Appendix 1. Glossary of Symbols

a	Fraction of convective cloud cover; also constant used in cloud microphysics
ABE	Available buoyant energy
A_T	The forcing terms of the thermodynamic equation that vary on the time-scale of the Rossby-waves
A_u, A_d, A_{tot}	Cloud Work Function for updraft, downdraft, and all of model cloud
A_v	The forcing terms of the v-momentum equation that vary on the time-scale of the Rossby-waves
A_v	The forcing terms of the u-momentum equation that vary on the time-scale of the Rossby-waves
A'	Parameter for heterogeneous freezing (K^{-1})
A	Antidiffusive flux
b	Backscattering coefficient; also fraction of total water vapor convergence used to moisten grid column (section 5.3.2.1); also constant (0.8) used in cloud microphysics computation
B	Planck function
B_u	Acceleration due to buoyancy
B'	Parameter for heterogeneous freezing ($m^{-3}s^{-1}$)
c_0	Rainfall conversion parameter (section 5.3.2.2)

c_m	Constant used in computation of D_m
c_i	Coefficients used in calculation of cloud effect on downward longwave radiation (Table 5.2)
c_p	Specific heat at constant pressure for dry air
c_{pm}	Specific heat at constant pressure for moist air
c^*	Net condensation rate averaged over grid volume
c_c^*	Net condensation rate in cumulus cloud (section 5.3.2.1)
C	Constant (2. m s^{-1} $K^{-1/2}$) used in computing convective velocity
C_g	Thermal capacity of slab per unit area (J m^{-2} K^{-1})
C_s	Heat capacity per unit volume (J m^{-3} K^{-1})
C_θ	Surface exchange coefficient for heat
C_u	Surface exchange coefficient for momentum; also total condensate in updraft (section 5.3.2.2)
C_D	Surface drag coefficient
C'_D	Component of surface-drag coefficient
C_{uN}	Value of surface momentum exchange coefficient under neutral stability conditions

$C_{\theta N}$	Value of surface heat exchange coefficient under neutral stability conditions
D	Mass divergence (hydrostatic split-explicit scheme); also horizontal deformation (section 5.1)
D_f	Diffusivity of water vapor in air
D_u, D_d, D_{tot}	Updraft, downdraft, and total cloud kinetic energy dissipation
	Distance between an observation and a given grid point (section 4)
D_i	Diameter of ice crystal (m)
D_α	Diffusion and PBL tendencies for variable α
D_m	Modified distance between an observation and a given grid point (section 4)
e	Horizontal Coriolis parameter (s^{-1})
e_s, e_{si}, e_{sw}	Saturation vapor pressure, over ice, over water (cb)
E	Efficiency of collection of cloud by precipitation; also vertical flux of water vapor
E_s	Flux of water vapor from surface into atmosphere
f	Coriolis parameter
f_1, f_2	Ventilation coefficients for rain or snow

F	Larger-scale forcing (section 5.3.2.2 and 5.3.2.3); also function of distance from lateral boundary (section 2.6.2)
FH, FL	Flux from high-order and low-order advective scheme
F_{bot}, F_{top}	Longwave radiative flux at bottom, top of model atmosphere (W m^{-2})
F_d, F_u	Downward, upward longwave radiative flux (W m^{-2})
$F_{H\alpha}$	Term representing contribution of horizontal diffusion of a variable α to the temporal rate of change of α
$F_{V\alpha}$	Term representing contribution of vertical diffusion of a variable α to the temporal rate of change of α
F_s	Flux of dry static energy (section 5.3.2.2); also Surface flux of heat, moisture or momentum
F_q	Flux of water vapor (section 5.3.2.2)
F_l	Flux of suspended cloud liquid water (section 5.3.2.2)
F_r	Deceleration
F_1, F_2	Amplitude factors used in computing lateral boundary conditions (section 2.6.2)
g	Acceleration of gravity (9.8 m s^{-2})
h	Moist static energy; also height of planetary boundary layer (m)
h_o	Local hour angle of sun

