Numerical analyses of explicit time-stepping methods for use in atmospheric forecasting models

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With thanks to: Alan Gadian (Leeds); Nigel Wood, Andrew Staniforth (UK Met Office); Hilary Weller (Reading)

NATURAL ENVIRONMENT RESEARCH COUNCIL

National Centre for Atmospheric Science
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Talk outline

- Some background
- Motivation for analyses
- Description of analysis framework
- Outline schemes for analysis
- Present some results of numerical and empirical analyses
- Conclusions and future work

GUNG-HO! (aka UM* dynamical core project)

G	lobally
U	niform
N	ext
G	eneration
H	ighly
O	ptizimed

UK Met Office; Science & Technology Facilities Centre;
Universities: Bath, Exeter, Imperial, Leeds, Manchester, Reading

Objective:
To research, design and develop a new dynamical core suitable for operational, global and regional, weather and climate simulation on massively parallel computers of the size envisaged over the coming 20 years.

Outline:
Phase 1 (2011-2013): exploration of alternative methods to address identified barriers to good performance on massively parallel computers in current UM:

- quasi-uniform horizontal grids
- conservative transport schemes
- time-stepping methods
- 2D testing

Phase 2 (2013 -2016): selected methods from Phase 1 will be combined with exploration of the vertical aspects and extended into a 3D model for extensive testing

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工合 "Gung-ho" = "Working together harmoniously"

*UM = Met Office "Unified Model"

Motivation for re-examining time-stepping

Problem:

Atmospheric (dry) dynamical equations: compressible, non-hydrostatic

=> include very wide range of wave frequencies:
acoustic, gravity, advection, Coriolis

=> inherently *stiff* system

And … stiffness is compounded in the numerical model:

Aim for future (2020) global model forecasts is for
horizontal grid-spacing of Δx ~ 1 km

while nature of dynamics *already* requires
variable vertical grid-spacing of Δz ~ 10 m (near surface)
to Δz ~ 1 km (near top)

Currently …

Current UM uses 3D semi-implicit time-stepping:

- handles fastest waves with implicit method

  ⇒ no stability constraints associated with fastest waves

  ⇒ achieve (very time-constrained) forecasts with long time-step

  BUT requires solution of a 3D Helmholtz problem

  ⇒ requires (multiple) global communications each time-step

  -> Q: looking to future (2020+) massively parallel architectures, can solvers be relied on to provide good scalability?

工合 GUNG-HO: Rob Scheichl, Bath University

Alternatively: consider explicit-based methods (no global communications)

How can explicit methods help?

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1D semi-implicit
$\Rightarrow$ tridiagonal problem
No inter-processor comms

How can explicit methods help?

Problem:

Atmospheric (dry) dynamical equations: compressible, non-hydrostatic

=> include very wide range of wave f[...]
acoustic, gravity, advection, Corio[...]

=> inherently *stiff* system

Explicit
⇒ accept stability constraint assoc. with fastest horizontal motion

And … stiffness is compounded in the numerical model:

Aim for future (2020) global model forecasts is for

horizontal grid-spacing of $\Delta x \sim 1$ km

while nature of dynamics *already* requires

variable vertical grid-spacing of $\Delta z \sim 10$ m (near surface)
to $\Delta z \sim 1$ km (near top)

1D semi-implicit
⇒ tridiagonal problem
No inter-processor comms

How can explicit methods help?

Problem:

Atmospheric (dry) dynamical equations: compressible, non-hydrostatic

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acoustic, gravity, advection, Corio

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Explicit
⇒ accept stability constraint assoc. with fastest horizontal motion

And … stiffness is compounded in the numerical model:

Aim for

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r surface)

to Δz ~ 1 km (near top)

Horizontally-explicit vertically-implicit
"HEVI" approach

1D semi-implicit
⇒ tridiagonal problem
No inter-processor comms

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Analysis framework – exact system:

- Consider the 3D linear equation set (incl. important processes – advection, Earth's rotation, pressure-gradient force, gravity and acoustic waves):

$$u_t + Uu_x + Vu_y + Wu_z - fv + \Phi_x = 0,$$
$$v_t + Uv_x + Vv_y + Wv_z + fu + \Phi_y = 0,$$
$$w_t + Uw_x + Vw_y + Ww_z + \Phi_z = 0,$$
$$\Phi_t + U\Phi_x + V\Phi_y + W\Phi_z + c_g^2 (u_x + v_y) + c_a^2 w_z = 0,$$

- Can consider coupled equations to be represented by the single equation:

$$F_t + i\tilde{\omega}F = 0$$

- Enhance the problem to reflect the high (K_V) and low (K_H) frequency contributions: $$\frac{\partial F}{\partial t} + iK_H F + iK_V F = 0$$

- Exact system has solution $F(t) = F_0 \mathrm{e}^{-\mathrm{i}(K_H + K_V)t}$

  which yields amplification factor, $A_0 = F(t+\Delta t)/F(t)$: $A_0 = \mathrm{e}^{-i(K_H + K_V)\Delta t}$

  with amplitude $|A_0|=1$ and phase $\theta_0 = -(K_H + K_V)\Delta t$,

  and group velocity $\mathrm{d}\omega/\mathrm{d}k = -\mathrm{d}\left(K_H + K_V\right)/\mathrm{d}k$

Analysis framework – discrete system:

- For the discrete system, construct amplification factors from

$F^{n+1} = AF^n$ where $A = |A|\,\mathrm{e}^{i\theta} = |A|\,(\cos\theta + i\sin\theta)$

Hence, can extract numerical amplitude and phase by

$$|A| = \left\{\Re(A)^2 + \Im(A)^2\right\}^{1/2}$$
$$\theta = \arctan\left(\frac{\Im(A)}{\Re(A)}\right).$$

Stability: $|A|>1$ ⇔ amplifying = unstable

$|A|<1$ ⇔ damping = stable

Phase: $\theta_{\mathrm{num}} > \theta_{\mathrm{exact}}$ ⇔ accelerating

$\theta_{\mathrm{num}} < \theta_{\mathrm{exact}}$ ⇔ decelerating

Group velocity: re-write as $A = \mathrm{e}^{\tilde{K}_{\Im}\Delta t}\mathrm{e}^{-i\tilde{K}_{\Re}\Delta t}$, then $\tilde{\omega} = -\tilde{K}_{\Re}$ and

$$\frac{\mathrm{d}\tilde{\omega}/\mathrm{d}k}{\mathrm{d}\omega/\mathrm{d}k} = \frac{\mathrm{d}\theta/\mathrm{d}K}{\mathrm{d}\theta_0/\mathrm{d}K}$$

For “good” behaviour, require that *direction* (sign) is always correct

Analysis framework - comparison:

- Compare exact and discrete systems

1. Examine performance over small Courant numbers

   i.e. $|K_H \Delta t|, |K_V \Delta t| < 2$

   Looking for stability and accuracy (not too damping, good phase)

2. Examine performance over large (vertical) Courant numbers

   i.e. $0 < |K_V \Delta t| < O(100)$

   reflecting range of Δz, e.g. $\frac{\Delta x}{100} \lesssim \Delta z \lesssim \Delta x$ ($\Delta x \sim 1$ km)

   Looking for stability and "good" behaviour (group velocity)

3. Consider errors accumulated over multiple steps:

   $$\Delta t_{\mathrm{implicit}} / \Delta t_{\mathrm{explicit}} = O(10)$$

   Hence, over M=O(10) steps, fair comparison with SI is

   $$|A_{\mathrm{implicit}}| \quad \text{versus} \quad |A_{\mathrm{explicit}}|^M$$
   $$\theta_{\mathrm{implicit}} \quad \text{versus} \quad M\theta_{\mathrm{explicit}}.$$

“Control” scheme: CN(ε)

Current UM semi-implicit scheme:

In simplest sense, equivalent to off-centered Crank-Nicholson, “CN(ε)”,

$$F^{n+1} = F^n - iK\Delta t \left\{ \frac{(1-\epsilon)}{2} F^n + \frac{(1+\epsilon)}{2} F^{n+1} \right\}$$

with off-centering parameter: $\epsilon = 0.1$

Analysis reveals the amplification factor $A = \dfrac{1 - \frac{1-\epsilon}{2} iK\Delta t}{1 + \frac{1+\epsilon}{2} iK\Delta t}$

which has amplitude $|A|^2 = 1 - \dfrac{\epsilon\,(K\Delta t)^2}{1 + \left(\frac{1+\epsilon}{2}\right)^2 (K\Delta t)^2}$

which is always stable for $\varepsilon>0$;

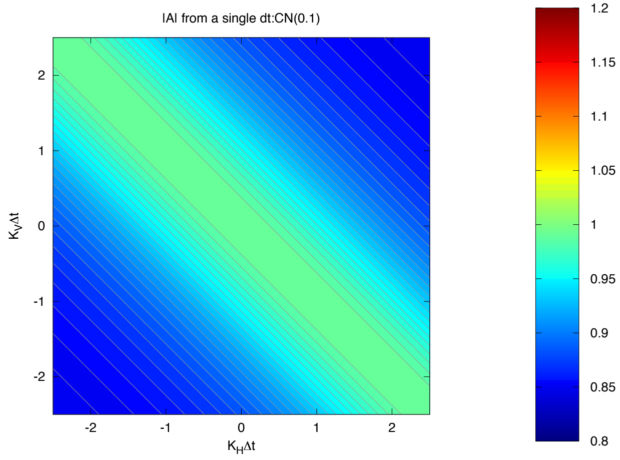
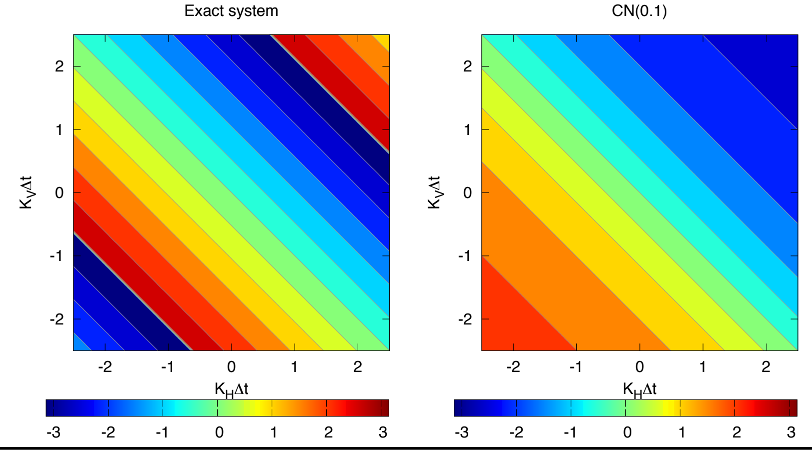
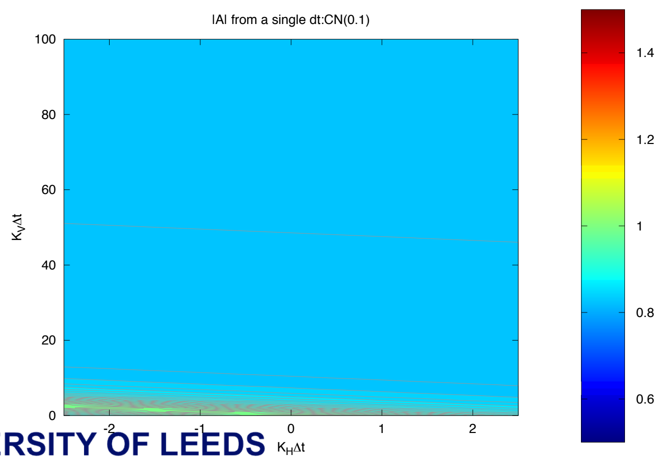
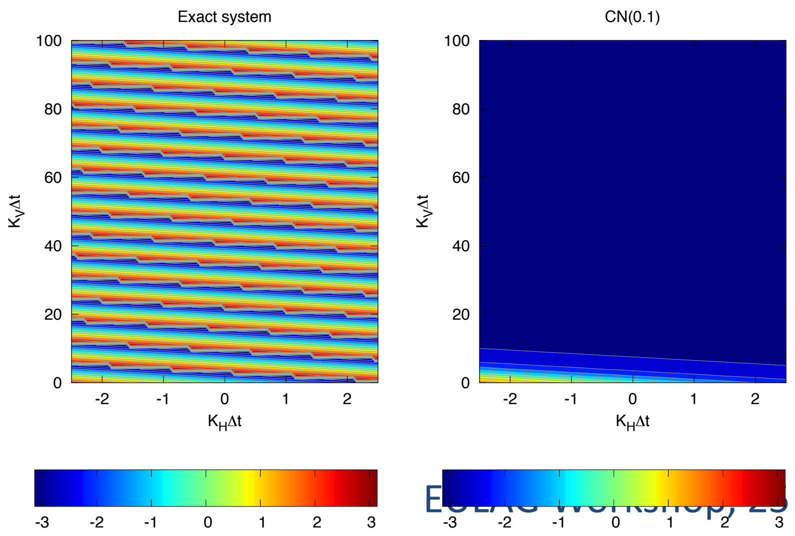
and phase $\theta \approx -K\Delta t \left(1 - \dfrac{1+3\epsilon^2}{12} (K\Delta t)^2\right)$

which is always decelerating for $|\varepsilon|>0$.

"Control" scheme: CN(0.1)

Alternative (graphical) approach for considering scheme characteristics

Generate the amplification factors empirically (e.g. Matlab), computing for range of $K\Delta t$ and plot:

	Amplitudes	Phase
Small $K_V Dt$		
Large $K_V Dt$		

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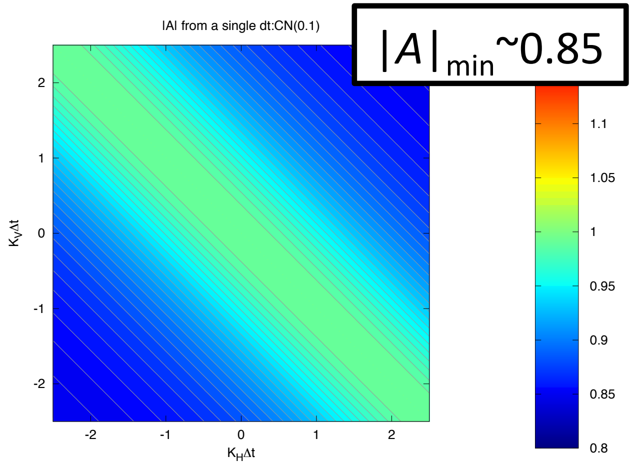
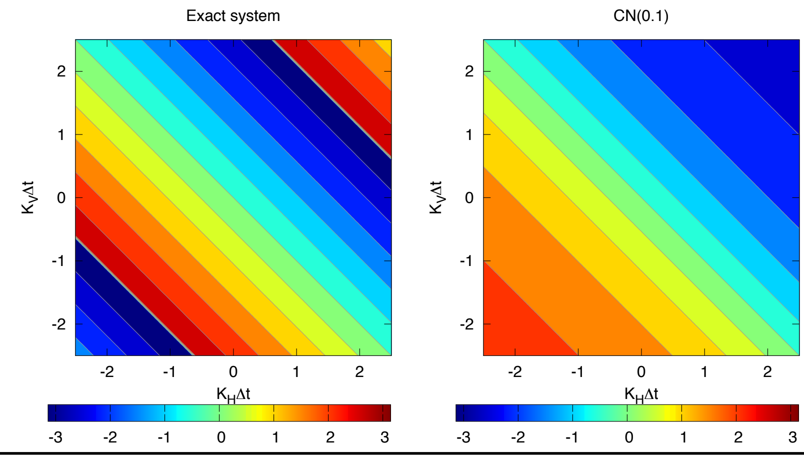
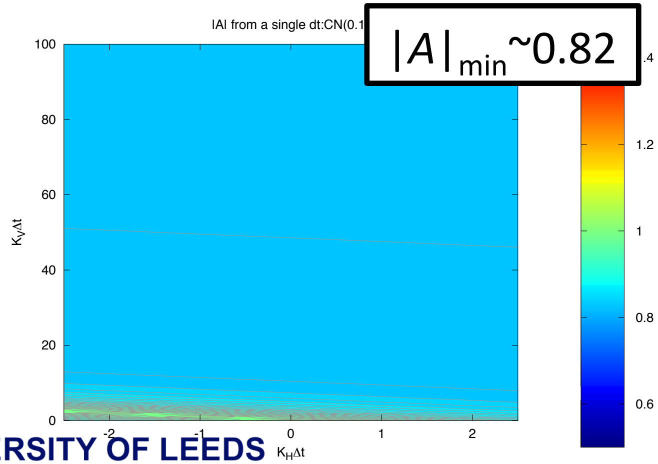
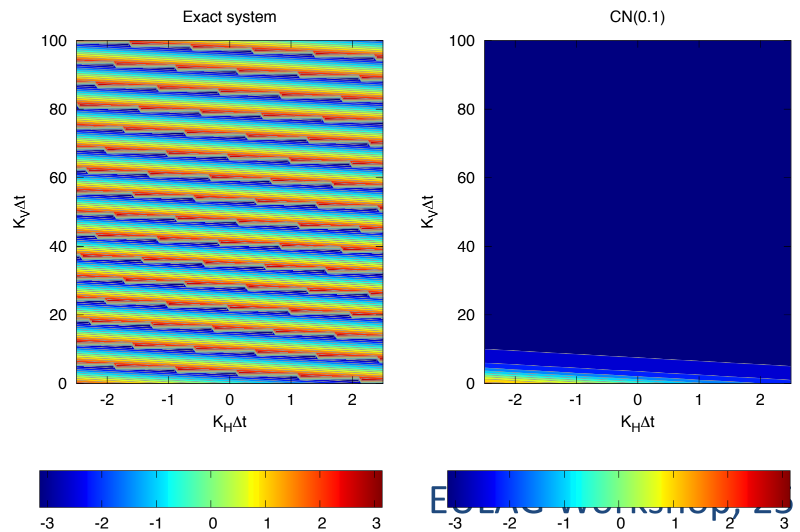
Small $K_V Dt$		
Large $K_V Dt$		

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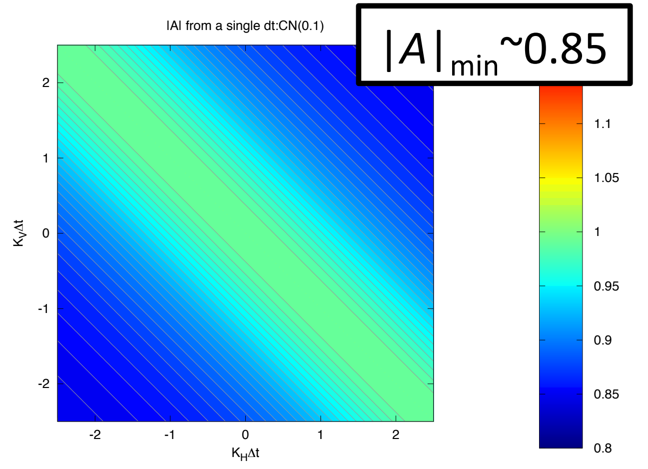
Alternative (graphical) approach for considering scheme characteristics

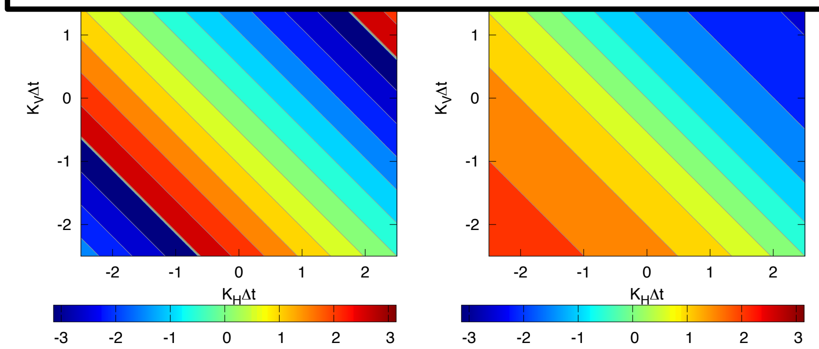
Generate the amplification factors empirically (e.g. Matlab), computing for range of $K\Delta t$ and plot:

Amplitudes

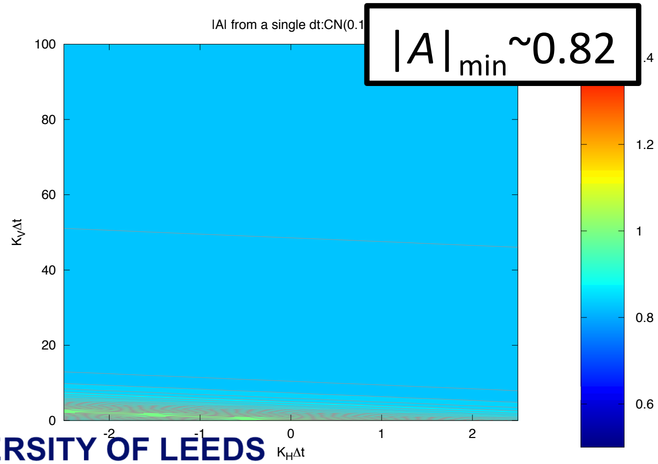
Phase: for any given $K\Delta t$, numerical scheme yields smaller (magnitude) phase, i.e. rotates more slowly around the unit circle, than exact => always decelerating

