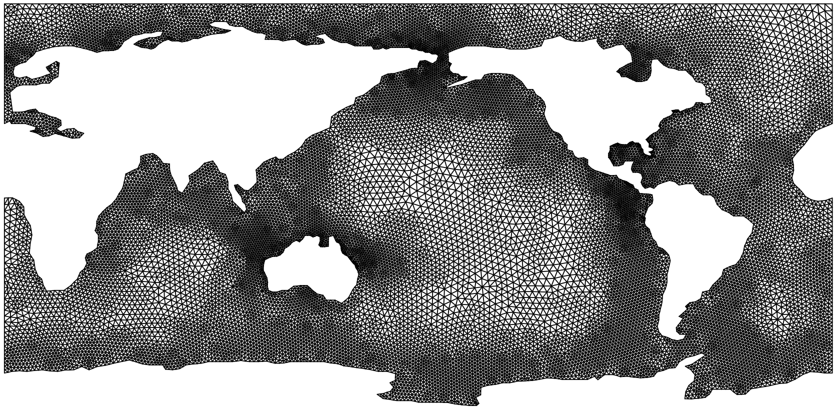
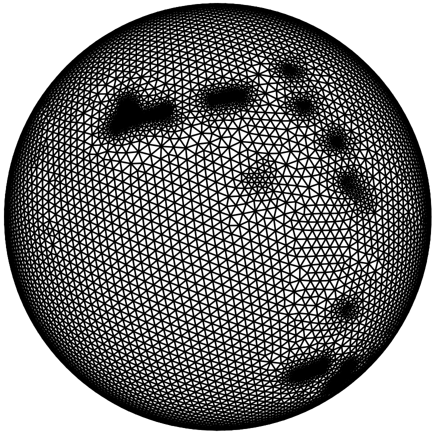
UNSTRUCTURED/ADAPTIVE MESH MODEL FOR STRATIFIRD TURBULENCE IN ATMOSPHERIC FLOWS

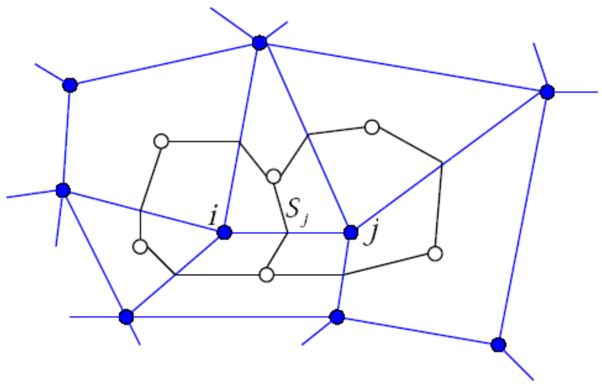
Joanna Szmelter [1] Piotr K Smolarkiewicz[2] Zhao Zhang[1]

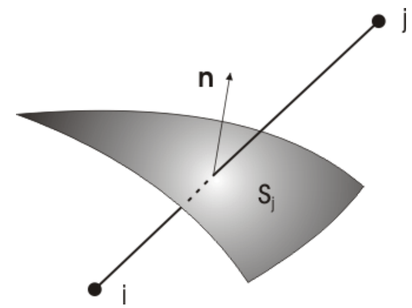
[1]*Loughborough University*
[2]*NCAR*

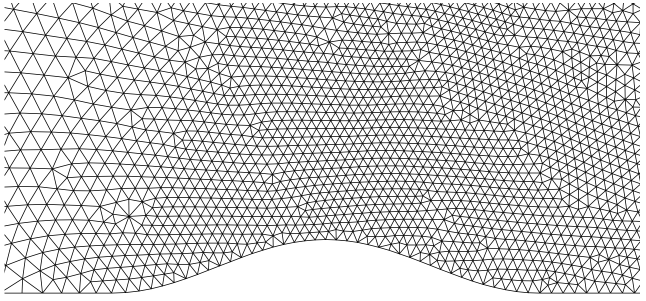




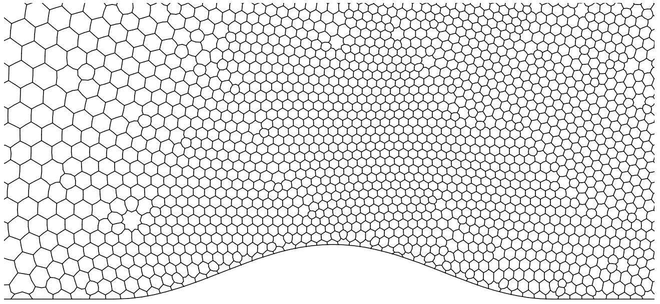
The Edge Based Finite Volume Discretisation







Edges



Median dual computational mesh
Finite volumes

A general NFT MPDATA unstructured mesh framework

$$\frac{\partial \Phi}{\partial t} + \nabla \bullet (\mathbf{V}\Phi) = \mathbf{R} \qquad \mathbf{\Phi}_i^{n+1} = \mathbf{\Phi}_i^{*} + 0.5\delta t \mathbf{R}_i^{n+1}$$

$$\mathbf{\Phi}^{*} \equiv \mathcal{A}(\mathbf{\Phi}^{n} + 0.5\delta t \mathbf{R}^{n}, \widehat{\mathbf{V}}^{n+1/2})$$

(Smolarkiewicz 91, Smolarkiewicz & Margolin 93; Mon. Weather Rev.)

