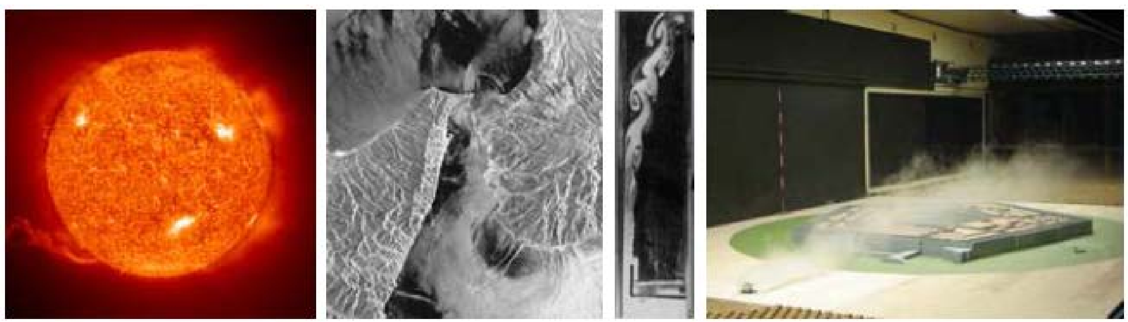
EULAG-MHD: simulation of global solar dynamo

Piotr K Smolarkiewicz*,

Geophysical turbulence; scales of motion $\mathcal{O}(10^7)$, $\mathcal{O}(10^4)$, and $\mathcal{O}(10^{-2})$ m.



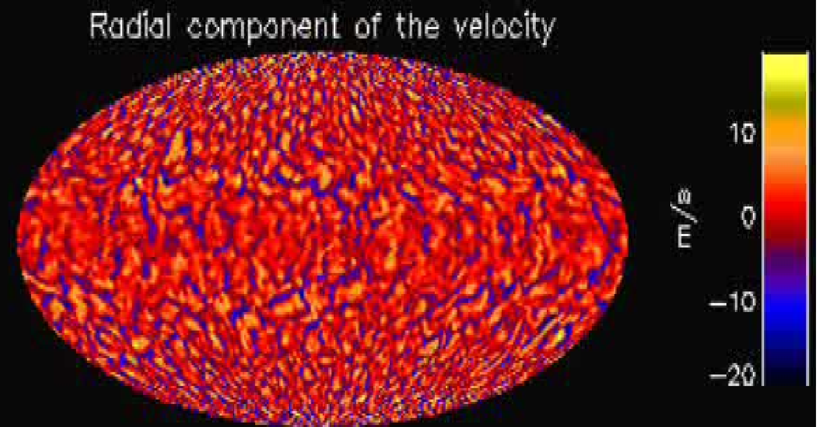
A range of applications

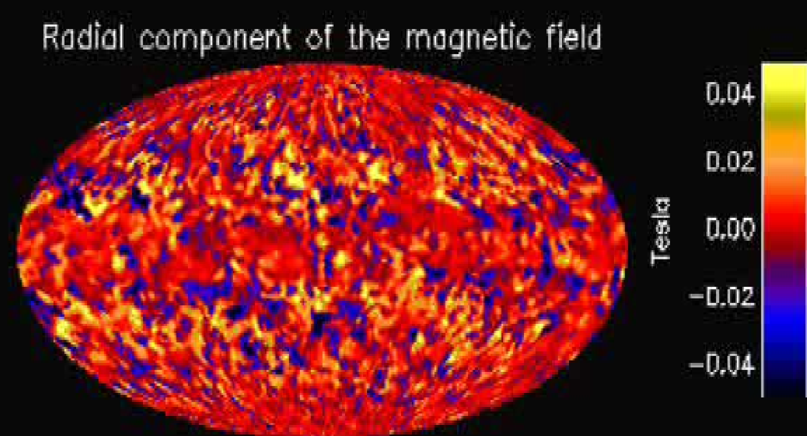
*National Center for Atmospheric Research, Boulder, Colorado, U.S.A.

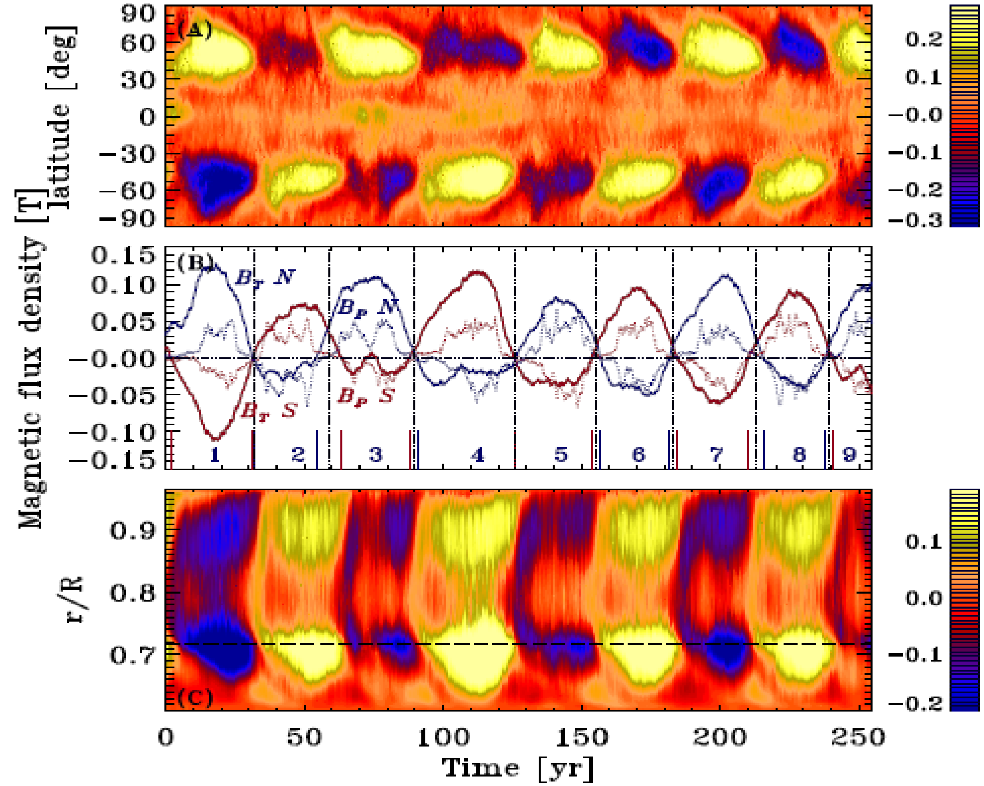
Mihai Ghizaru, Paul Charbonneau, PK Smolarkiewicz,
MAGNETIC CYCLES IN GLOBAL LARGE-EDDY SIMULATIONS OF SOLAR CONVECTION,
The Astrophysical Journal Letters, 715:L133–L137, 2010;

