NFT INTEGRATION ON UNSTRUCTURED MESHES: FLOWS PAST OBSTACLES

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Lipps-Hemler anelastic system

Mass, momentum and entropy conservation laws:

$$\nabla \cdot (\mathbf{V}\rho_o) = 0$$

$$\frac{\partial \rho_o V_I}{\partial t} + \nabla \cdot (\mathbf{V}\rho_o V_I) = -\rho_o \frac{\partial \varphi}{\partial x_I} + \rho_o g \frac{\theta'}{\theta_o}\delta_{I3} + (\nabla \cdot \boldsymbol{\tau})_I$$

$$\frac{\partial \rho_o \theta'}{\partial t} + \nabla \cdot (\mathbf{V}\rho_o \theta') = -\mathbf{V} \cdot \nabla \theta_\mathbf{e}$$

$$ds = c_p d \ln \theta$$

$$\varphi = c_p \theta_o (\pi - \pi_e) \quad \pi \equiv (p/p_o)^{R/c_p}$$

$$\theta' = \theta - \theta_e$$

$$\theta_e(x_3) = \theta_o + S_o x_3$$

Incompressible Navier-Stokes equations

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Mass and momentum conservation laws:

$$\nabla \cdot (\mathbf{V}\rho_o) = 0$$

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Deviatoric stress tensor $\tau_{IJ} = (\mu + \mu_T)\left(\frac{\partial V_I}{\partial x_J} + \frac{\partial V_J}{\partial x_I}\right)$

$$\varphi = (p - p_e)/\rho_o$$

Dynamic eddy viscosity $\mu_T = \mu_t$ for RANS; $\mu_T = \mu_{sgs}$ for LES

DES hybrid approach for simulating turbulent incompressible flows

Particularly, μ_t is computed by Spalart-Allmaras model: $\mu_t = \rho_0 \hat{\nu} f_{v1}$

$$\frac{\partial \hat{\nu}}{\partial t} + \mathbf{V} \cdot \nabla \hat{\nu} = c_{b1}\hat{S}\hat{\nu} + \frac{1}{\sigma}\nabla \cdot (\nu + \hat{\nu})\nabla \hat{\nu} + \frac{c_{b2}}{\sigma}(\nabla \hat{\nu})^2 - c_{w1} f_w \left(\frac{\hat{\nu}}{d}\right)^2$$

NFT- Finite Volume integration

Time and space discretization

Generalized equation

$$\frac{\partial \rho_o \xi}{\partial t} + \nabla \cdot (\boldsymbol{\mathcal{V}} \rho_o \xi) = \rho_o R$$

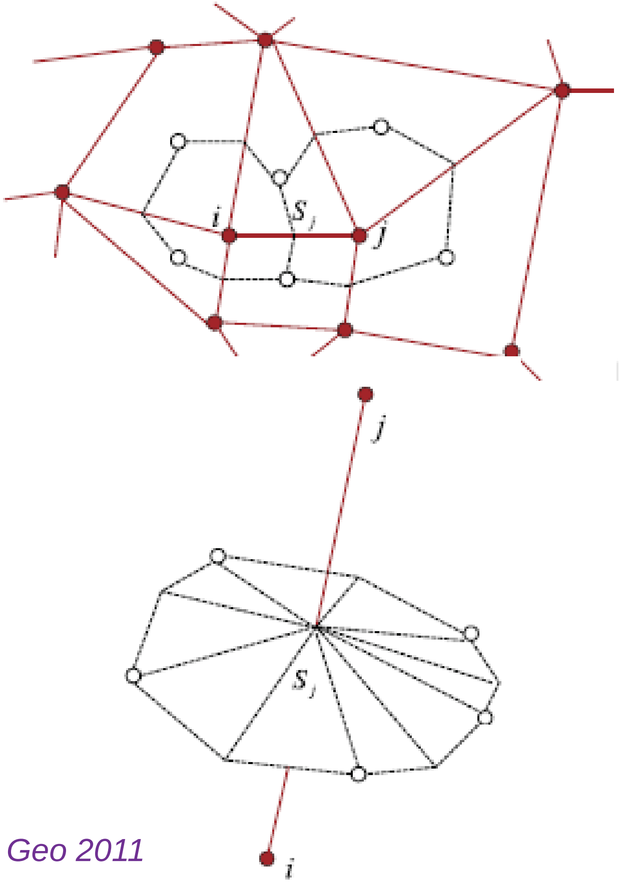
Semi-Implicit NFT based solver

$$\xi_P^{n+1} = \mathcal{A}_P \left(\widetilde{\xi}, \boldsymbol{\mathcal{V}}^{n+1/2}, \rho_o\right) + 0.5\delta t R_P^{n+1}$$

Advection using MPDATA

BVP (Poisson equation) solved using GCR Solver

Median dual meshes with edge-based connectivity



Smolarkiewicz & Szmelter, Acta Geo 2011

Summary of the procedure

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1. Linear extrapolation of the advective components to the time $t^{n+1/2}$

$$\rho_o V_I^{n+1/2} = 1.5\rho_o V_I^n - 0.5\rho_o V_I^{n-1}$$

2. Computation of auxiliary variables according to previous time steps

$$\widetilde{\mathbf{V}} = (\mathbf{V} + 0.5\delta t\, R^{\mathbf{V}})^n \quad \widetilde{\theta'} = (\theta' + 0.5\delta t\, R^{\theta'})^n$$

$$\widetilde{\nu} = \hat{\nu} + 0.5\delta t\, 2R^{\nu}$$

3. MPDATA transports the auxiliary variables $\tilde{V}$ and $\widetilde{\theta}'$. For solution using DES, MPDATA completes the solution of $\hat{\nu}^{n+1}$

4. The solution of elliptic BVP leads to an updated value of pressure φ^{n+1}

$$\forall\ P \quad \left\{ \sum_{J=1,3} \frac{\partial}{\partial x_J} \left[\rho_o \left(\widehat{\widehat{V_J}} - 0.5\delta t\, \beta_J^{-1} \frac{\partial \varphi}{\partial x_J}^{n+1} \right) \right] \right\}_P = 0$$

5. Evaluation of updated V_I^{n+1} and θ'^{n+1}

$$V_I^{n+1} = \widehat{\widehat{V_I}} - 0.5\delta t\, \beta_I^{-1} \frac{\partial \varphi}{\partial x_I}^{n+1} \qquad \theta'^{\,n+1} = \hat{\theta}' - 0.5\delta t \frac{\partial \theta_e}{\partial x_3} V_3^{n+1}$$

6. Update the forcings using V_I^{n+1} and θ'^{n+1}. For solution using DES, Spalart-Allmaras model retrieve μ_t using $\hat{\nu}^{n+1}$

