The direct modeling of subgrid-scale stresses and its relevance for medium range weather prediction

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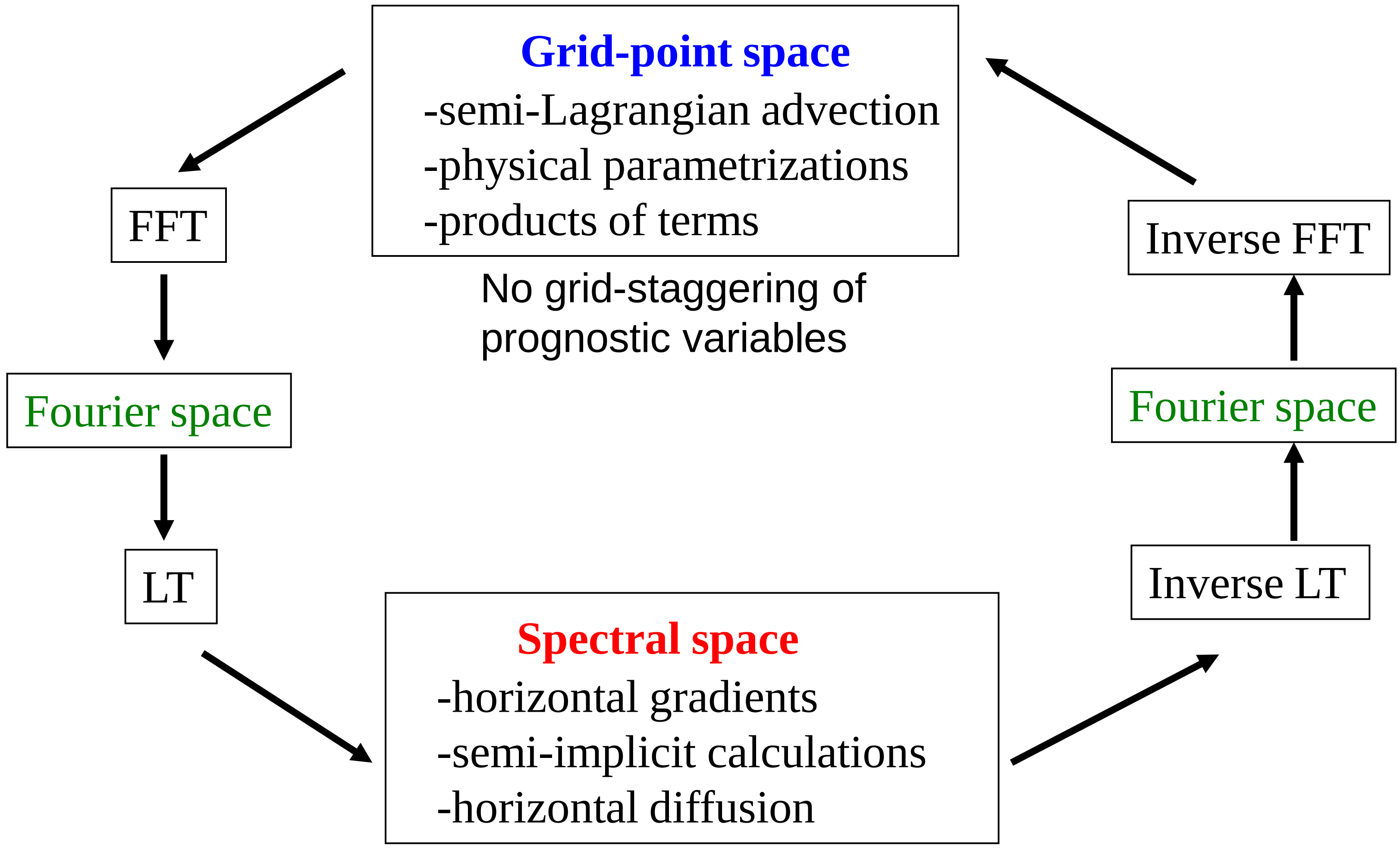
Outline

- The spectral transform method: de-aliasing of spectral computations on the linear grid
- The role of horizontal diffusion
- Non-linear diffusion

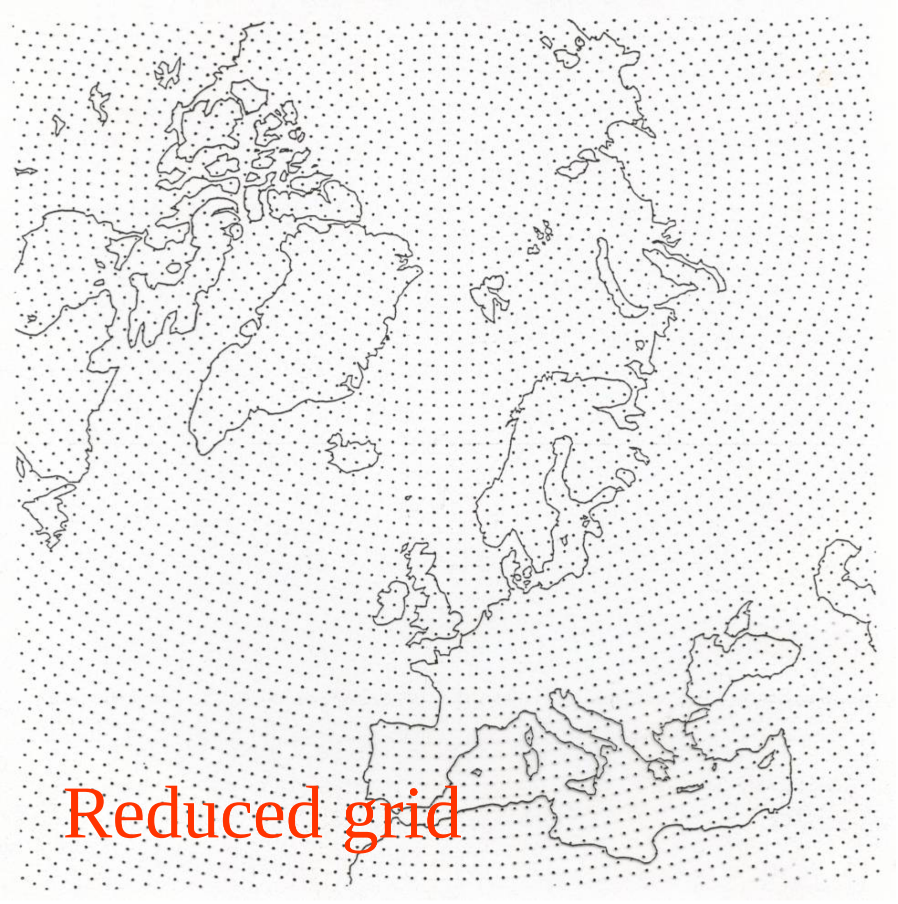
Schematic description of the spectral transform method in the ECMWF IFS model



FFT: Fast Fourier Transform, LT: Legendre Transform

The Gaussian grid

About 30% reduction in number of points



Reduction in the number of Fourier points at high latitudes is possible because the associated Legendre polynomials are very small near the poles for large *m*.

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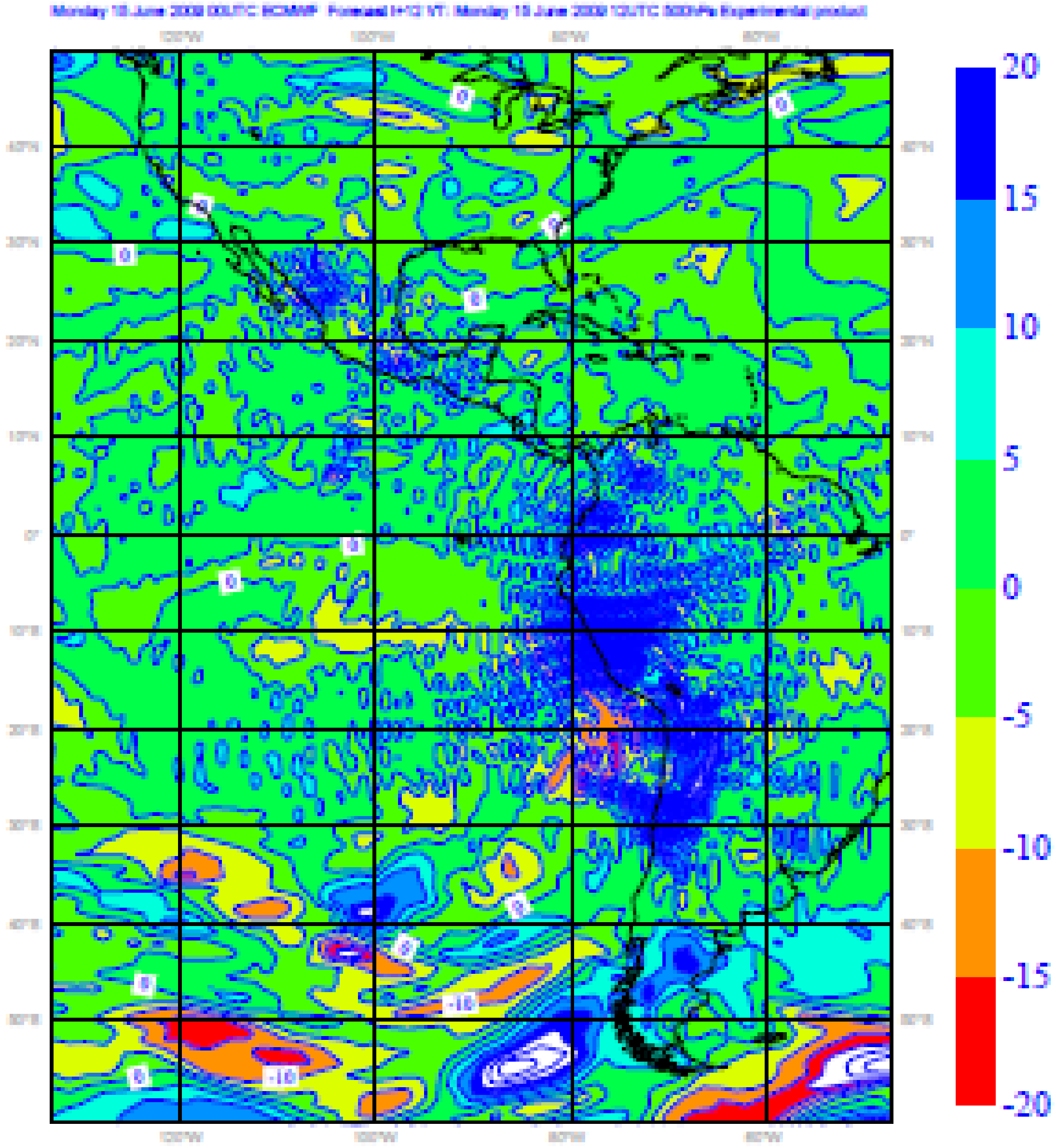
De-aliasing

- **Aliasing of quadratic terms on the linear grid (2N+1 gridpoints per N waves), where the product of two variables transformed to spectral space cannot be accurately represented with the available number of waves (as quadratic terms would need a 3N+1 ratio).**
- **Absent outside the tropics in E-W direction due to the design of the reduced grid (obeying a 3N+1 ratio) but present throughout (and all resolutions) in N-S direction.**
- **By subtracting the difference between a specially filtered and the unfiltered pressure gradient term at every time-step the stationary noise patterns can be removed at a cost of approx. 5% at T1279 (2 extra transforms).**

De-aliasing

E-W

500hPa adiabatic zonal wind tendencies (T159)



