current SL trajectories for departure point coordinates
- use the same information than the ones needed for the current SL weights computation

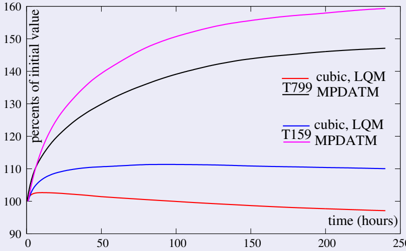
method 1 with a donor scheme = linear interpolations
method 1 with 3rd order MPDATM+NonOsc $\simeq$ cubic interpolations+QM

3D adiabatic transport

Total mass O_3
10 day evolution
no chemistry
adiabatic
no diffusion scheme



Total mass cloud content
10 day evolution
adiabatic
no diffusion scheme



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2D transport Cartesian model

Method 2 is technically not easy to implement in the IFS (trajectories of corners):
$\Rightarrow$ 2D (Cartesian, biperiodic) transport model for comparison of Eulerian and SL schemes

Advection Schemes

- 2D Eulerian MPDATM (as in EULAG)
- SL with linear or cubic interpolation as in IFS
- SL with method 1
- SL with method 2

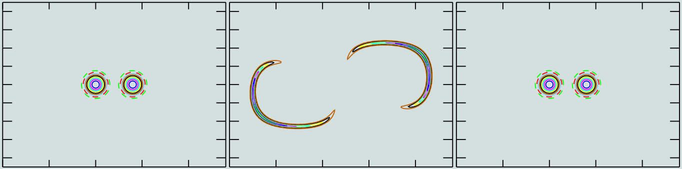
Constant winds

- Method 2 with NonOsc filter $\rightarrow$ conservative and stable for very high CFL
- unlike
  - Eulerian scheme $\rightarrow$ stable only if CFL< 1
  - Cubic interpolations or method 1 with NonOsc filter $\rightarrow$ non-conservative

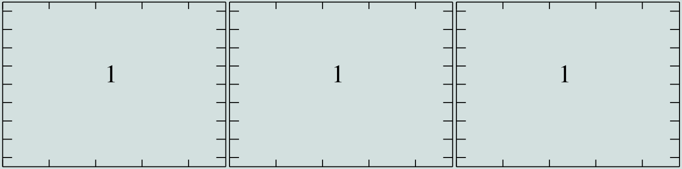
Test cases (Nair and Lauritzen, 2010)

Non-divergent but deformational flow

Tracer



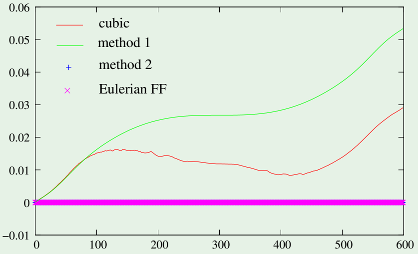
Total mass



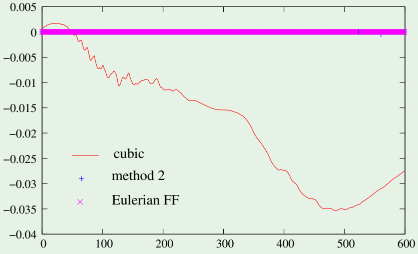
Small CFL (0.3) and limited deformation in one time step

The Eulerian advection scheme and Method 2 are equivalent. They are conservative unlike the other methods, with and without monotonous filters.