Incompressible viscous flow past a sphere

Setup of experiment

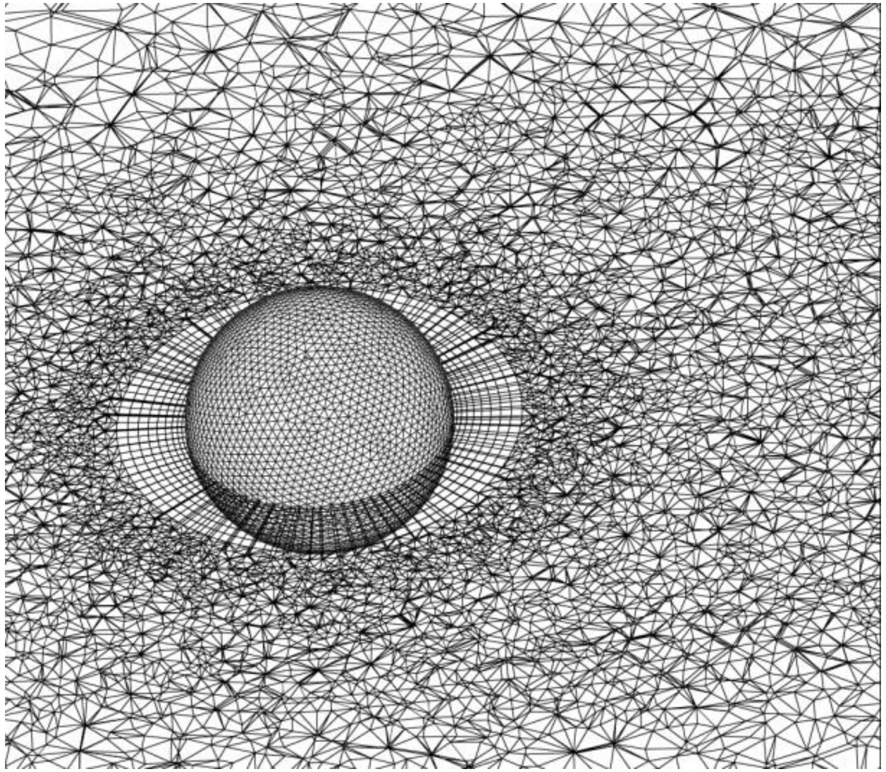
Computational domain: 20 x 20 x 20 cube
Sphere in the centre having diameter D=1

Meshes adopted

- Prismatic layers in the proximity of the sphere
- Tetrahedral elsewhere

Ambient state is the constant velocity flow $\boldsymbol{V}_e = (1,0,0)$, and potential flow determines the initial condition of Navier-Stokes equations

Variable Reynolds number



Re = 200

Number of nodes: 190327

Prismatic elements:

- 9 layers of increasing thickness
- within 0.6D from the boundary of sphere

Results

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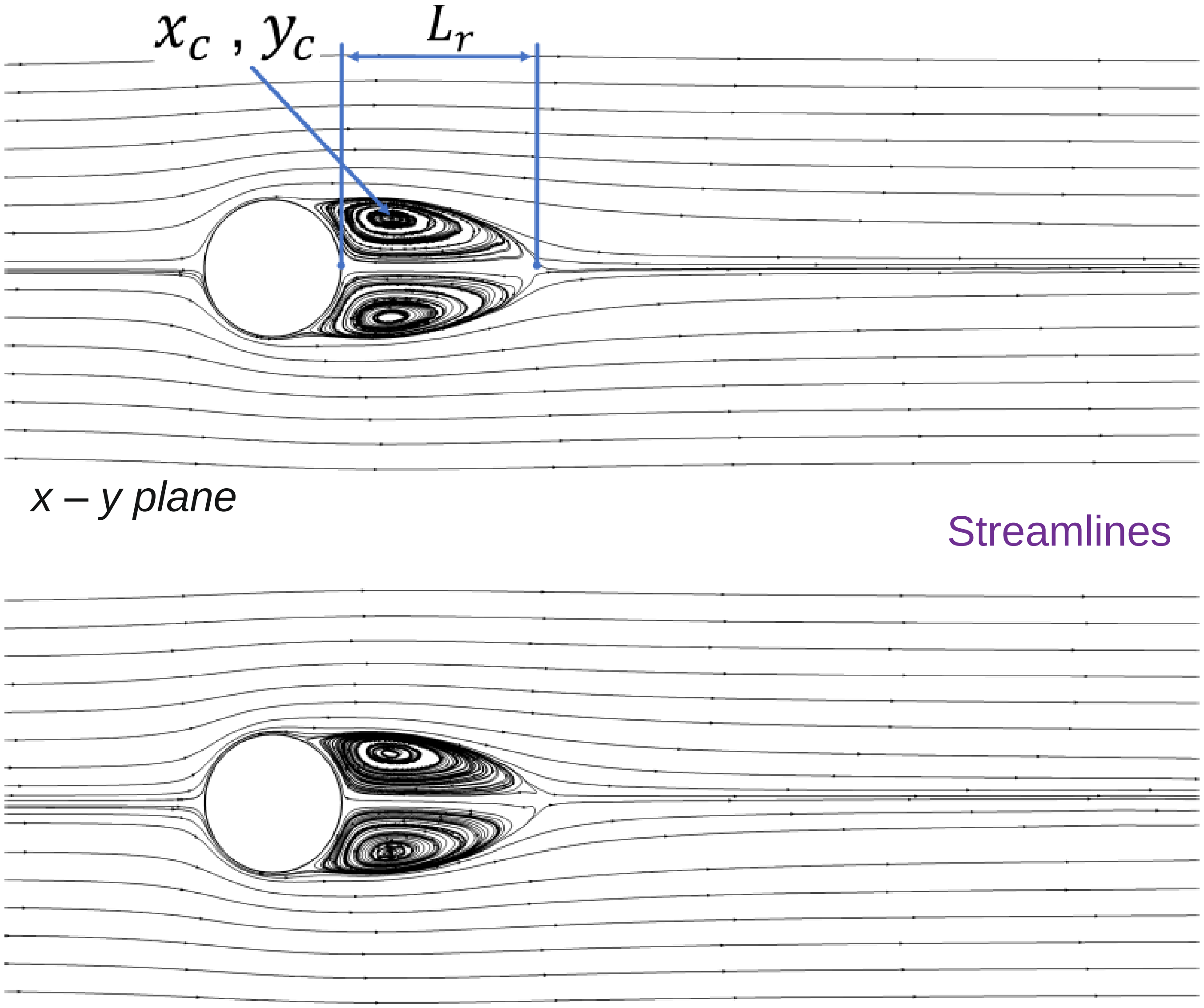
Steady flow: circulation behind the sphere is axisymmetric to the x-directed axis

Agreement with previous results, both numerical and experimental:

- drag coefficient C_d
- recirculation length L_r
- separation angle ϕ_s



x – y plane

Streamlines

x – z plane

Szmelter et al., under review JCP

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Szmelter et al., under review JCP

	C_d	L_r	ϕ_s
NFTFV	0.774	1.429D	116.6°
[34] (experiment)	-	-	116.5°
[3]	0.77	1.43D	116.3°
[70]	-	1.429D	116.1°
[14]	0.768	1.436D	-
[17]	0.776	1.427D	116.2°
[4]	0.775	1.430D	116.7°
[30]	0.749	-	114.3°
[7]	-	1.436D	116.3°
[67]	0.771	-	-
[62]	0.784	1.310D	118°
[25]	0.772	-	-

Stratified laminar flow past a sphere

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Setup of experiment

Computational domain: 20 x 20 x 20 cube
Sphere in the centre having diameter D=1

Meshes adopted

- 24 prismatic layers in the proximity of the sphere
- Tetrahedral elsewhere

Ambient state is the constant velocity flow $\boldsymbol{V}_e = (V_0, 0{,}0) = (1{,}0{,}0)$ m/s, and kinematic viscosity $\nu = 5 \cdot 10^{-3} m^2 s^{-1}$ is chosen to give *Re* = 200