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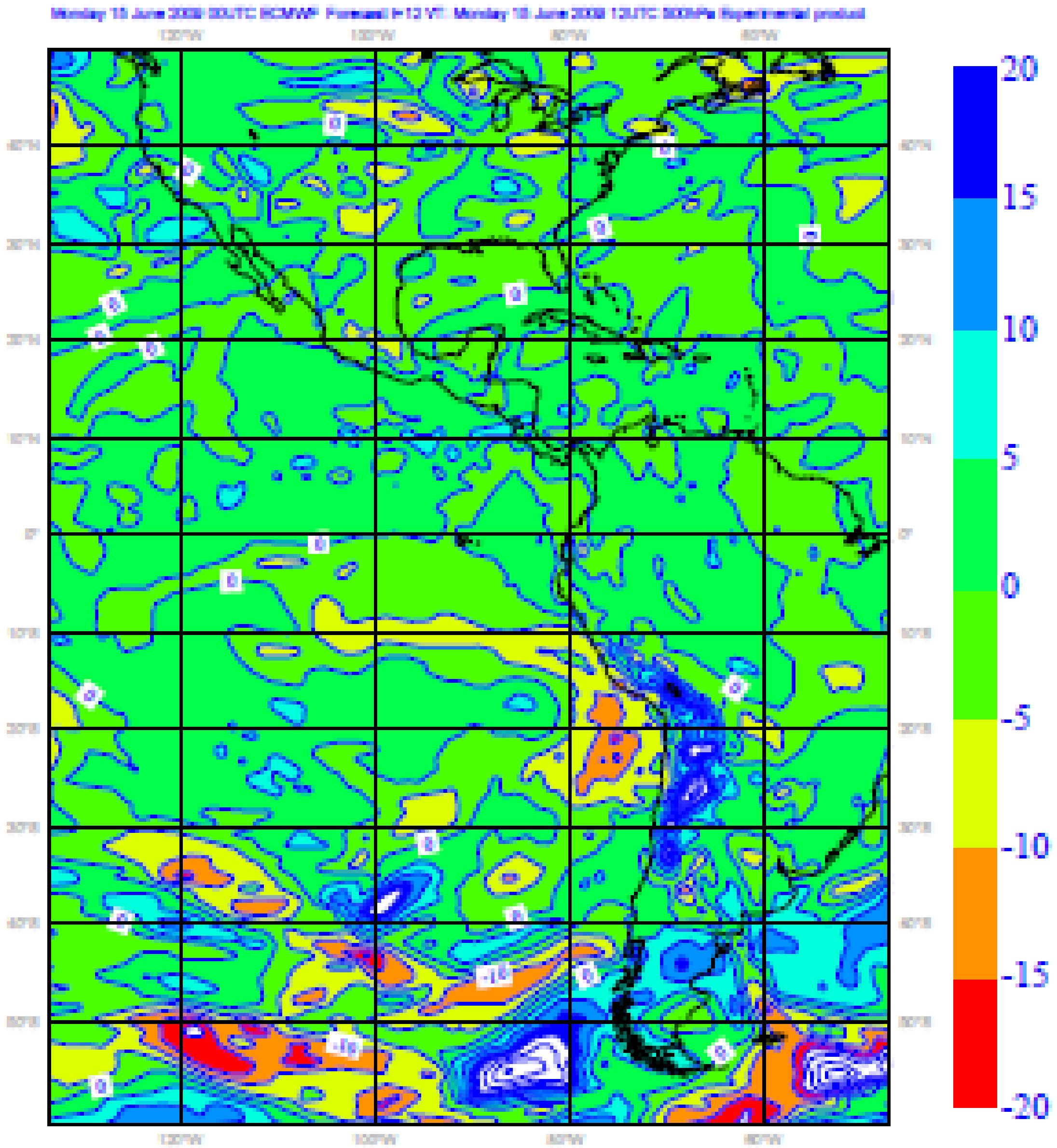
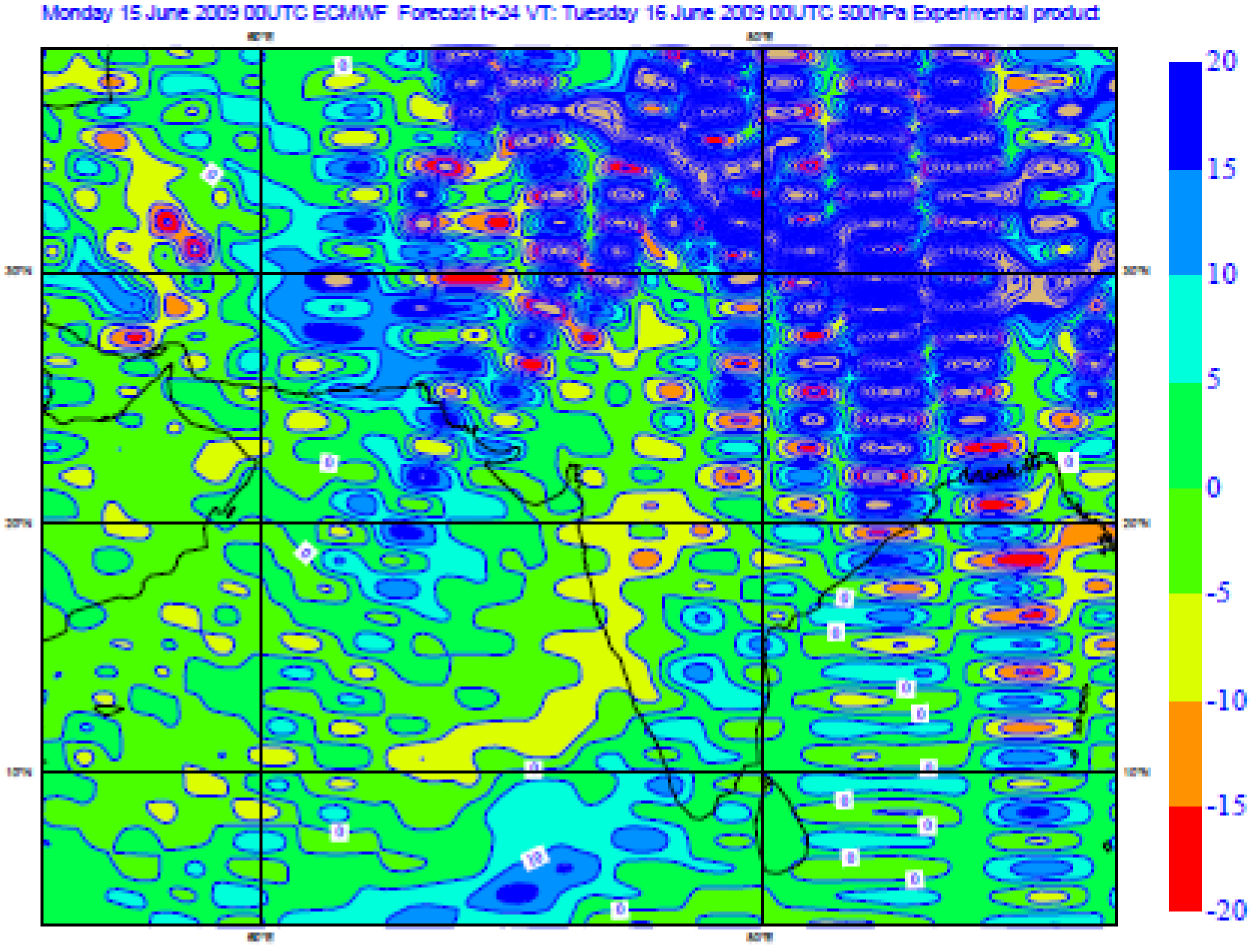
De-aliasing



De-aliasing

N-S

500hPa adiabatic meridional wind tendencies (T159)



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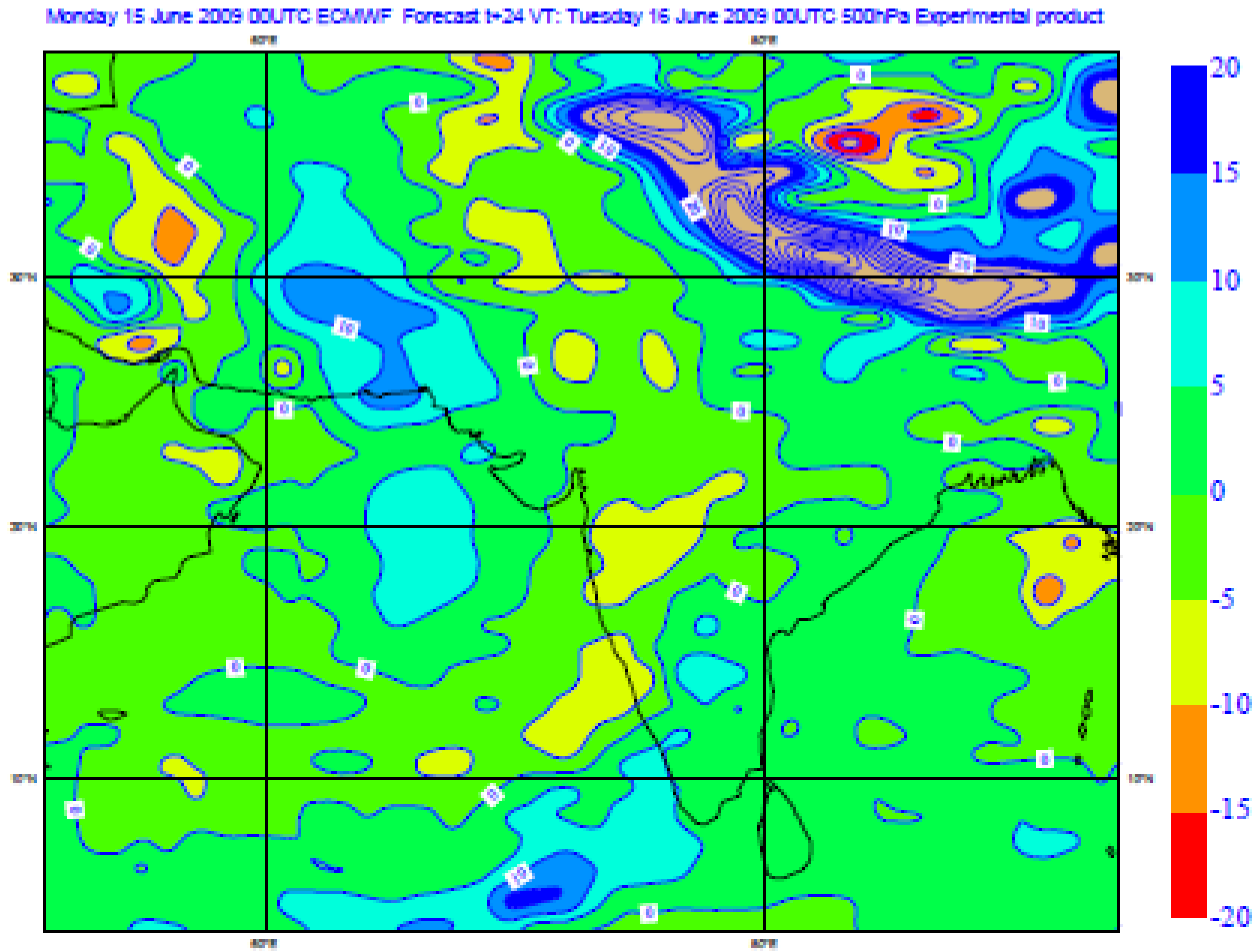
De-aliasing



