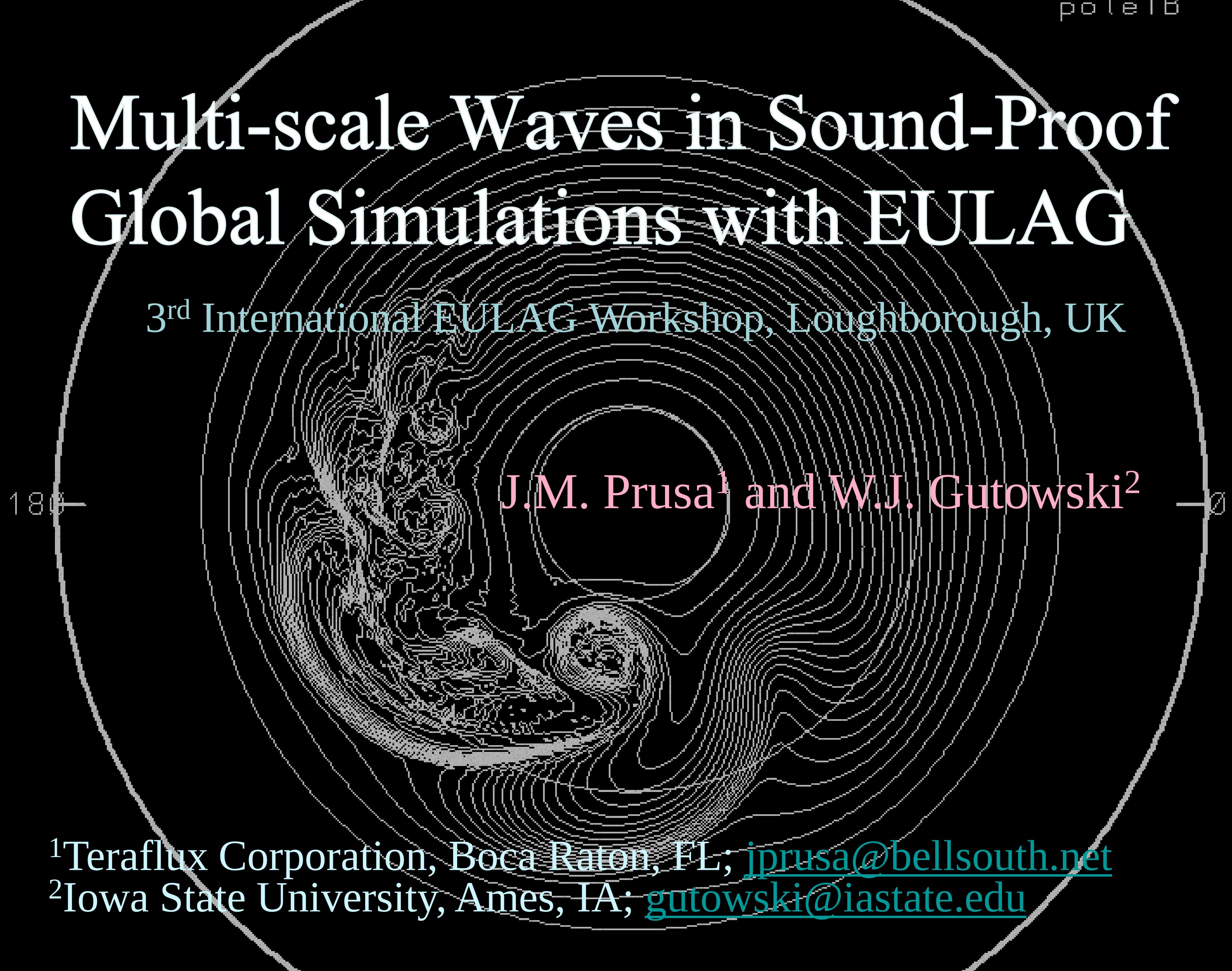

Multi-scale Waves in Sound-Proof Global Simulations with EULAG

3rd International EULAG Workshop, Loughborough, UK

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EULAG Computational Model

(see Prusa, Smolarkiewicz, and Wyszogrodzki, *J. Comp. Fluids* 2008 for review)

- **NFT integration algorithm**
- SL or fully conservative **Eulerian** advection
- Robust, preconditioned non-symmetric Krylov solver for pressure
- **Implicit integration of θ perturbation**
- Nonhydrostatic, deep moist **anelastic** equations
- Demonstrated scalability to thousands of PE’s
- GA via continuous remapping of coordinates
- Turbulence model options: DNS, LES, or **ILES**

JW Baroclinic Instability Test

(Jablonowski and Williamson, QJRMS 2006)

- Idealized dry global baroclinic instability test
- Balanced initial state with prescribed environmental profiles
- Gaussian perturbation in zonal wind introduced to seed a perturbation to “grow” baroclinic instability
- Instability grows linearly for first 8 days. Characterized by (i) distinct waves, and (ii) amplitudes that grow exponentially in time
- Nonlinear interactions after 8 days and wave breaking after 10 days

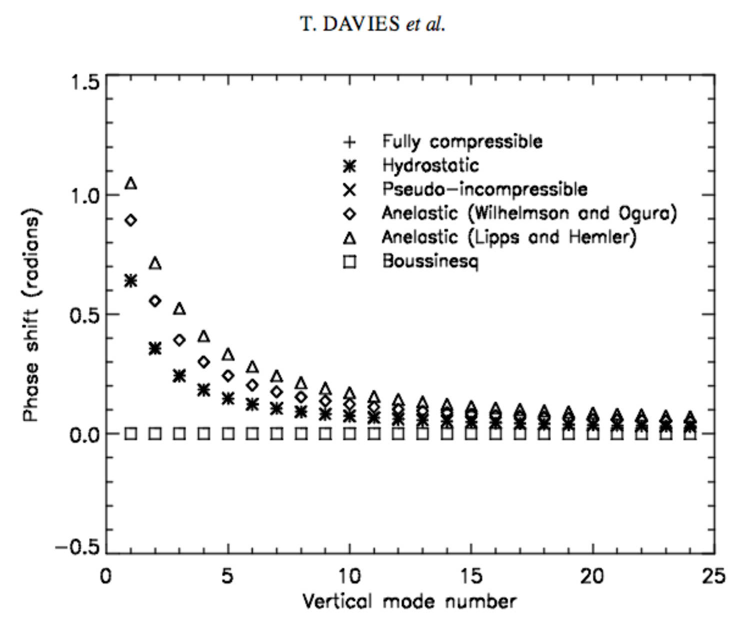
Linear Modal Analyses:

- Davies et al, *QJRMS* 2003 – 2D normal mode *f*-plane, isothermal analysis used to rigorously examine sound-proof systems:

(i) "merit for small-scale motions is well recognized"

(ii) Anelastic not good for amplitudes and height scales of external planetary modes, nor for finite amplitude Lamb (acoustic) waves

(iii) Anelastic introduces phase error for deep wave modes



- Arakawa and Konor (*MWR* 2009) – analysis of Davies et al. extended to β-plane with similar conclusions.

Balanced initialization for EULAG is *not given* by JW initialization

JW test 2

–> A good match in linear regime will not occur unless the parameters controlling the waves for the balanced state match those of JW.

An elementary *2-layer, geostrophic model* (Holton, 2004) – based upon the pioneering works of Charney (1947) and Eady (1949), demonstrates that linear wave properties are determined primarily by the **mean wind** $\langle u_e \rangle$ and **thermal wind** u_T for representative values of **static stability** σ.

Balance, cont.

JW test 3

The thermal wind effect is secondary on *phase speeds*, but dominates *disturbance growth rates*.

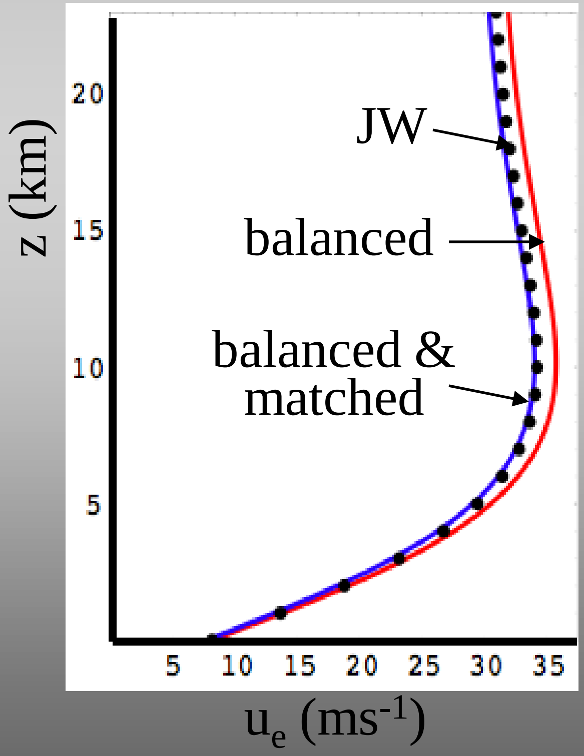
$$c_x = c_o \pm \sqrt{D}, \text{ where } \lambda = \lambda(\sigma)$$

$$c_o = < \mathrm{u_e} > - \frac{\beta(k^2 + \lambda^2)}{k^4(k^2 + 2\lambda^2)}$$

$$D = \frac{\beta^2 \lambda^4}{k^4(k^2 + 2\lambda^2)^2} - \frac{\mathrm{u_T^2}(2\lambda^2 - k^2)}{k^2 + 2\lambda^2}$$

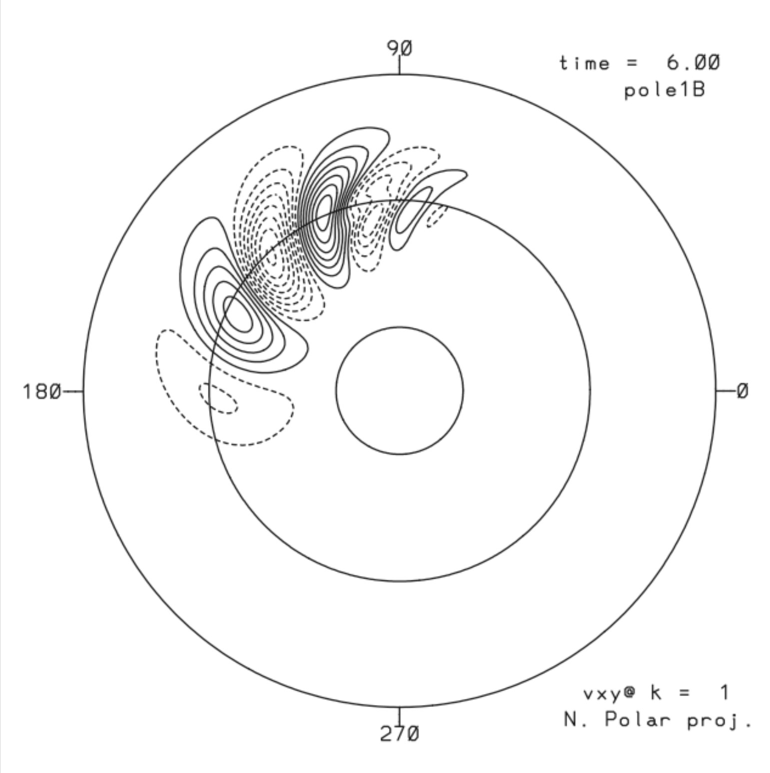
If $D < 0$, then $D^{1/2} = \mathrm{i}\zeta$ and $\tau = 1/(k\ \zeta)$ is the growth rate