R. Bhattacharyya, B.C. Low, and P.K. Smolarkiewicz
On spontaneous formation of current sheets: Untwisted magnetic fields.
PHYSICS OF PLASMAS 17, 112901 2010

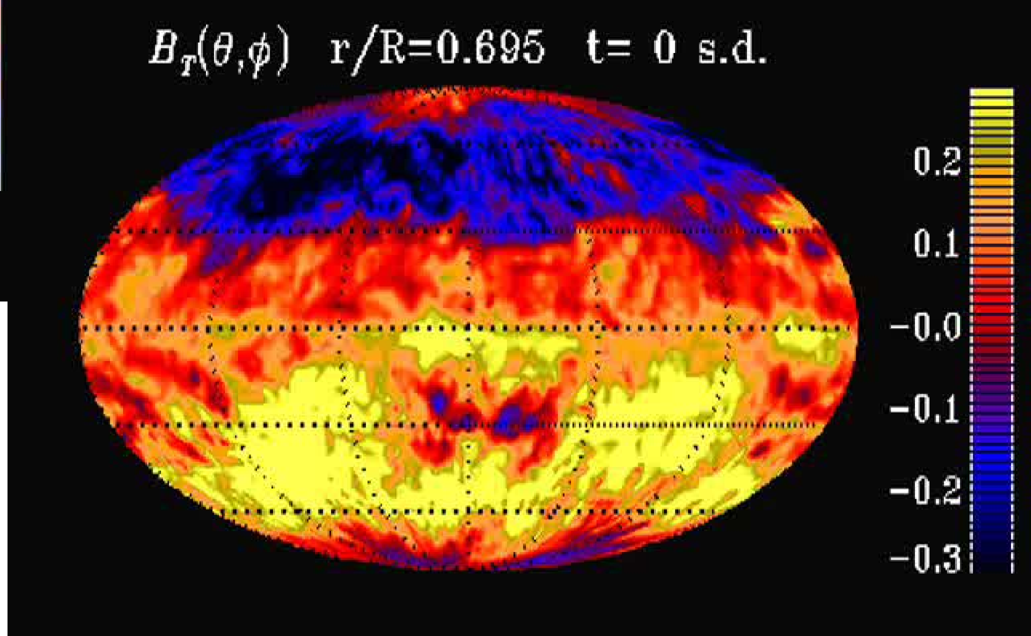
Etienne Racine, P Charbonneau, M Ghizaru, Amelie Bouchat, PK Smolarkiewicz,
ON THE MODE OF DYNAMO ACTION IN A GLOBAL LARGE-EDDY SIMULATION OF SOLAR CONVECTION,
The Astrophysical Journal, 735:46 (22pp), 2011;







Toroidal component of **B** in the uppermost portion of the stable layer underlying the convective envelope at r/R≈0 .7 ➔



Standard anelastic equations of solar magnetohydrodynamics

NCAR

Brun, Miesch & Toomre, The Astrophysical Journal, 614:1073–1098, 2004 October 20

$$\nabla \cdot (\bar{\rho} \boldsymbol{v}) = 0, \tag{1}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{2}$$

$$\bar{\rho}\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} + 2\Omega_0 \times \boldsymbol{v}\right] = -\nabla P + \rho \boldsymbol{g} + \frac{1}{4\pi}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \nabla \cdot \boldsymbol{\mathcal{D}} - \left(\nabla \bar{P} - \bar{\rho}\boldsymbol{g}\right), \tag{3}$$

$$\bar{\rho}\bar{T}\frac{\partial S}{\partial t} + \bar{\rho}\bar{T}\boldsymbol{v} \cdot \nabla\left(\bar{S} + S\right) = \nabla \cdot \left[\kappa_r \bar{\rho} c_p \nabla\left(\bar{T} + T\right) + \kappa \bar{\rho}\bar{T}\nabla\left(\bar{S} + S\right)\right] + \frac{4\pi\eta}{c^2}\boldsymbol{j}^2 + 2\bar{\rho}\nu\left[e_{ij}e_{ij} - \frac{1}{3}(\nabla \cdot \boldsymbol{v})^2\right] + \bar{\rho}\epsilon, \tag{4}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times (\eta \nabla \times \boldsymbol{B}), \tag{5}$$

$$\mathcal{D}_{ij} = -2\bar{\rho}\nu\left[e_{ij} - \tfrac{1}{3}(\nabla \cdot \boldsymbol{v})\delta_{ij}\right]$$

$$\boldsymbol{j} = (c/4\pi)(\nabla \times \boldsymbol{B})$$

Elemental EULAG formulation

NCAR

$$\frac{d\mathbf{u}}{dt} = -\nabla\pi' - \mathbf{g}\frac{\Theta'}{\Theta_o} + 2\mathbf{u}\times\mathbf{\Omega} + \frac{1}{\mu\rho_o}\mathbf{B}\cdot\nabla\mathbf{B} \ ,$$

$$\frac{d\Theta'}{dt} = -\mathbf{u}\cdot\nabla\Theta_e + \frac{1}{\rho_o}\mathcal{H}(\Theta') - \alpha\Theta' \ ,$$

$$\frac{d\mathbf{B}}{dt} = \mathbf{B}\cdot\nabla\mathbf{u} - \mathbf{B}\nabla\cdot\mathbf{u} \ ,$$