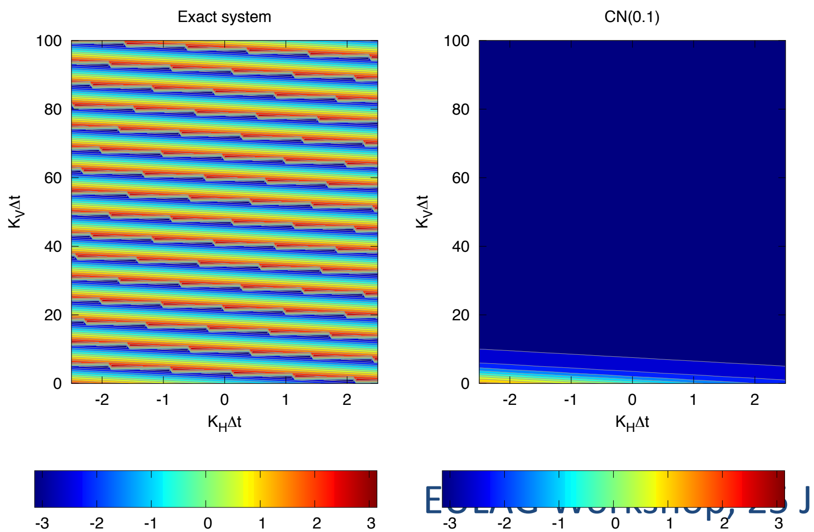
Small $K_V Dt$





Large $K_V Dt$





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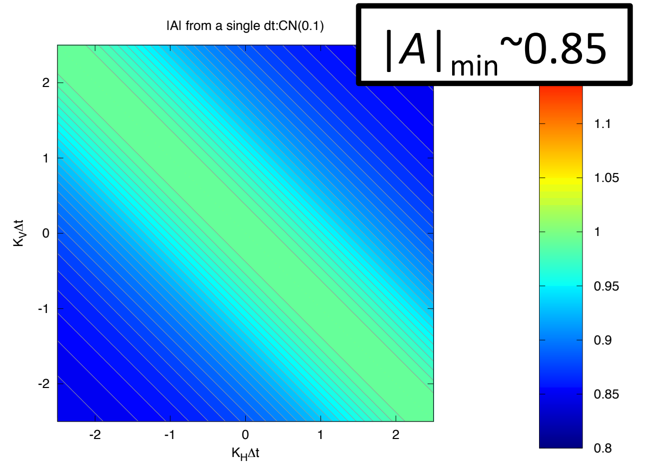
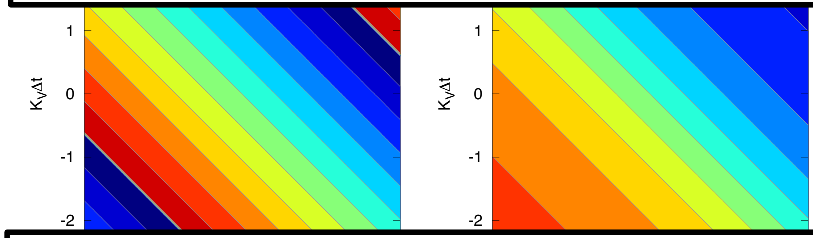


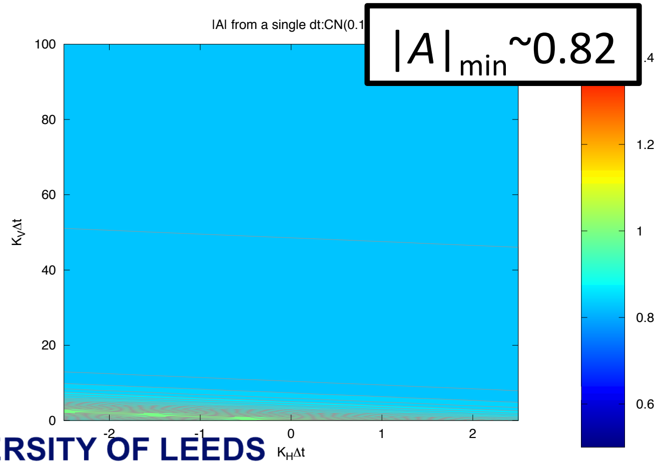


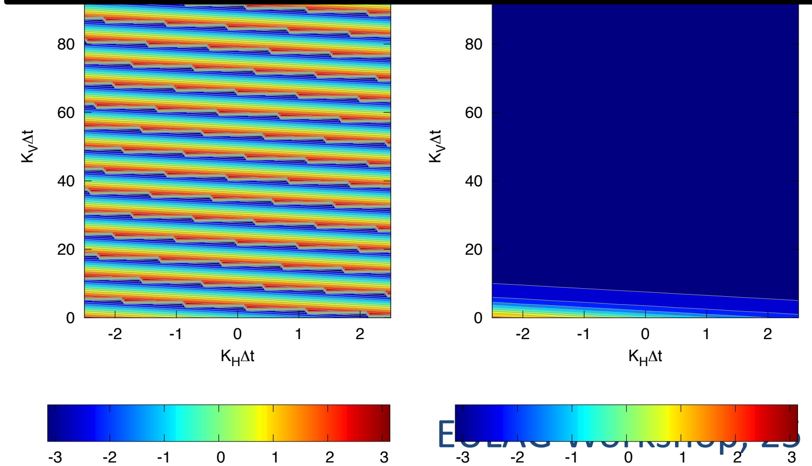
Group velocity: $d\omega/dk$ ~ $d\theta/dK$

For “good” behaviour, signs must agree

=> requires that sign of *gradients* must agree

Large $K_V Dt$





Potential new schemes:

- Initially, considered multi-step (e.g. leapfrog) schemes, but suffer computational (parasitic) modes
- Currently, focusing on single-step, multi-stage (Runge-Kutta) schemes
- Runge-Kutta (RK) IMEX schemes can be efficiently & usefully described by double Butcher tableau, e.g.

3rd-order RK (RK3) combined with CN(ε) presented

in full:

$$
\begin{aligned}
F^{(1)} &= F^n \\
F^{(2)} &= F^n - \frac{1}{3} i K_H \Delta t F^{(1)} \\
F^{(3)} &= F^n - \frac{1}{2} i K_H \Delta t F^{(2)} \\
F^{*} &= F^n - i K_H \Delta t F^{(3)} \\
F^{n+1} &= F^{*} - i K_V \Delta t \left\{ \frac{(1-\epsilon)}{2} F^{*} + \frac{(1+\epsilon)}{2} F^{n+1} \right\}
\end{aligned}
$$

and as double Butcher tableau:

Explicit tableau

$$
\begin{array}{c|ccccc}
0 & 0 & & & & \\
1/3 & 1/3 & 0 & & & \\
1/2 & 0 & 1/2 & 0 & & \\
1 & 0 & 0 & 1 & 0 & \\
1 & 0 & 0 & 1 & 0 & 0 \\
\hline
 & 0 & 0 & 1 & 0 & 0
\end{array}
$$

Implicit tableau

$$
\begin{array}{c|ccccc}
0 & 0 & & & & \\
0 & 0 & 0 & & & \\
0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & \\
1 & 0 & 0 & 0 & (1-\epsilon)/2 & (1+\epsilon)/2 \\
\hline
 & 0 & 0 & 0 & (1-\epsilon)/2 & (1+\epsilon)/2
\end{array}
$$

Potential new schemes:

RK IMEX combinations based on:

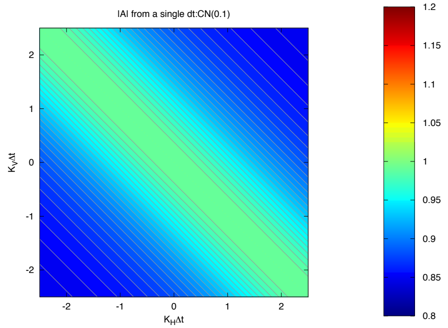
- 2nd-order RK (RK2)
- 3rd-order RK (RK3)
- from literature: SSP (strong stability preserving), DIRK (diagonally implicit RK)

Identify them as

- fully "split"
- unsplit

$$
\begin{array}{c|ccccc}
0 & 0 & & & & \\
1/3 & 1/3 & 0 & & & \\
1/2 & 0 & 1/2 & 0 & & \\
1 & 0 & 0 & 1 & 0 & \\
1 & 0 & 0 & 1 & 0 & 0 \\
\hline
 & 0 & 0 & 1 & 0 & 0
\end{array}
\qquad
\begin{array}{c|ccccc}
0 & 0 & & & & \\
0 & 0 & 0 & & & \\
0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & \\
1 & 0 & 0 & 0 & (1-\epsilon)/2 & (1+\epsilon)/2 \\
\hline
 & 0 & 0 & 0 & (1-\epsilon)/2 & (1+\epsilon)/2
\end{array}
$$

RK3-based schemes (fully split)