Env. zonal wind

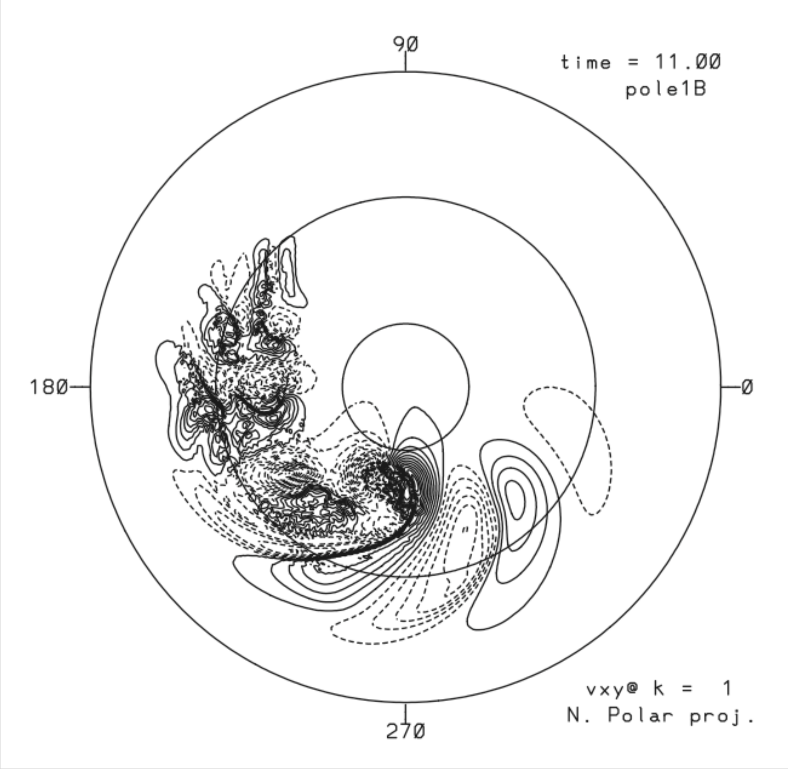


Baroclinic Instability Test: Linear and wave-breaking regimes (Jablonowski and Williamson, QJRMS 2006)

Meridional wind field (global 0.7^o resolution)



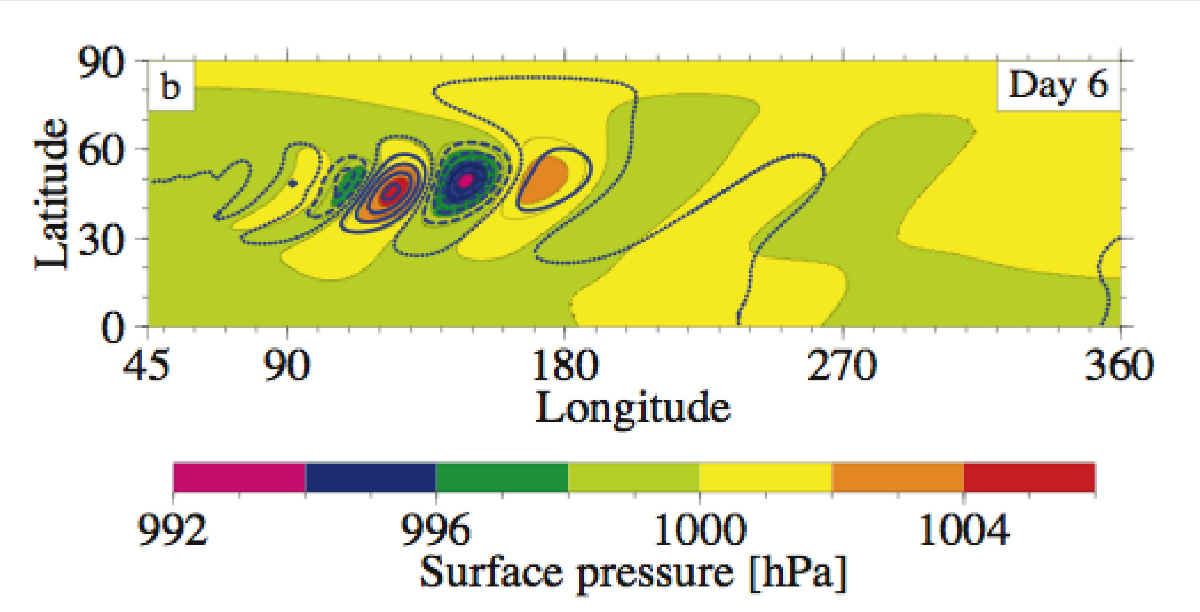
(mx, mn, cnt)=(6, -6, 0.8) ms^{-1}



(mx, mn, cnt)=(40, -40, 3) ms^{-1}

Surface Pressure Comparisons

JW test 5

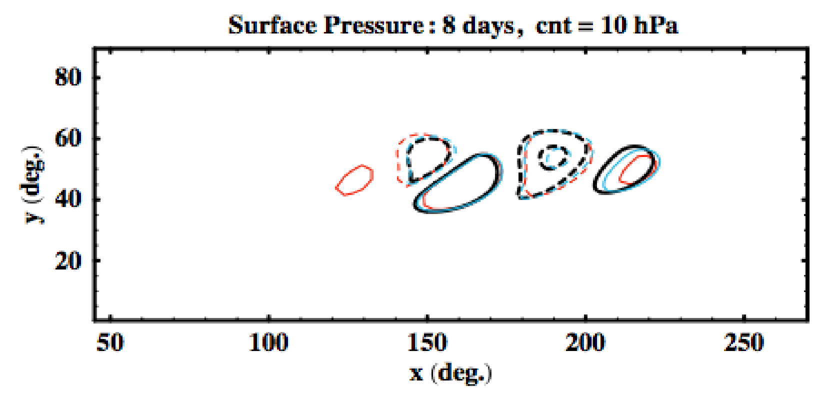


BELOW:
resolution effects for EULAG simulations

Red - 2.8^o resolution
Aqua - 1.4^o
Dark Blue - 0.7^o

ABOVE:
Color - JW results using CAM
FV dycore
Blue dots/dashes - EULAG

$\Delta c_x / c_x \sim 0.3\%$



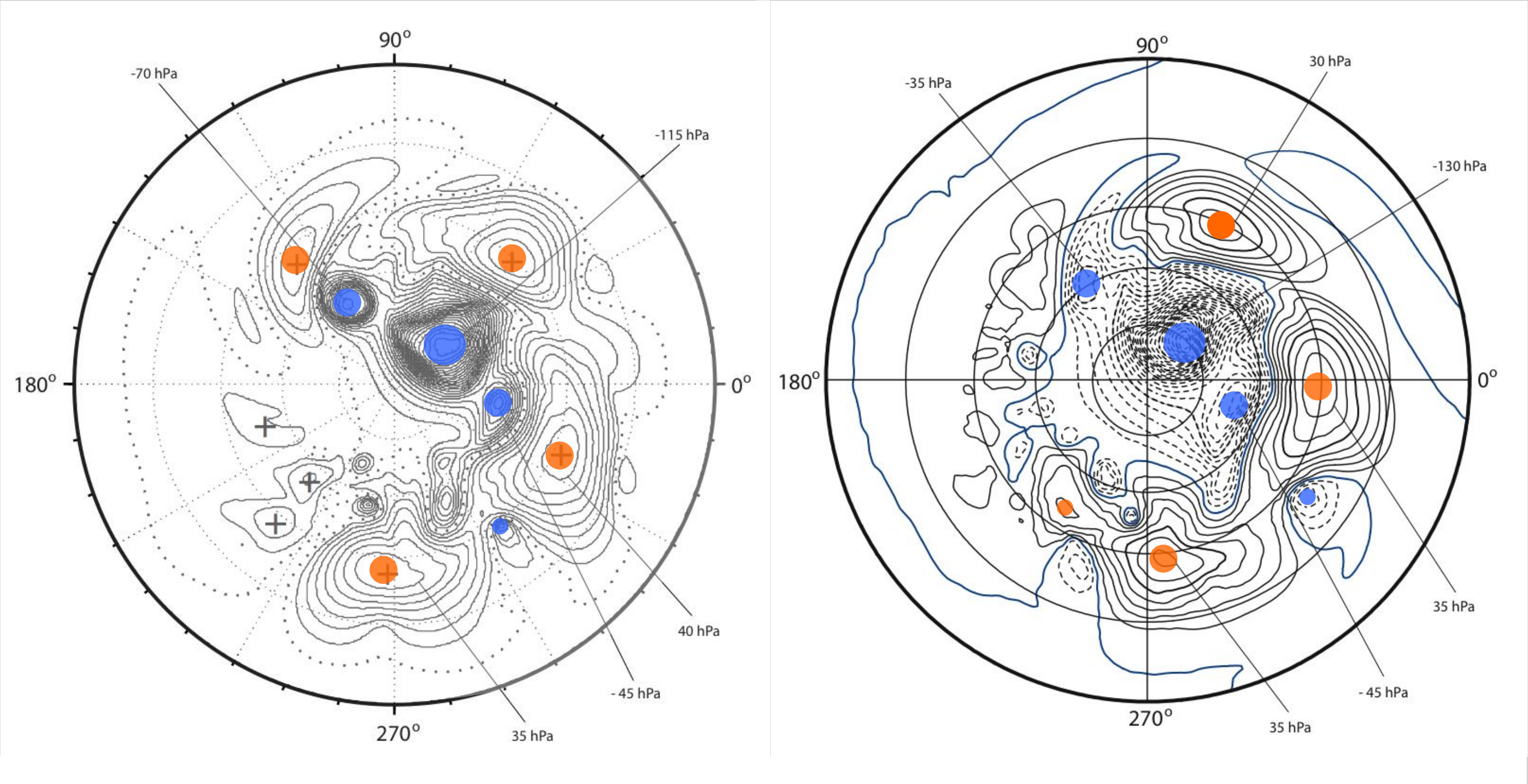
Surface Pressure:

High resolution grid: 16 days

JW test 6

JW results using CAM Eulerian spectral dycore (T172)

EULAG results using Eulerian advection (0.7°)



How consistent are these results with compressible linear modal analyses?

- Dispersion equation for 2D β-plane approximation, assuming static, isothermal environment (AK 2009):

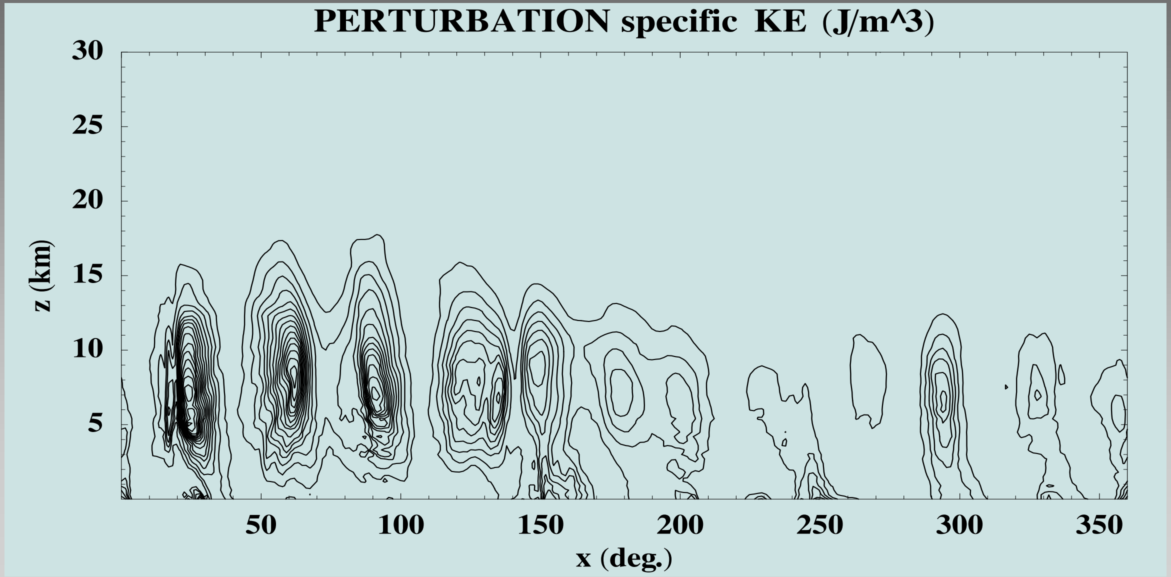
$$\omega = - \beta k / \{ k^2 + f_o^2 [c_s^{-2} + H M^2 (\kappa g)^{-1}] \}$$