$$\nabla\cdot\rho_o\mathbf{u} = 0 \ ,$$

$$\nabla\cdot\mathbf{B} = 0 \ .$$

The underlying ambient states assumes mean Sun

$$0 = -\frac{\partial}{\partial r}\left(\frac{p_e - p_o}{\rho_o}\right) + g\frac{\Theta_e - \Theta_o}{\Theta_o} \ ,$$

$$0 = \mathcal{H}(\Theta_e) + \mathcal{H}^* \ ,$$

Coordinate dependent form

NCAR

$$(\overline{t},\overline{\mathbf{x}}) \equiv (t,\ F(\mathbf{x}))$$

$$dx = r\cos\phi\, d\lambda,\ dy = r\, d\phi \text{ and } z = r - R_0$$

$$\overline{x} = R_0\lambda,\ \overline{y} = R_0\phi, \text{ and } \overline{z} = z$$

$$\frac{d\mathbf{u}}{dt} = -\widetilde{\mathbf{G}}\overline{\nabla}\pi' - \mathbf{g}\frac{\Theta'}{\Theta_o} + 2\mathbf{u}\times\boldsymbol{\Omega} + \mathcal{M}(\mathbf{u},\mathbf{u}) + \frac{1}{\mu\rho_o}\overline{\mathbf{B}}^* \cdot \overline{\nabla}\mathbf{B} - \frac{1}{\mu\rho_o}\mathcal{M}(\mathbf{B},\mathbf{B}) ,$$

$$\widetilde{\mathbf{G}} \propto (\partial\overline{\mathbf{x}}/\partial\mathbf{x})$$

$$\widetilde{G}^1_1 = [\Gamma\cos(\overline{y}/R_0)]^{-1}$$
$$\widetilde{G}^2_2 = \Gamma^{-1}$$
$$\widetilde{G}^3_3 = 1$$
$$\overline{\mathcal{G}} = \Gamma^2\cos(\overline{y}/R_0)$$

$$\frac{d\Theta'}{dt} = -\overline{\mathbf{u}}^* \cdot \overline{\nabla}\Theta_e + \frac{1}{\rho^*}\overline{\mathcal{H}}(\Theta') - \alpha\Theta' ,$$

$$\frac{d\mathbf{B}}{dt} = \overline{\mathbf{B}}^* \cdot \overline{\nabla}\mathbf{u} - \mathbf{B}\frac{1}{\overline{\mathcal{G}}}\overline{\nabla}\cdot\overline{\mathcal{G}}\overline{\mathbf{u}}^* + \mathcal{M}(\mathbf{u},\mathbf{B}) - \mathcal{M}(\mathbf{B},\mathbf{u}) ,$$

$$\frac{1}{\rho^*}\overline{\nabla}\cdot\rho^*\overline{\mathbf{u}}^* = 0 ,$$

$$\frac{1}{\overline{\mathcal{G}}}\overline{\nabla}\cdot\overline{\mathcal{G}}\,\overline{\mathbf{B}}^* = 0 .$$

$$\overline{\mathbf{u}}^* = \widetilde{\mathbf{G}}^T\mathbf{u} ,\quad \overline{\mathbf{B}}^* = \widetilde{\mathbf{G}}^T\mathbf{B} ,$$

Numerical approximations; preliminaries

NCAR

$$\frac{\partial \rho^* \Psi}{\partial \bar{t}} + \overline{\nabla} \cdot (\mathbf{V}^* \Psi) = \rho^* \mathrm{R} \ ,$$

$$\Psi = \{\mathbf{u}, \Theta', \mathrm{B}\}^T$$

$$\mathrm{R} = \{\mathrm{R}_{\mathbf{u}}, R_{\Theta'}, \mathrm{R}_{\mathrm{B}}\}^T$$

$$\overline{\mathbf{B}}^* \cdot \overline{\nabla} \mathbf{B} = \frac{1}{\overline{\overline{\mathcal{G}}}} \overline{\nabla} \cdot \overline{\mathcal{G}}\, \overline{\mathbf{B}}^* \mathbf{B} \ , \quad \overline{\mathbf{B}}^* \cdot \overline{\nabla} \mathbf{u} = \frac{1}{\overline{\overline{\mathcal{G}}}} \overline{\nabla} \cdot \overline{\mathcal{G}}\, \overline{\mathbf{B}}^* \mathbf{u}$$

$$\Psi_{\mathrm{i}}^{n} = \mathcal{A}_{\mathrm{i}} \left(\widetilde{\Psi},\ \widetilde{\mathbf{V}}^*,\ \rho^* \right) + 0.5\delta t \mathbf{R}_{\mathrm{i}}^{n} \equiv \widehat{\Psi}_{\mathrm{i}} + 0.5\delta t \mathbf{R}_{\mathrm{i}}^{n}$$

$$\Psi_{\mathrm{i}}^{n,\nu} = \widehat{\Psi}_{\mathrm{i}} + 0.5\delta t\ \mathbf{L}\Psi|_{\mathrm{i}}^{n,\nu} + 0.5\delta t\ \mathbf{N}(\Psi)|_{\mathrm{i}}^{n,\nu-1} - 0.5\delta t\ \widetilde{\mathbf{G}}\overline{\nabla}\Phi|_{\mathrm{i}}^{n,\nu}$$

$$\Phi \equiv (\pi',\ \pi',\ \pi',\ 0,\ \pi^*,\ \pi^*,\ \pi^*)$$

$$\Psi_{\mathrm{i}}^{n,\nu} = [\mathbf{I} - 0.5\delta t \mathbf{L}]^{-1} \left(\widehat{\widehat{\Psi}} - 0.5\delta t \widetilde{\mathbf{G}} \overline{\nabla} \Phi^{n,\nu} \right) \Big|_{\mathrm{i}}$$

$$\widehat{\widehat{\Psi}} \equiv \widehat{\Psi} + 0.5\delta t \mathbf{N} \Psi|^{n,\nu-1}$$