RK3-CN(0)*

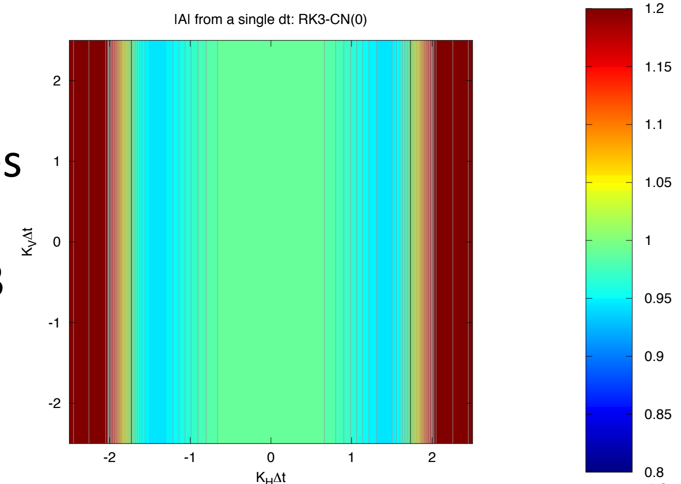
$$
\begin{array}{c|ccccc}
0 & 0 & & & & \\
1/3 & 1/3 & 0 & & & \\
1/2 & 0 & 1/2 & 0 & & \\
1 & 0 & 0 & 1 & 0 & \\
1 & 0 & 0 & 1 & 0 & 0 \\
\hline
 & 0 & 0 & 1 & 0 & 0
\end{array}
\qquad
\begin{array}{c|ccccc}
0 & 0 & & & & \\
0 & 0 & 0 & & & \\
0 & 0 & 0 & 0 & & \\
0 & 0 & 0 & 0 & 0 & \\
1 & 0 & 0 & 0 & 1/2 & 1/2 \\
\hline
 & 0 & 0 & 0 & 1/2 & 1/2
\end{array}
$$

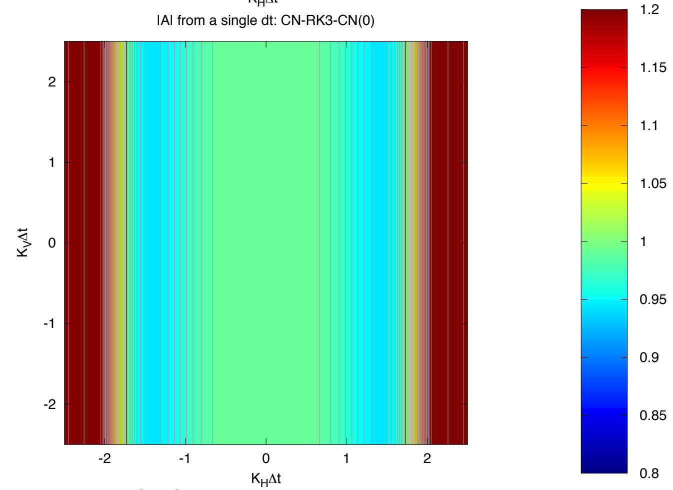
*Equivalent to "Strang Carryover" scheme of Ullrich & Jablonowski (MWR, 2012)

Amplitudes (small Courant numbers)

Two schemes:

- yield identical amplitudes
- good stability properties
- only limited by $(K_H\Delta t)^2<3$
- only small damping in stable region

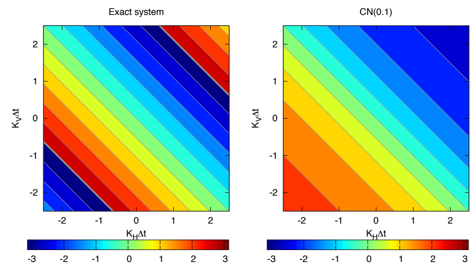




CN(0)-RK3-CN(0)+

$$
\begin{array}{c|cccccc}
0 & 0 & & & & & \\
0 & 0 & 0 & & & & \\
1/3 & 0 & 1/3 & 0 & & & \\
1/2 & 0 & 0 & 1/2 & 0 & & \\
1 & 0 & 0 & 0 & 1 & 0 & \\
1 & 0 & 0 & 0 & 1 & 0 & 0 \\
\hline
 & 0 & 0 & 0 & 1 & 0 & 0
\end{array}
\qquad
\begin{array}{c|cccccc}
0 & 0 & & & & & \\
1/2 & 1/4 & 1/4 & & & & \\
1/2 & 1/4 & 1/4 & 0 & & & \\
1/2 & 1/4 & 1/4 & 0 & 0 & & \\
1/2 & 1/4 & 1/4 & 0 & 0 & 0 & \\
1 & 1/4 & 1/4 & 0 & 0 & 1/4 & 1/4 \\
\hline
 & 1/4 & 1/4 & 0 & 0 & 1/4 & 1/4
\end{array}
$$

+Strang-splitting



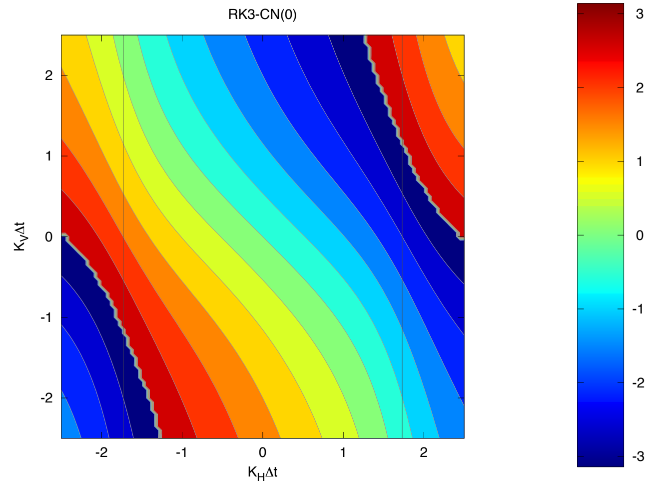
RK3-based schemes (fully split)

RK3-CN(0)

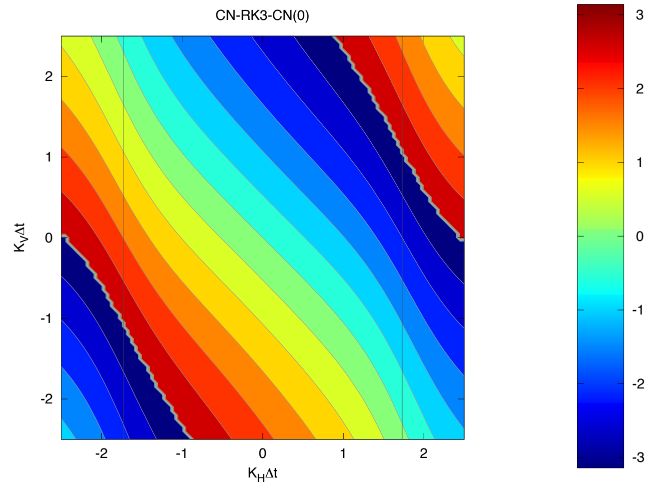
0	0				
1/3	1/3	0			
1/2	0	1/2	0		
1	0	0	1	0	
1	0	0	1	0	0
	0	0	1	0	0

0	0				
0	0	0			
0	0	0	0		
0	0	0	0	0	
1	0	0	0	1/2	1/2
	0	0	0	1/2	1/2

Phase (small Courant numbers)



CN(0)-RK3-CN(0) yields better phase representation for small Courant numbers

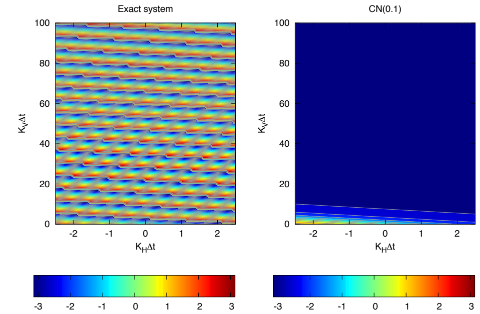


CN(0)-RK3-CN(0)

0	0					
0	0	0				
1/3	0	1/3	0			
1/2	0	0	1/2	0		
1	0	0	0	1	0	
1	0	0	0	1	0	0
	0	0	0	1	0	0

0	0					
1/2	1/4	1/4				
1/2	1/4	1/4	0			
1/2	1/4	1/4	0	0		
1/2	1/4	1/4	0	0	0	
1	1/4	1/4	0	0	1/4	1/4
	1/4	1/4	0	0	1/4	1/4

RK3-based schemes (fully split)