$\tilde{h}, h_c, h_u, h_d, \tilde{h}^*$	Moist static energy in environment, cloud, updraft, downdraft, and saturation value in environment
H	Vertical flux of sensible heat (W m^{-2})
H_m	Heat flux into substrate (W m^{-2})
H_s	Sensible heat flux from surface into atmosphere (W m^{-2})
I	Function of static stability and surface friction velocity; also horizontal grid-index in y-direction
$IMAX$	Maximum value of grid-index in y-direction
I_s	Net longwave iradiance at surface (wm^{-2})
$I \uparrow$	Outgoing longwave radiation from surface (W m^{-2})
$I \downarrow$	Downward longwave radiation absorbed at surface (W m^{-2}) under clear skies
$I \downarrow'$	Downward longwave radiation absorbed at surface (W m^{-2}) in presence of clouds
I_1	Normalized condensate in updraft (section 5.3.2.2)
I_2	Normalized evaporate in updraft (section 5.3.2.2)
J	Horizontal grid index in x-direction
$JMAX$	Maximum value of grid index in x-direction

k	Dimensionless x-wavenumber for upper radiative scheme; also von Karman constant (0.4)
$\hat{k}$	Dimensionless effective x-wavenumber for upper radiative scheme
k_1	Constant used in formula for computing autoconversion of cloud drops to rain drops
K	Total horizontal wavenmumber (m^{-1}) also Kernels
K_a	Background molecular diffusivity ($2.4 \times 10^{-5} m^2 s^{-1}$); also thermal conductivity of air (J m^{-1} s^{-1} K^{-1})
K_H	Horizontal eddy diffusivity (m^2 s^{-1})
K'_H	Coefficient used in fourth-order diffusion (s^{-1})
K_{HO}	Background value of horizontal eddy diffusivity (m^2 s^{-1})
K_m	Coefficient of heat transfer from ground into substrate (s^{-1})
$KMAX$	Maximum value of index in vertical direction
K_z	Coefficient of vertical diffusivity (m^2 s^{-1})
K_{z0}	Background value of coefficient of vertical diffusivity (m^2 s^{-1})
KE_u, KE_d, KE_{tot}	Kinetic energy for updraft, downdraft, and all of model cloud
L	Hydrostatic term due to liquid water loading; also Monin-Obukhov length
L_m	Latent heat of fusion (0.35×10^6 J kg^{-1})

L_s	Latent heat of sublimation (2.85×10^6 J kg^{-1})
L_v	Latent heat of condensation (2.5×10^6 J kg^{-1})
l	Dimensionless y-wavenumber for upper radiative scheme; also vertical mixing length
$\hat{l}$	Dimensionless effective y-wavenumber for upper radiative scheme
M_i	Mass of ice crystal (kg)
M_{max}	Maximum mass of ice crystal (kg)
M_0	Initial mass of ice crystal (kg)
m	Mass flux (updraft and downdraft) in convective parameterization cloud (5.3.2.2); also map scale factor
$\bar{m}$	Mixing coefficient used in free-convective regime of high-resolution PBL model
m_b	Cloud base mass flux (section 5.3.2.2)
m_0	Downdraft base mass flux (section 5.3.2.2)
m_u	Updraft mass flux in convective parameterization cloud (5.3.2.2)
m_d	Downdraft mass flux in convective parameterization cloud (5.3.2.2)
M	Surface moisture availability

M_t	Vertical integral of horizontal convergence of water vapor
n	Fraction of cloud
n_0	Cloud microphysics parameter
n_c	Number concentration of ice crystals (kg^{-1})
N	Brunt-Vaisälä frequency (s^{-1})
N_c	Number concentration of cloud droplets per unit volume (10^{10} m^{-3})
N_h	Nondimensional function for vertical profile of convective heating
N_m	Nondimensional function for vertical profile of convective moistening
N_0	Cloud microphysics parameter (8×10^6 m^{-4} for rain 2×10^7 m^{-4} for snow)
p	Pressure (cb)
p_b	Pressure (cb) at convective cloud base
p_s	Surface pressure (cb)
p_t	Pressure (cb) at top of model
p_u	Pressure (cb) at top of convective cloud
p_{LCL}	Pressure (cb) at lifting condensation level

p^*	$p_s - p_t$ (cb)
p_d^*	Dot-point p^* (cb)
p_0	Reference-state pressure
p'	Perturbation pressure (Pa)
	Pressure value representing the free atmosphere, where terrain influences are small (in FDDA)
$\hat{p}$	Fourier transform of p' for upper radiative boundary condition
P_{CON}	Condensation of water vapor or evaporation of cloud drops (kg kg^{-1} s^{-1})
P_{RA}	Accretion of cloud drops by rain drops (kg kg^{-1} s^{-1})
P_{RC}	Autoconversion of cloud drops to rain drops (kg kg^{-1} s^{-1})
P_{RE}	Evaporation of rain drops (kg kg^{-1} s^{-1})
P_{CI}	Heterogeneous freezing of cloud water (kg kg s^{-1})
P_{ID}	Deposition of vapor onto ice crystals (kg kg s^{-1})
P_{II}	Initiation of ice crystals (kg kg s^{-1})
P_{MF}	Melting/freezing of cloud and precipitation due to advection (kg kg s^{-1})