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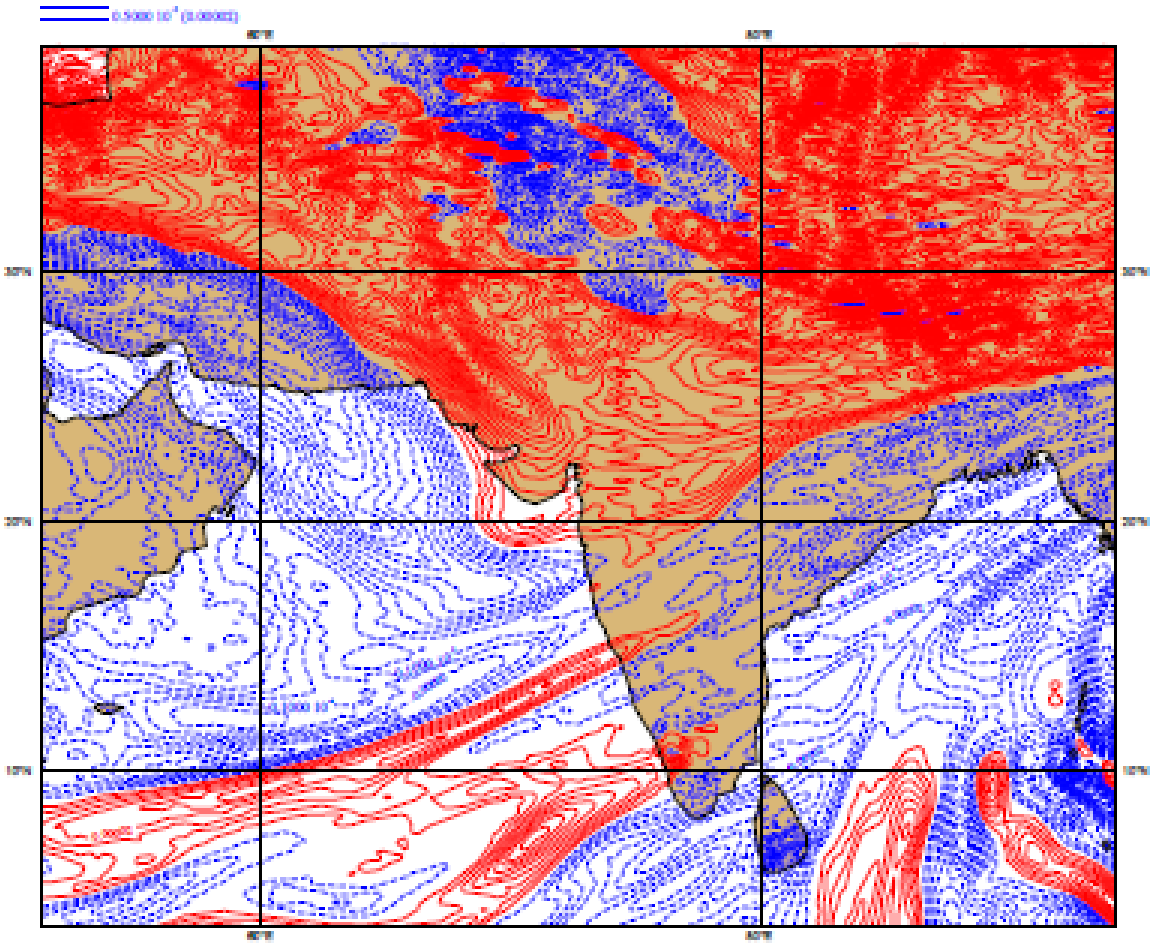
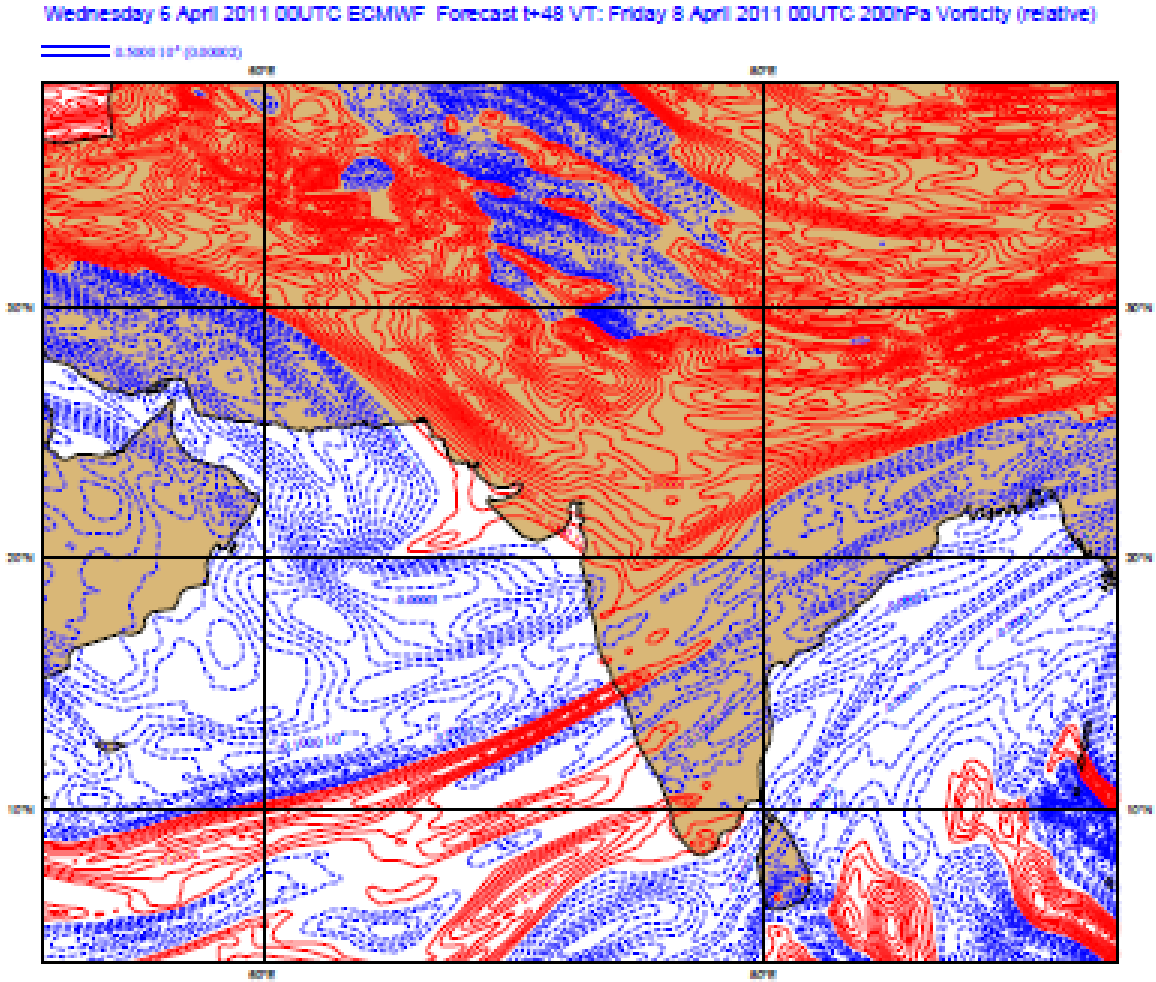
de-aliasing: vorticity noise

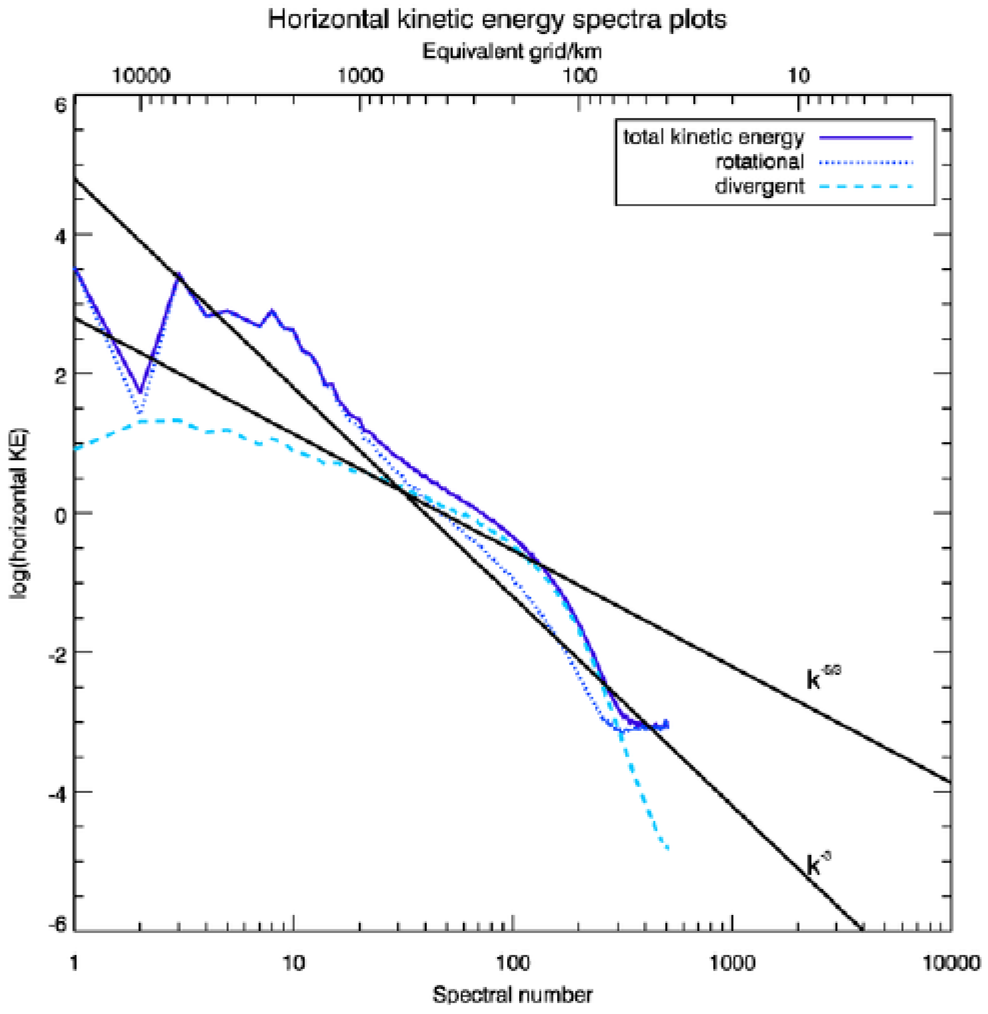
Wednesday 6 April 2011 00UTC ECMWF Forecast t+48 VT: Friday 8 April 2011 00UTC 200hPa Vorticity (relative)



De-aliasing: vorticity noise



Kinetic Energy Spectra – 100 hPa



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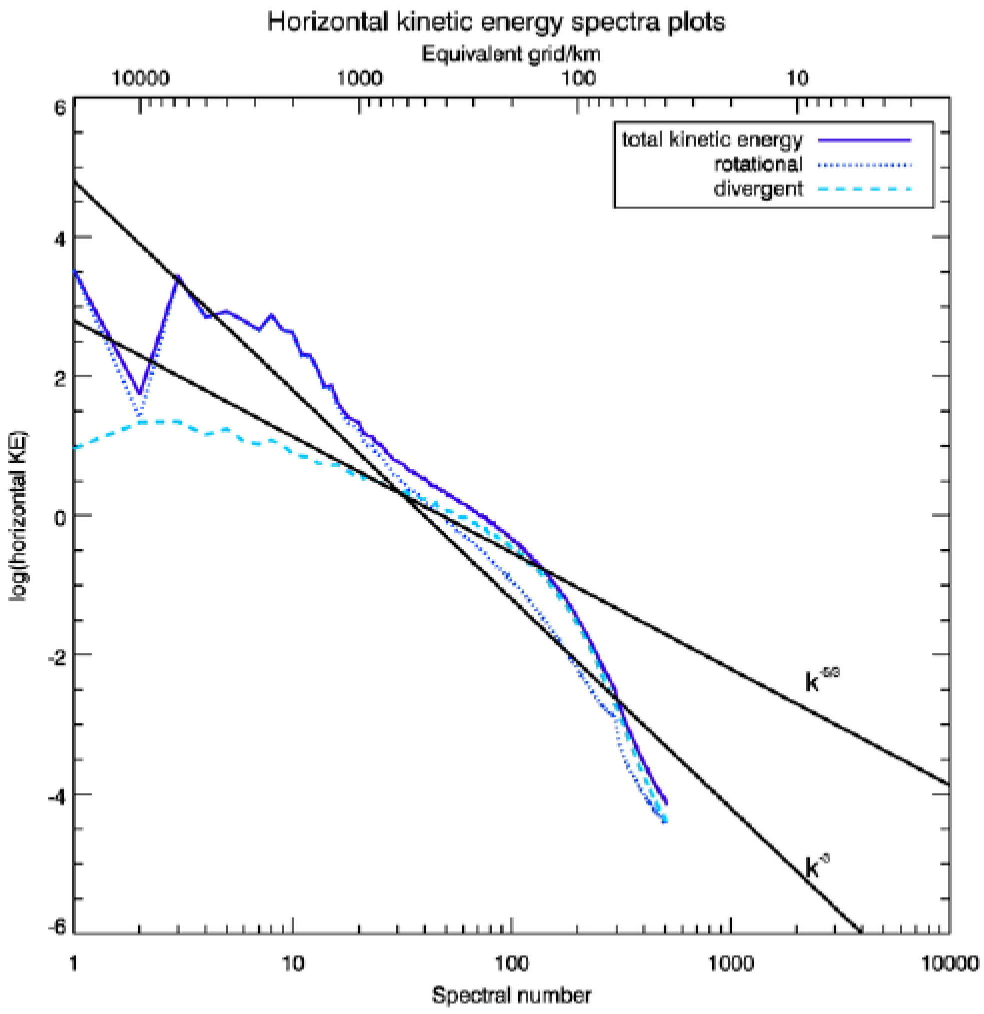
Kinetic Energy Spectra – 100 hPa



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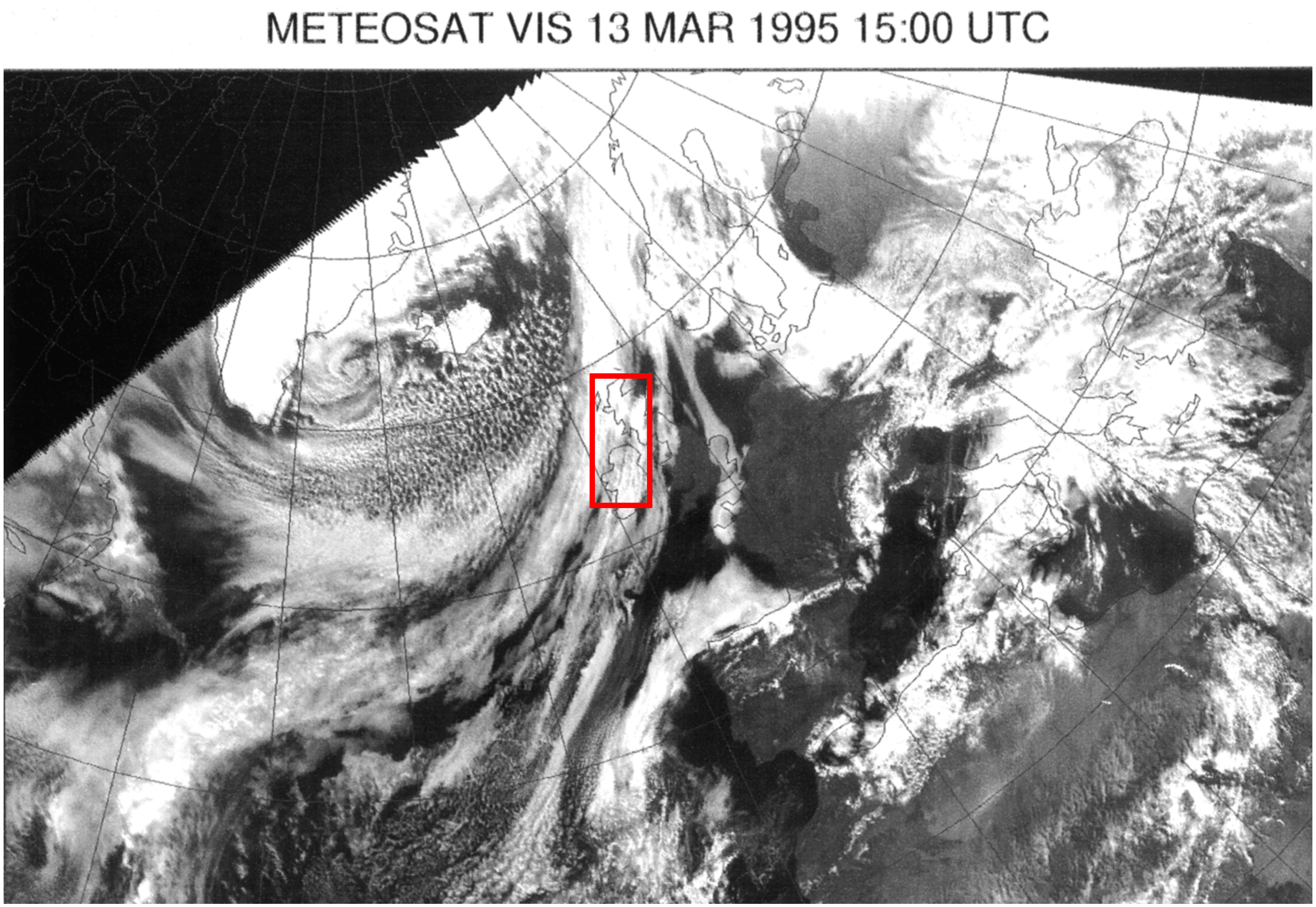
Horizontal Diffusion