➔ elliptic problems for potentials π

Specialized MHD implementation

NCAR

Hydrodynamic block

$$\mathrm{B}_{\mathrm{i}}^{\nu-1/2} = \widehat{\mathrm{B}}_{\mathrm{i}} + \delta_h t\Big[(\mathrm{B}^{\nu-1/2}\cdot\widetilde{\mathbf{G}}\overline{\nabla}\mathbf{u}^{\nu-1}) - \mathrm{B}^{\nu-1/2}tr\{\widetilde{\mathbf{G}}\overline{\nabla}\mathbf{u}^{\nu-1}\} + \delta\mathcal{M}(\mathbf{u}^{\nu-1},\mathrm{B}^{\nu-1})\Big]_{\mathrm{i}} \;;$$

$$\mathbf{u}_{\mathrm{i}}^{\nu} = \widehat{\mathbf{u}}_{\mathrm{i}} + \frac{\delta_h t}{\mu\rho_o}\left(\frac{1}{\overline{\overline{\mathcal{G}}}}\overline{\nabla}\cdot\overline{\mathcal{G}}\,\overline{\mathrm{B}}^{*}\mathrm{B}\right)_{\mathrm{i}}^{\nu-1/2} + \frac{\delta_h t}{\mu\rho_o}\mathcal{M}(\mathrm{B},\mathrm{B})_{\mathrm{i}}^{\nu-1/2} + \delta_h t\mathcal{M}(\mathbf{u},\mathbf{u})^{\nu-1} - \delta_h t\left[\mathrm{g}\frac{\Theta'}{\Theta_o} - 2\mathbf{u}\times\boldsymbol{\Omega}\right]_{\mathrm{i}}^{\nu} - \delta_h t\left(\widetilde{\mathbf{G}}\overline{\nabla}\pi'\right)_{\mathrm{i}}^{\nu} \;,$$

$$\Theta'|_{\mathrm{i}}^{\nu} = \widehat{\Theta'}_{\mathrm{i}} - \delta_h t\left[\mathbf{u}\cdot\widetilde{\mathbf{G}}\overline{\nabla}\Theta_e\right]_{\mathrm{i}}^{\nu} \;,$$

$$0 = \frac{1}{\rho_{\mathrm{i}}^{*}}\left(\overline{\nabla}\cdot\rho^{*}\overline{\mathbf{u}}^{*}\right)_{\mathrm{i}}^{\nu} \;.$$

NCAR

Magnetic block

$$\mathbf{B}_{\mathrm{i}}^{\nu-1/4} = \widehat{\mathbf{B}}_{\mathrm{i}} + \delta_h t\Big[(\mathbf{B}^{\nu-1/4}\cdot\widetilde{\mathbf{G}}\overline{\nabla}\mathbf{u}^{\nu}) - \mathbf{B}^{\nu-1/4}tr\{\widetilde{\mathbf{G}}\overline{\nabla}\mathbf{u}^{\nu}\} + \delta\mathcal{M}(\mathbf{u}^{\nu},\mathbf{B}^{\nu-1/2})\Big]_{\mathrm{i}}$$

$$\mathbf{B}_{\mathrm{i}}^{\nu} = \widehat{\mathbf{B}}_{\mathrm{i}} + \delta_h t\left(\frac{1}{\overline{\overline{\mathcal{G}}}}\overline{\nabla}\cdot\overline{\mathcal{G}}\,\overline{\mathbf{B}}^{*}|^{\nu-1/4}\mathbf{u}^{\nu} + \mathbf{B}^{\nu-1/4}w^{\nu}\frac{d\ln\rho_o}{dr}\right)_{\mathrm{i}} + \delta\mathcal{M}(\mathbf{u}^{\nu},\mathbf{B}^{\nu-1/4})\Big]_{\mathrm{i}} - \delta_h t\left(\widetilde{\mathbf{G}}\overline{\nabla}\pi^{*}\right)_{\mathrm{i}}^{\nu}\ ,$$

$$0 = \frac{1}{\overline{\overline{\mathcal{G}}}_{\mathrm{i}}}\left(\overline{\nabla}\cdot\overline{\mathcal{G}}\,\overline{\mathbf{B}}^{*}\right)_{\mathrm{i}}^{\nu}\ .$$

Thermodynamics

$$\Theta'|_{\mathrm{i}}^{\nu} = \left(\widehat{\Theta'} - \delta_h t\mathbf{u}^{\nu}\cdot\widetilde{\mathbf{G}}\overline{\nabla}\Theta_e\right)_{\mathrm{i}}$$

is not evaluated until the last iteration $\nu = m$.

Model setups

spherical shell $(0.62 \leq r/R_\odot \leq 0.96)$ spanning 3.4 density scale heights