(Smolarkiewicz & Szmelter, J. Comput. Phys. 2009
Szmelter & Smolarkiewicz, J. Comput. Phys. 2010)

Nonhydrostatic Boussinesq mountain wave

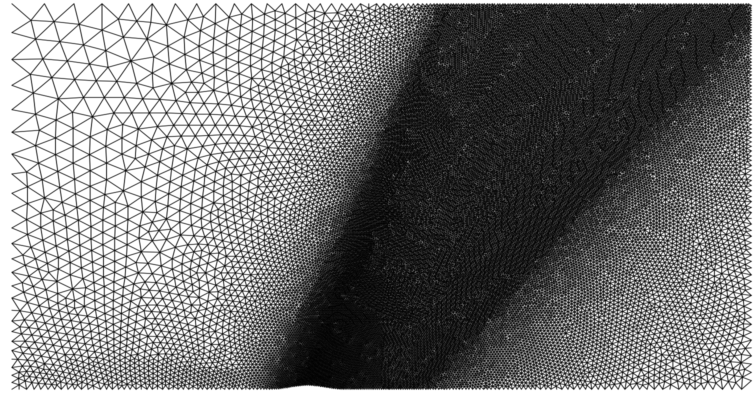
Szmelter & Smolarkiewicz , *Comp. Fluids*, 2011

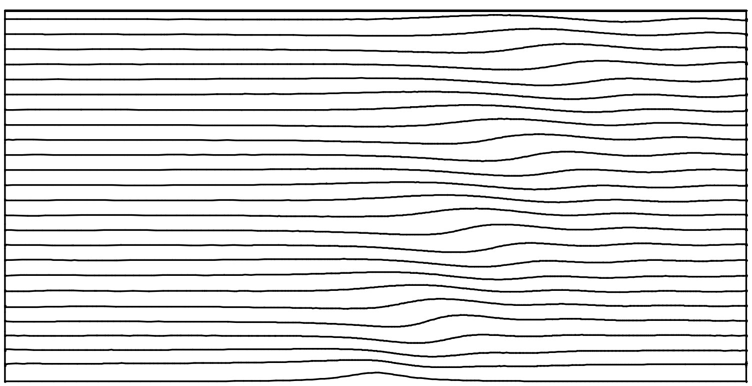
$$\frac{\partial \Phi}{\partial t} + \nabla \bullet (\mathbf{V}\Phi) = \mathbf{R}$$

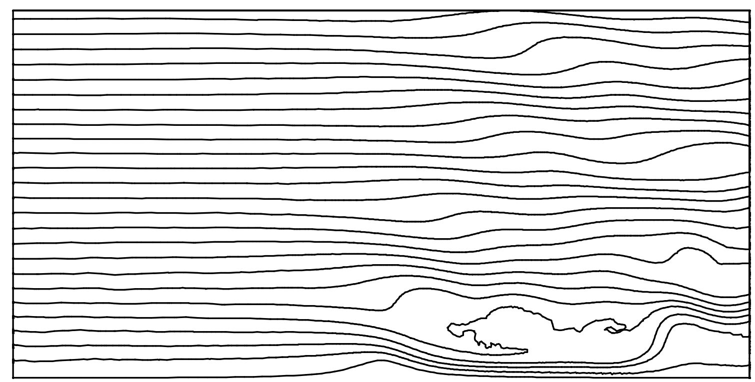
$$\nabla \bullet (\mathbf{V}\rho_o) = 0 \ ,$$

$$\frac{\partial \rho_o V^I}{\partial t} + \nabla \bullet (\mathbf{V}\rho_o V^I) = -\rho_o \frac{\partial \tilde{p}}{\partial x^I} + g\rho_o \frac{\theta'}{\theta_o}\delta_{I2}$$

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \bullet (\mathbf{V}\rho_o \theta) = 0 \ .$$





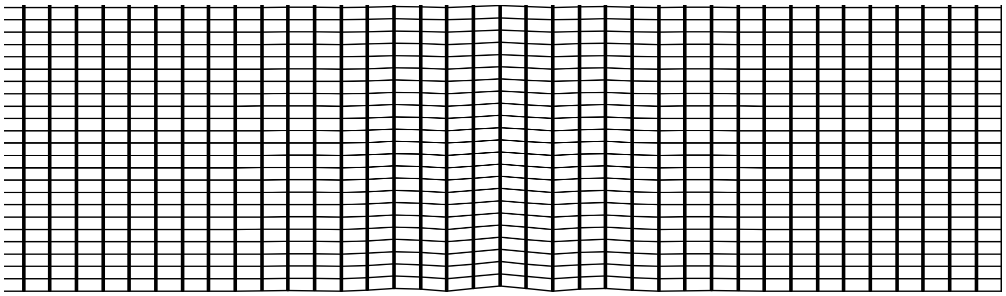


$Fr \lesssim 2$ $\quad$ $NL/U_o = 2.4$ $\quad$ $Fr \lesssim 1$;

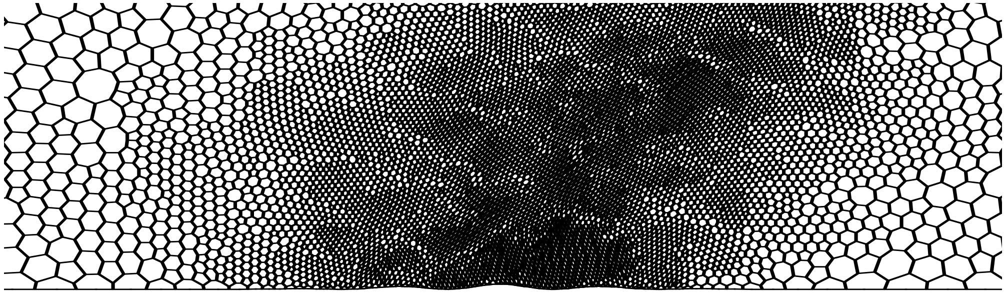
Comparison with the EULAG's results --- very close
with the linear theories (Smith 1979, Durran 2003):
over 7 wavelenghts : 3% in wavelength; 8% in propagation angle; wave amplitude loss 7%