- **Four reasons for horizontal diffusion:**
  - **“Sponge” for vertically propagating gravity waves**
  - **Dampen the accumulation of KE and enstrophy at the smallest resolved scales**
  - **Represent unresolved subgrid-scale mixing**
  - **Make tangent-linear and non-linear evolution more similar (relevant for variational data-assimilation)**
- **Given the removal of aliasing, horizontal diffusion can now be reduced or alternative schemes applied to represent unresolved subgrid-scale mixing**
  - **=> Non-linear horizontal diffusion**

Satellite picture of N. Atlantic



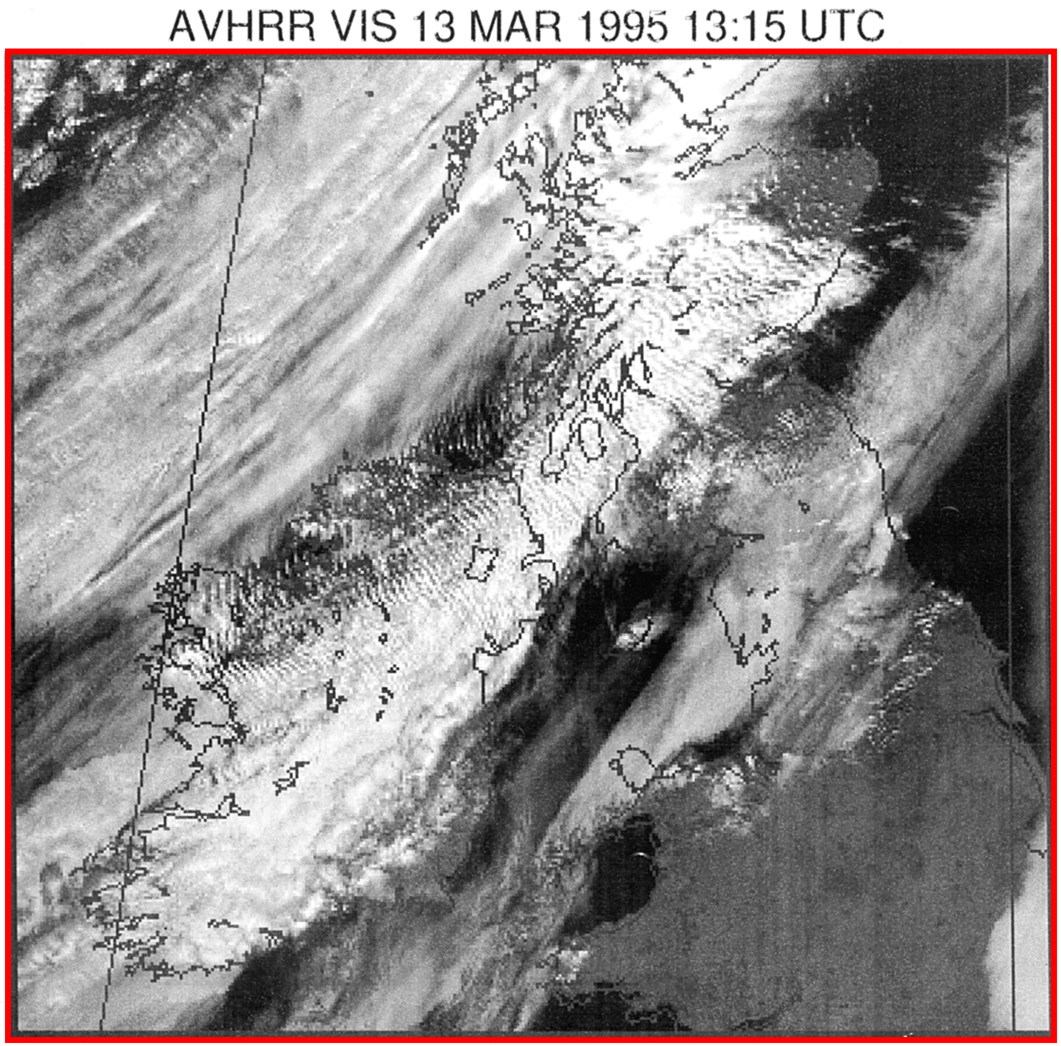
Large-Scale:

Frontal systems on scale of N.Atlantic.

Cellular convection S. of Iceland.

2d vortex shedding S. of Greenland.

Enlarged picture over U.K.



More detail …
Gravity-wave train running SW-NE over UK.

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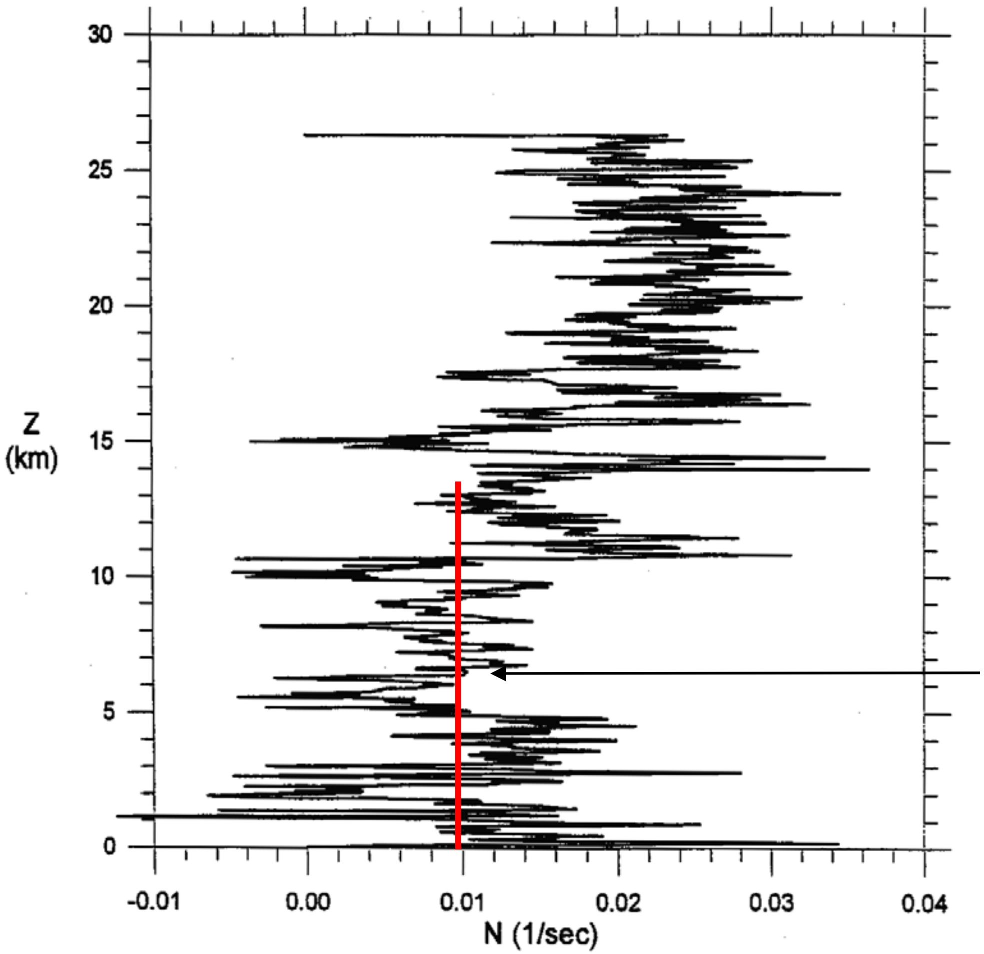
Balloon measurements of static stability



$$S = \frac{N^2}{g} = \frac{1}{\theta}\frac{\partial \theta}{\partial z}$$

Static stability S
Brunt-Väisällä frequency N
Potential temperature θ

Idealized reference or initial state often assumed in idealized simulations

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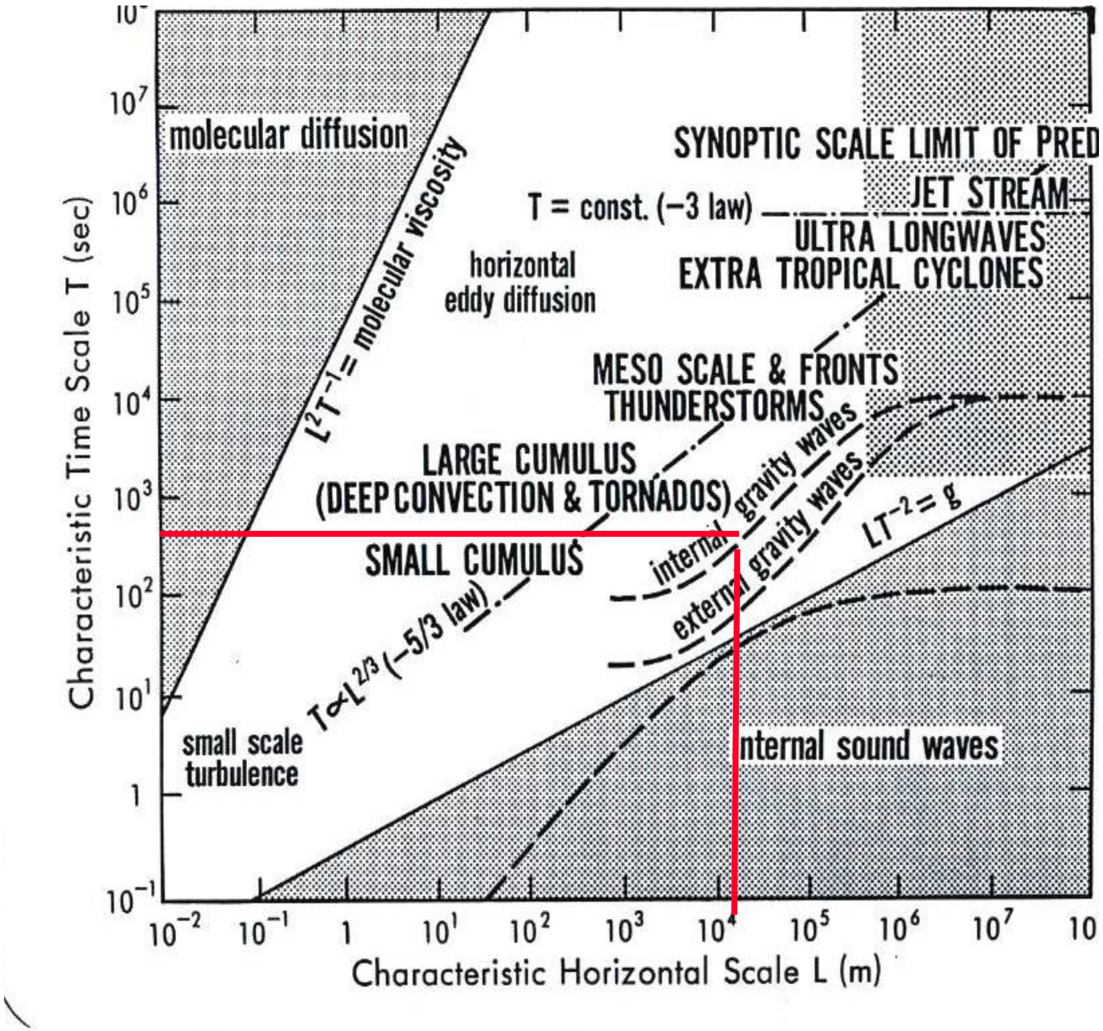
Scales of atmospheric phenomena



Practical *averaging* scales do not necessarily correspond to a physical scale separation.