$$\Omega_\odot = 2.69 \times 10^{-6}\,\mathrm{rad\ s}^{-1}$$

Θ_e stable up to $r/R_\odot = 0.718$ and unstable stable aloft

$$\alpha = 2 \cdot 10^{-8}\mathrm{s}^{-1}$$

$$(N_r \times N_\theta \times N_\phi = 47 \times 64 \times 128)$$

Power of ILES

NCAR

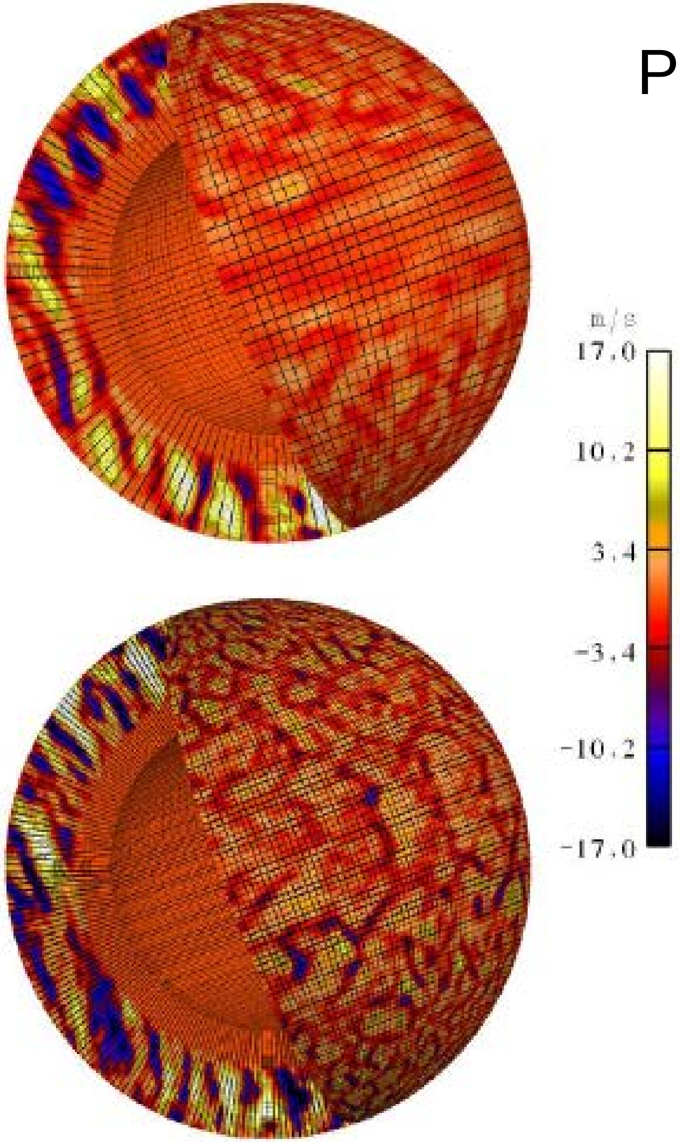
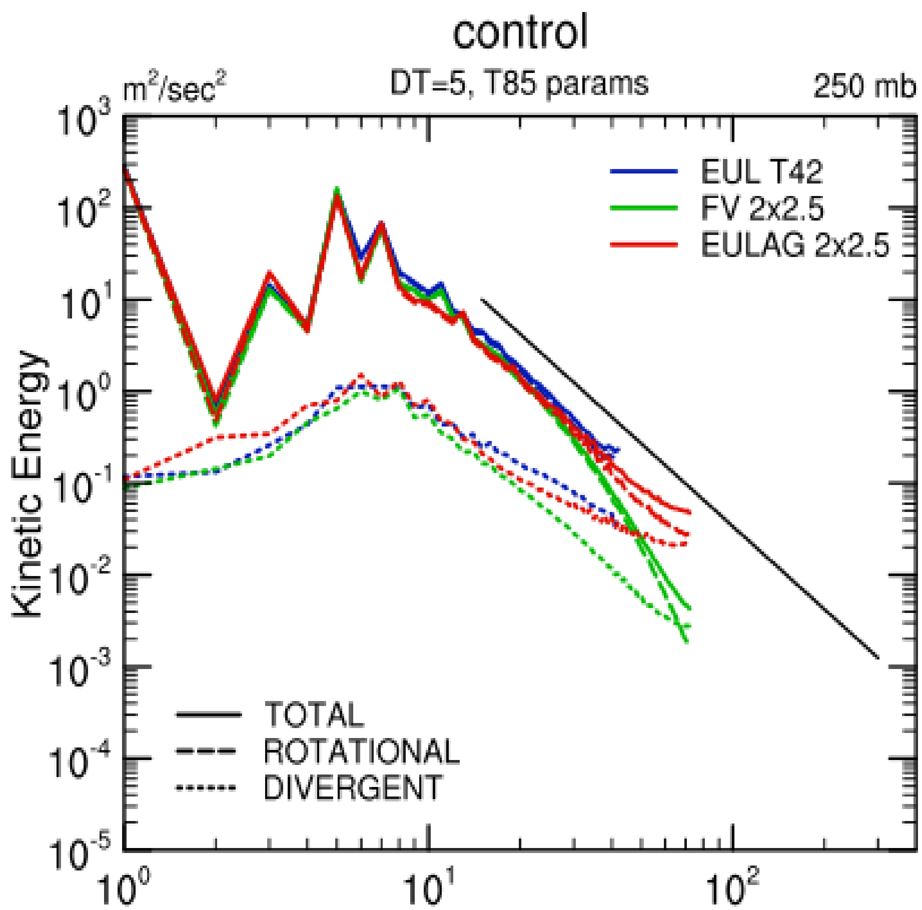
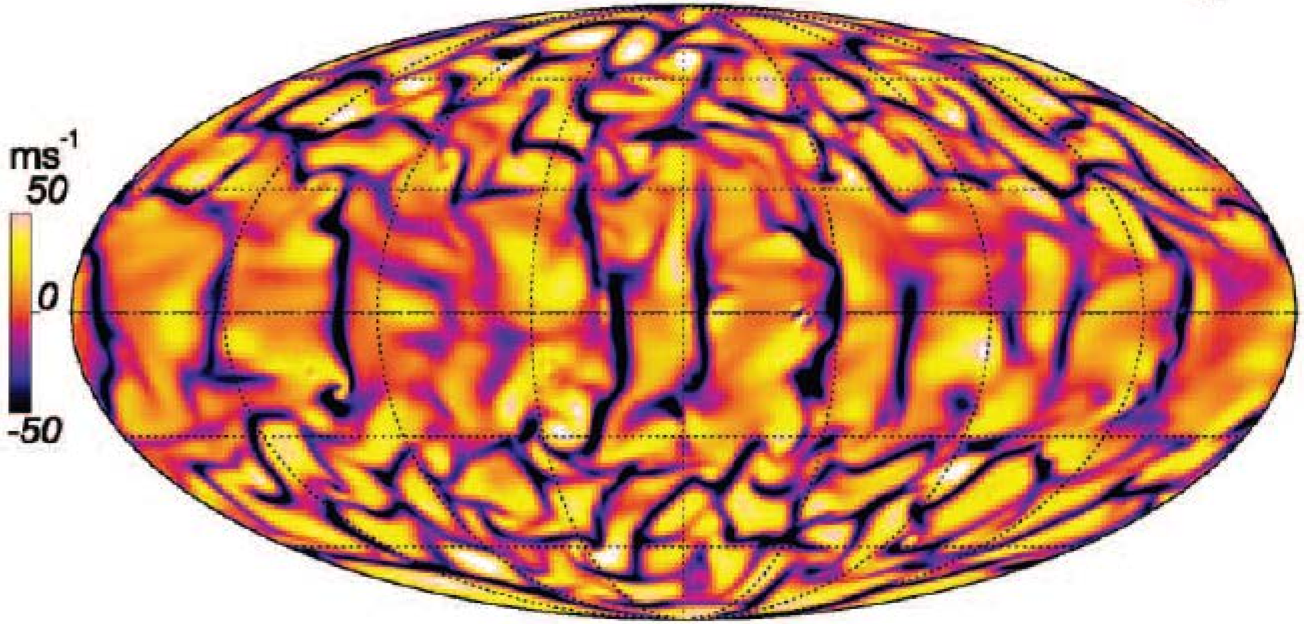
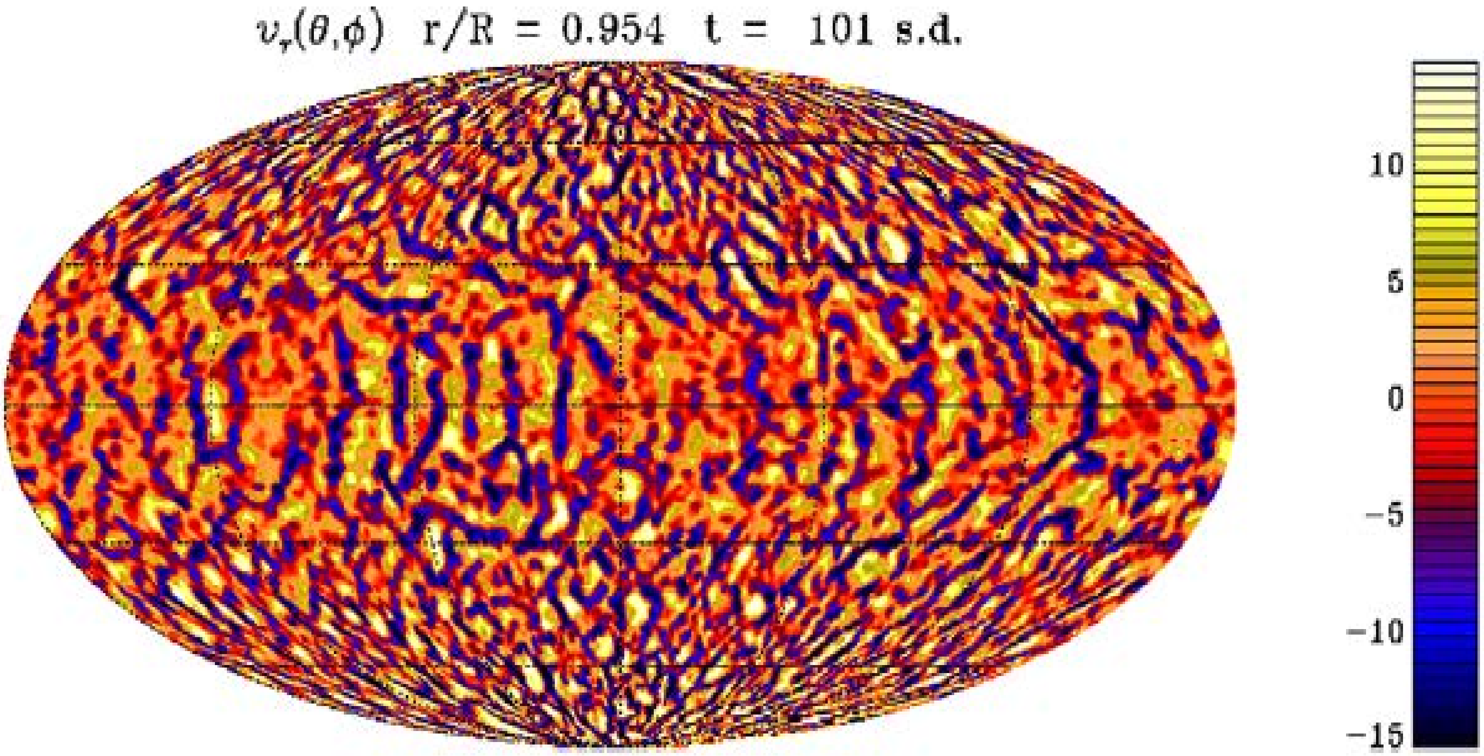