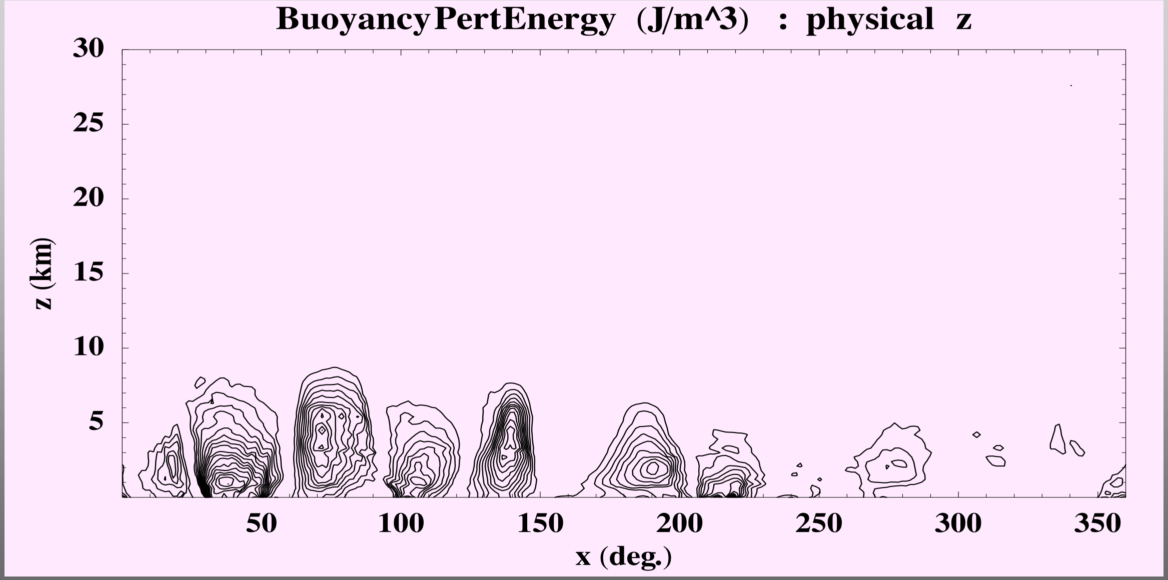
where $M^2 = m^2 + \mu^2 ; \quad \mu = (1/2 - \kappa)/H$

- Requisite parameters: (β, f_o, c_s, κ, H) <– environment

- Anelastic model: $c_s \rightarrow \infty$ and $\mu \rightarrow 1 / 2H$

define phase error = $1-\omega_{an} / \omega_{com}$

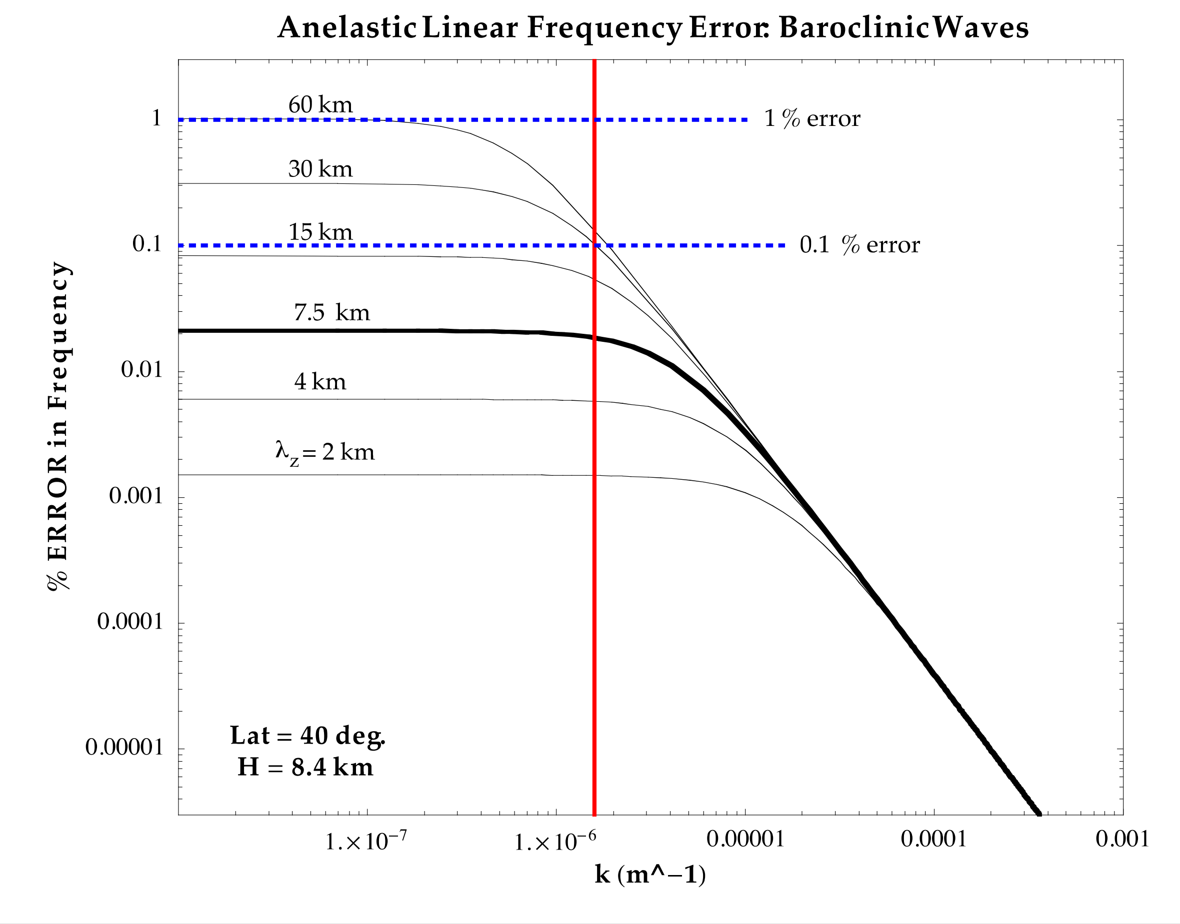




What is appropriate vertical scale for baroclinic modes?

HS simulations
→ ~ 15 km

Consistency with compressible modes (AK2009)



BAROCLINIC:

Shown: β-plane –

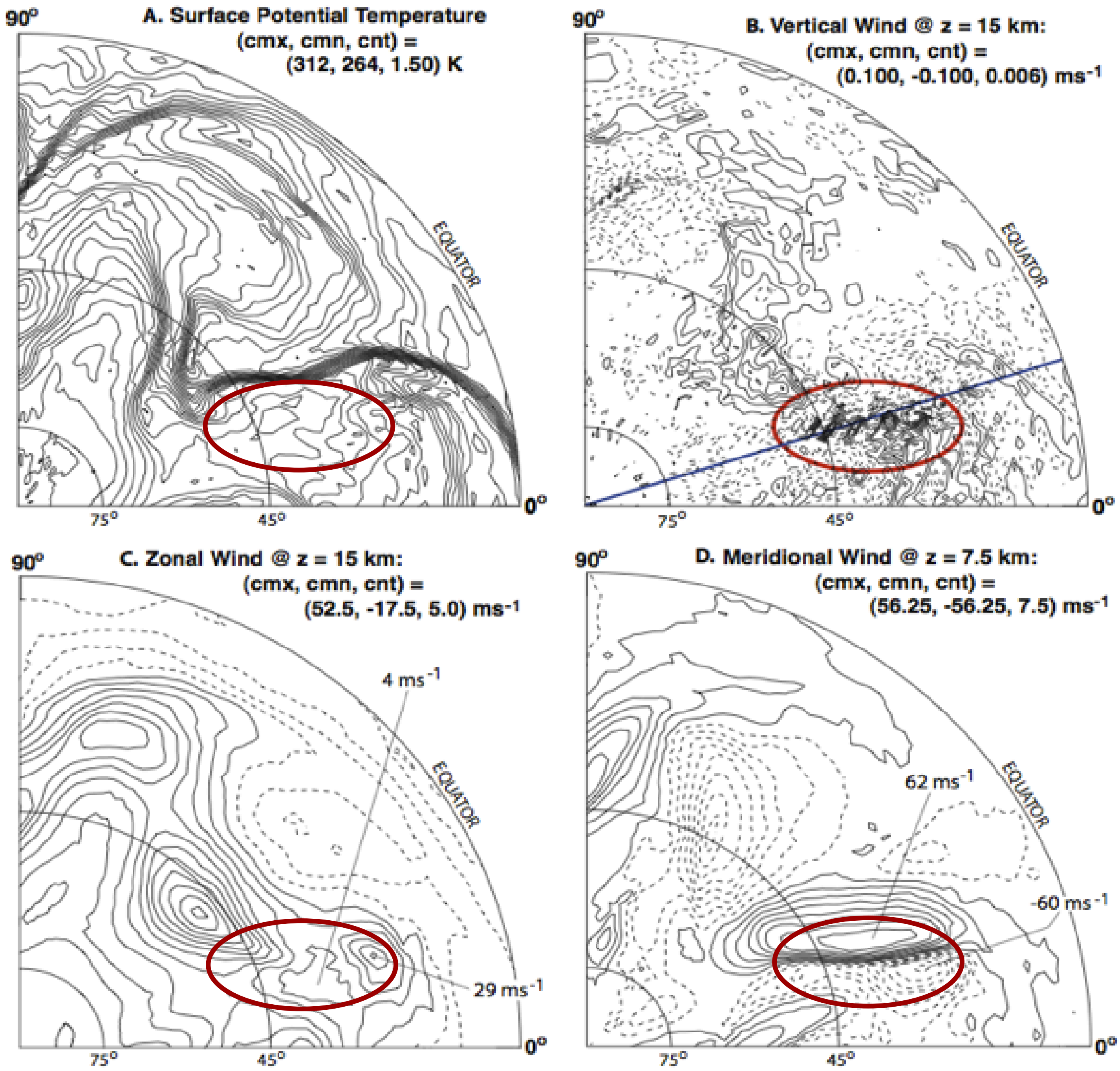
Red line

λ = 3950 km

HELD-SUAREZ FLOWS

(Held and Suarez, *BAMS* 1994)

- Idealized dry global climate simulation
- Prescribes Rayleigh damping of low level winds to approximate PBL
- Prescribes Newtonian relaxation of temperature to emulate radiative heat transfer
- Held-Suarez (HS) climate develops in an approximately stationary, quasi-geostrophic state that replicates essential climate features