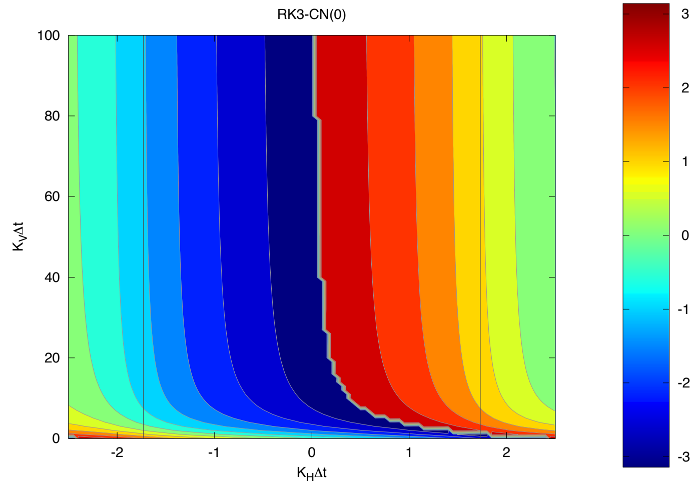


RK3-CN(0)

0	0				
1/3	1/3	0			
1/2	0	1/2	0		
1	0	0	1	0	
1	0	0	1	0	0
	0	0	1	0	0

0	0				
0	0	0			
0	0	0	0		
0	0	0	0	0	
1	0	0	0	1/2	1/2
	0	0	0	1/2	1/2

Phase (large vertical Courant numbers)

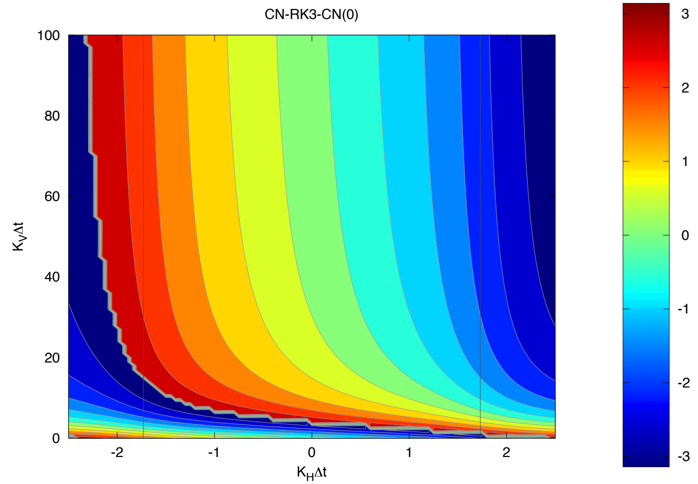


Better phase representation from CN(0)-RK3-CN(0) persists to larger $K_V\Delta t$ but both schemes indicate strong decelerating nature. No sign of poor group velocity behaviour.

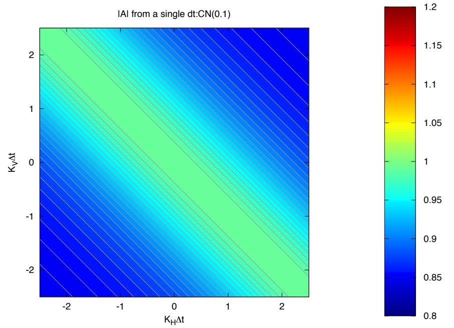
CN(0)-RK3-CN(0)

0	0					
0	0	0				
1/3	0	1/3	0			
1/2	0	0	1/2	0		
1	0	0	0	1	0	
1	0	0	0	1	0	0
	0	0	0	1	0	0

0	0					
1/2	1/4	1/4				
1/2	1/4	1/4	0			
1/2	1/4	1/4	0	0		
1/2	1/4	1/4	0	0	0	
1	1/4	1/4	0	0	1/4	1/4
	1/4	1/4	0	0	1/4	1/4



RK2-based schemes (unsplit)



ENDG-2

2-iteration Heun-CN + initial stage

$$\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ \delta_H & \delta_H & 0 & 0 & 0 \\ 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \hline & \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{array} \qquad \begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ \delta_V & \delta_V\frac{(1-\alpha)}{2} & \delta_V\frac{(1+\alpha)}{2} & 0 & 0 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \hline & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{array}$$

Initial stage: α = off-centering parameter

δ_H, δ_V = 0,1

For pure explicit initial stage:

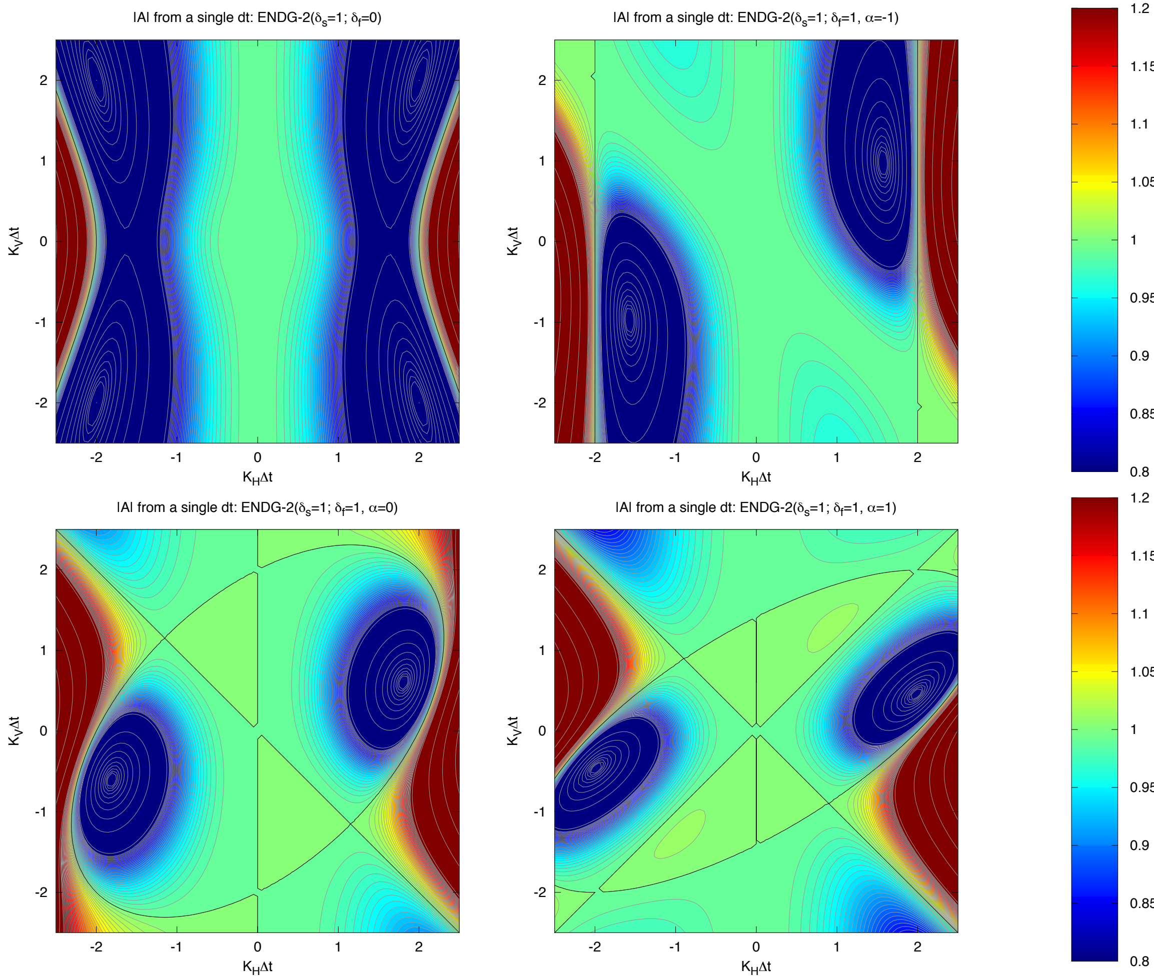
- stability conditional (only) on $K_H\Delta t$