If equations are averaged, there may be strong interactions between resolved and unresolved scales.

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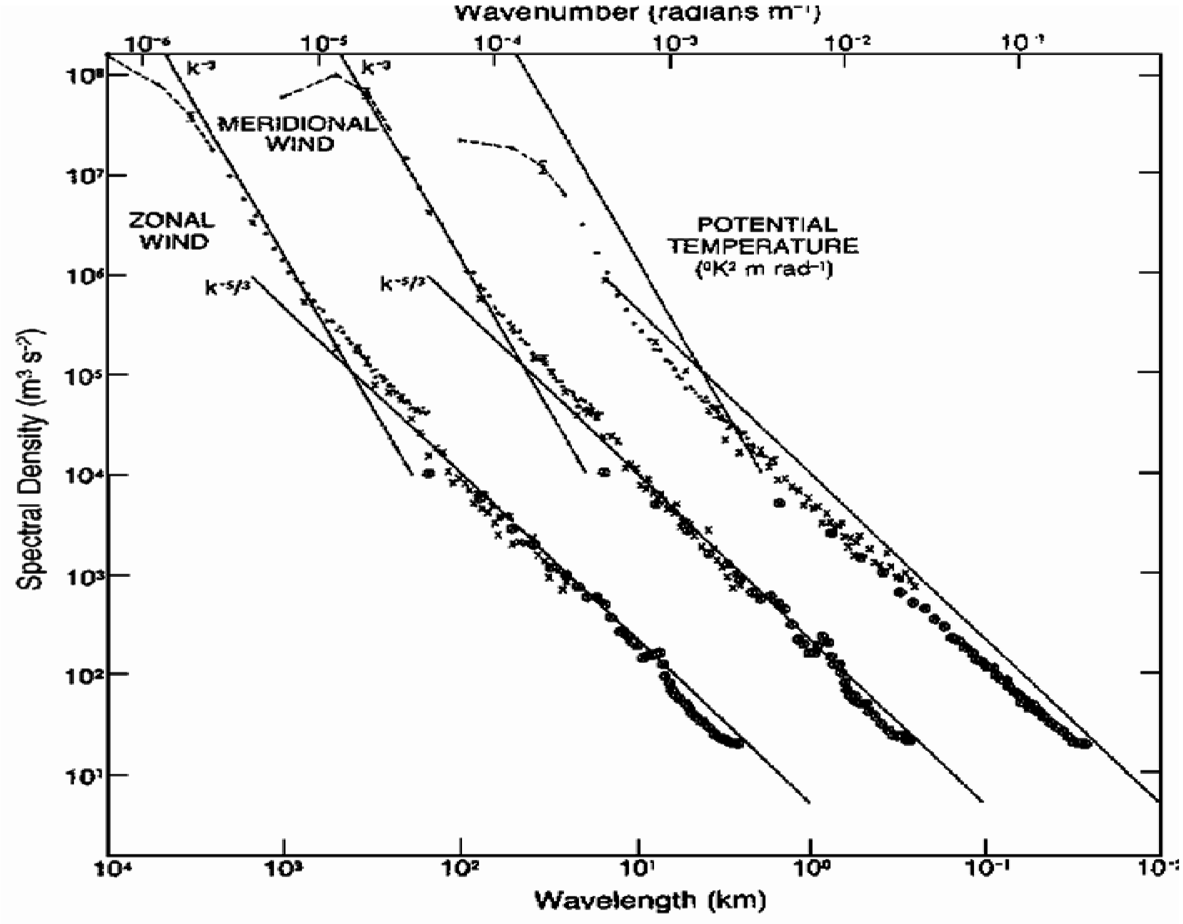
“Averaged” equations

- **The equations as used in an operational NWP model represent the evolution of a space-time average of the true solution.**
- **The *sub-grid* model represents the effect of the unresolved scales on the averaged flow expressed in terms of the resolved flow variables which represent an averaged state.**

(Aircraft) observed spectra of motions in the upper troposphere



Spectral slope near k^{-3} for wavelengths >500km. Near $k^{-5/3}$ for shorter wavelengths.

"It is remarkable that no scientific consensus yet ... whether the energy cascade through the mesoscale range is upscale or downscale. "

(Vallgren et al., 2012)

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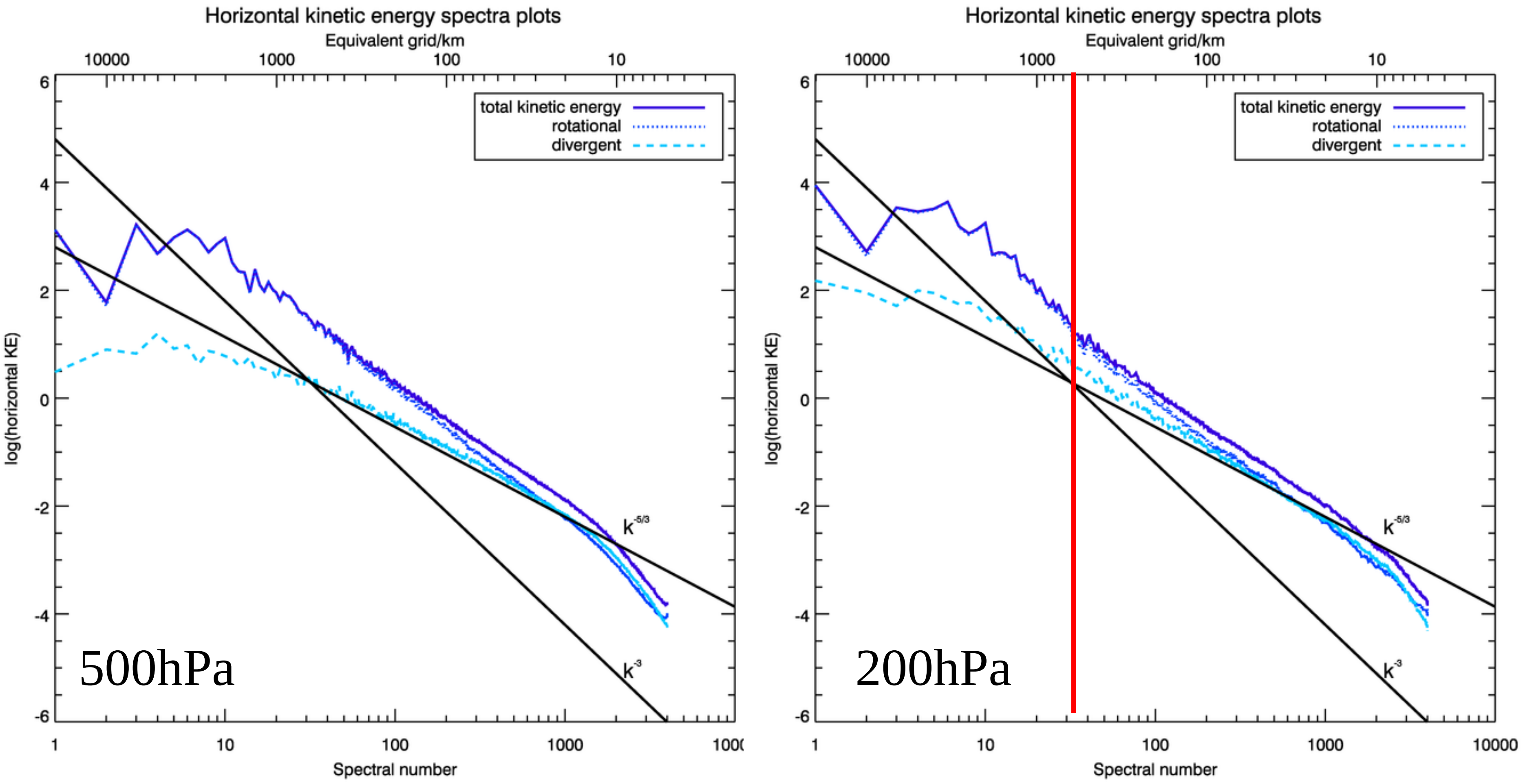
2D vs 3D turbulence

- **So far horizontal correlations are simply neglected and only the vertical correlations of subgrid-scale fluctuations are parameterized.**
- **based on scale arguments accelerations of the larger scale flow caused by eddy motions …**
  - **… in 2D turbulence *decrease* as the scale of motion *decreases* (i.e. as the model resolution *increases*)**
  - **… in 3D turbulence *increase* as the scale of motion decreases (i.e. as the model resolution *increases*)**
- **So as long as the resolved flow regime behaves like 2D turbulence the neglect of horizontal eddy fluxes in NWP is okay …** *(Tennekes , 1978)*

rotational/divergent energy of T3999 10 day fc



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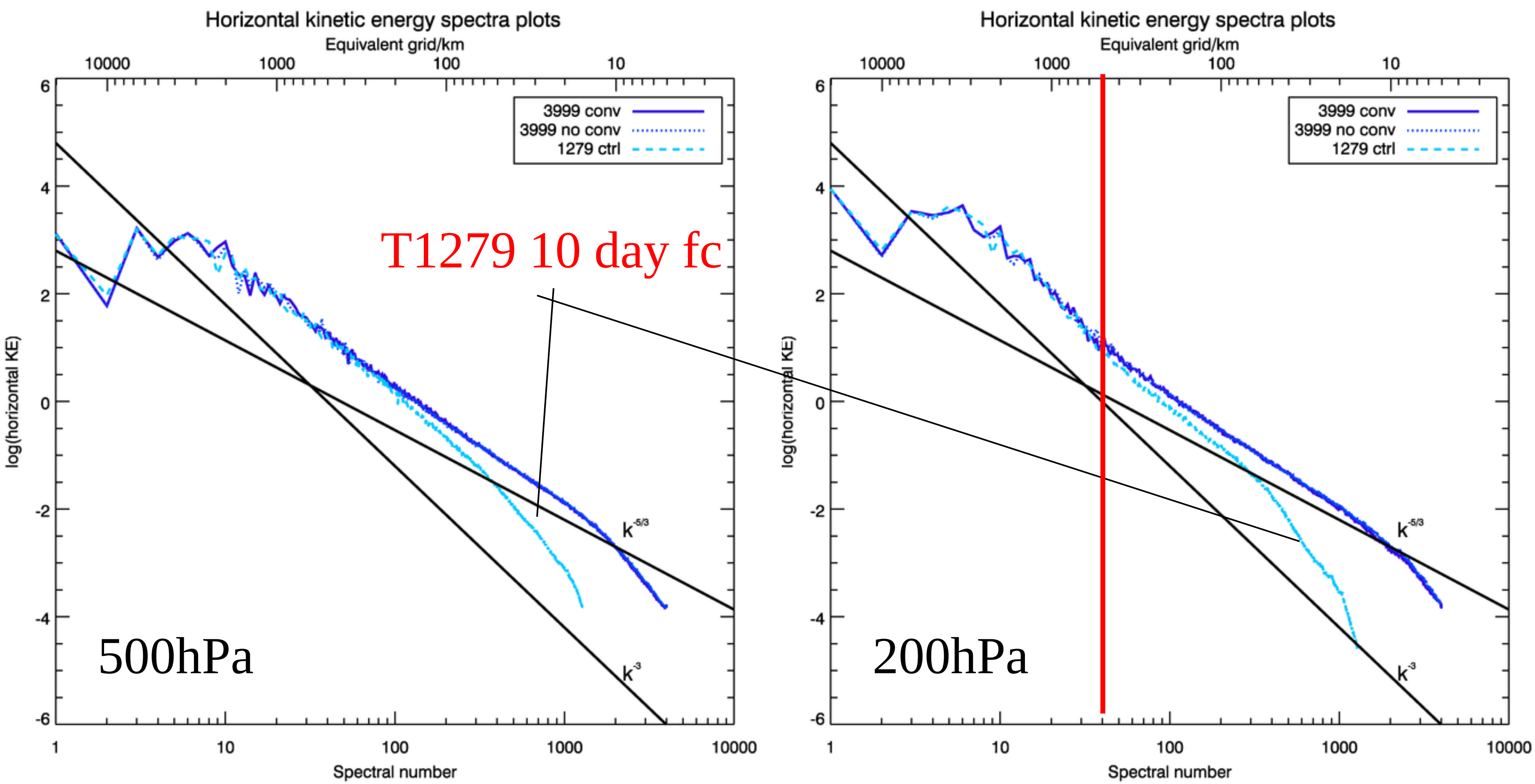
Kinetic Energy of T3999 10 day fc



Attempt to apply LES techniques to global NWP

- **Take into account horizontal and vertical non-linear interactions of scales that are still represented on the grid.**
- **Utilize similarity of spectral slope over a wide range of wavenumbers while assuming self-similar behaviour of nonlinear interactions at the smallest resolved scales and at the largest unresolved (subgrid) scales.**