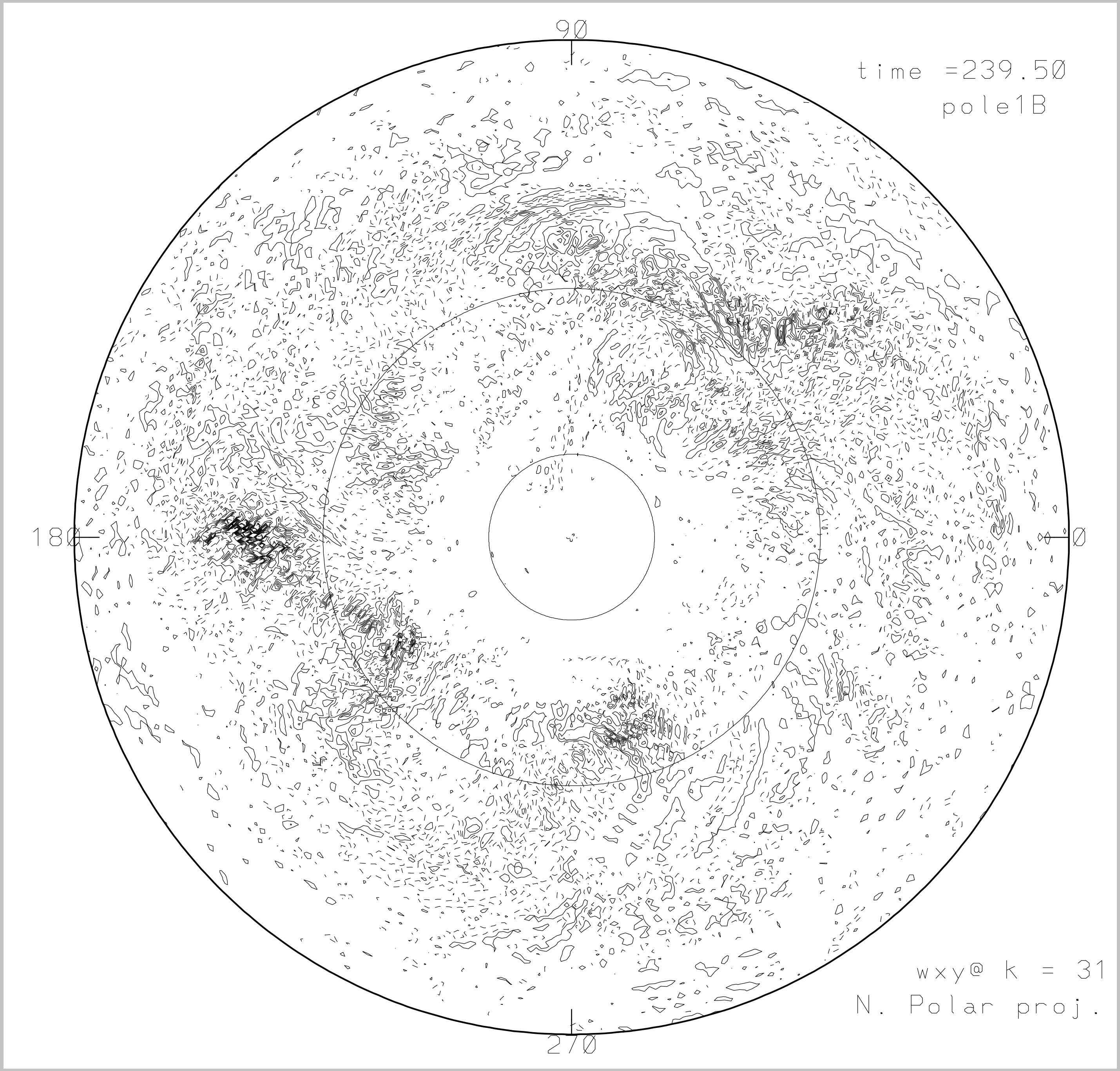


Event
NW5 @
277.50
days

N. Polar
proj.

Vertical Wind Field @ 15 km for 0.7° deg grid

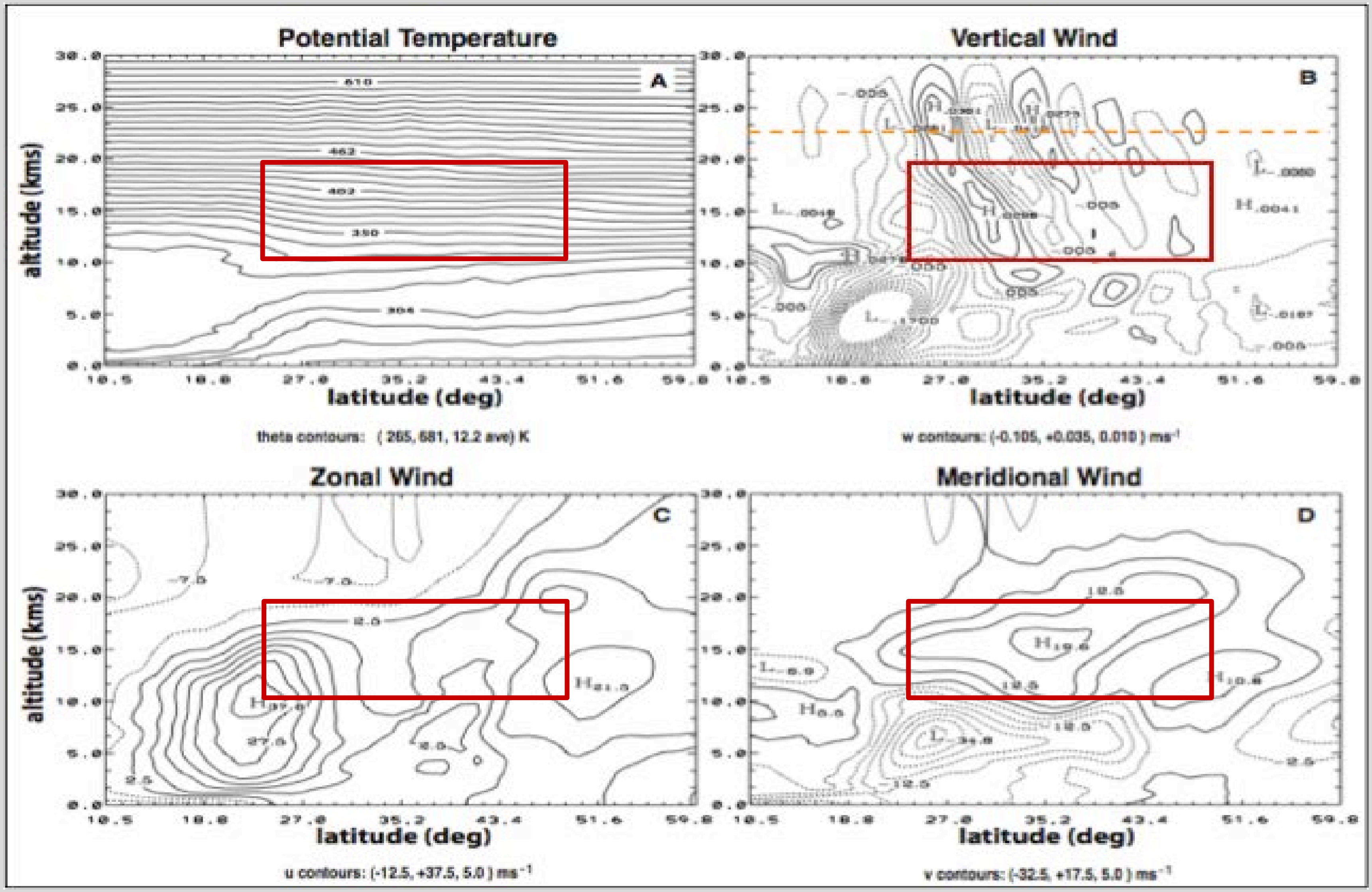
N. Polar proj.



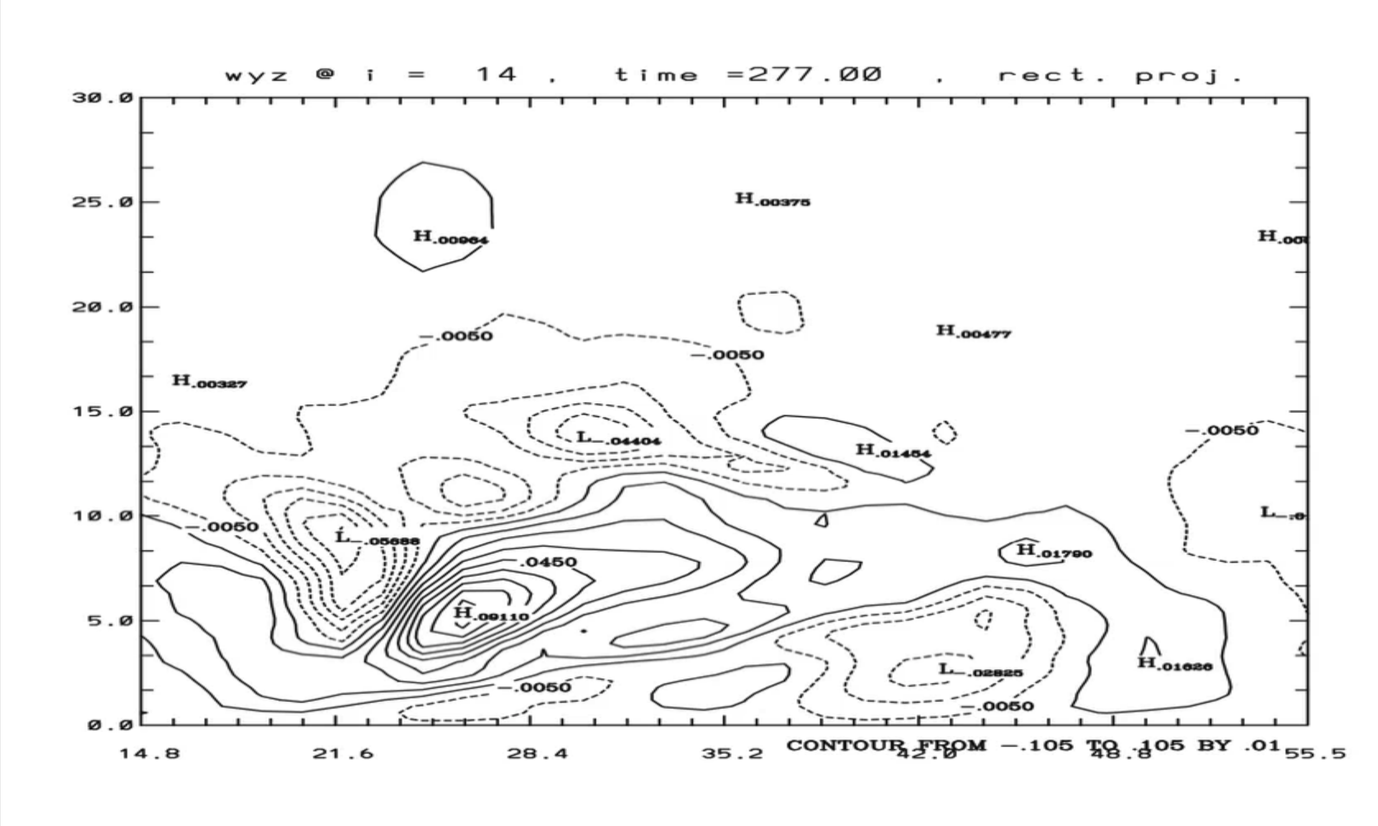
cmx = 0.21
cmn = -0.21
cnt = 0.02

NW5 – Vertical x NS cross section

HS, cont.



NW5 animation:
meridional slice @ 18 deg. Long.



HS, concluded

NW5 event: Analysis

- Observed properties

P_i = 5.3 hr
λ_x = 550 ± 60 km
λ_y = 620 ± 60 km
λ_z = 14.3 ± 1.1 km

c_x = -31 ± 15 ms^{-1}
c_y = -32 ± 5 ms^{-1}
c_{gx} = 9. ± 1. ms^{-1}
c_{gy} = -2 ± 6 ms^{-1}

U_e = 12.6 ± 8 ms^{-1}
V_e = 14.9 ± 15 ms^{-1}

- Deduced properties

(via dispersion eq. for IGW's)

$$\omega_{rel}^2 = f^2 + \frac{N^2(k^2+l^2)}{(K^2+1/4H_\rho^2)}$$

Caveat: fields are not uniform in space or time

→ Choose negative root

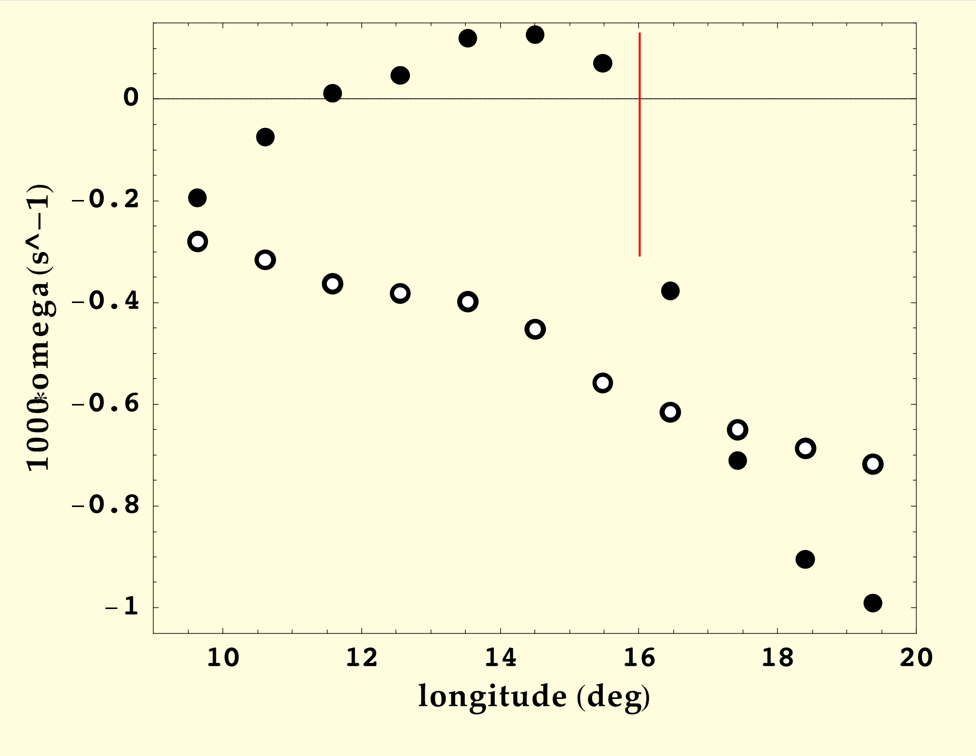
$$\omega_{rel} = -2\pi/2.815\ \text{hr}^{-1},\ \omega = -2\pi/5.33$$

c_x = -28.9 ms^{-1} , c_y = -32.6 ms^{-1}

c_{gx} = -18.0 ms^{-1} , c_{gy} = -12.3 ms^{-1}

WKB analysis: critical surface forms to the SW and blocks predicted wave propagation