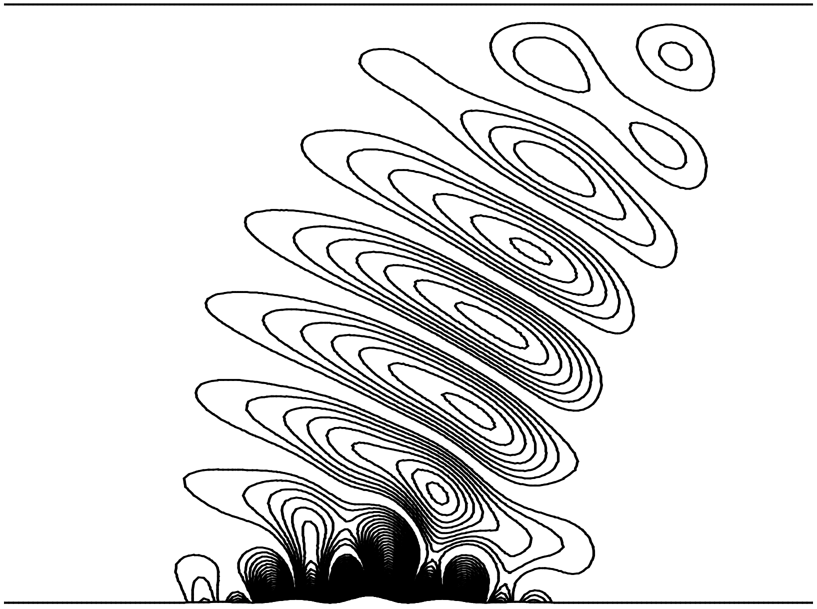
Static mesh adaptivity with MPDATA based error indicator

Schaer problem



Coarse initial mesh 3600 and solution





Adapted mesh 8662 points and solution

Notion of MPDATA

Iterative upwind

(Smolarkiewicz & Szmelter, J. Comput. Phys. 2005

$$\frac{\partial \Psi}{\partial t} = -\nabla \cdot (\mathbf{v}\Psi)$$

$$\Psi_i^{n+1} = \Psi_i^n - \frac{\delta t}{\mathcal{V}_i} \sum_{j=1}^{l(i)} F_j^{\perp} S_j$$

$$F_j^{\perp} = [v_j^{\perp}]^{+} \Psi_i^n + [v_j^{\perp}]^{-} \Psi_j^n$$

$$[v]^{+} := 0.5(v + |v|) \quad , \quad [v]^{-} := 0.5(v - |v|)$$

FIRST ORDER UPWIND (DONOR CELL)

$$F_j^{\perp} = v_j^{\perp} \Psi|_{s_j}^{n+1/2} + Error \qquad \tilde{v} := -\frac{1}{\Psi} Error$$

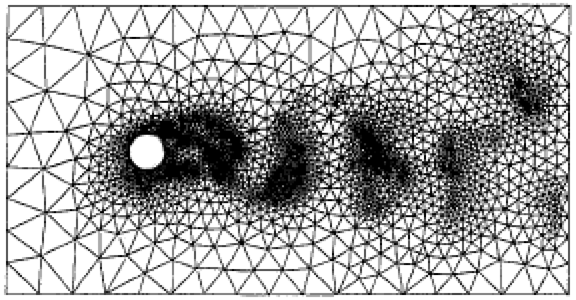
compensating velocity

$$Error = -0.5|v_j^{\perp}| \frac{\partial \Psi}{\partial r}|_{s_j}^{*} (r_j - r_i) + 0.5 v_j^{\perp} \frac{\partial \Psi}{\partial r}|_{s_j}^{*} (r_i - 2r_{s_j} + r_j)$$
$$+ 0.5\delta t v_j^{\perp} (\mathbf{v}\nabla\Psi)|_{s_j}^{*} + 0.5\delta t v_j^{\perp} (\Psi\nabla \cdot \mathbf{v})|_{s_j}^{*} + \mathcal{O}(\delta r^2, \delta t^2, \delta t \delta r)$$

Szmelter & Smolarkiewicz IJNMF 2006

$$(h_e)^{\text{new}} = h_e / \xi_e^{1/p} \qquad \xi_e = \frac{\| e \|_e}{\bar{e}_m}. \qquad \bar{h}_{\min} \le h_e \le \bar{h}_{\max}.$$

Wu, Zhu, Szmelter & Zienkiewicz , Comp Mech 1990



$$\frac{\partial \Phi}{\partial t} + \nabla \bullet (\mathbf{V}\Phi) = \mathbf{R}$$

Gravity wave breaking in an isothermal stratosphere

$$\nabla \cdot (\bar\rho \mathbf{v}) = 0\,, \quad \frac{D\theta}{Dt} = 0\,, \quad \frac{D\mathbf{v}}{Dt} = -\nabla\Phi' - \mathbf{g}\frac{\theta'}{\bar\theta}\,,$$ ***Lipps & Hemler***

$$\nabla \cdot (\bar\rho\bar\theta \mathbf{v}) = 0\,, \quad \frac{D\theta}{Dt} = 0\,, \quad \frac{D\mathbf{v}}{Dt} = -c_p\theta\nabla\pi' - \mathbf{g}\frac{\theta'}{\bar\theta}$$ ***Durran***