Varying Froude number

$$Fr = \frac{2V_0}{ND}$$

where N is the buoyancy frequency function of the stratification S_0

$$N = \sqrt{\frac{g}{S_0}}$$

$$\theta_e(x_3) = \theta_o + S_o x_3$$

Stratification is modified to have resulting Froude numbers equals to

$$Fr \nearrow \infty \quad Fr = 1 \quad Fr = 0.25$$

Stratified laminar flow past a sphere

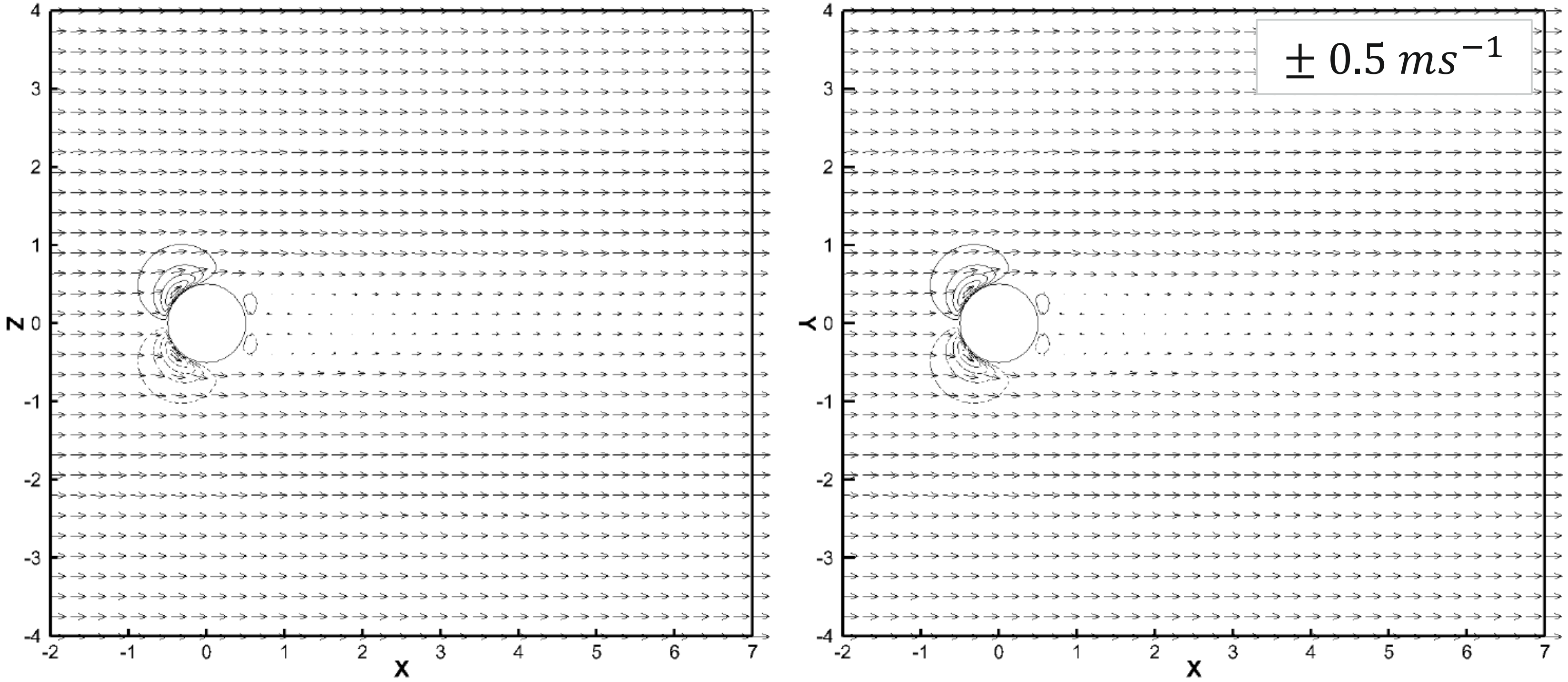
$Fr \nearrow \infty$
Flow neutrally stratified, axisymmetric circulation

$Fr = 1$
Buoyancy-induced gravity waves,
Dividing streamline height $h_s = h(1 - Fr)$

$Fr = 0.25$
Large horizontal eddies

Good agreement with linear theory, λ of gravity waves
Smith R.B., 1988

$Fr \nearrow \infty$



z-component velocity   *y-component velocity*

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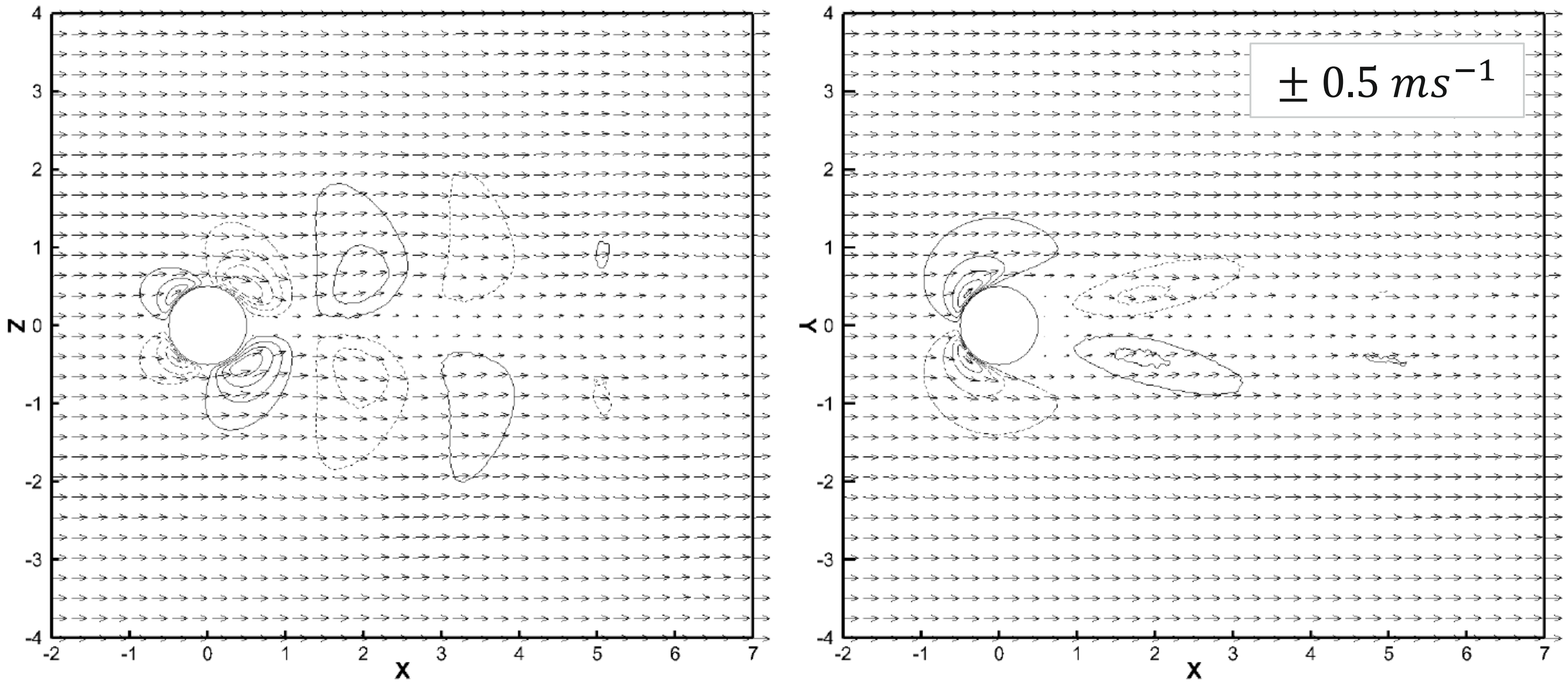