Vertical structure equation indicates assumption of uniform wind may be serious error → nonlinear analysis required

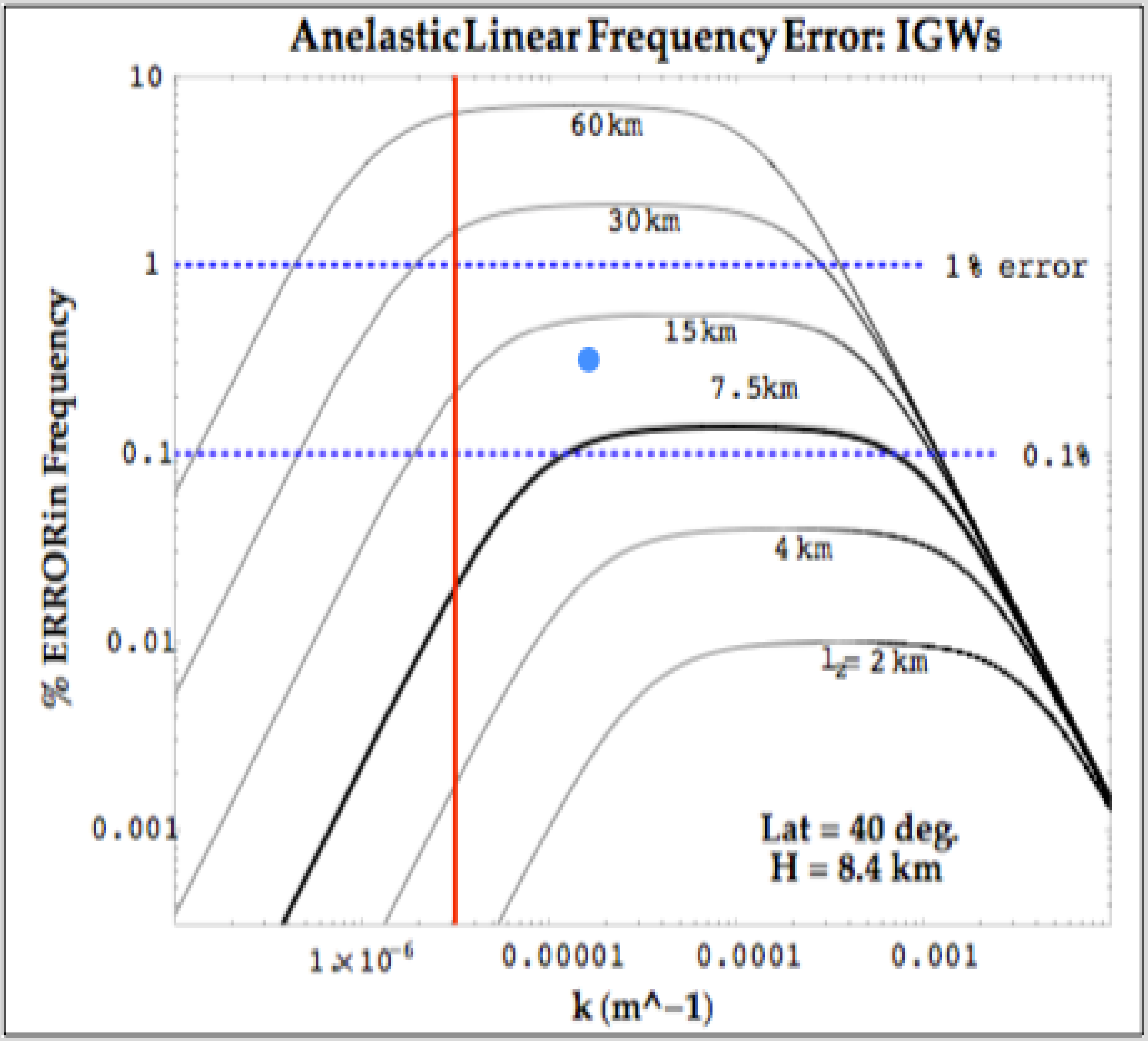


WKB theory: handles effects of “weak” time and space gradients

Critical Surface defined by $\omega_i = 0$ (Bretherton 1966, 1971)

Figure: ω_i along 215^o ray into SW predicted by leading order WKB

Consistency with compressible modes (AK2009)



BAROCLINIC:
Shown: *f*-plane
-
blue dot is NW5
→ 0.3% phase error

β-plane – 0.1% phase error for λ~ 4000 km

BAROTROPIC:
0.3% phase err. for λ ~ 1500 km

Remarks

- Baroclinic wave test results show excellent agreement with published results of JW (2006) in linear wave regime. REQUIRED: initialization balanced for EULAG; and MATCHING of env. mean wind, thermal wind, and stability. In wavebreaking regime, differences arise in details.

- Global HS simulations show (some) localized packets of internal gravity waves in lower stratosphere. Linear modal analysis predicts all properties of a representative wave packet except group velocity, which WKB analysis shows depends strongly upon wind gradients in local environment of waves. → Linear analyses do not give correct group velocities due to nonlinear synoptic scale wave interactions.

Why Baroclinic Motion?

- “Baroclinic instability is the most important form of instability in the atmosphere, as it is responsible for mid-latitude cyclones” (Houghton, 1986)
- Baroclinic wave breaking radiates gravity waves, which act to restore geostrophic balance (Holton, 2004).

Why Study Baroclinic Dynamics?

- "Baroclinic instability is the most important form of instability in the atmosphere, as it is responsible for mid-latitude cyclones" (Houghton, 1986)
- Baroclinic wave breaking radiates gravity waves, which act to restore geostrophic balance (Holton, 2004) –> multi-scale physics
- J. Charney (*J. Meteor* 1947), E. Eady (*Tellus* 1949), and J. Smagorinsky (*MWR* 1963)
- Good test case for multi-scale global atmospheric models

Baroclinic Dynamics

- J. Charney (*J. Meteor* 1947), E. Eady (*Tellus* 1949), and J. Smagorinsky (*MWR* 1963)
- Baroclinicity is due to a horizontal temperature gradient: $(\nabla\rho \times \nabla p)/\rho^3 \neq 0$ is the baroclinicity vector, **Ba**
- Induced horizontal gradient in density creates yz circulation, in NH rising air moves N vs. sinking air moves S -> SLANTWISE CONVECTION
- $\frac{\partial U_g}{\partial z} = -\frac{g}{Tf}\frac{\partial T}{\partial y}$ and $\frac{\partial V_g}{\partial z} = \frac{g}{Tf}\frac{\partial T}{\partial x}$ are the resulting geostrophically balanced, thermal winds.
- Baroclinic instability grows planetary waves by converting APE associated with the horizontal temperature gradients that are required for thermal wind balance (Holton, 2004)

ANELASTIC elementary scale analysis– continuity (AK2009):

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{u}) = 0$$

$$\rho = \rho(p, \theta) \longrightarrow \Delta\rho \approx \left.\frac{\partial \rho}{\partial p}\right|_{\theta} \Delta p + \left.\frac{\partial \rho}{\partial \theta}\right|_{p} \Delta\theta$$

(i) Acoustic term: $\left.\frac{\partial \rho}{\partial p}\right|_{\theta} \Delta p = \left(\frac{\rho}{\gamma p}\right)\Delta p = \frac{\rho_o}{\gamma}\left(\frac{\rho}{\rho_o}\right)\left(1 - \frac{p_o}{p}\right)$

$$p\rho^{-\gamma} = c$$

reversible
gas dynamics

$$\rho / \rho_o = \left[1 - (\gamma - 1) M^2 / 2\right]^{-1/(\gamma-1)}$$

$$\longrightarrow \left.\frac{\partial \rho}{\partial p}\right|_{\theta} \Delta p = -\rho_o M^2 / 2 + \ldots \quad = -\rho_o \delta_\theta$$

(ii) Thermobaric term: $\left.\dfrac{\partial\rho}{\partial\theta}\right|_p \Delta\theta \approx \Delta\rho\big|_p$

Baroclinic dynamics

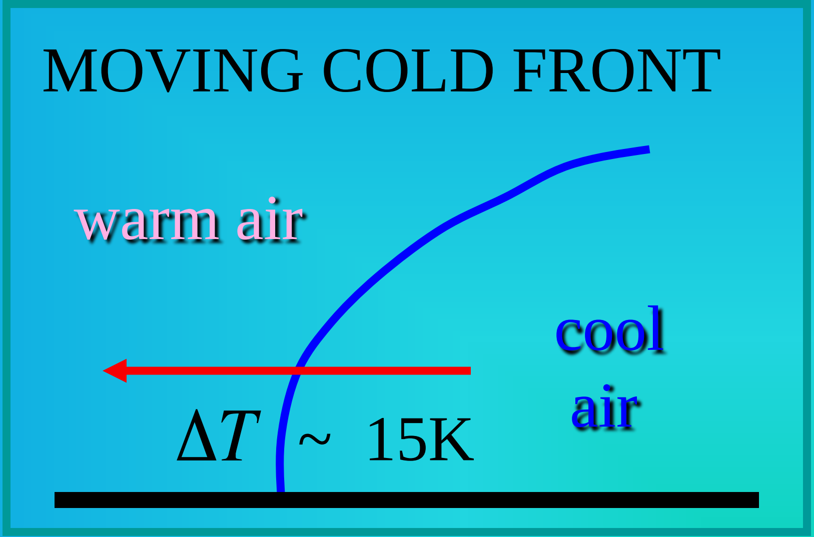
$$p = \rho RT, \qquad \theta = T(p_o/p)^{\kappa}$$

$$\rightarrow \Delta\rho/\rho_o \approx -\Delta T/T_o = -\Delta\theta/\theta_o$$

$$\rightarrow \left.\frac{\partial\rho}{\partial\theta}\right|_p \Delta\theta \approx \rho_o\delta_p$$