Fig. 1. Color rendering of the radial component of the flow velocity in two simulations carried out on a longitude-latitude-radius spatial mesh of size 128 × 64 × 47 (top) and 256 × 128 × 96 (bottom), with the mesh lines overlaid as black lines. Both snapshots are taken many tens of convective eddy turnover times after convection has reached a statistically stationary state. The tendencies for convective flow structures to align themselves parallel to the rotation axis at low latitudes is typical of global simulations of solar convection, with or without magnetic fields, operating in this parameter regime (see, e.g., Fig. 1 in [6]; Fig. 2 in [28]).



NCAR





Magnetic cycles

NCAR

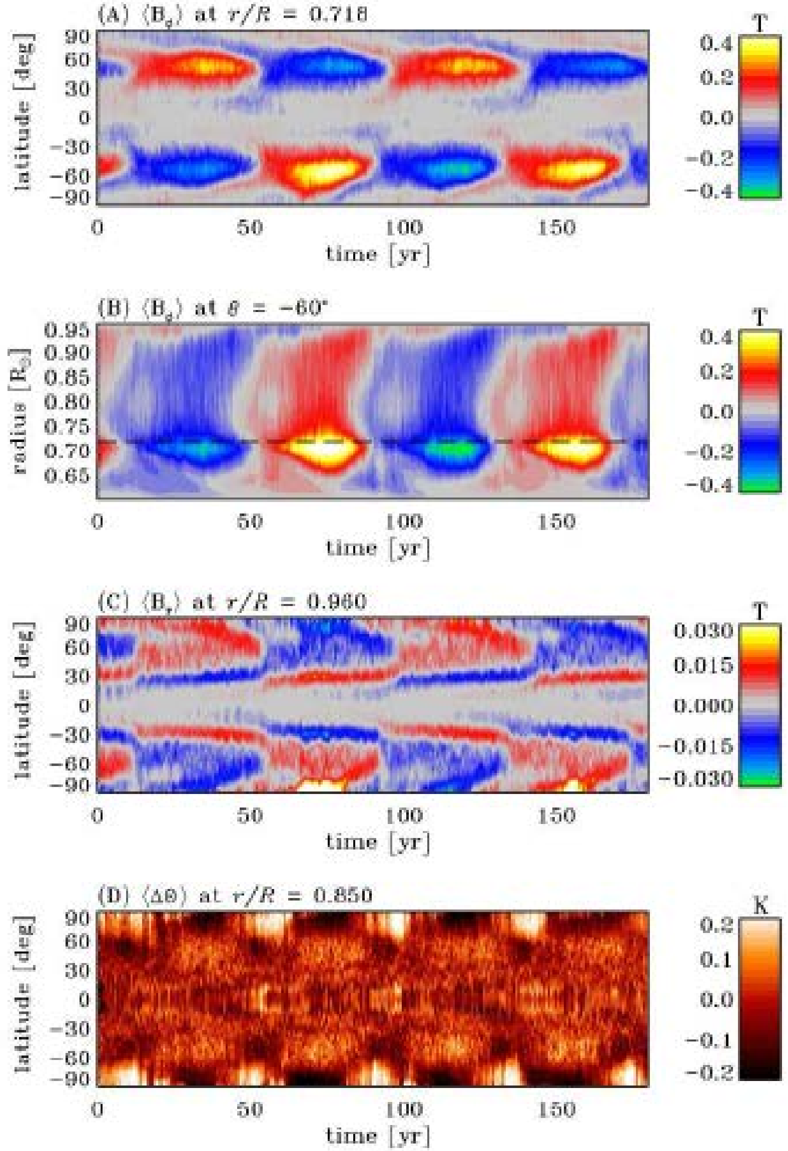


Fig. 4. Four views of a 180 yr segment of a simulation carried out at low spatial resolution, $128 \times 64 \times 47$. (A) Time-latitude cut of the zonally-averaged toroidal magnetic field at the core-envelope interface ($r/R_\odot = 0.718$); (B) time-radius cut of the same at mid-latitude in the Southern hemisphere; (C) time-latitude cut of the zonally-averaged radial magnetic field component on the top layer of the simulation ($r/R_\odot = 0.96$); (D) a time-latitude cut of the potential temperature residual with respect to a zonally and temporally averaged latitudinal temperature profile, at mid-depth in the convective layers ($r/R_\odot = 0.85$).

Mode of large-scale dynamo action

NCAR

$$\mathbf{u}'(r,\theta,\phi,t) = \mathbf{u}(r,\theta,\phi,t) - \langle\mathbf{u}\rangle(r,\theta,t)$$
$$\mathbf{B}'(r,\theta,\phi,t) = \mathbf{B}(r,\theta,\phi,t) - \langle\mathbf{B}\rangle(r,\theta,t)$$

$$\frac{\partial\langle\mathbf{B}\rangle}{\partial t} = \nabla \times \langle\mathbf{u}\rangle \times \langle\mathbf{B}\rangle + \nabla \times \langle\mathbf{u}' \times \mathbf{B}'\rangle$$