z-component velocity
y-component velocity

Stratified laminar flow past a sphere

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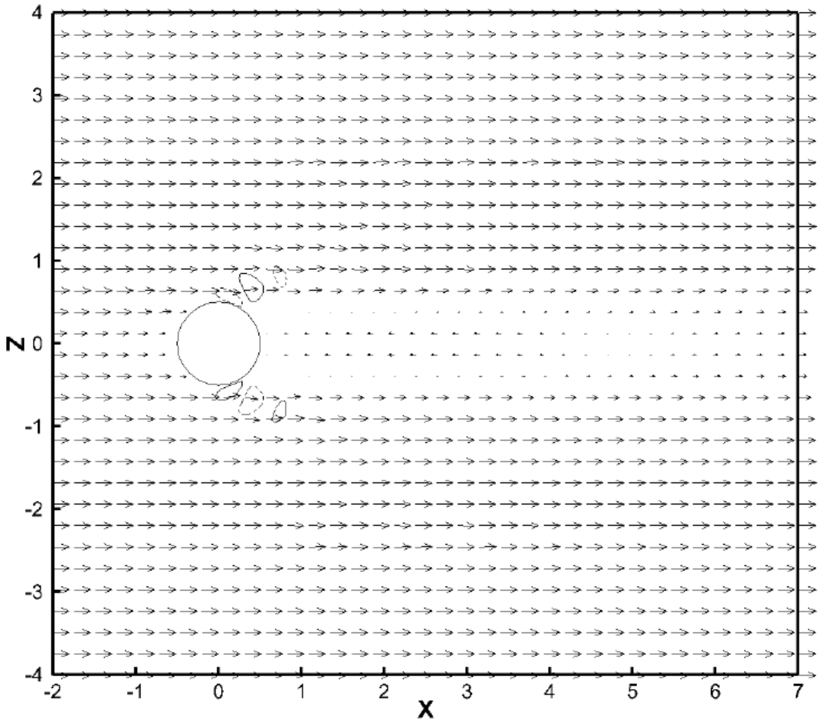
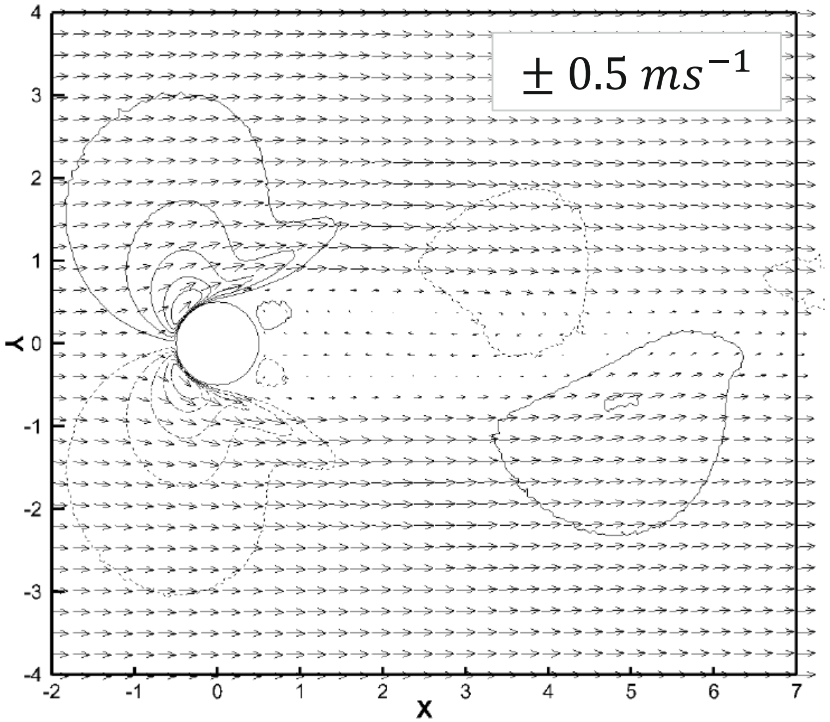


z-component velocity



y-component velocity

Effects of stratification on drag coefficient

Drag coefficient of flow compared to neutral stratified flow

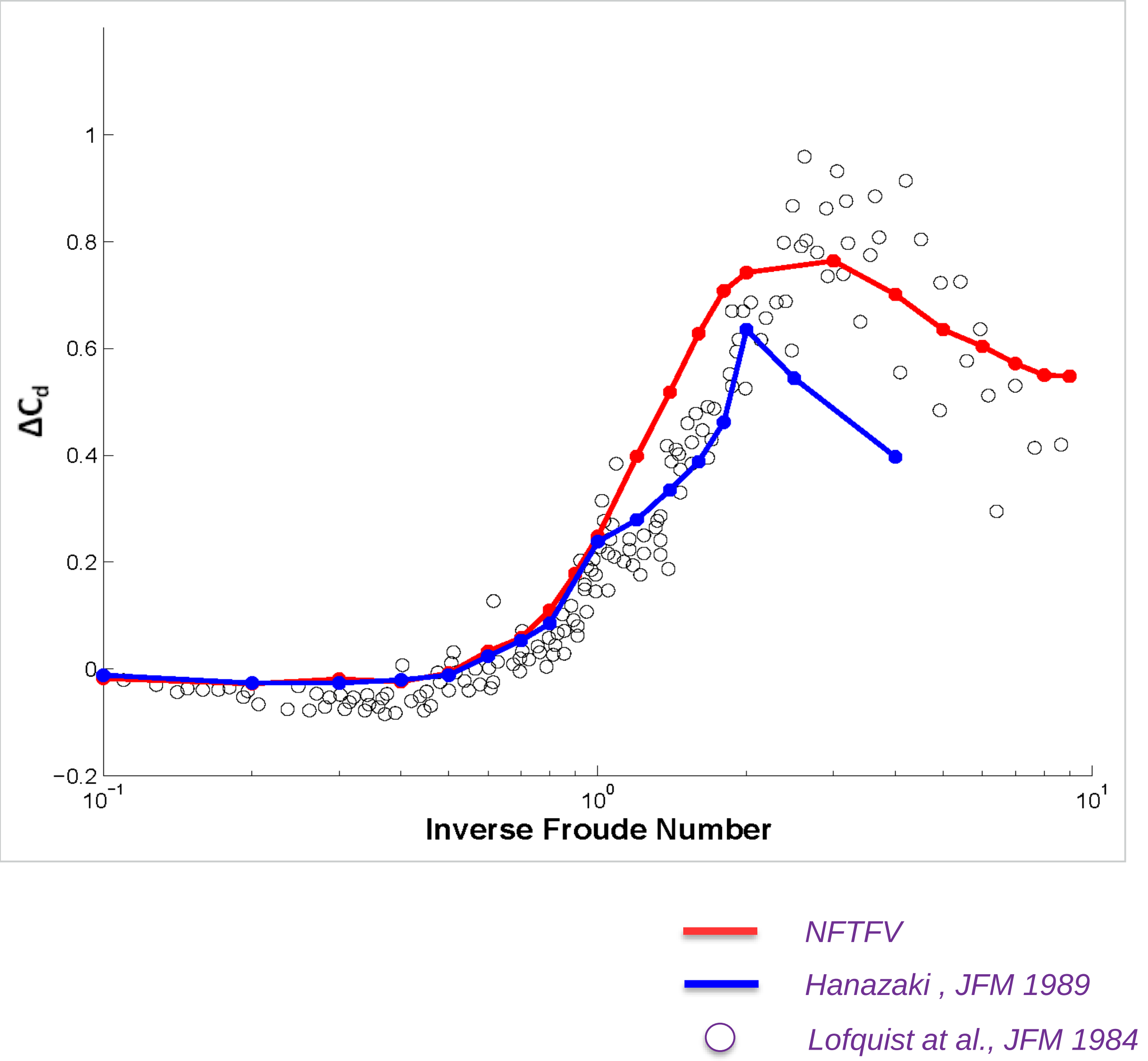
$$\Delta C_d = C_d(Re, 1/Fr) - C_d(Re, 0)$$