For implicit initial stage:

- regions of instability extending to $K_H\Delta t = 0$

Not shown here: for large $K_V\Delta t$, all have $|A| \rightarrow 1$

Amplitudes (small Courant numbers)



RK2-based schemes (unsplit)

ENDG-2

2-iteration Heun-CN + initial stage

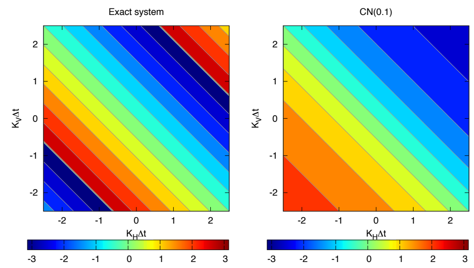
$$
\begin{array}{c|cccc}
0 & 0 & 0 & 0 & 0 \\
\delta_H & \delta_H & 0 & 0 & 0 \\
1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\
1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\
\hline
 & \frac{1}{2} & 0 & \frac{1}{2} & 0
\end{array}
\qquad
\begin{array}{c|cccc}
0 & 0 & 0 & 0 & 0 \\
\delta_V & \delta_V\frac{(1-\alpha)}{2} & \delta_V\frac{(1+\alpha)}{2} & 0 & 0 \\
1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\
1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\
\hline
 & \frac{1}{2} & 0 & 0 & \frac{1}{2}
\end{array}
$$

Initial stage: α = off-centering parameter

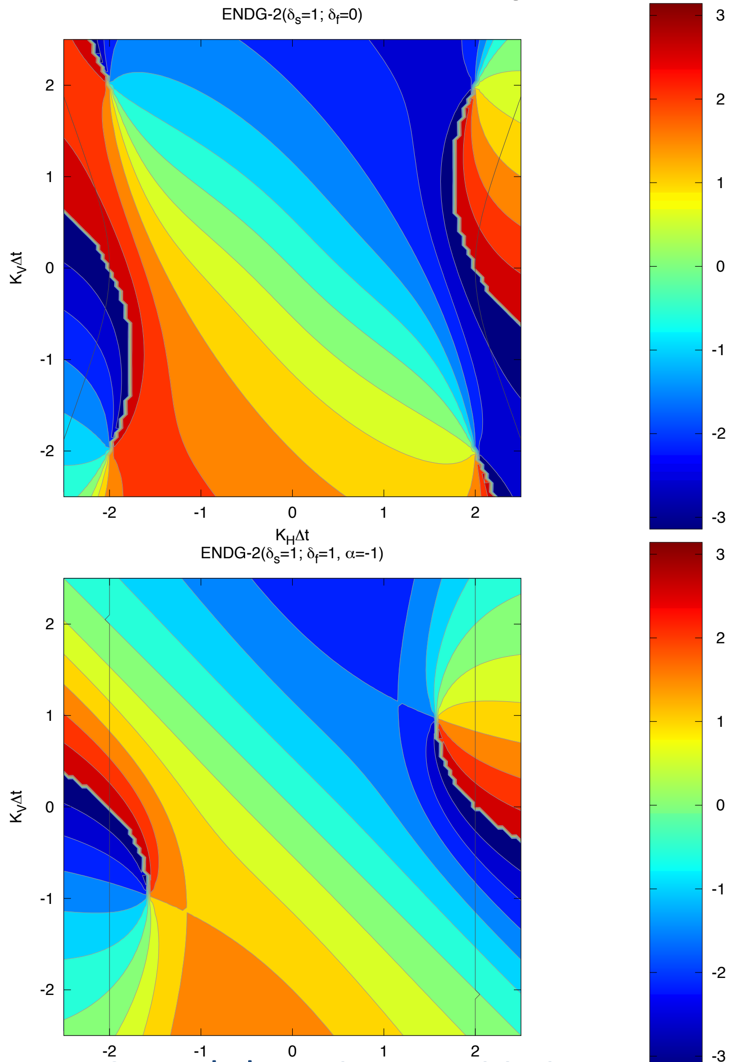
δ_H, δ_V = 0,1

For pure explicit initial stage:

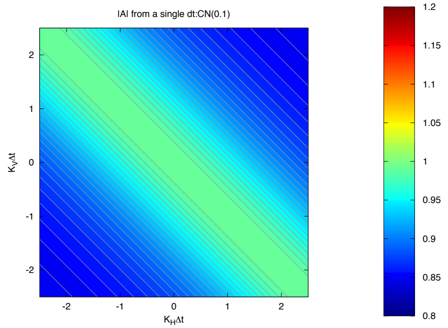
- regions of acceleration and deceleration
- considering sign of the gradients (indicating direction of group velocity), there is evidence of "poor" behaviour within stable limits
- not shown here: for large $K_V\Delta t$, no evidence of further poor behaviour



Phase (small Courant numbers)



RK2-based schemes (unsplit)

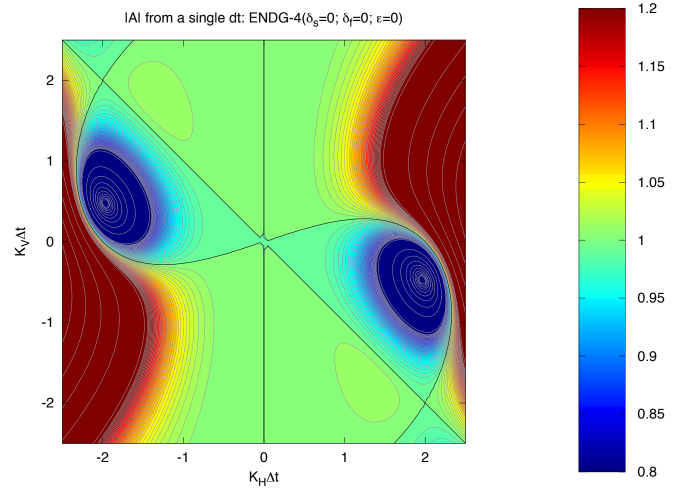


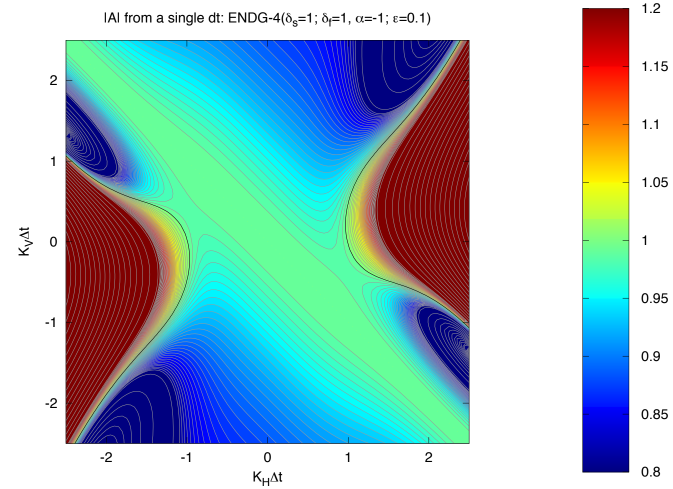
ENDG-4

4-iteration Heun-CN + initial stage

$$
\begin{array}{c|cccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_H & \delta_H & 0 & 0 & 0 & 0 & 0 \\
1 & \frac{1-\epsilon}{2} & \frac{1+\epsilon}{2} & 0 & 0 & 0 & 0 \\
1 & \frac{1-\epsilon}{2} & 0 & \frac{1+\epsilon}{2} & 0 & 0 & 0 \\
1 & \frac{1-\epsilon}{2} & 0 & 0 & \frac{1+\epsilon}{2} & 0 & 0 \\
1 & \frac{1-\epsilon}{2} & 0 & 0 & 0 & \frac{1+\epsilon}{2} & 0 \\
\hline
 & \frac{1-\epsilon}{2} & 0 & 0 & 0 & \frac{1+\epsilon}{2} & 0
\end{array}
\qquad
\begin{array}{c|cccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_V & \delta_V\frac{1-\alpha}{2} & \delta_V\frac{1+\alpha}{2} & 0 & 0 & 0 & 0 \\
1 & \frac{1-\epsilon}{2} & 0 & \frac{1+\epsilon}{2} & 0 & 0 & 0 \\
1 & \frac{1-\epsilon}{2} & 0 & 0 & \frac{1+\epsilon}{2} & 0 & 0 \\
1 & \frac{1-\epsilon}{2} & 0 & 0 & 0 & \frac{1+\epsilon}{2} & 0 \\
1 & \frac{1-\epsilon}{2} & 0 & 0 & 0 & 0 & \frac{1+\epsilon}{2} \\
\hline
 & \frac{1-\epsilon}{2} & 0 & 0 & 0 & 0 & \frac{1+\epsilon}{2}
\end{array}
$$