$$D\psi/Dt = R$$

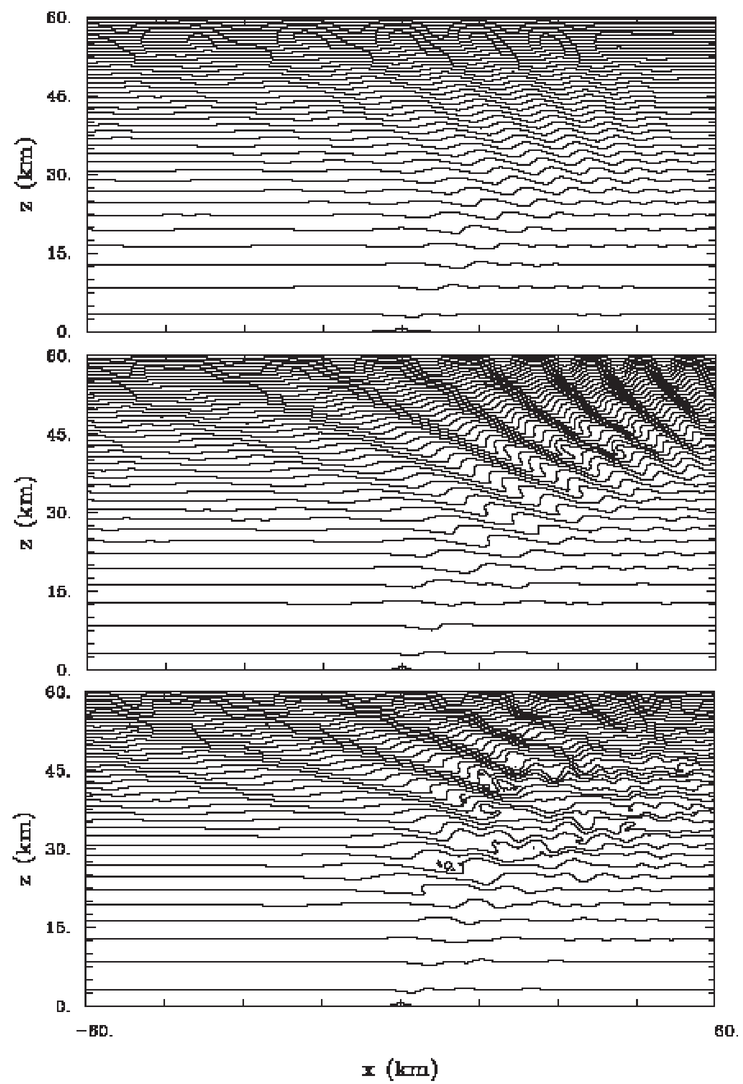
by combining $\rho^* \cdot \left(D\psi/Dt = R\right)$ with $\psi \cdot \left(\nabla\rho^*\mathbf{v} = 0\right)$,

$$\frac{\partial \rho^* \psi}{\partial t} + \nabla \cdot (\rho^* \mathbf{v} \psi) = \rho^* R\,.$$

$$\psi_i^{n+1} = \mathcal{A}_i(\tilde\psi, \mathbf{v}^{n+1/2}, \rho^*) + 0.5\delta t R_i^{n+1}$$

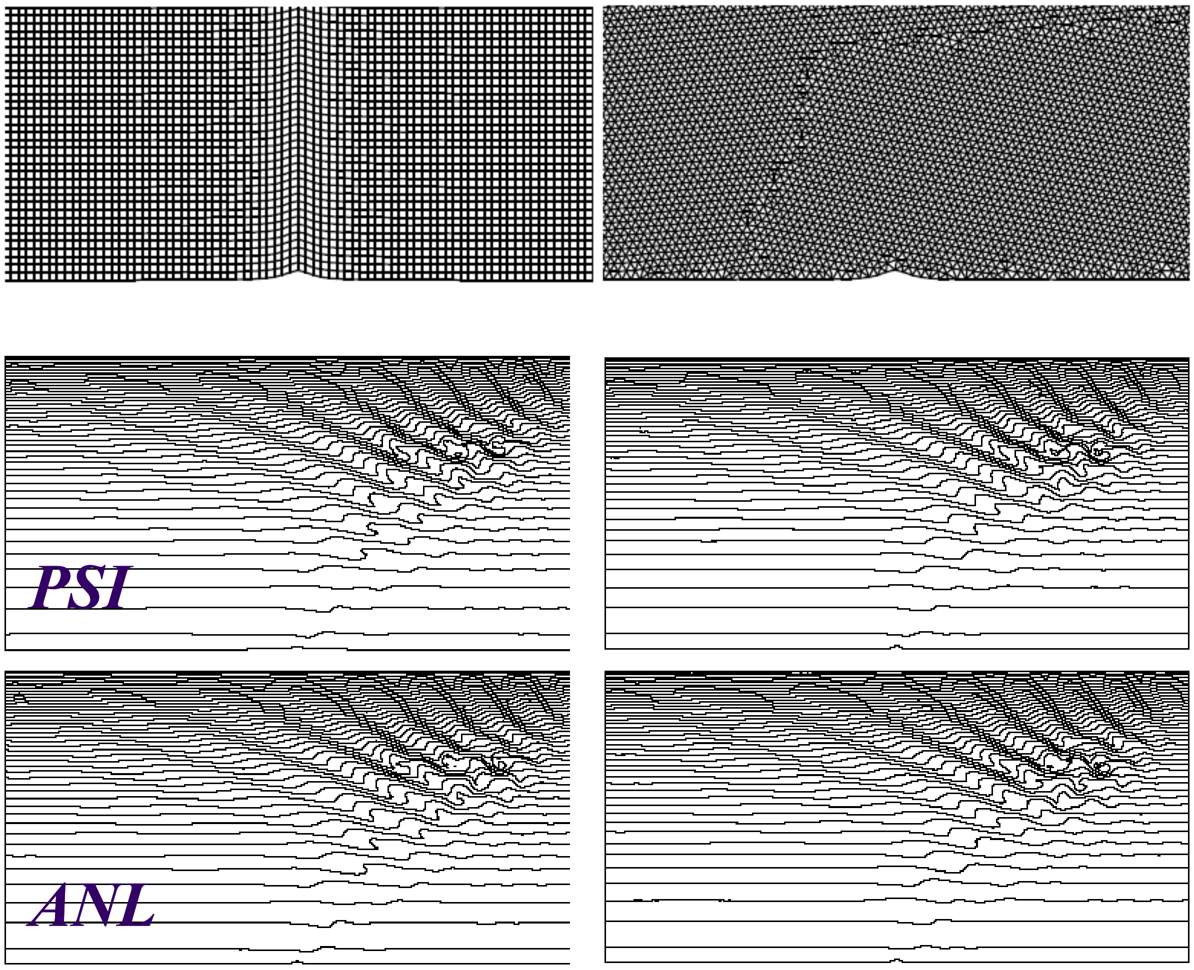
$$S_\theta = d\ln\bar\theta/dz = 4.4\cdot 10^{-5}\ \mathrm{m}^{-1}$$