$$C_d = \frac{F_d}{0.5\rho_0 V_0^2 A}$$

- NFTFV solutions — $Re = 200$
- Early numerical solutions
- Laboratory experiments — $10^2 < Re < 10^4$

Reasonable agreement between numeric solutions and experimental data

Sharp transition related to the concept of dividing streamline height



Effects of stratification on drag coefficient

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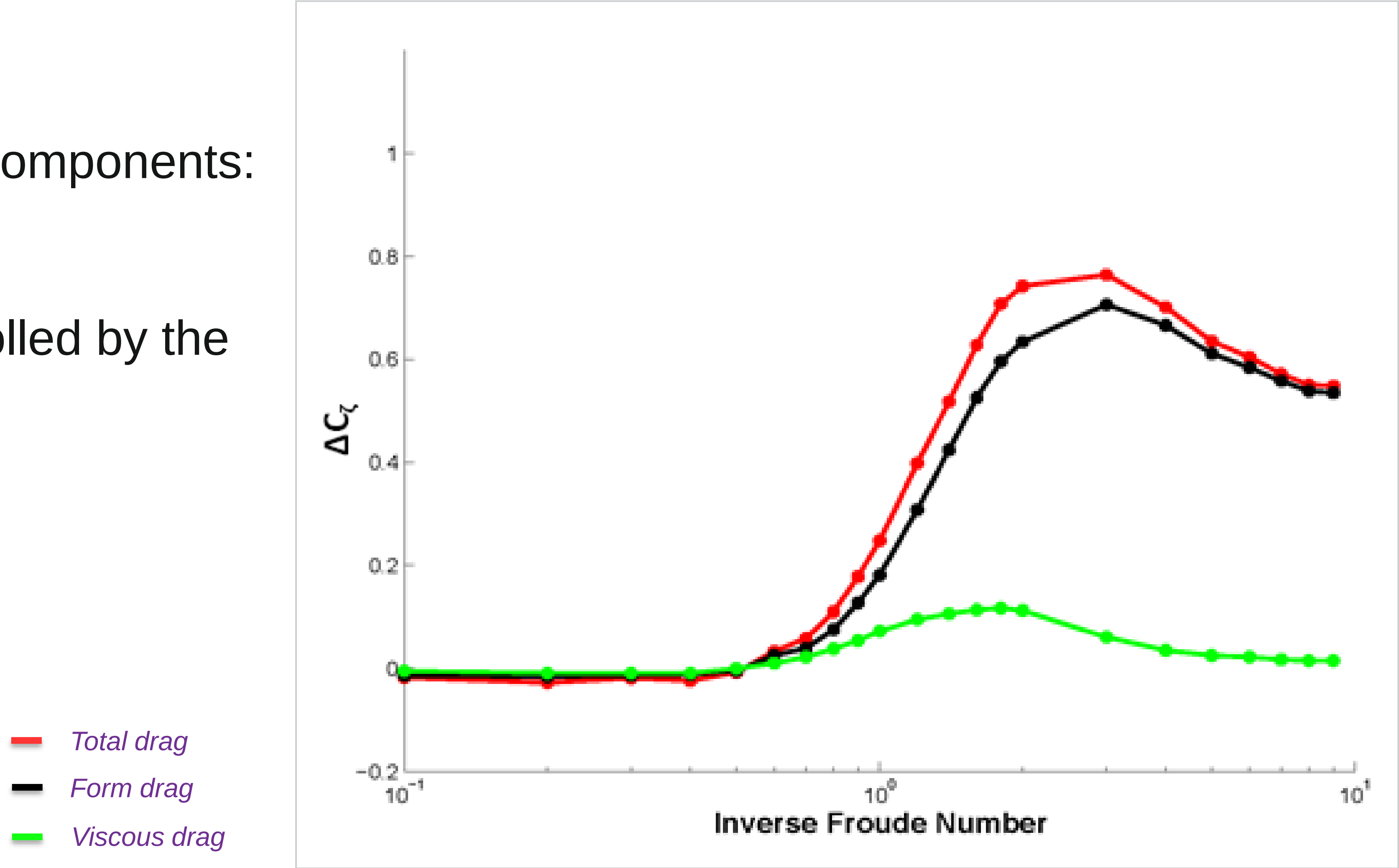
Study on drag coefficient components:

Flow organization is controlled by the stratification



Stratified flow past a steep isolated hill

Stratified flow past a steep isolated hill

- Hill definition

$$h(x,y) = \begin{cases} h_0 \cos^2\left(\frac{\pi r}{2L}\right), & \text{if } r \geq L \\ 0, & \text{otherwise} \end{cases}$$

$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2}, \quad L = 3000\,m\,, \quad h_0 = 1500\,m$$