Two parts to the SGS stress tensor

Domaradzki and Adams (2002)

The result of non-linear interactions of scales actually represented on the numerical grid

truly subgrid-scale

$$\frac{d}{dt}\overline{v}_i = \ldots - \frac{\partial}{\partial x_j}\tau_{ij}^{rep} - \frac{\partial}{\partial x_j}\tau_{ij}^{nrp},$$

$$\frac{d}{dt}\overline{T} = \ldots - \frac{\partial}{\partial x_j}h_i^{rep} - \frac{\partial}{\partial x_j}h_i^{nrp}$$

Truly subgrid-scale

== Model with spectral viscosity in IFS

e.g. Gelb and Gleeson (2001)

In EULAG ==
ILES of MPDATA ?

$$F_{\zeta_n^m}(\epsilon,\hat{q}) = -\epsilon\hat{q}^2\left(\frac{n^2(n+1)^2}{a^4}\right)\zeta_n^m$$

$$\hat{q} = \begin{cases} 0 & n \leq n_c \ , \\ \exp\left(-(n-N)^2/(n-n_c)^2\right) & n_c < n \leq N \ , \end{cases}$$

$$\epsilon = 2a^3/N^3 \quad \text{and} \quad n_c = N/2,$$

SGS Stress tensor - similarity model

$$\tau_{ij}^{rep} = \overline{v_i \; v_j} - \overline{v}_i \; \overline{v}_j,$$

$$\tau_{ij}(x) = \int v_i(x')v_j(x')G_\Delta(k,l,x-x')dx' - \overline{v}_i(x)\overline{v}_j(x).$$

$$\overline{v}_i(x) = \int v_i(x')G_\Delta(k,l,x-x')dx'.$$

De-convolution operation to recover the “true” velocity

$$v_i(x) = \int \overline{v}_i(x')G_\Delta^{-1}(k,l,x-x')dx'$$

$$\tau_{ij}(x) = \int \overline{v}_i(x')\overline{v}_j(x')G_\Delta^{-1}(k,l,x-x')dx' - \overline{v}_i(x)\overline{v}_j(x).$$

The Gaussian filter

$$G_\Delta(k,l) = \exp\left\{-\frac{\Delta^2 n(n+1)}{24a^2}\right\}$$

e.g. $\Delta_1 = 6\Delta\lambda, \quad \Delta_2 = 8\Delta\lambda.$

Has the same shape in physical and in wave space !

SGS Stress tensor - similarity model

Based on the idea of similar behaviour between the smallest (well) resolved scales and the actual subgrid scales

$$\tau_{ij}^{rep} = C_L L_{ij}$$

Secondary filter

$$L_{ij} = \widetilde{\overline{v}_i \, \overline{v}_j} - \widetilde{\overline{v}_i} \, \widetilde{\overline{v}_j},$$

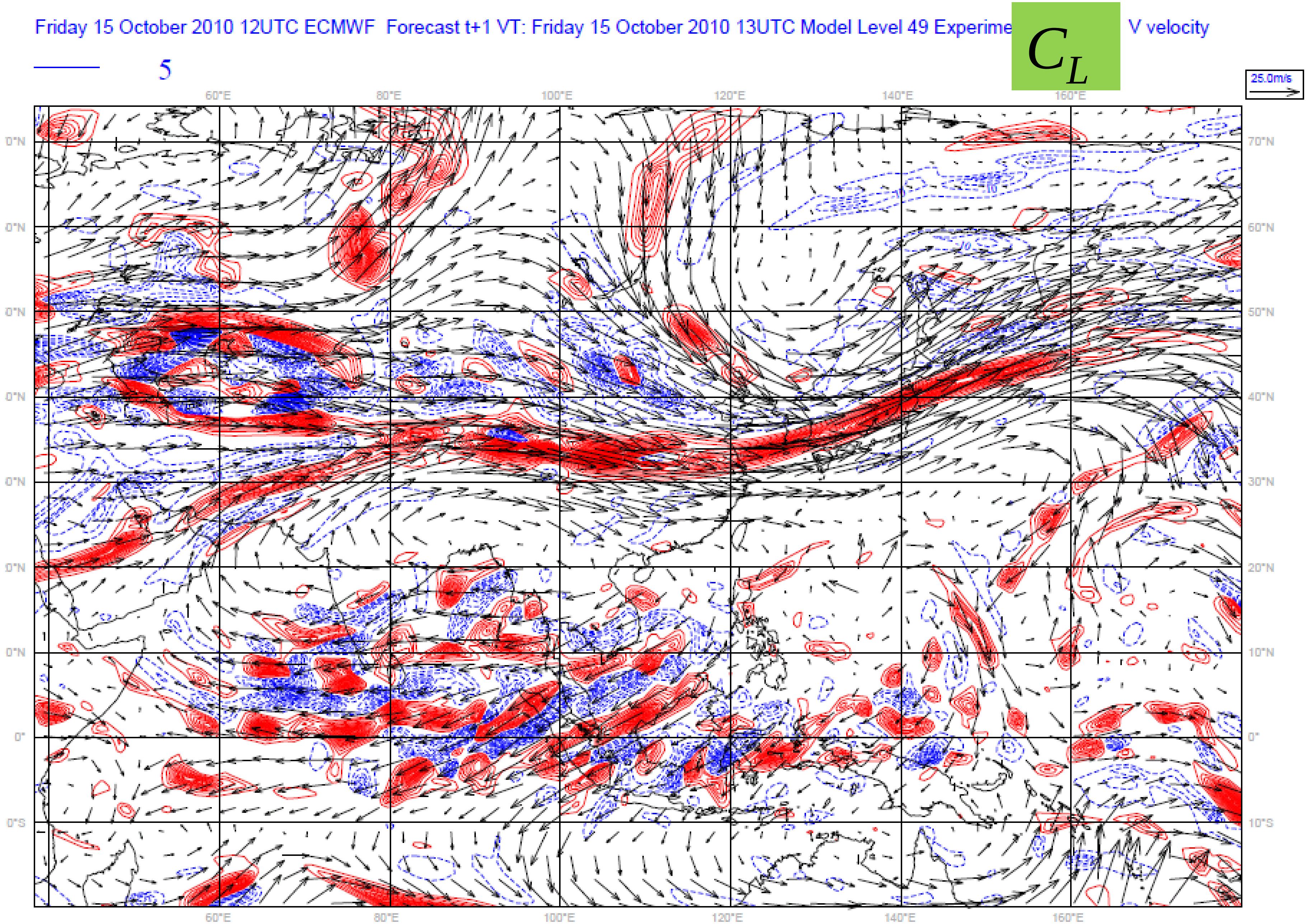
Minimize error norm (Lilly 1991)

$$f(C_L) = ||Q||_F^2 = \sum_{i,j} (\tau_{ij} - C_L L_{ij})^2.$$

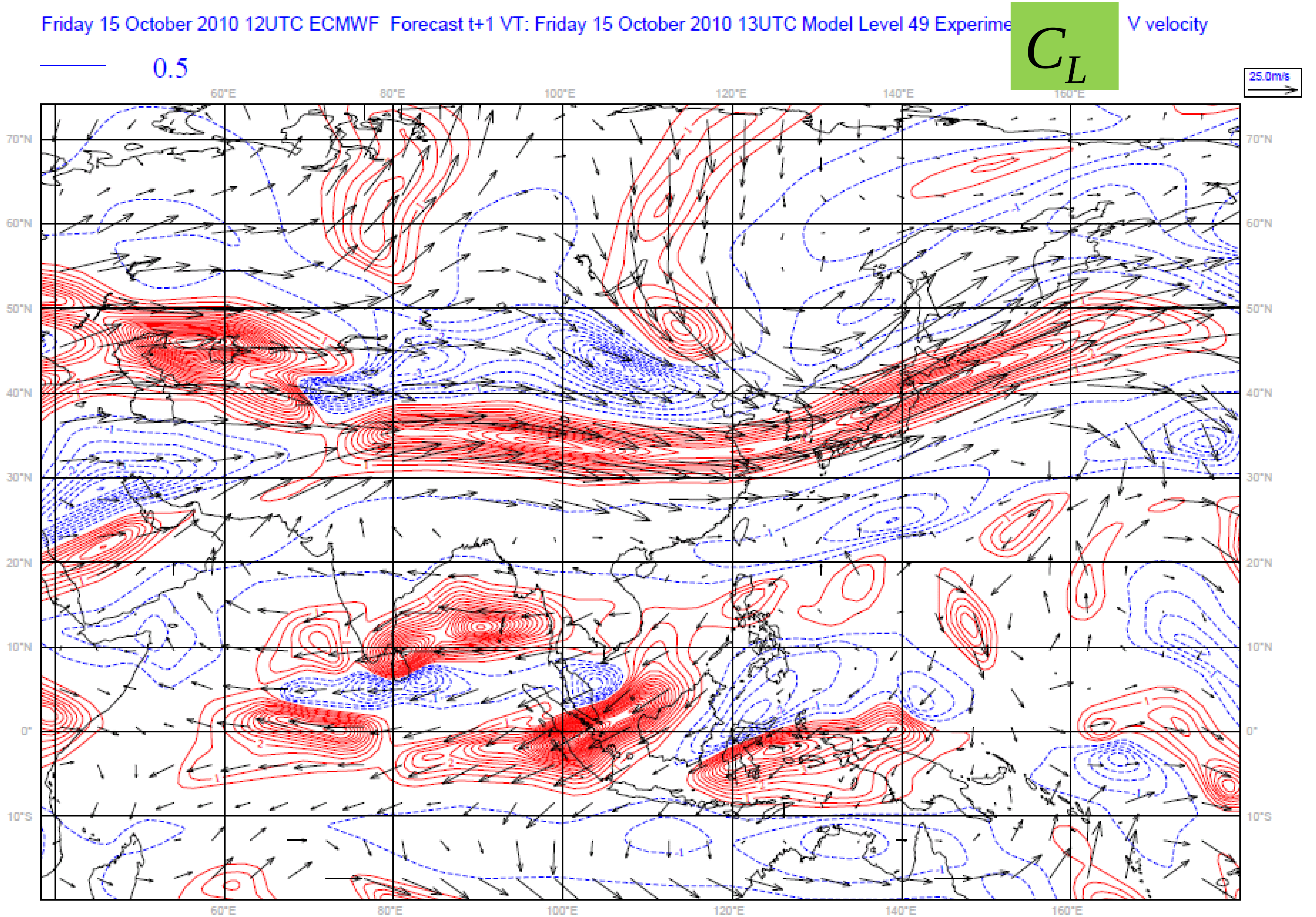
$$C_L = \frac{\sum_{i,j} \tau_{ij} L_{ij}}{\sum_{i,j} L_{ij}^2}.$$

$$\sum_{i,j} \tau_{ij} L_{ij} = \sum_i \tau_{ii} L_{ii} + 2 \sum_{i<j} \tau_{ij} L_{ij}.$$

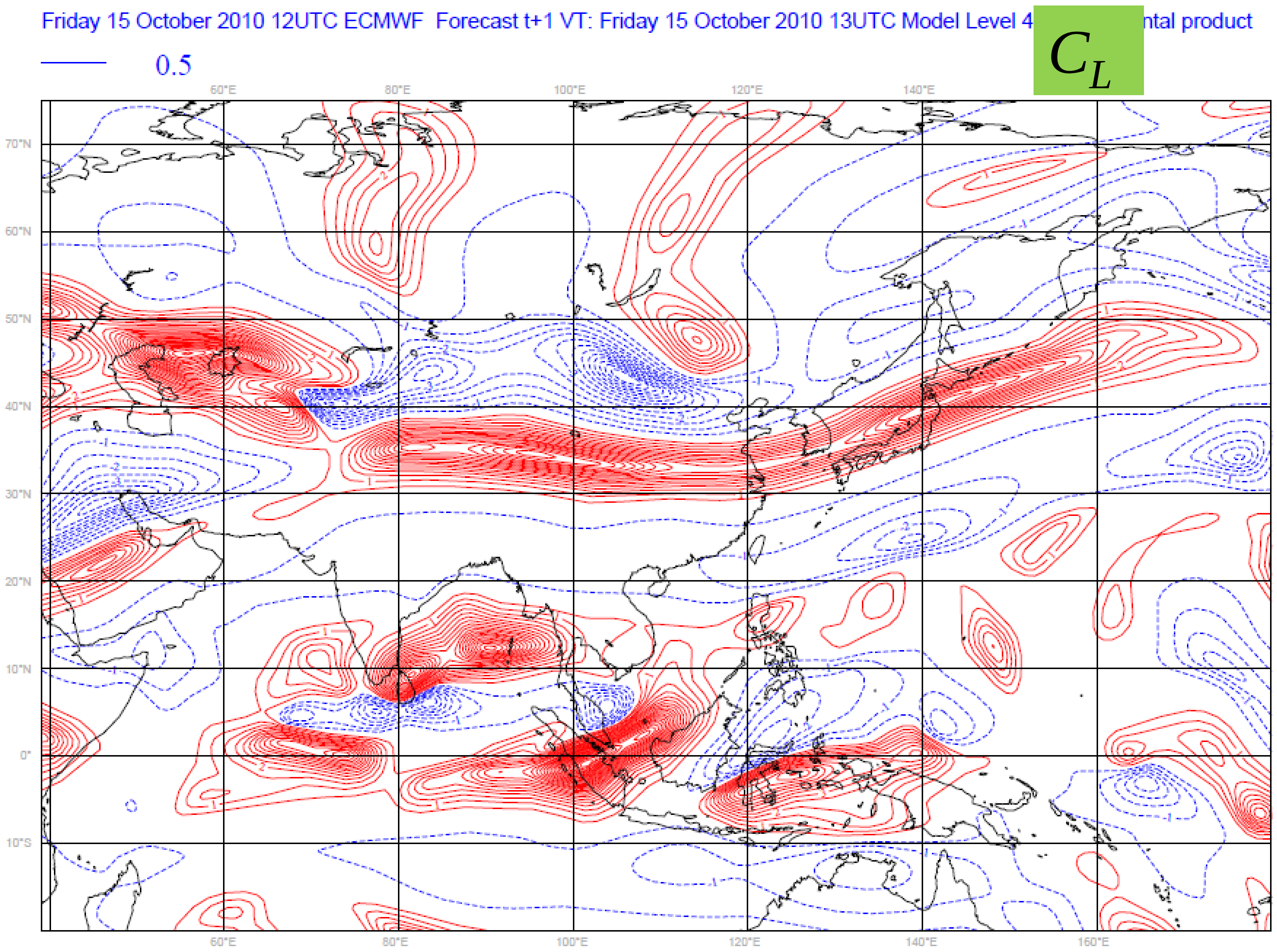
$$\sum_{i,j} L_{ij}^2 = \sum_i L_{ii}^2 + 2 \sum_{i<j} L_{ij}^2$$