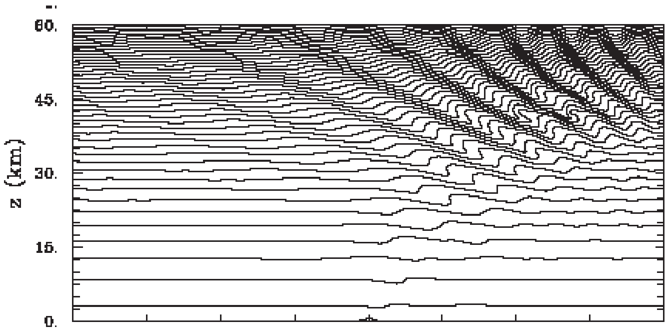
$$\mathbf{v}_e = (u_e, 0) \quad u_e = U = 20\mathrm{ms}^{-1}$$



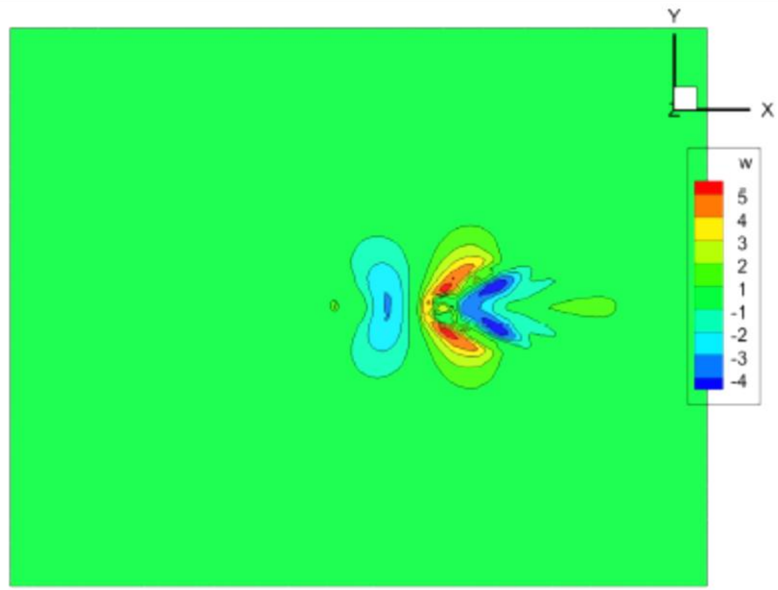
(Prusa et al JAS 1996,
Smolarkiewicz & Margolin, Atmos. Ocean 1997
Klein, Ann. Rev. Fluid Dyn., 2010)

