A VOLUME-PRESERVING SEMI-LAGRANGIAN TRAJECTORY SCHEME

Jean-Francois Cossette[1] and Piotr K. Smolarkiewicz[2]

[1]Univserité de Montréal, Montréal, Canada

[2]National Center for Atmospheric Research
PO Box 3000, Boulder, CO, 80307, U.S.A.
e-mail: smolar@ucar.edu

ABSTRACT

Demanding the compatibility of semi-Lagrangian trajectory schemes with the fundamental Euler expansion formula leads to the Monge-Ampère (MA) nonlinear second-order partial differential equation [1]. Given standard estimates of the departure points of flow trajectories, solving the associated MA problem provides a corrected solution satisfying a discrete Lagrangian form of the mass continuity equation to round-off error. In reference [1], we studied the impact of the MA enhancement in simulations of passive tracer advection and a fully developed turbulence and showed that the MA correction can improve computational results.

Current work extends the theoretical foundations of the MA problem that were presented in [1] to encompass questions of existence and uniqueness of its solutions. In particular, closed-form solutions have been found in the contexts of elemental rotational and deformational motions. We find that a necessary condition for the existence of the trajectory correction in the case of elemental rotation is the ellipticity of the MA equation and revisit the experiments of [1] to put this condition to test inside arbitrary chaotic flows.
New applications include the gravitational collapse of material surfaces in a stratified flow and the formation of current sheets in nonlinear MHD.

[1] Cossette and Smolarkiewicz, *Comput. Fluids*, **46 (**2011) 180-185