NCAR

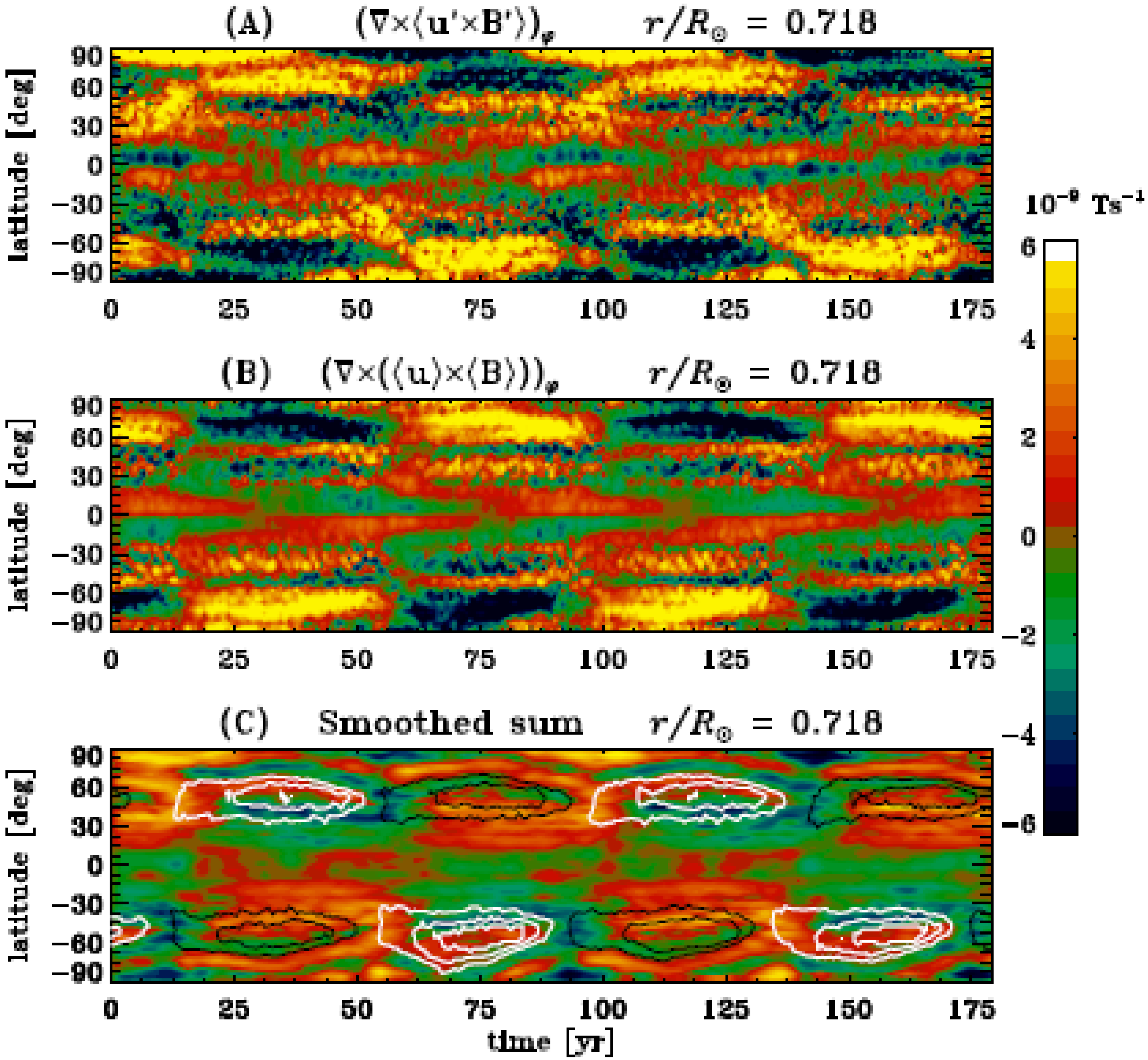


Fig. 5. The two inductive contributions to the zonally-averaged large-scale zonal magnetic component, for the same simulation and over the same time interval as in Fig. 4. (A) induction by the turbulent electromotive force; (B) Induction by the large scale flow; (C) the sum of the above two on the same color scale, with a few isocontours of mean zonal field overplotted (black/white for negative/positive).

Remarks

NCAR

Notwithstanding some departures from the real solar climate, global MHD simulations of solar-like cycles have landed.

EULAG-MHD offers an outstanding virtual MHD laboratory allowing to address quantitatively a number of questions that could until now only be speculated upon on the basis of simplified model formulations.

There is much to be learned. For example, our calculations suggest that the entire magnetic variability involves less then 10% of solar luminosity ➔ connection to physics of other “dissipative structures” (QBO, MJO, ….)

Deriving the entropy equation

NCAR

$$\frac{d\Theta}{dt} = \frac{\Theta_o}{\rho_o T_o}\left[\nabla\cdot\left(\kappa_r\rho_o\nabla\frac{T_o}{\Theta_o}\Theta\right) + \nabla\cdot\left(\kappa\rho_o\frac{T_o}{\Theta_o}\nabla\Theta\right)\right] ,$$

$$\frac{\partial\rho_o\Theta}{\partial t} + \nabla\cdot\rho_o\mathbf{u}\Theta = \frac{\Theta_o}{T_o}\left[\nabla\cdot\left(\kappa_r\rho_o\nabla\frac{T_o}{\Theta_o}\Theta\right) + \nabla\cdot\left(\kappa\rho_o\frac{T_o}{\Theta_o}\nabla\Theta\right)\right]$$

$$\frac{d}{dr}\rho_o\langle w\Theta\rangle = \frac{\Theta_o}{T_o}\left[\frac{d}{dr}\left(\kappa_r\rho_o\frac{d}{dr}\frac{T_o}{\Theta_o}\langle\Theta\rangle\right) + \frac{d}{dr}\left(\kappa\rho_o\frac{T_o}{\Theta_o}\frac{d}{dr}\langle\Theta\rangle\right)\right]$$

where $\langle..\rangle$ = denotes horizontal-time average

Decomposing w and Θ into $\langle..\rangle + \breve{..}$, and identifying $\langle\Theta\rangle$ with ambient profile Θ_e

$$\begin{aligned} 0 &\equiv \frac{d}{dr}\rho_o\langle w\rangle\Theta_e \\ &= \frac{\Theta_o}{T_o}\left[\frac{d}{dr}\left(\kappa_r\rho_o\frac{d}{dr}\frac{T_o}{\Theta_o}\Theta_e\right) + \frac{d}{dr}\left(\kappa\rho_o\frac{T_o}{\Theta_o}\frac{d}{dr}\Theta_e\right)\right] + \mathcal{H}^* \end{aligned}$$

where $\mathcal{H}^* \equiv -d/dr\left(\rho_o\langle\breve{w}\breve{\Theta}\rangle\right)$

$\tau_\alpha \sim \mathcal{O}(10^8)$s is assumed

What is *A* operator?

NCAR

Multidimensional positive definite advection transport algorithm (MPDATA):

$$\frac{\partial \phi}{\partial t} = -\nabla \bullet (\mathbf{V}\phi) \ , \qquad \phi_i^{n+1} = \phi_i^n - \frac{\delta t}{\mathcal{V}_i} \sum_{j=1}^{l(i)} F_j^{\perp} S_j$$

$$F_j^{\perp}(\phi_i, \phi_j, V_j^{\perp}) = [V_j^{\perp}]^+ \phi_i + [V_j^{\perp}]^- \phi_j \ , \qquad [V]^+ \equiv 0.5(V + |V|) \ \ , \ \ [V]^- \equiv 0.5(V - |V|) \ ,$$

$$\phi_i^{(k)} = \phi_i^{(k-1)} - \frac{\delta t}{\mathcal{V}_i} \sum_{j=1}^{l(i)} F_j^{\perp} \left(\phi_i^{(k-1)}, \phi_j^{(k-1)}, V_j^{\perp,(k)} \right) S_j$$

with $k = 1, .., IORD$ such that

$$\phi^{(0)} \equiv \phi^n \ \ ; \ \ \phi^{(IORD)} \equiv \phi^{n+1}$$

$$V^{\perp,(k+1)} = V^{\perp} \left(\mathbf{V}^{(k)}, \phi^{(k)}, \nabla \phi^{(k)} \right) \ \ ; \ \ V_j^{\perp,(1)} \equiv V^{\perp}|_j^{n+1/2}$$

$$V^{\perp}\Big|_{s_j}^{(k+1)} = \Bigg\{ \ 0.5|V^{\perp}| \left(\frac{1}{|\phi|} \frac{\partial |\phi|}{\partial r} \right) (r_j - r_i) \ - 0.5 V^{\perp} \left(\frac{1}{|\phi|} \frac{\partial |\phi|}{\partial r} \right) (r_i - 2r_{s_j} + r_j)$$

$$- 0.5 \delta t V^{\perp} \left(\mathbf{V} \bullet \frac{1}{|\phi|} \nabla |\phi| \right) - 0.5 \delta t V^{\perp} (\nabla \bullet \mathbf{V}) \Bigg\} \Bigg|_{s_j}^{(k)}$$