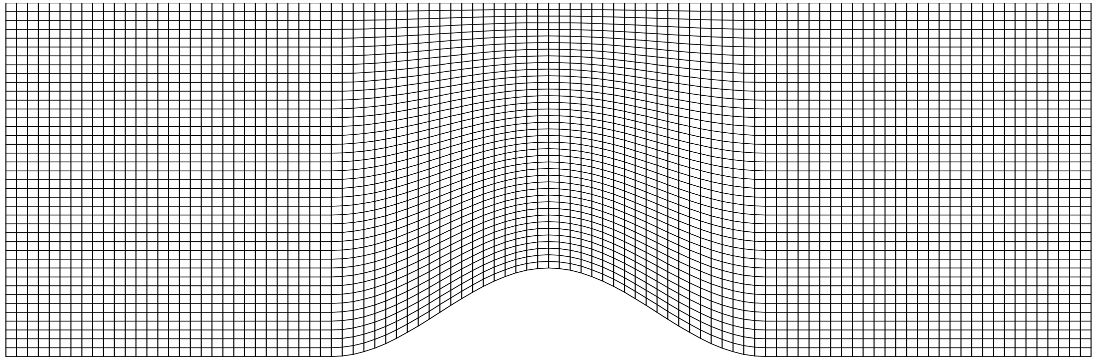
- Vertical mesh construction

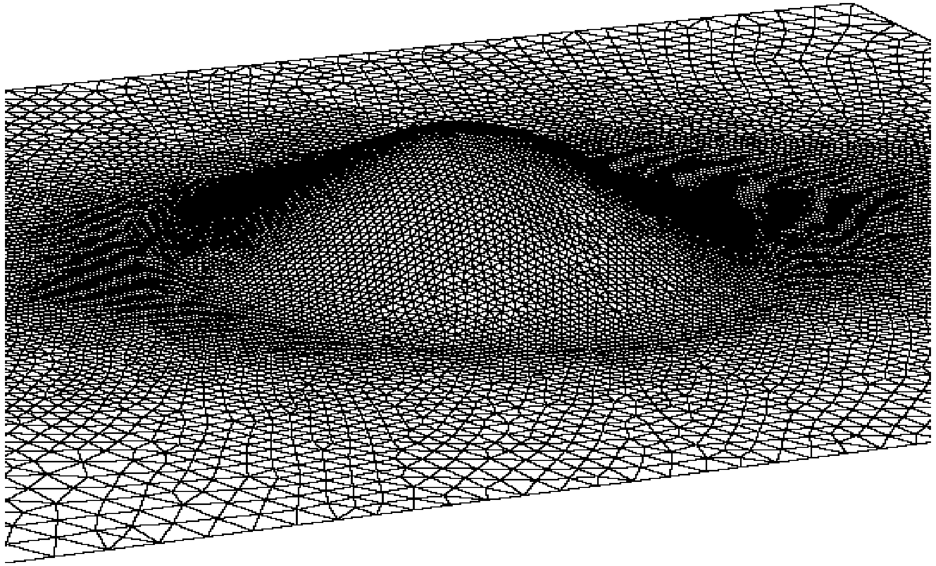
$$z_{ijk} = (k-1)\delta z * \left(1 - \frac{h_{ij}}{H}\right) + h_{ij}$$

- Flow features

$$Fr = \frac{1}{3}\,, \quad N = 0.01\,s^{-1}\,, \quad u_e = 5\,ms^{-1}$$

Computational meshes





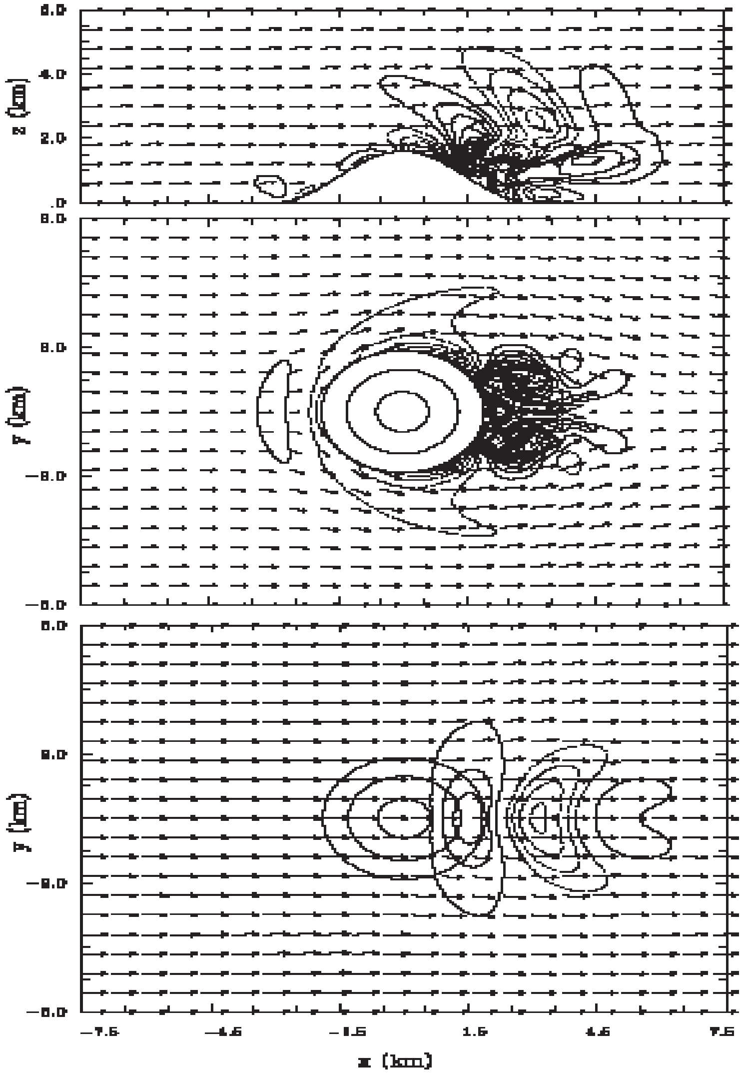
Smolarkiewicz et. al., *JCP* 2013

Stratified flow past a steep isolated hill

Previous results

Smolarkiewicz et al., JCP 2013

- Turbulent wake on the lee side
- Gravity-wave response above dividing streamline height



Vertical velocity and flow vectors are reported

z = 500 m

z = 2500 m

MPI-Parallelization of the code

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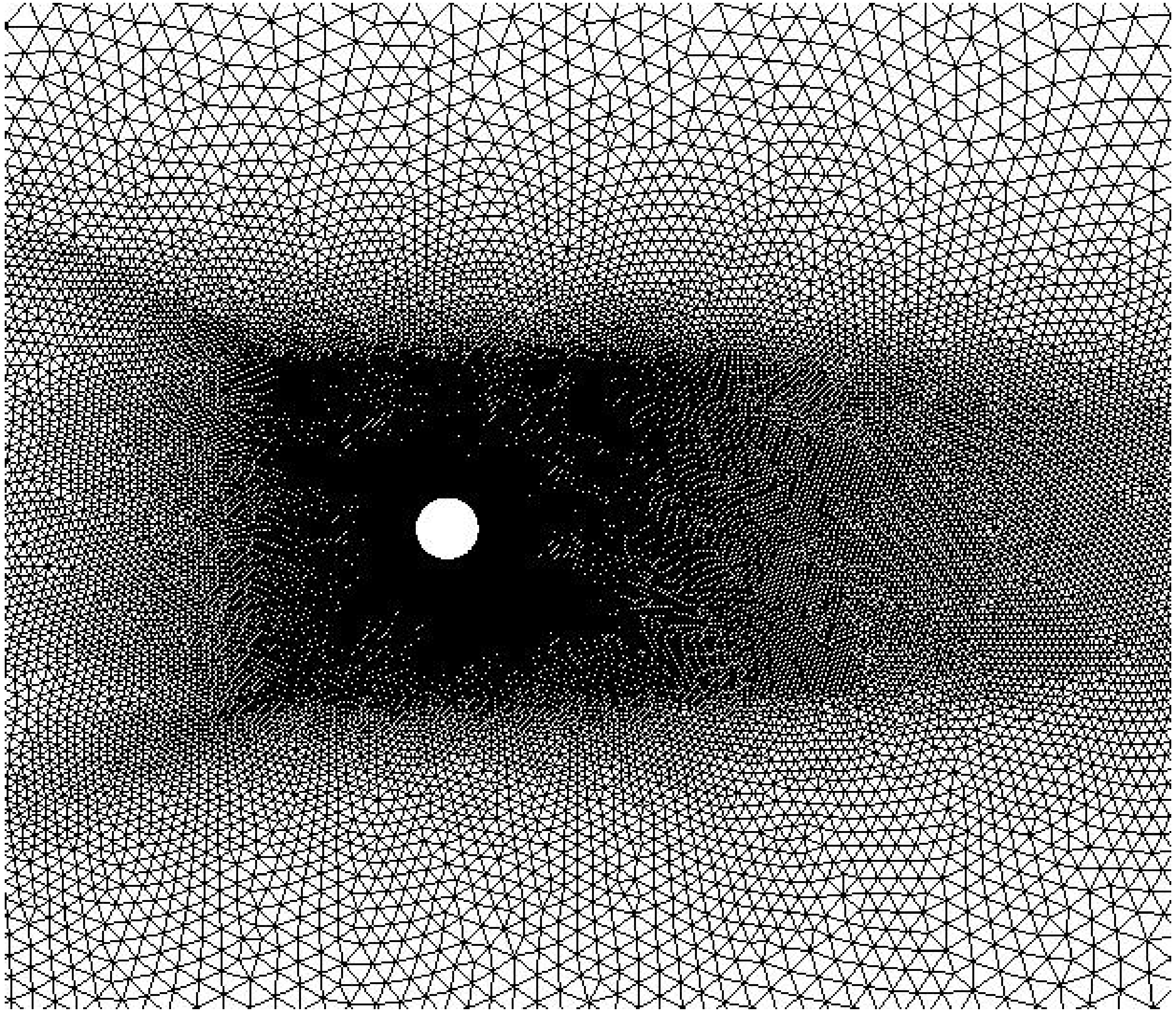
MPI library specification for distributed memory model