T511 simulation with 6,8 dx filters ~ 240km



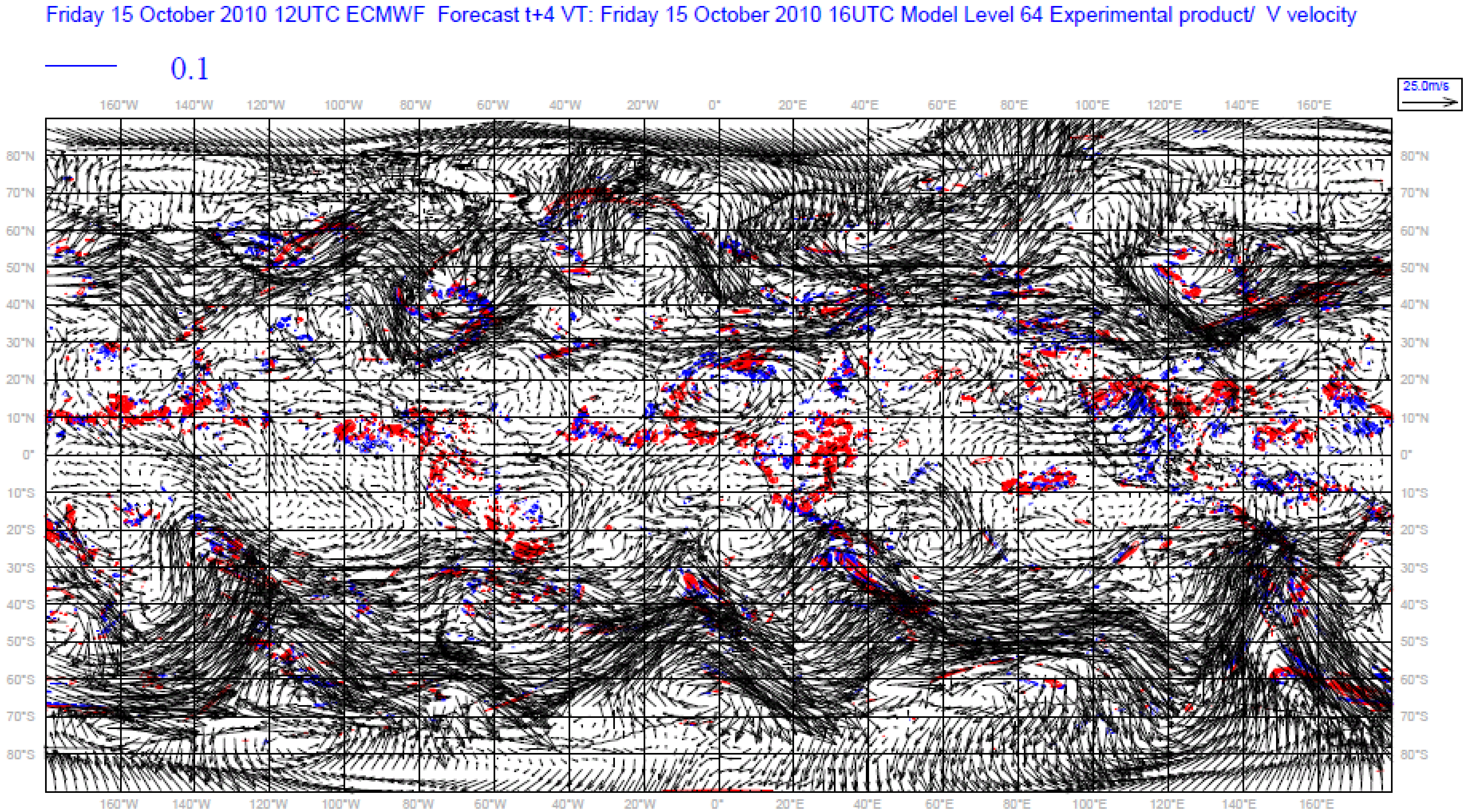
T159 simulation with 6,8 dx filters ~750km



T511 simulation with 19,25 dx filters ~750km

Total physics tendency Δu

T511 simulation



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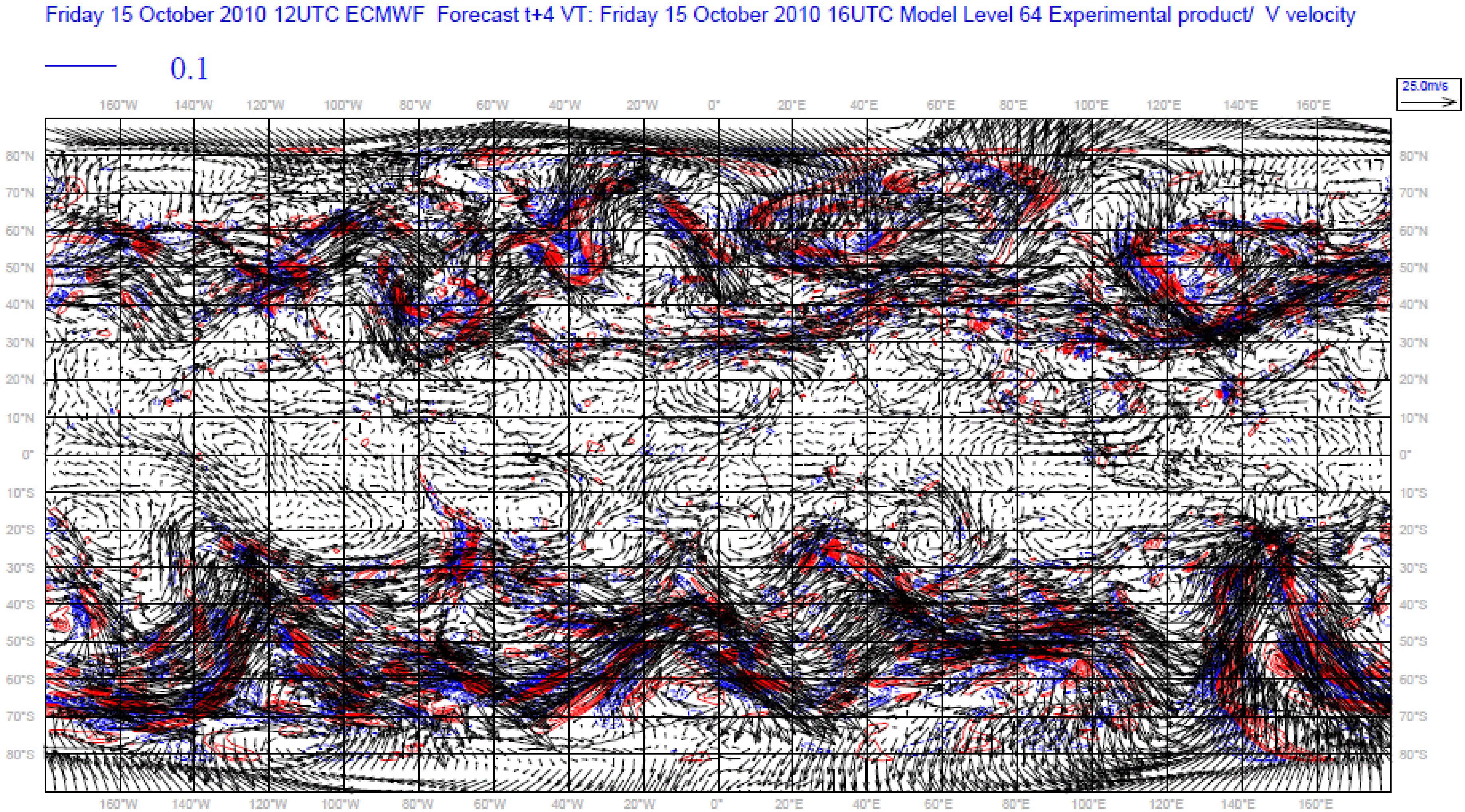
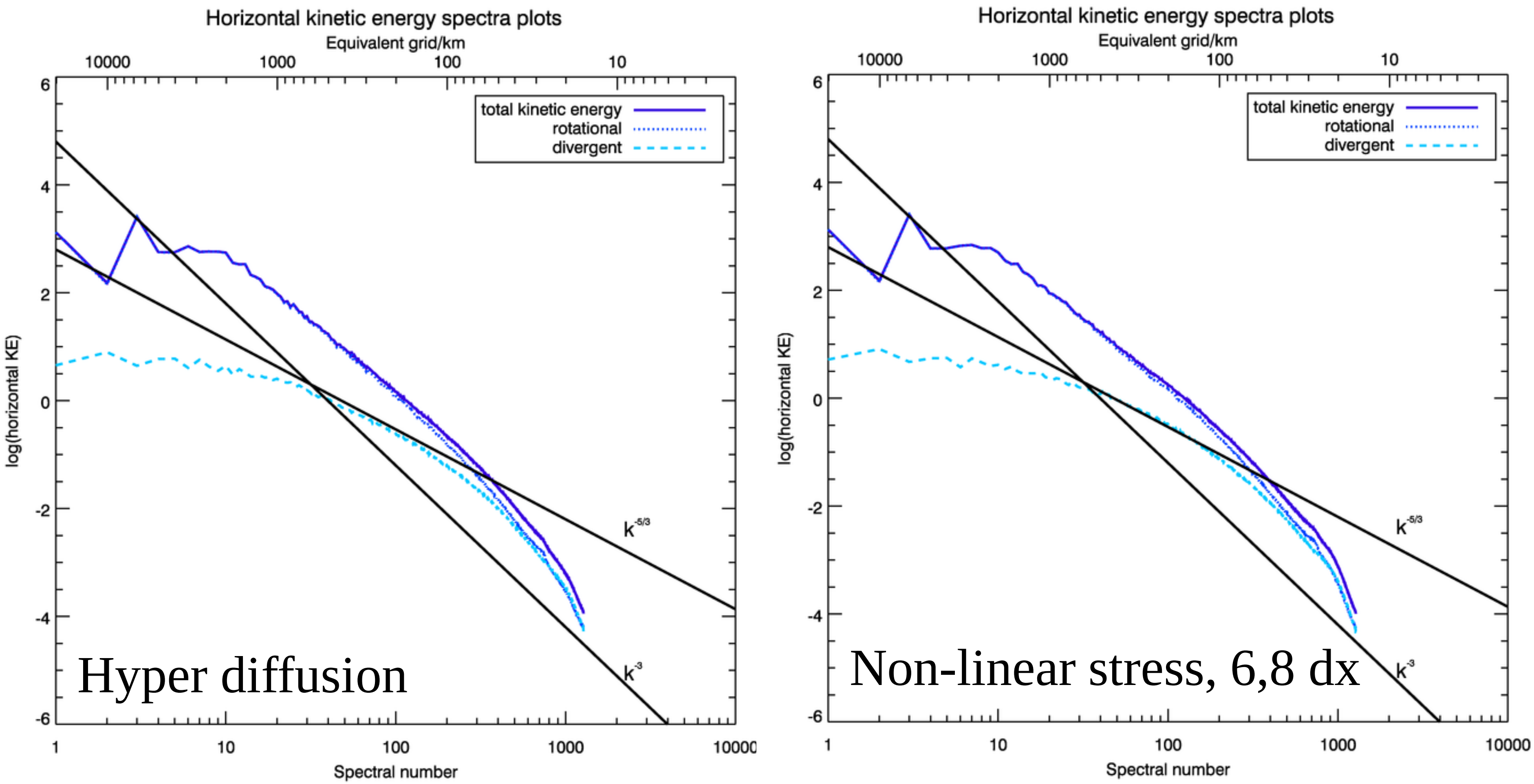
New stress Δu

T511 simulation



Spectra T1279



But no change in the spectra ...