where

$$\delta_p = -\Delta\theta/\theta_o \sim 0.05$$


MOVING COLD FRONT

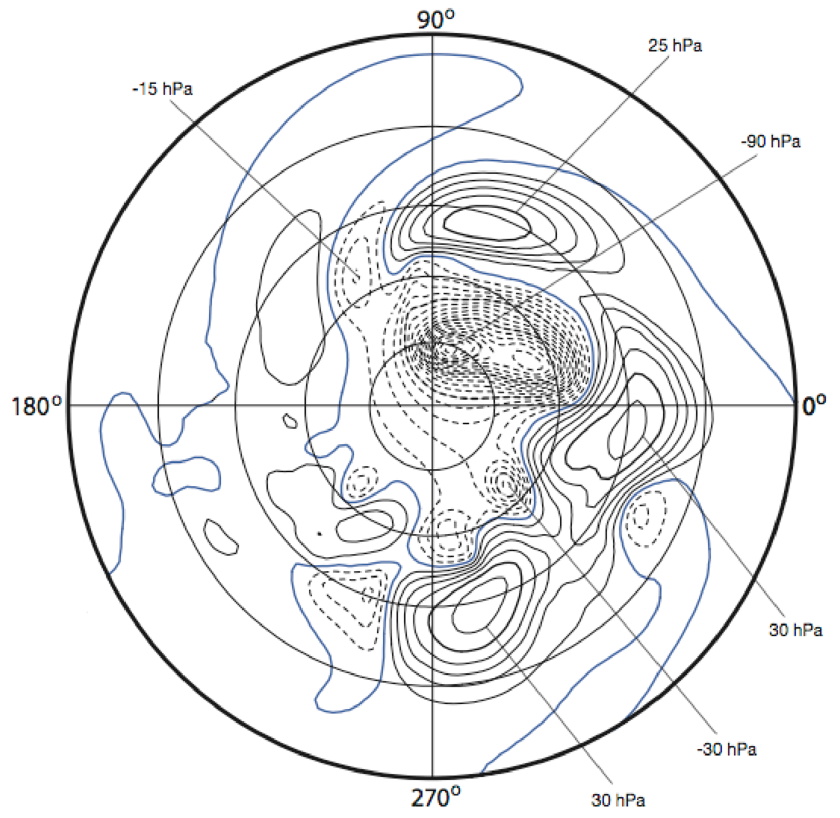
Baroclinic concerns…

- The anelastic system of equations are known to accurately represent “smaller scale” atmospheric gravity waves
- But there are concerns about longer scale **planetary waves, acoustic modes,** and **baroclinicity…**
- Let $\rho = \rho_o(z) + \rho'(t,x,y,z)$, where $\rho_o(z)$ is the anelastic *basic state* density. The basic state provides a hydrostatic reference that underlies the anelastic system.
- Then **Ba** ~ $\frac{\partial \rho_o}{\partial z}\left(\frac{\partial p}{\partial x}\mathbf{i}+\frac{\partial p}{\partial y}\mathbf{j}\right)+\cancelto{0}{\frac{\partial \rho'}{\partial x}\frac{\partial p}{\partial z}}\mathbf{i}+\cancelto{0}{\frac{\partial \rho'}{\partial x}\frac{\partial p}{\partial y}}\mathbf{k}+...$
- Twisting/tilting of horizontal baroclinic vorticity will produce effects in the vertical
- $\rho'/\rho_o = p'/(\rho_o g H_\rho) - \theta'/\theta_c$ (Bannon, *JAS* 1996a).

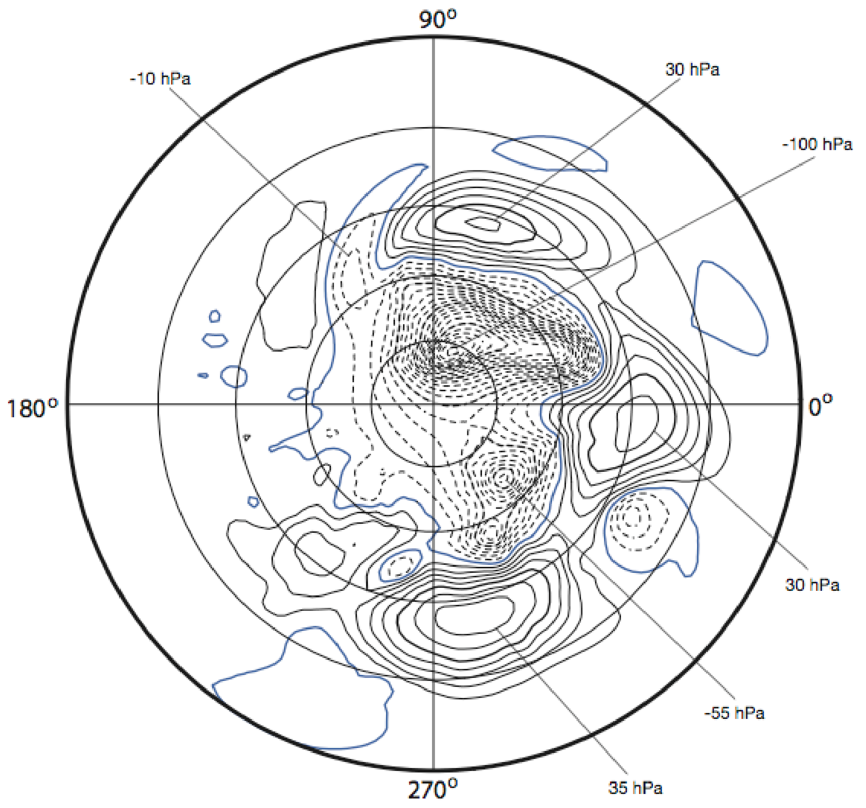
Surface Pressure: 16 days

JW, concluded

EULAG results (1.4^{o}) using Semi-Lagrangian advection



EULAG results (1.4^{o}) using Eulerian advection



How consistent are these results with compressible linear modal analyses?

- Dispersion equation for 2D *f*-plane approximation, assuming static, isothermal environment (AK 2009):

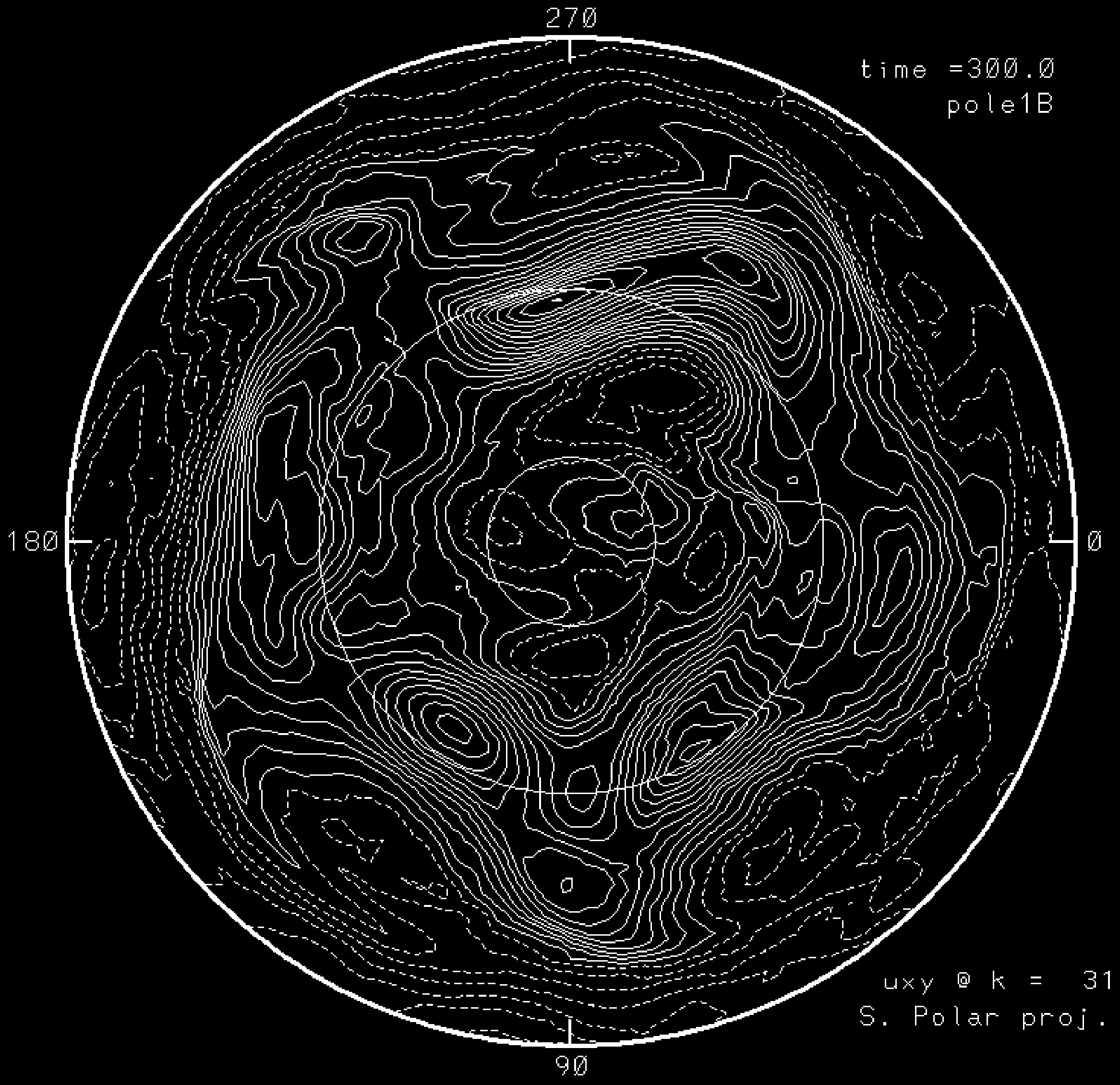
$$\omega^4 - \left\{ N^2 + f^2 + k^2 c_s^2 (k^2 + M^2) \right\} \omega^2$$
$$+ \left\{ N^2 f^2 + c_s^2 (N^2 k^2 + f^2 M^2) \right\} = 0$$

where $M^2 = m^2 + \mu^2$; $\mu = (1/2 - \kappa)/H$

- Requisite parameters: (N, f, c_s, κ, H) <– environment
- Anelastic model: $c_s \rightarrow \infty$ and $\mu \rightarrow 1 / 2H$

define phase error = $1-\omega_{an} /\omega_{com}$

Held-Suarez flow



Isolines of constant u show "westerly jets" @ 15 km alt.

typical jet core $\Delta u_{max} \sim 65\ \mathrm{ms}^{-1}$ over synoptic scales

(mx, mn, cnt)=(48, -24, 4.2 ms^{-1})

HS, cont. 1

Isolines of
constant θ
show "fronts"
on surface

typical ΔT_{max} ~
20K over
mesoscales
(surface fronts)

Errors of anelastic arise in density (AK2009)

$$\rho = \rho(p,\theta) \rightarrow \Delta\rho \approx \left.\frac{\partial\rho}{\partial p}\right|_{\theta} \Delta p + \left.\frac{\partial\rho}{\partial\theta}\right|_{p} \Delta\theta$$

(i) Acoustic term:

$$\rightarrow \left.\frac{\partial\rho}{\partial p}\right|_{\theta} \Delta p = -\rho_o M^2/2 + \ldots \ = -\rho_o\delta_\theta$$