Domain decomposed using METIS library

Communication using double halos

Hydra HPC architecture:

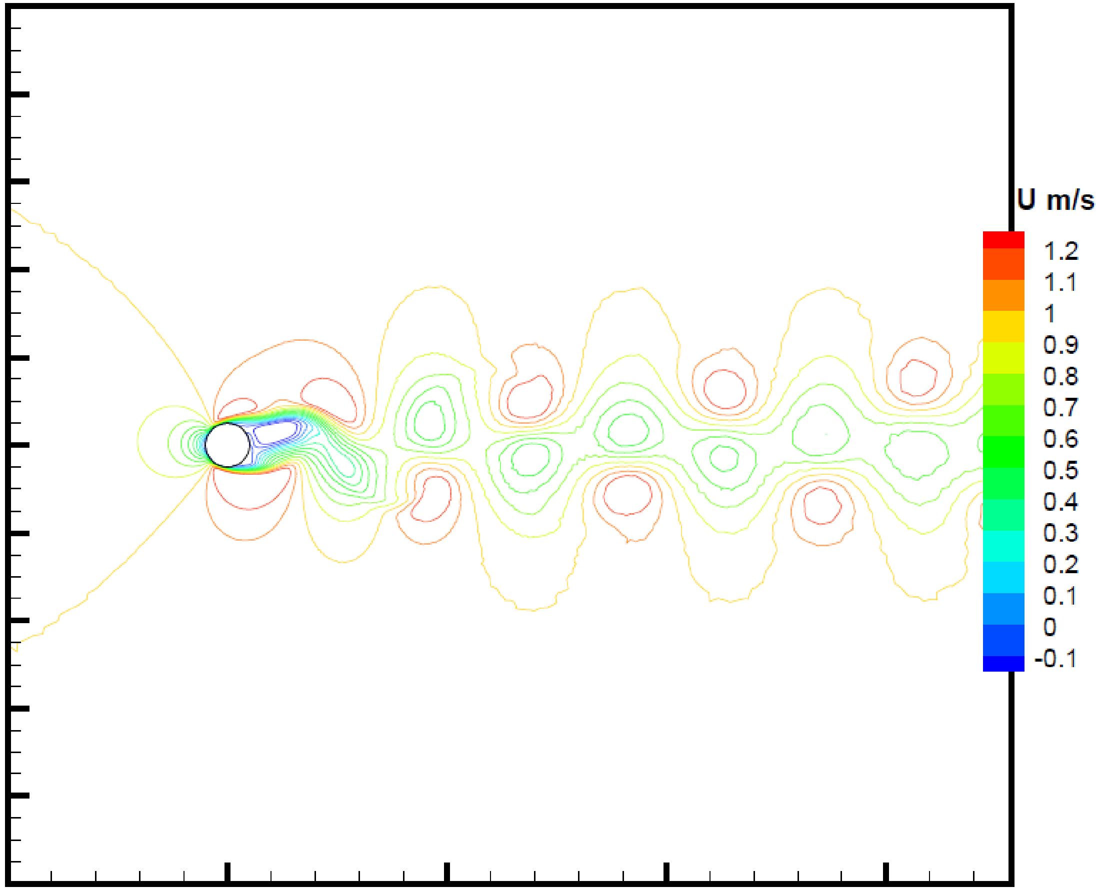
- 2460 cores
- 64-bit Intel Xenon CPUs



Void 2D-cylinder in uniform flow

Setup of experiment

- Domain 90D x 20D, where D is the diameter of the cylindrical void – 22524 points
- Velocity of the uniform flow *u = (1,0)*
  Re = 200
- Incompressible Navier-Stokes equations



Performance

- Speedup ratio $S_N = \frac{T_1}{T_N} \cdot 100$
- Good scaling until 24 cores, then scaling deteriorates
- Parallel efficiency is affected by relatively small number of points

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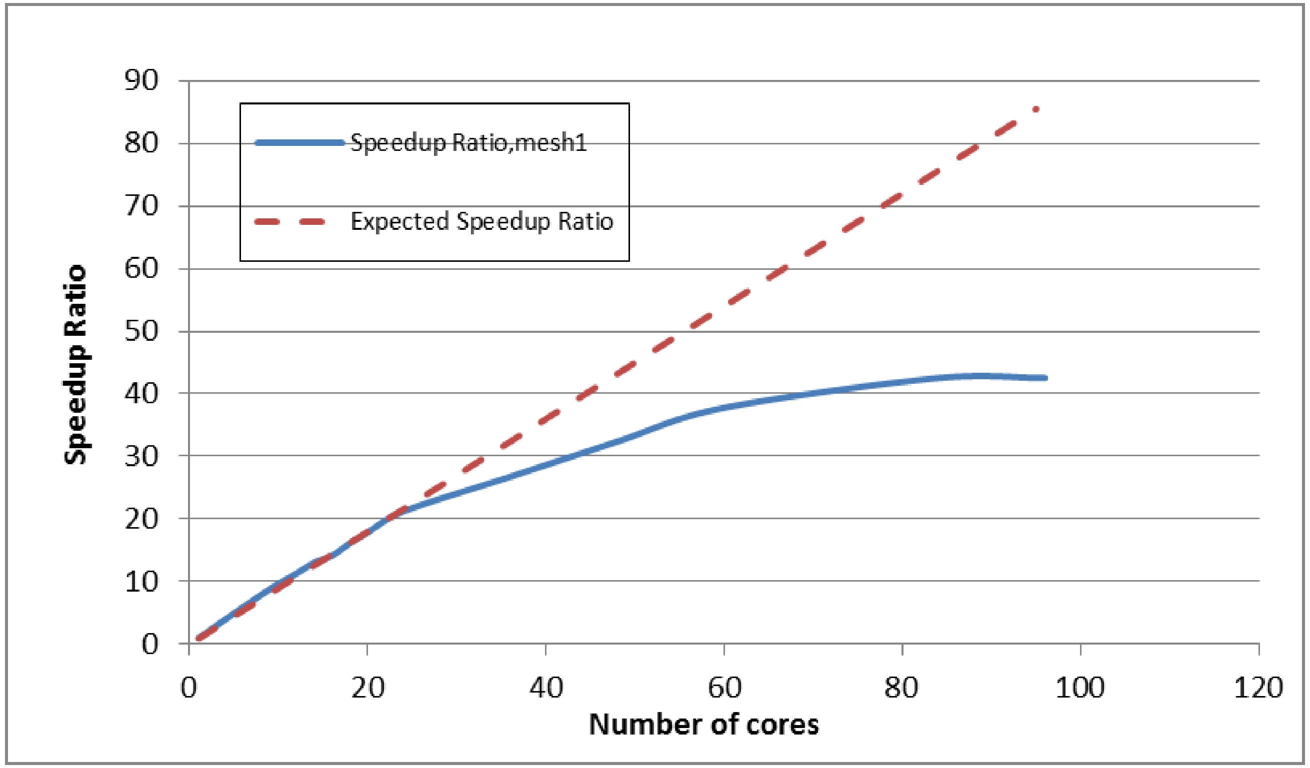