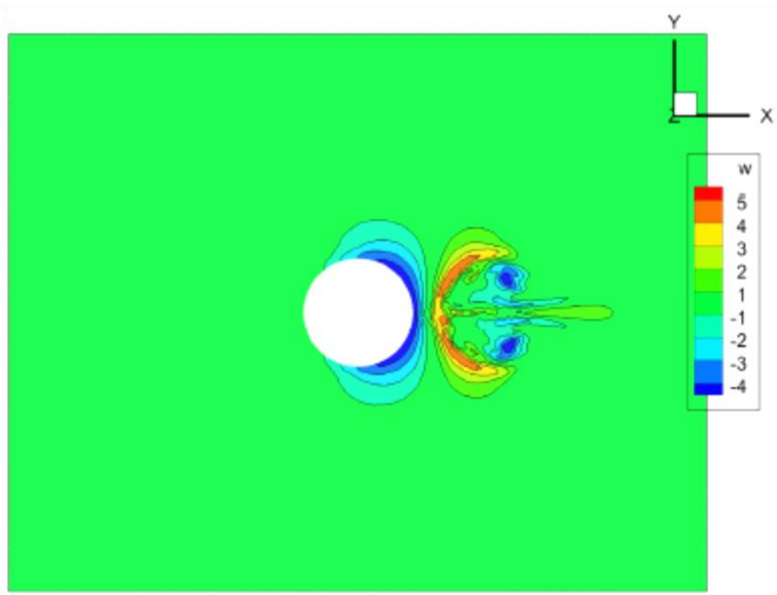
EULAG CV/GRID

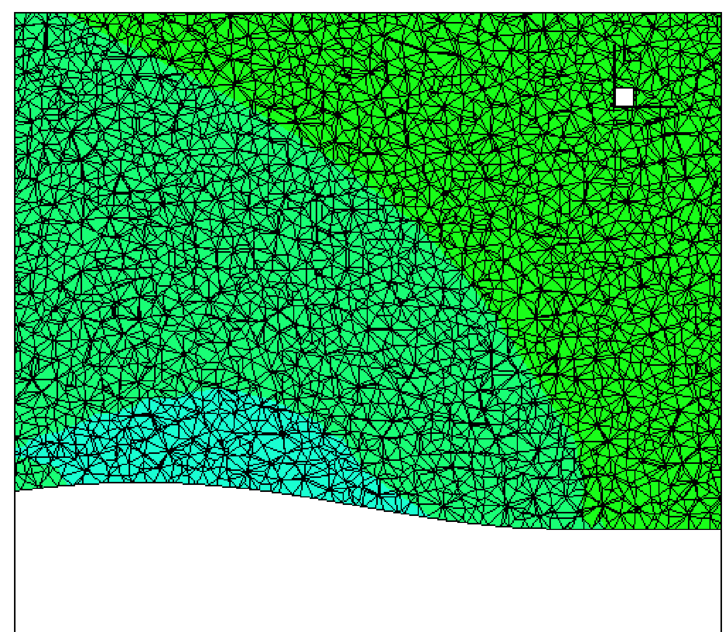




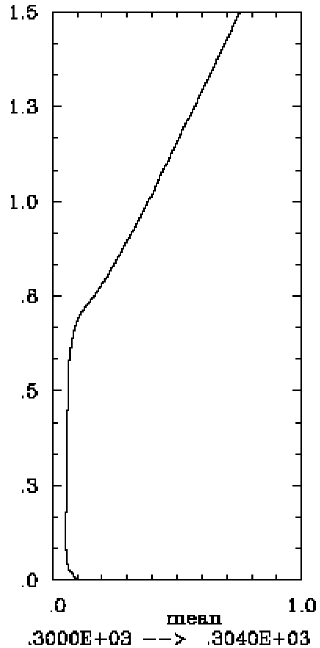
Low Froude Number Flow
Past a Three-Dimensional Hill