Increment of global mass of tracer(%), no QM filter



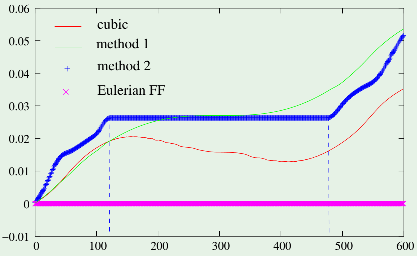
Increment of global mass of tracer(%), QM filter



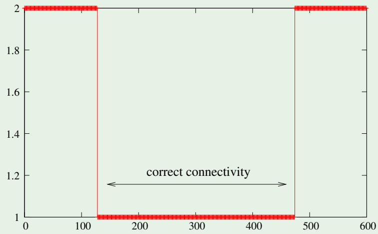
Medium CFL (0.65) and medium deformation in one time step

When the deformation/shear (?) is too strong, the "nearest point" cells do not form a connected grid. Some fluxes at the edge of the cells are only "one way". Exchanges between cells are not conservative any more.

Increment of global mass of tracer(%)



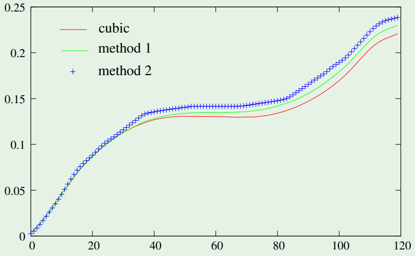
max number of time one grid point is used as nearest grid point



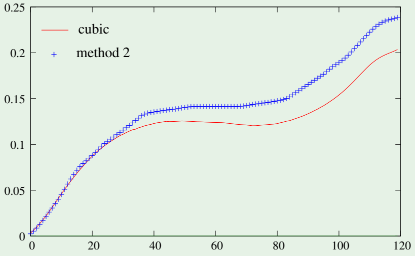
Large CFL (3.2) and very large deformation in one time step

When the CFL increases, the method 2 remains stable but the conservation is not insured any more.

Increment of global mass of tracer(%), no QM filter



Increment of global mass of tracer(%), QM filter



Outline

2D transport Cartesian model

2D Cartesian model

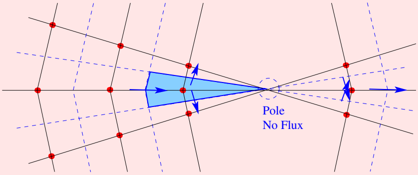
Results with Method 2 and Method 3 are similar.

IFS shallow water on the sphere

- regular Gaussian grid for connected cells definition
- transport equation for $q_x * h$ (total column of x species)
- non-parallellized implementation (all globe available without communication)
- nearest grid point=arrival point (connectivity is preserved) $\rightarrow$ CFL< 1
- Donor scheme, MPDATM

Shallow Water on the sphere

Finite difference discretisation on the Gaussian grid



Finite difference : $\surd$

$$D = \frac{1}{a\overline{\cos(\varphi)}} \frac{\left((U\psi)_{+1/2} - (U\psi)_{-1/2}\right)}{\Delta\lambda} + \frac{1}{a} \frac{\left((\cos(\varphi)V\psi)_{+1/2} - (\cos(\varphi)V\psi)_{-1/2}\right)}{\Delta\mu}$$

with $\overline{\cos(\varphi)} = 1/2(\cos(\varphi)_{+1/2} + \cos(\varphi)_{-1/2})$ or

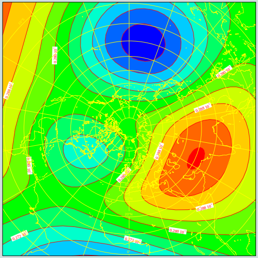
$\overline{\cos(\varphi)} = \Delta\mu/\Delta\varphi$

Shallow Water on the sphere: validation

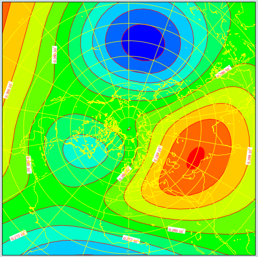
T159, regular Gaussian grid ($\Delta x = 125$ km at the equator)

Comparison between the h field given by the continuity equation of the shallow water in the IFS (spectral, SI, SL) and the field $q * h$ with $q = 1$ transported with the new scheme:

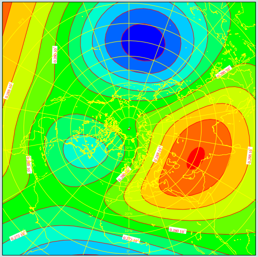
shallow water variable gh



passive scalar $q * gh$ with donor scheme

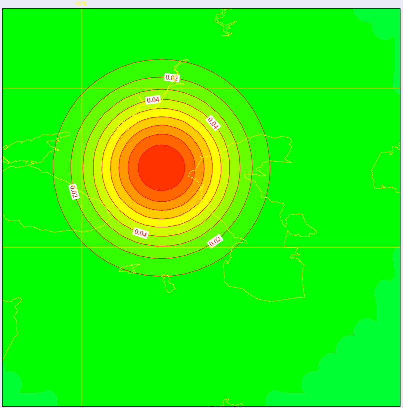


passive scalar $q * gh$ with MPDATM

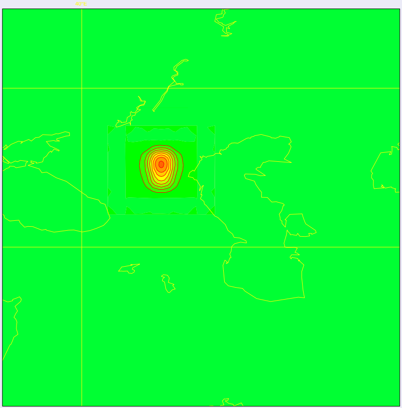


Shallow Water on the sphere: global conservation

500 km radius bell shape



100 km radius bell shape



Shallow Water on the sphere: global conservation

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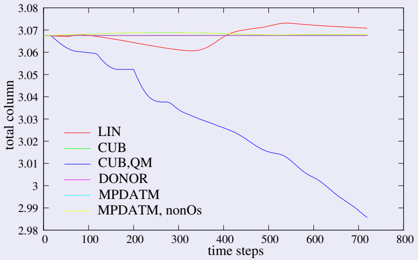
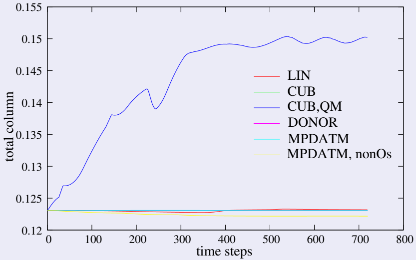
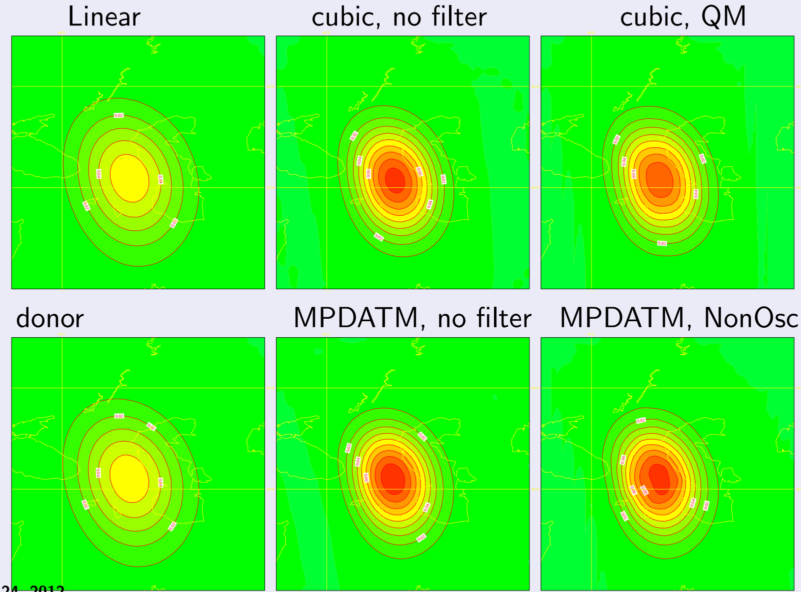