Performance

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- Good scaling until 24 cores, then scaling deteriorates
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Conclusions

- Capability of NFTFV schemes to accurately simulate viscous and stratified flows at a range of Reynolds and Froude numbers has been demonstrated.

- Parallel implementation in 2D shows a promising speed up.
  For the 2D-cylinder benchmark the parallel efficiency is about 90% when using up to 24 cores.

THANK YOU FOR YOUR ATTENTION

Preconditioners adopted

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Richardson-iteration preconditioner

For any initial guess ϕ^0, subsequent $\phi^{\nu+1}$ are evaluated by

$$\frac{\partial \phi_i^\nu}{\partial \tau} = \mathcal{L}(\phi_i^\nu) - r_i$$

$$\frac{\phi_i^{\nu+1} - \phi_i^\nu}{\beta} = \mathcal{L}(\phi_i^\nu) - r_i$$

$$\phi_i^{\nu+1} = \phi_i^\nu + \beta(\mathcal{L}(\phi_i^\nu) - r_i)$$

where pseudo-time β is defined minimizing the distance between points of the primary mesh ($\widetilde{dx}$) and applying

$$\beta = \frac{1}{2}\widetilde{dx}^2$$

Jacobi smoother as preconditioner

Starting from Poisson equation

$$\mathcal{L}(\phi_i^\nu) = r_i$$

$$\mathcal{L}(\phi_i^\nu) - \mathcal{D}_i\phi_i^\nu + \mathcal{D}_i\phi_i^{\nu+1} = r_i$$

For any initial guess ϕ^0, subsequent $\phi^{\nu+1}$ are evaluated by

$$\phi_i^{\nu+1} = \phi_i^\nu - (\mathcal{L}(\phi_i^\nu) - r_i)\mathcal{D}_i^{-1}$$

where $\mathcal{D}_i$ aims to consider the diagonal part of Laplacian operator

$$\mathcal{D}_i = \frac{1}{4\mathcal{V}_i}\sum_{j=1}^{l(i)}\left[\frac{S_{x\,j}^2}{\mathcal{V}_j} + \frac{S_{y\,j}^2}{\mathcal{V}_j} + \frac{S_{z\,j}^2}{\mathcal{V}_j}\right]$$

ESCAPE

Preconditioners adopted

Mixed preconditioner

Starting from Richardson preconditioner

$$\phi_i^{\nu+1} = \phi_i^\nu + \beta(\mathcal{L}(\phi_i^\nu) - r_i)$$

$$\phi_i^{\nu+1} = \phi_i^\nu + \beta\left(\mathcal{L}(\phi_i^\nu) - r_i - \mathcal{D}_i\phi_i^\nu + \mathcal{D}_i\phi_i^{\nu+1}\right)$$

For any initial guess ϕ^0, subsequent $\phi^{\nu+1}$ are evaluated by

$$\phi_i^{\nu+1} = \phi_i^\nu + \frac{\beta}{1 - \beta\mathcal{D}_i}(\mathcal{L}(\phi_i^\nu) - r_i)$$

where pseudo-time β and $\mathcal{D}$ operator are mesh-dependent and can be pre-computed

Relaxation

Elliptic solver could also benefit from relaxing (or over-relaxing) the solution of preconditioner operator, e.g.

$$\phi_i^{\nu+1} = (1 - \omega)\phi_i^\nu + \omega\beta(\mathcal{L}(\phi_i^\nu) - r_i)$$

	Richardson	Jacobi	Mixed
ω	1.5	0.6	1.5

Multigrid Preconditioning

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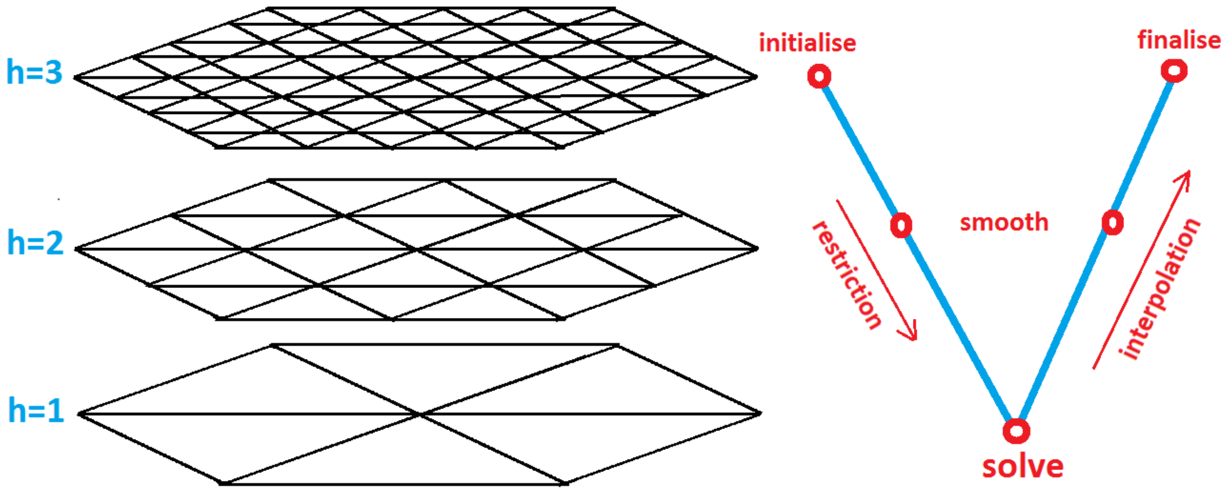
V-Cycle

The aim is to minimize the solution error acting on length-waves quickly detected by coarser meshes



Example on 2-levels multigrid

Smooth p_i^f on the finest grid

Calculate residuals $rr_i^f = rr_i^f - \mathcal{L}(p_i^f)$

$\rightarrow$ Restrict $rr_j^c = I_f^c rr_i^f$

Solution for e_j^c

$\leftarrow$ Interpolate $e_i^f = I_c^f e_j^c$

Add coarse grid error $p_i^f = p_i^f + e_i^f$

Smooth p_i^f on the finest grid

Preliminary/Intermediate results

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- Flow past a steep hill
- Cartesian grid
- Relaxation factor ω = 1.5

Considering the potential flow

- Number of iterations of the flow solver VS number of iterations of preconditioner

- Comparison between Richardson preconditioner and Richardson preconditioner implemented with 2-levels multigrid (5 loops on the coarser grid)

- No gain in terms of elapsed time