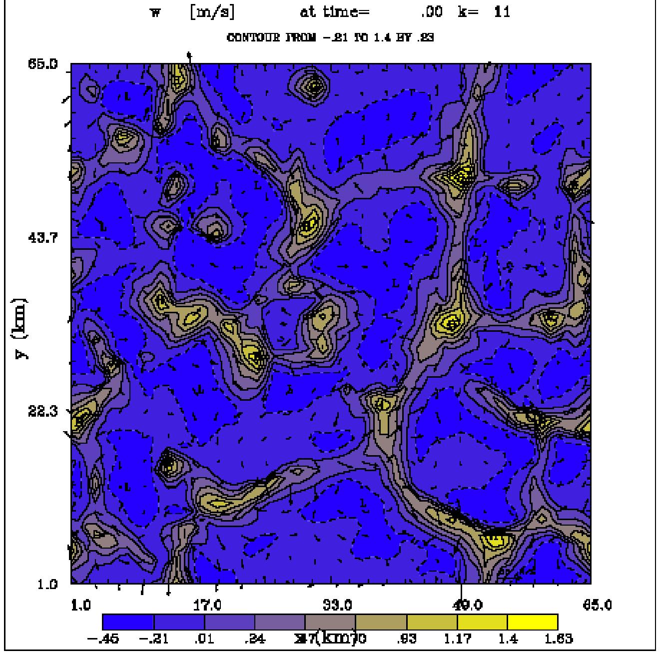


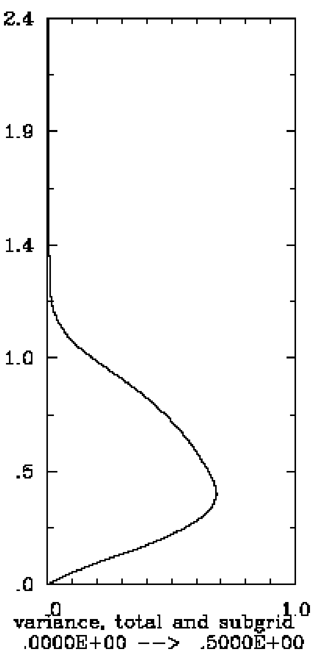


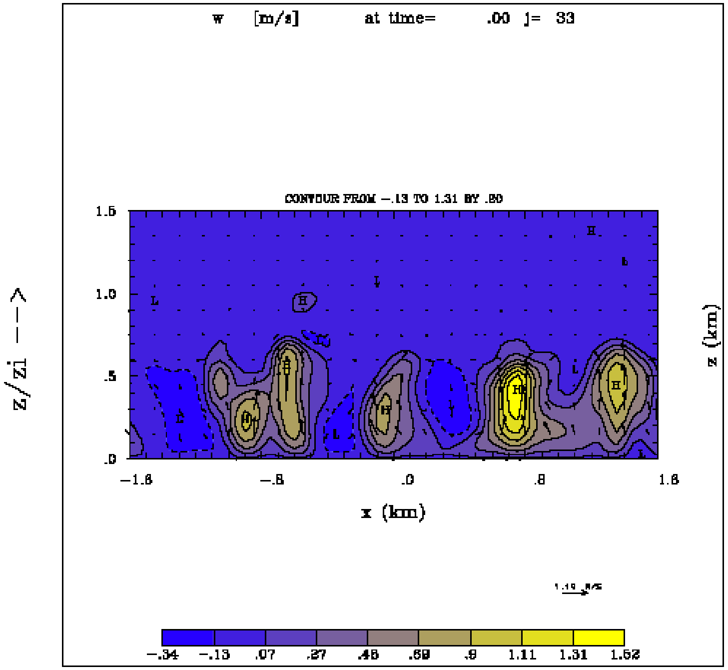


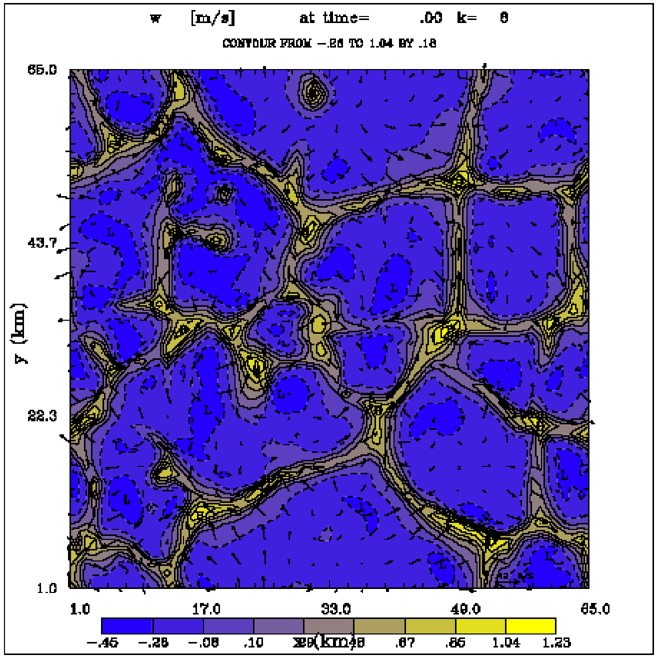
Convective Planetary Boundary Layer









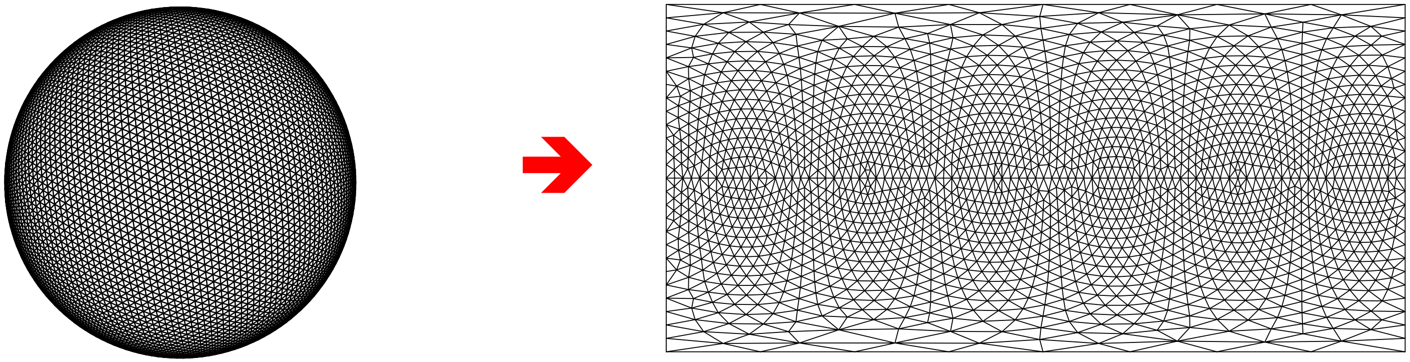


Unstructured-mesh framework for atmospheric flows

Smolarkiewicz ↻ Szmelter, pubs in *JCP, IJNMF,Comp. Fluids*, 2005-2011

- Differential manifolds formulation $$\frac{\partial G\mathbf{\Phi}}{\partial t} + \nabla \cdot (\mathbf{V\Phi}) = G\boldsymbol{\mathcal{R}}\ , \quad \mathrm{V}(\mathrm{x},t) := G\dot{\mathrm{x}}$$
- Finite-volume NFT numerics with a fully unstructured spatial discretization, heritage of EULAG and its predecessors (*A = MPDATA*)

$$\mathbf{\Phi}_i^{n+1} = \mathcal{A}_i(\mathbf{\Phi}^n + 0.5\delta t\,\boldsymbol{\mathcal{R}}^n,\ \mathrm{V}^{n+1/2}, G) + 0.5\delta t\,\boldsymbol{\mathcal{R}}_i^{n+1}$$