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SGS Stress tensor – similarity model

- **The turbulent stress clearly not "random" and complimentary to physical parameterizations**
- **Similarity constant appears to be a property of the flow, quite independent of the model resolution while it's value is influenced by the chosen physical length-scale of the flow.**
- **Similarity constant naturally defines (dynamically consistent) *backscatter* (positive and negative C_L) that may be used in generating EPS perturbations.**

Summary

- De-aliasing improved mass conservation by 50 %, improved energy spectra and removed unphysical noise "seen" (and compensated in part) by the physical parametrizations.
- Potential for weaker and/or alternative schemes for horizontal (hyper-)diffusion
- Potential for the use of a dynamic similarity model to compute non-linear velocity correlations, in particular in the context of ensemble prediction systems
- More work required to assess the impact of different filter choices on the resulting kinetic energy spectra

Additional slides

The final additional contributions to the IFS equations

$$\frac{du}{dt} = \ldots - \left[\nabla_\eta \cdot \begin{pmatrix} C_L L_{11} \\ C_L L_{12} \end{pmatrix} + \frac{1}{RT} \nabla_\eta \phi \cdot \pi \frac{\partial}{\partial \pi} \begin{pmatrix} C_L L_{11} \\ C_L L_{12} \end{pmatrix} - \frac{g}{RT} \pi \frac{\partial}{\partial \pi} C_L L_{13} \right],$$

$$\frac{dv}{dt} = \ldots - \left[\nabla_\eta \cdot \begin{pmatrix} C_L L_{12} \\ C_L L_{22} \end{pmatrix} + \frac{1}{RT} \nabla_\eta \phi \cdot \pi \frac{\partial}{\partial \pi} \begin{pmatrix} C_L L_{21} \\ C_L L_{22} \end{pmatrix} - \frac{g}{RT} \pi \frac{\partial}{\partial \pi} C_L L_{23} \right],$$

$$\frac{dT}{dt} = \ldots - \left[\nabla_\eta \cdot \begin{pmatrix} C_{LH} H_1 \\ C_{LH} H_2 \end{pmatrix} + \frac{1}{RT} \nabla_\eta \phi \cdot \pi \frac{\partial}{\partial \pi} \begin{pmatrix} C_{LH} H_1 \\ C_{LH} H_2 \end{pmatrix} - \frac{g}{RT} \pi \frac{\partial}{\partial \pi} C_{LH} H_3 \right]$$

Direct spectral transform

Fourier transform:

$$\zeta_m(\theta) = \frac{1}{2\pi}\int_0^{2\pi} \zeta(\lambda,\theta)e^{-im\lambda}d\lambda$$

FFT (fast Fourier transform)
using
$N_F \geq 2N+1$
points (linear grid)
(3N+1 if quadratic grid)

Legendre transform:

$$\zeta_n^m = \frac{1}{2}\int_{-1}^{1} \zeta_m \overline{P_n^m}(cos(\theta))dcos(\theta).$$

$$\zeta_n^m = \sum_{k=1}^{K} w_k \zeta_m(x_k)\overline{P_n^m}(x_k)$$

Legendre transform
by Gaussian quadrature
using $N_L \geq (2N+1)/2$
"Gaussian" latitudes (linear grid)
((3N+1)/2 if quadratic grid)

$$w_k = \frac{2N+1}{[\overline{P_N^{m=1}}(x_k)]^2}.$$

Inverse spectral transform

(normalized) associated Legendre polynomials

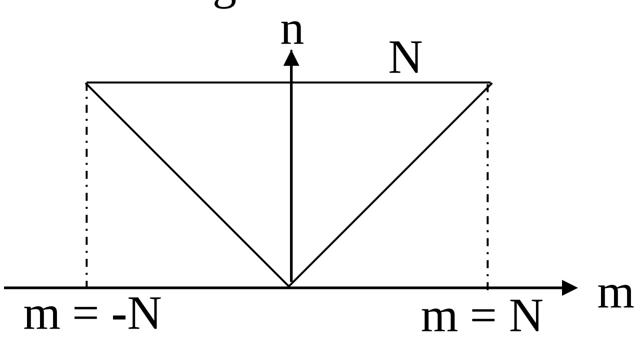
$$\zeta(\theta,\lambda) = \sum_{m=-N}^{N} e^{im\lambda} \sum_{n=|m|}^{N} \zeta_n^m \overline{P_n^m}(cos(\theta)),$$

$$\zeta(\lambda,\mu,\eta,t) = \sum_{m=-N}^{N} \sum_{n=|m|}^{N} \zeta_n^m(\eta,t) Y_n^m(\lambda,\mu)$$

Triangular truncation (isotropic)

Spherical harmonics

Triangular truncation:



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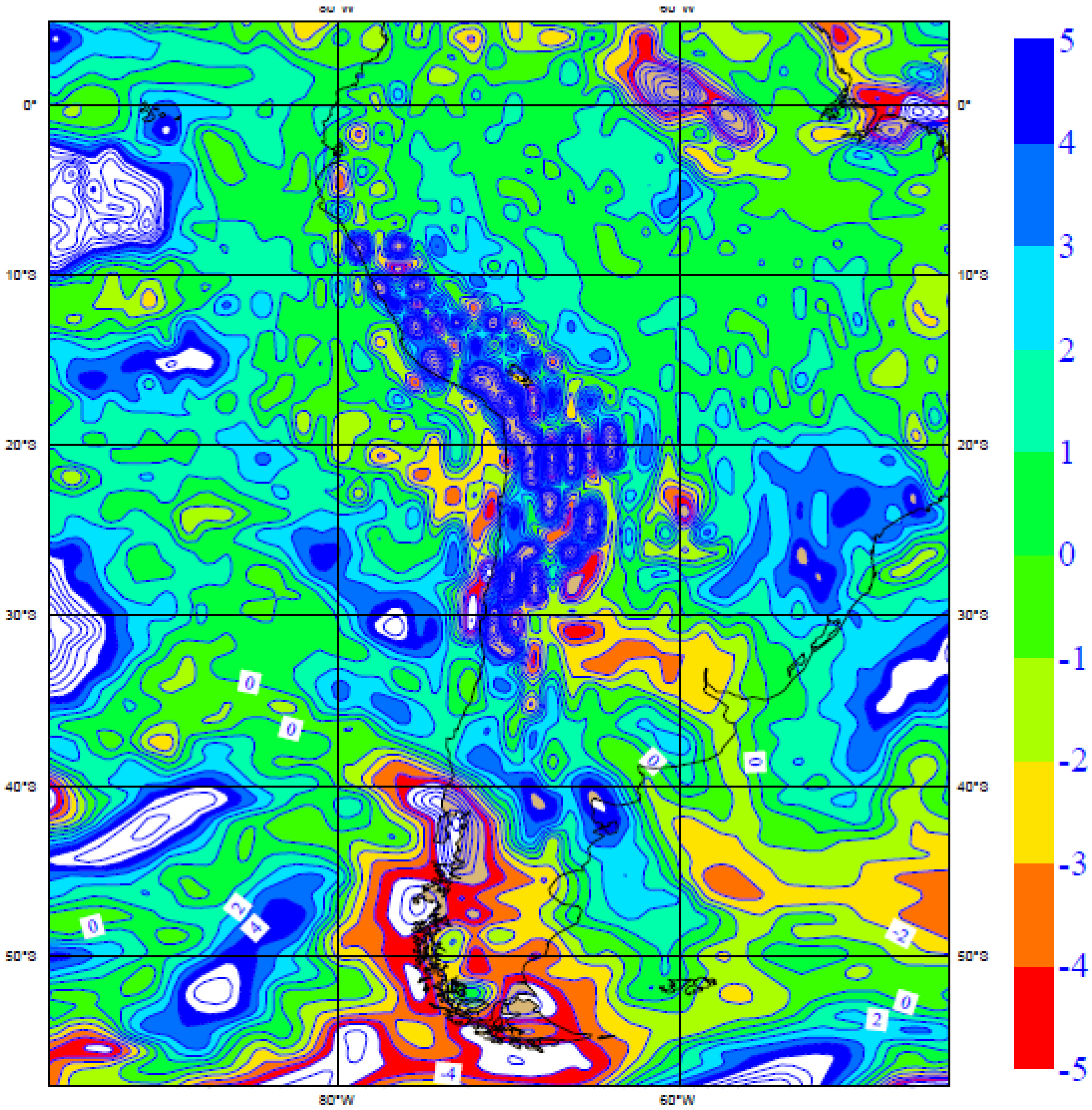
Heat

$$h_i = C_{LH} H_i.$$

$$H_i = \widetilde{\overline{v}_i \overline{T}} - \widetilde{\overline{v}_i}\;\widetilde{\overline{T}},$$

$$C_{LH} = \frac{\sum_i h_i H_i}{\sum_i H_i^2},$$

Temperature (Control)



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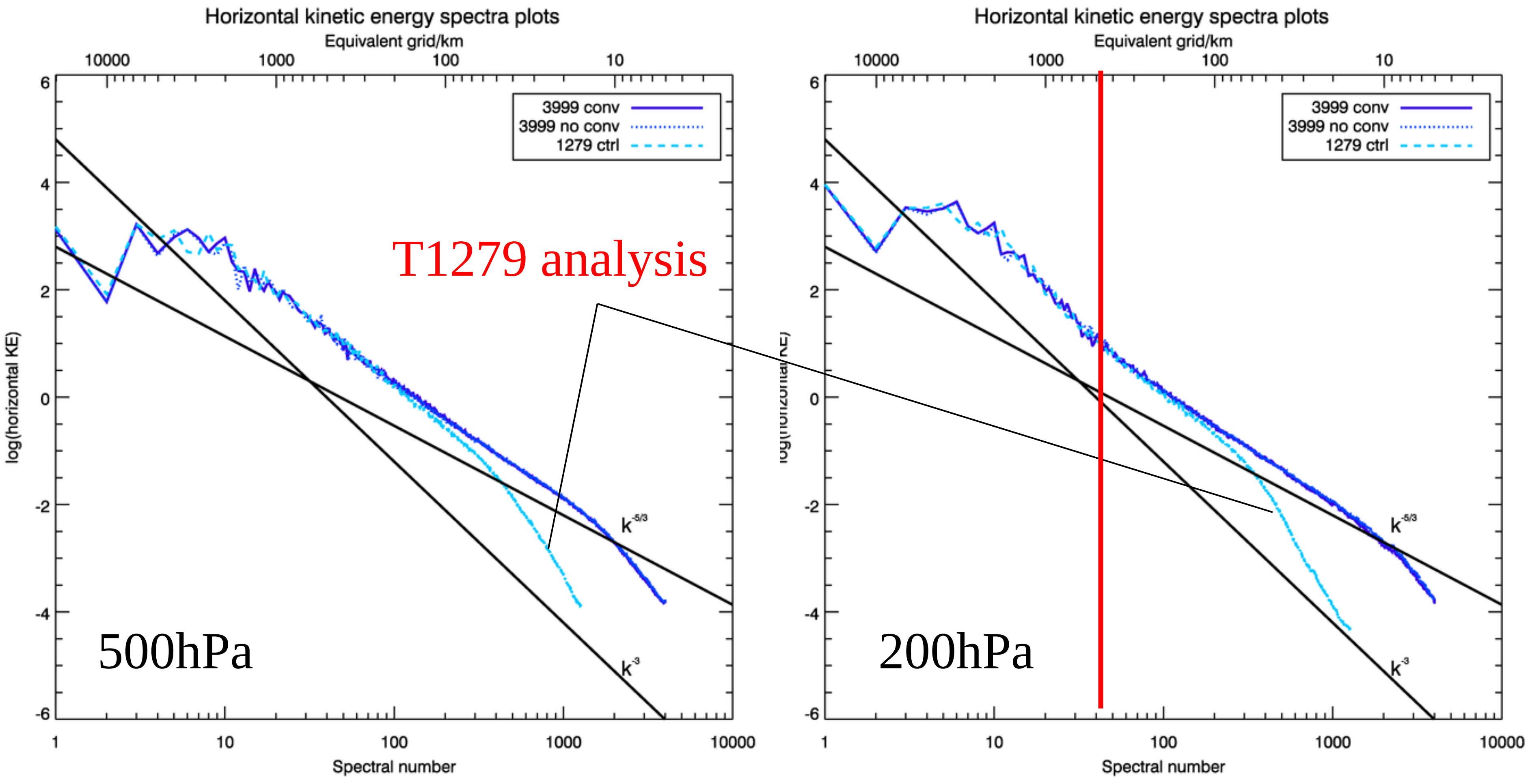
Kinetic Energy of T3999 10 day fc



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Temperature variance