Amplitudes (small Courant numbers)



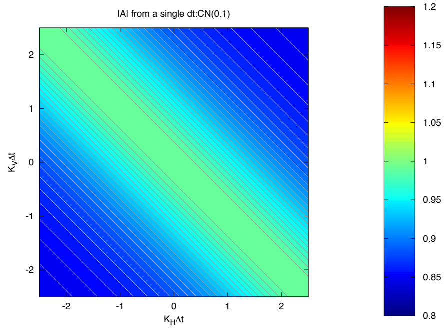


Yields some weird and whacky stability constraints!

- "best"-looking combination from pure explicit initial stage and small off-centering in later stages

SSP-RK schemes (partially-split)

Pareschi & Russo (2005, J. Sci. Comp.)



Amplitudes (small Courant numbers)

SSP2(2,2,2)

$$\begin{array}{c|cc} 0 & 0 & 0 \\ 1 & 1 & 0 \\ \hline & 1/2 & 1/2 \end{array} \qquad \begin{array}{c|cc} \gamma & \gamma & 0 \\ 1-\gamma & 1-2\gamma & \gamma \\ \hline & 1/2 & 1/2 \end{array}$$

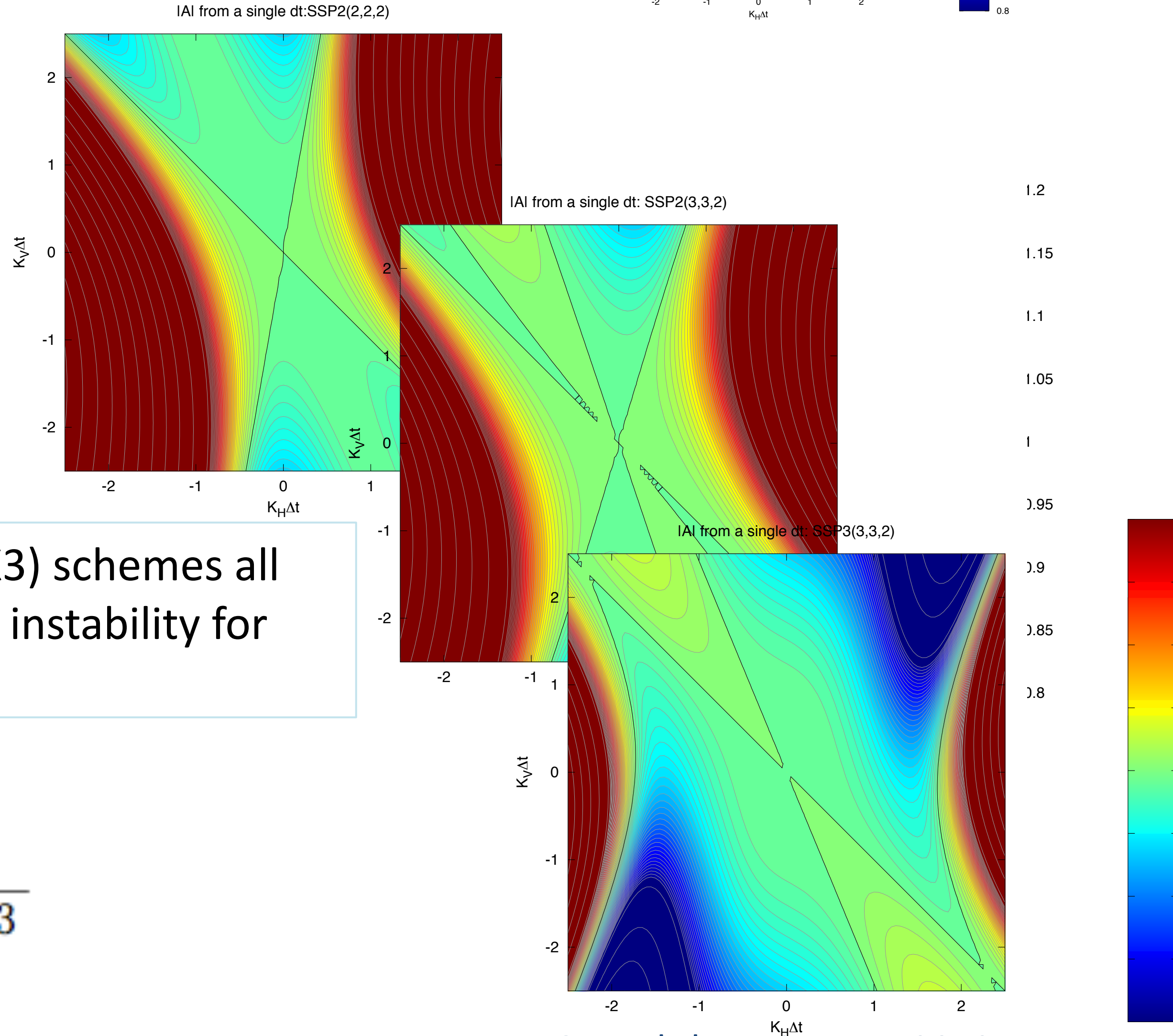
SSP2(3,3,2)

$$\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ \hline & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array} \qquad \begin{array}{c|ccc} \frac{1}{5} & \frac{1}{5} & 0 & 0 \\ \frac{3}{10} & \frac{1}{10} & \frac{1}{5} & 0 \\ 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \hline & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}$$

SSP3(3,3,2)

$$\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1/2 & 1/4 & 1/4 & 0 \\ \hline & 1/6 & 1/6 & 2/3 \end{array} \qquad \begin{array}{c|ccc} \gamma & \gamma & 0 & 0 \\ 1-\gamma & 1-2\gamma & \gamma & 0 \\ 1/2 & 1/2-\gamma & 0 & \gamma \\ \hline & 1/6 & 1/6 & 2/3 \end{array}$$

$$\gamma = 1 - \frac{1}{\sqrt{2}}$$



SSP (RK2 & RK3) schemes all indicate some instability for small $K_H\Delta t$

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Conclusions & Future work

Simple 1D analyses have

- identified good candidate schemes, e.g. CN-RK3-CN
- highlighted potential concerns (instabilities/poor behaviour) of some established schemes, e.g. (not shown here) leapfrog-CN(ε>0) instability (demonstrated in Durran & Blossey, 2012)

BUT experience suggests:

- fully split schemes (e.g. CN-RK3-CN) introduce errors due to the splitting
- from recent testing, some RK IMEX schemes are more stable in practice than indicated by 1D analyses

➔ Next steps:

- Extend analyses to - 1D coupled (atmospheric) system
  - 2D coupled system
- Perform numerical testing of some RK IMEX schemes (H. Weller, Reading)

Numerical analyses of explicit time-stepping methods for use in atmospheric forecasting models

Sarah-Jane Lock

Thanks for your attention!

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