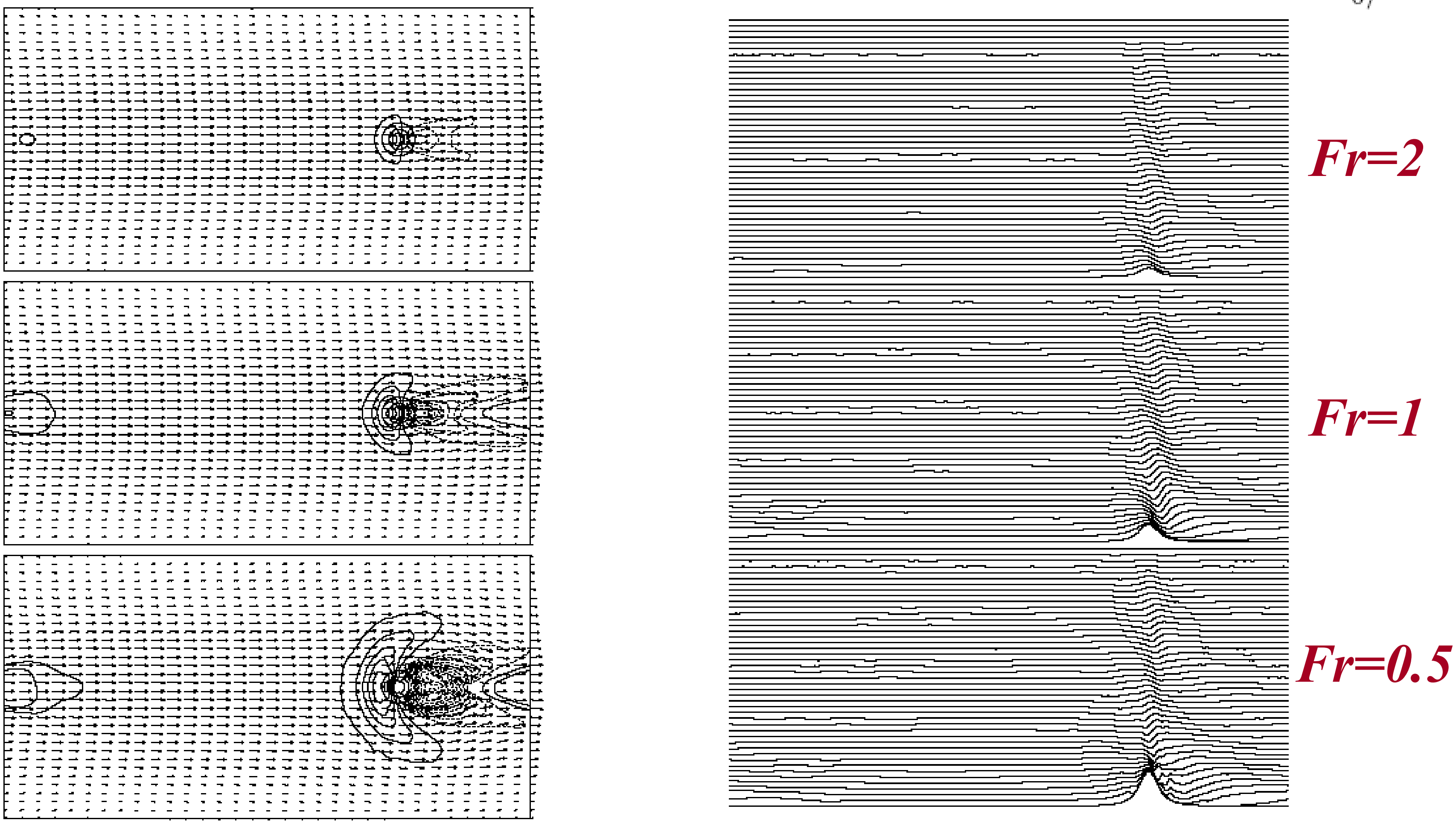


- Focus (so far) on wave phenomena across a range of scales and Mach, Froude & Rossby numbers

Stratified (mesoscale) flow past an isolated hill on a reduced planet

4 hours

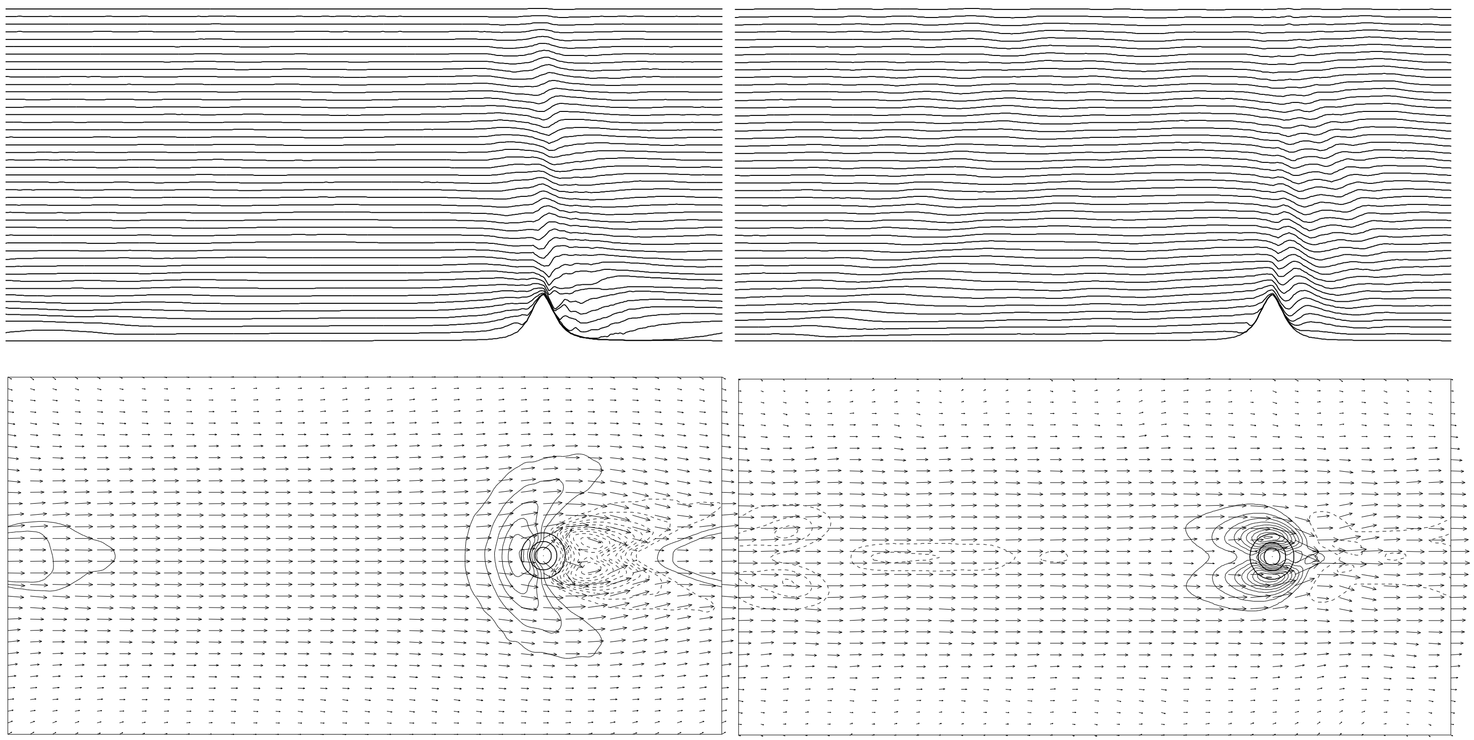
$Fr = U_0/Nh$



Hunt & Snyder J. Fluid Mech. 1980; Smolar. & Rotunno, J. Atmos. Sci. 1989 ; Wedi & Smolar., QJR 2009

Fr=0.5

$Ro \gg 1$ $\quad$ $Ro \gtrsim 1$



$h_{\zeta}=8^{-1}\lambda_o$

Smith, Advances in Geophys 1979; Hunt, Olafsson & Bougeault, QJR 2001

CONCLUSIONS

The paper demonstrates applicability of the general NFT framework to simulation of atmospheric flows using fully unstructured meshes. Novel numerical illustrations confirm that the edge-based discretisation sustains accuracy of structured grids.