100 km radius bell shape



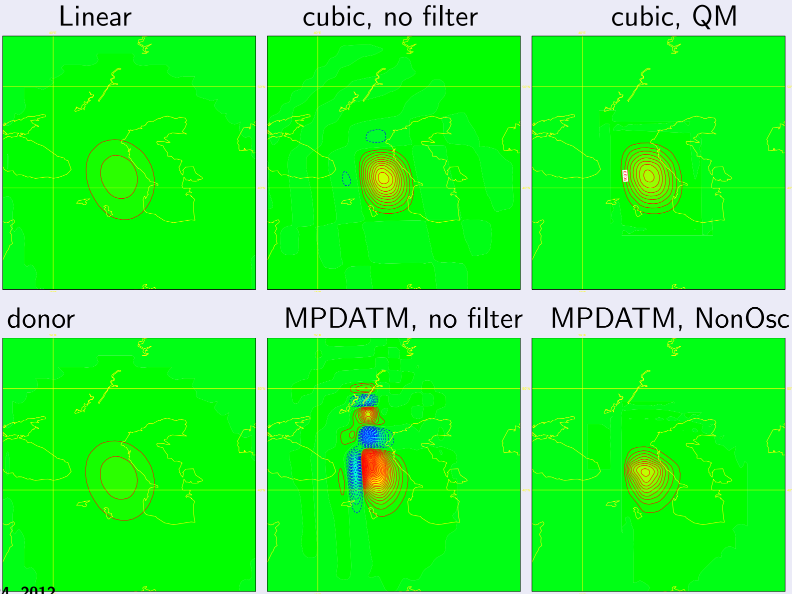
Shallow Water on the sphere: shape conservation

500 km radius bell shape



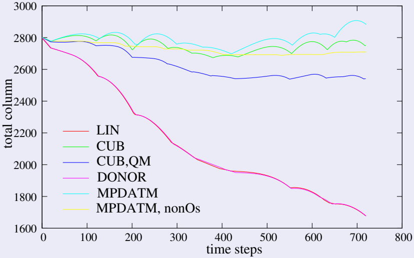
Shallow Water on the sphere: shape conservation

100 km radius bell shape

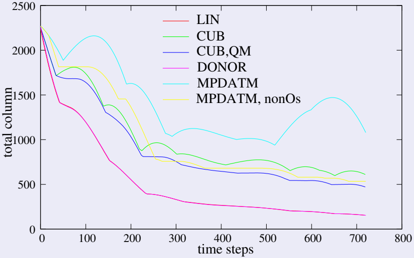


Shallow Water on the sphere: “peak” conservation

500 km radius bell shape



100 km radius bell shape



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Conclusion

3 methods for field reconstruction at departure point of a semi-Lagrangian scheme

- Method 1: move departure point to nearest grid point (Smolarkiewicz and Pudykiewicz, 1992). Implemented into the IFS, but not better than current interpolations.
- Method 2: move departure cell into nearest grid cell (Hill and Szmelter, 2010). Difficult to implement in the IFS.
- Method 3: move departure point to nearest grid point and built field of pseudo-wind at the edges. Implemented in the IFS-shallow water (2D horizontal) with the donor scheme and MPDATM.

Conclusion

- Method 2 and 3:
  - non-deformational flows: conservative with very high CFL
  - deformational flows: conservative if the connectivity of the "nearest grid cell" grid is preserved.
- Method 3 in the IFS: very expensive
  - regular Gaussian grid (connectivity) – [but Rasch, 1994?]
  - small time step (CFL<1, for security: nearest grid point = arrival point)
  - need to change prognostic variables in the IFS
  - Scalability? (more communications)
- Applicable to all equations in the IFS?
- 3D version?
- Interest of the semi-Lagrangian formalism versus an Eulerian Flux-Form scheme?