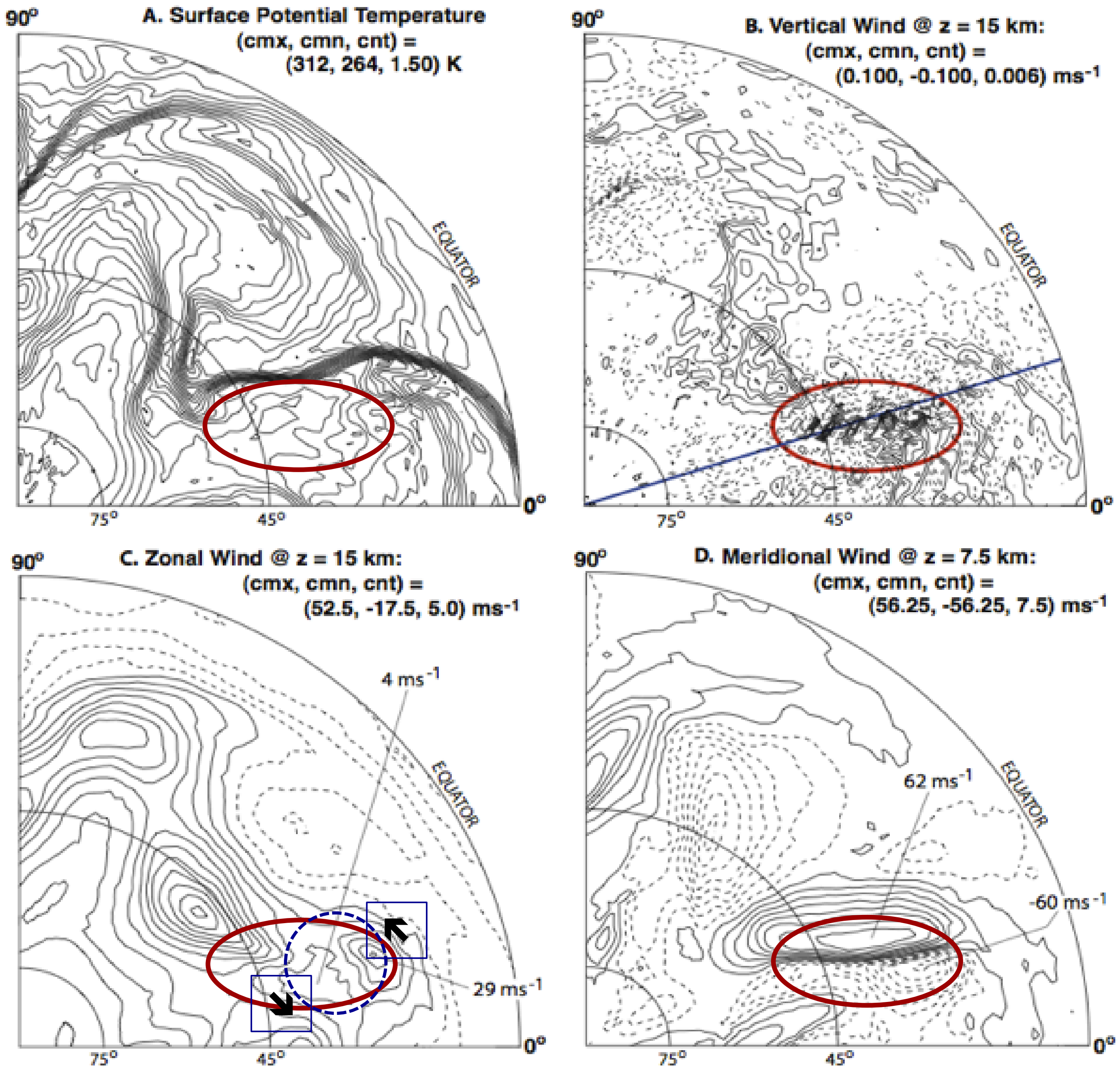
NOTE: 65 ms^{-1} → $M = 0.2$ → $\delta_\theta = 0.02$

(ii) Thermobaric term:

$$\rightarrow \left.\frac{\partial\rho}{\partial\theta}\right|_{p} \Delta\theta \ = \ - \rho_0 \ \Delta T / T_o = - \rho_0 \ \delta_p$$

NOTE: $\Delta T = 20$ K → $\delta_p = 0.06$

→ Compare with $\omega/N = 0.03$ for synoptic waves

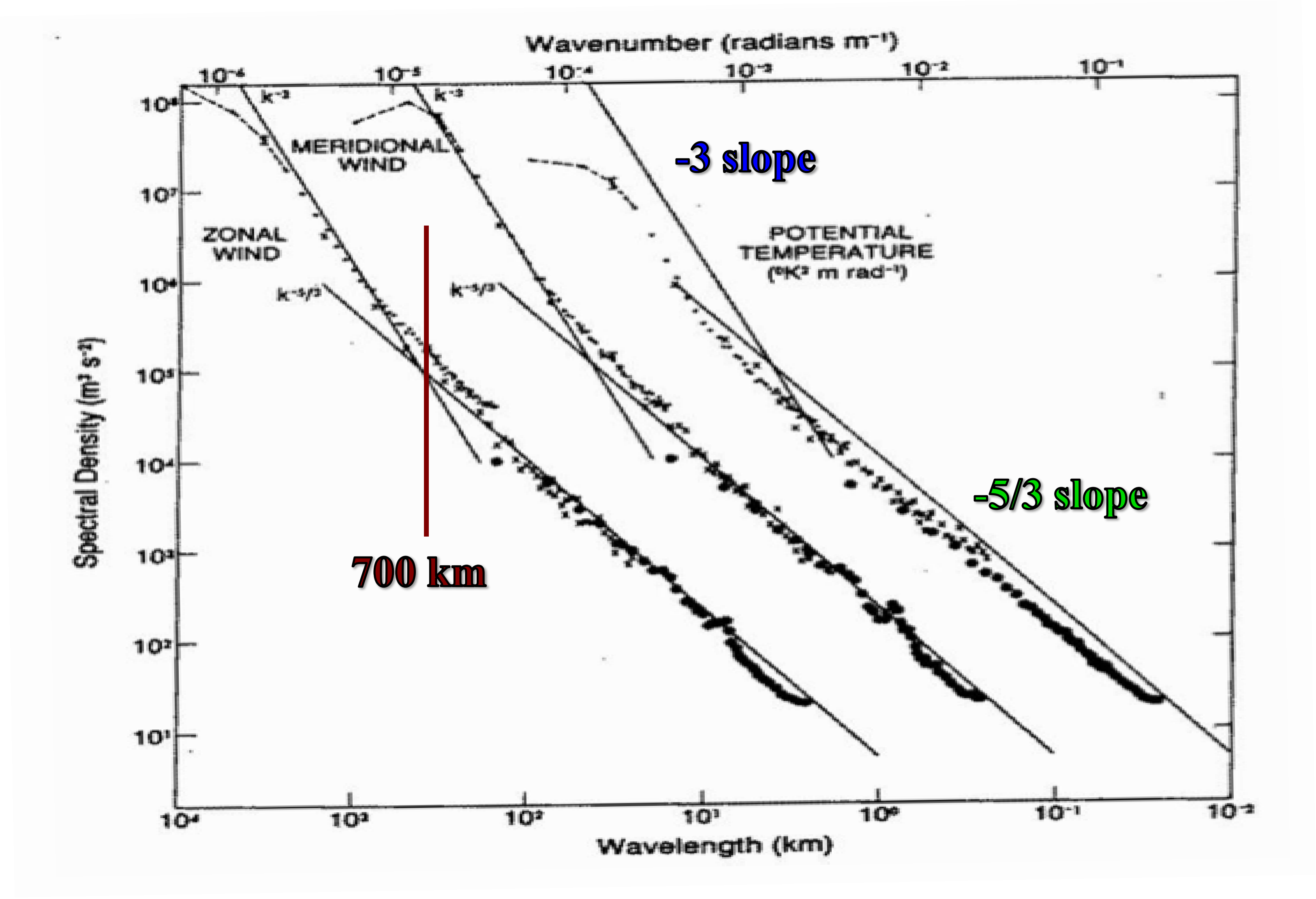


Event
NW5 @
277.50
days

N. Polar
proj.

Universal Atmospheric Power Spectrum

(Gage and Nastrom *JAS* 1986)



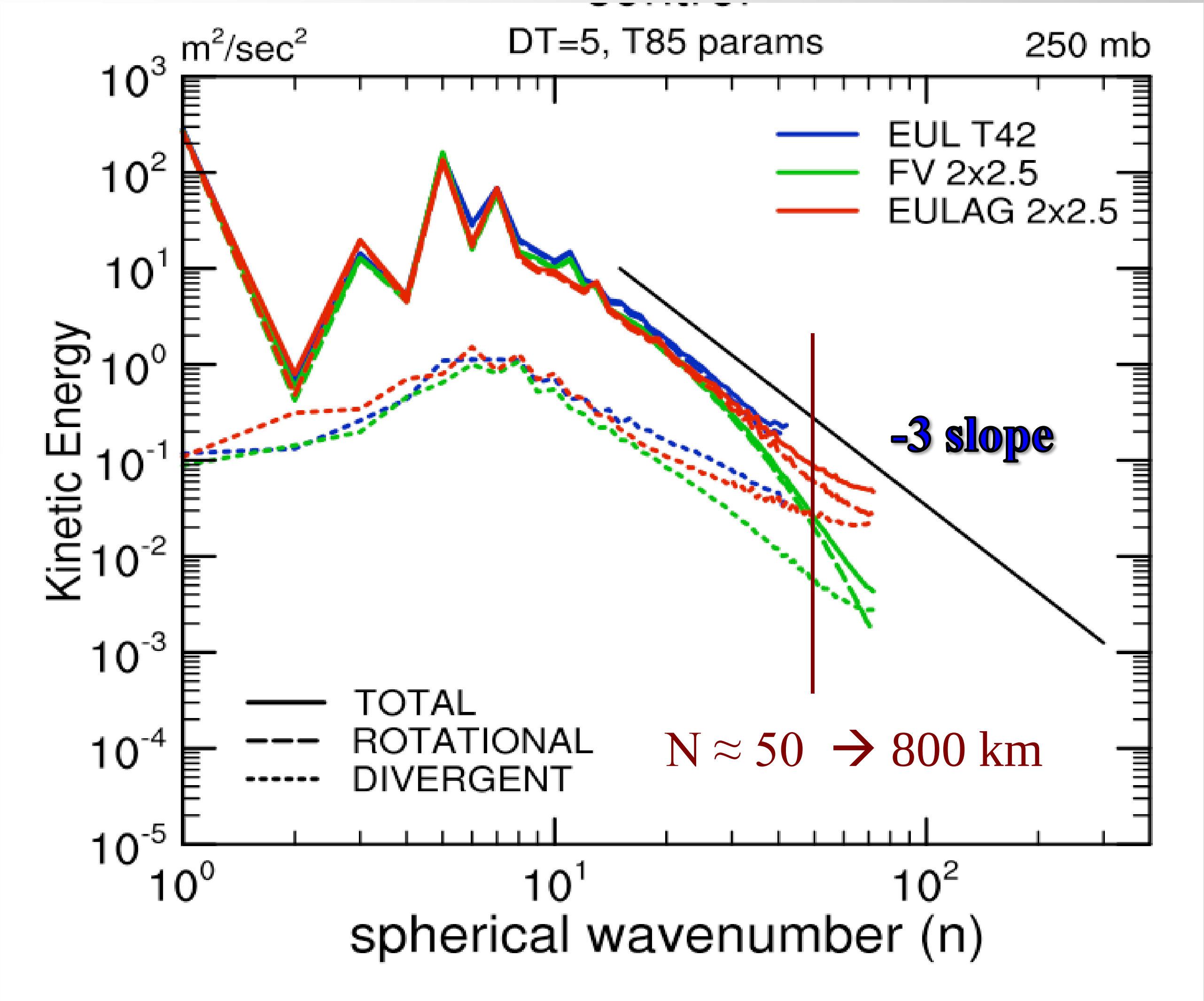
-3 slope

-5/3 slope

700 km

CAM Aqua-planet power spectra

(courtesy of Williamson NCAR, using results from Abiodun et el. *Climate Dyn*. 2008)



-3 slope

N ≈ 50 → 800 km

CAM-EULAG (CEU) result is in red. Cutoff n ~ 70 is Nyquist:

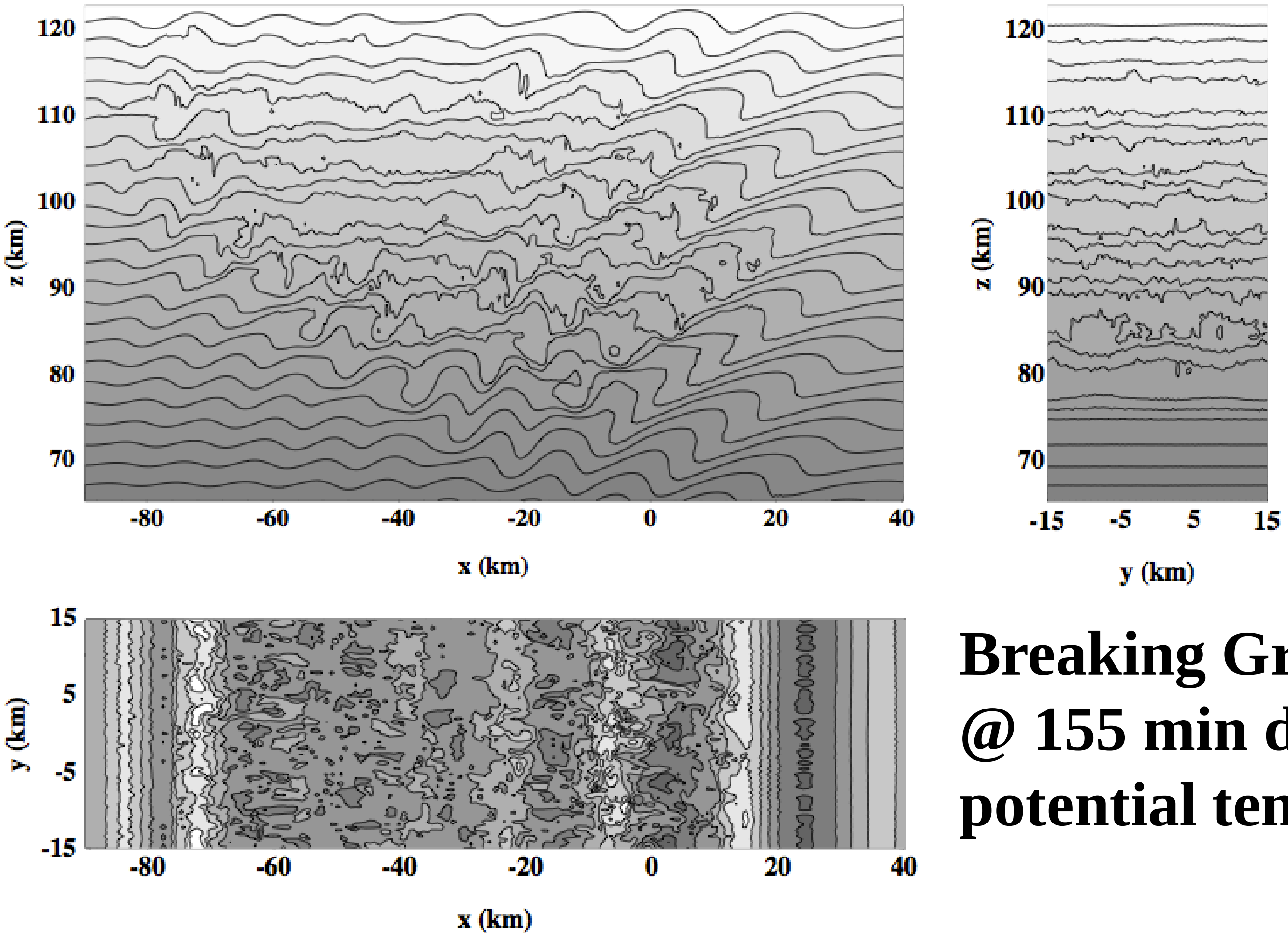
$2\pi R_o / n_{nyq}$

≈ 550 km

CAM-EUL and CEU match well to n ~ 40

CAM-FV match is to n ~ 25

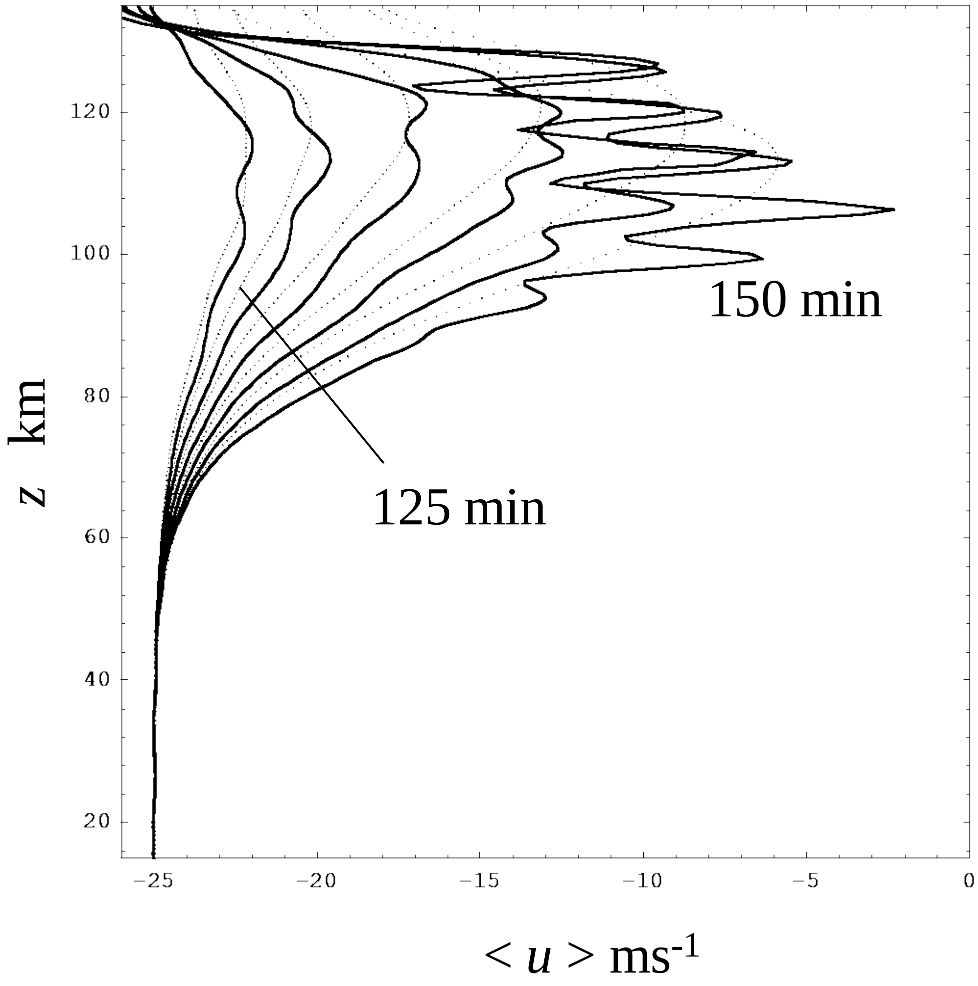
Homogeneity? Isotropy?



Breaking Gravity Waves @ 155 min depicted by potential temperature.

(Smolarkiewicz and Prusa, Chapter 8 Turbulent Flow Computation, Kluwer Academic Publishers, 2002)

Reverse cascade forcing of < *u* >



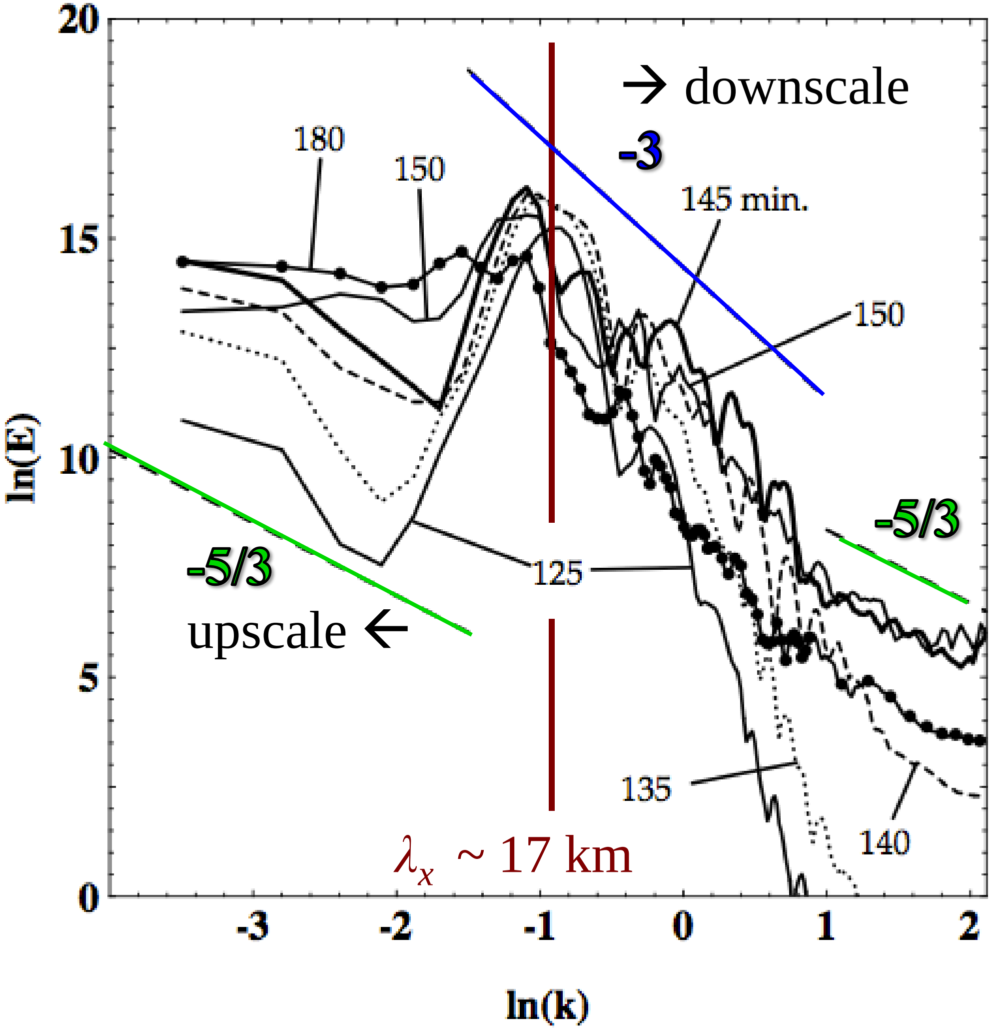
Initial domain averaged zonal wind is -25 ms^{-1}

Profiles shown every 5 min.

OBS: meridional flow near mesopause is opposite of what is expected from rad. cooling → requires forcing of 10-20 ms^{-1}/day

Model results for EP flux divergence → ~ 2 cm s^{-2} over 1 hr duration of maximum wavebreaking: APPEARS SUFFICIENT!

→ Multiple scale interaction can be upscale as well as downscale



Power Spectra of breaking gravity waves in upper atmosphere:

left side: ln(k) = -4 →
340 km (n ~ 115)

right side: ln(k) = 2.1 is Nyquist wavenumber
→ λ_{nyq} = 760 m

(Prusa et al. *Int. J. Math. Comput. Sci.* 2001)

less obvious, cont.

TURBULENCE CLOSURE:

- Large Eddy Simulation (LES): may be based upon 3D, isotropy, homogeneity, etc.,; and as a result may generate dissipation terms that are not consistent with turbulence.

- hyperviscosity: may suppress intermittency (Novikov – conjecture, *Proceedings* Monte Verita, 1993), thus altering multiscale interactions; observed that resulting 2D cascades very sensitive to parameter settings (Gkioulekas & Tung *J. Low Temp Phys* 2006)

- **high-resolution models** have **nonlinear** leading order dissipation terms that are consistent with turbulent flow (Rider, *IJNMF* 2006)

→ offer effective resolutions of 3-5 Δx, vs. ~10 Δx commonly accepted criterion

A Few Select Studies…

- Davies et al, *QJRMS* 2003 – 2D normal mode *f*-plane analysis used to rigorously examine sound-proof systems:

  (i) Anelastic works well for all gravity wave frequencies

  (ii) **Anelastic not good for amplitudes and height scales of external planetary modes, nor for finite amplitude Lamb (acoustic) waves**

  (iii) **Anelastic introduces phase error for deep wave modes**

- Klein et al., *JFM* 2010 – multiple parameter, singular perturbation analysis that examines multiscale interaction between planetary and gravity wave modes:

  (i) The anelastic sytem "*gets it right*", with differences from elastic systems being asymptotically small, of order ~ O($M^{2/3}$)

  (ii) This translates into stratification increases of ~ 10% over a pressure scale height

Numerical Results from EULAG…

Select studies, concluded

- HS simulations (Smolarkiewicz et. al., *JAS* 2001):

zonally averaged fields compare well to those of Held and Suarez (1994)

- Aqua-planet simulations – EULAG is coupled to CAM physics (Abiodun et. al., *Clim. Dyn.* 2008a,b):

(i) Zonally averaged fields compare favorably to those in Neale and Hoskins (Atm. Sci. Lett. 2000a,b)

(ii) Good comparisons with standard CAM dycores in baroclinic modes