P_{RM}	Melting of falling precipitation (kg kg s^{-1})
P_{SM}	Melting of falling snow (kg kg s^{-1})
$\tilde{q}, q_u, q_d, \tilde{q}^*$	Water vapor mixing ratio in environment, updraft, downdraft, and saturation value in environment
q_c	Mixing ratio of cloud water; also water vapor mixing ratio in cloud (section 5.3.2.2)
q_{c0}	Critical value of mixing ratio of cloud water
q_r	Mixing ratio of rain water
q_l	Suspended liquid water vapor mixing ratio inside updraft
q_v	Mixing ratio of water vapor
q_{vc}	Mixing ratio of water vapor in cumulus cloud
q_{vs}	Saturation mixing ratio of water vapor
Q	Diabatic heating rate per unit mass (J kg^{-1} s^{-1})
Q_s	Net short wave irradiance at the surface (W m^{-2})
R	Rainfall (convective-scale sink of cloud water, 5.3.2.2); also ideal gas constant for dry air (287 J kg^{-1} K^{-1})
RH	Relative humidity
R_i	Richardson number

R_n	Net radiation
R_{iB}	Bulk Richardson number
R_{ic}	Critical value of bulk Richardson number; also critical value of Richardson number
R_v	Gas constant for water vapor (461.5 J kg^{-1} K^{-1})
R_T	Radiative heating rate (K s^{-1})
r	Radius of convective parameterization cloud (sections 5.3.2.2)
S	Supersaturation; also source or sink term (section 5.3.2.2); also square of the vertical wind shear
S_c	Schmidt number
S_o	Solar constant (1395.6 W m^{-2})
S_u	Source or sink term in updraft (section 5.3.2.2)
S_d	Downward solar flux (W m^{-2}); also source or sink term in downdraft (section 5.3.2.2)
S_i	Supersaturation over ice
s	Dry static energy
t	Time (s)

T	Temperature (K)
T_c	Longwave transmissivity due to cloud
T_d	Dewpoint temperature (K)
T_g	Temperature (K) of ground
T_p	Longwave transmissivity due to precipitation
T_v	Virtual temperature (K); also longwave transmissivity due to vapor
T_*	Surface friction temperature (K)
T_0	Reference-state temperature (K)
T'	Perturbation temperature (K)
u	Component of wind velocity in eastward direction (m s^{-1}); also water vapor path (g m^{-2})
u_*	Surface friction velocity (m s^{-1})
u_c, u_p	Liquid water path for cloud, precipitation (g m^{-2})
v	Component of wind velocity in northward direction (m s^{-1})
v_t	Mass weighted mean terminal velocity of rain drops (m s^{-1})
V	Fall speed of a precipitation particle (m s^{-1}); also modified horizontal wind velocity in PBL

$\mathbf{V}$	Horizontal wind vector
V_a	Horizontal windspeed at lowest model layer
V_c	Convective PBL velocity(m s^{-1})
V_{qf}	Divergence of vertical eddy flux of water vapor due to convective clouds
w	Vertical velocity (m s^{-1}); also weight function for reducing mixing near top of mixed layer
w_n	Weight function for blending model tendencies and large-scale tendencies near lateral boundaries (section 2.6.1)
w_p	Precipitable water (cm)
w_u	Vertical velocity in updraft
$\hat{w}$	Fourier transform of w
x	Horizontal grid coordinate increasing generally eastward
X	Horizontal coordinate on earth surface increasing generally eastward
X_c	Multiple-reflection factor in cloudy air
X_d	Distance vector
X_R	Multiple-reflection factor in clear air

y	Horizontal grid coordinate increasing generally northward
Y	Horizontal coordinate on earth surface increasing generally northward
z	Height above surface (m)
z_a	Height of lowest layer in model (m)
z_b	Height of updraft originating level(section 5.3.2.2) (m)
z_0	Height of downdraft originating level(section 5.3.2.2); also surface roughness length(m)
z_{oc}	Background value of surface roughness length over water (10^{-4}m)
z_l	Depth of molecular layer
z_{LCL}	Height of lifting condensation level (m)
z_T	Height of updraft top (section 5.3.2.2)
α	Coefficient array for upper radiative boundary condition (m s^{-1} Pa^{-1}); also any thermodynamic variable (section 5.3.2.2)
$\tilde{\alpha}$	Any thermodynamic variable in environment
α_u	Any thermodynamic variable in updraft
α_d	Any thermodynamic variable in downdraft
α_c, α_p	Longwave absorption coefficients for cloud, precipitation (m^2 g^{-1})

β	Parameter in sound-wave temporal differencing; also precipitation efficiency parameter in section 5.3.2.2
Γ	Gamma function
Γ_d	Dry adiabatic lapse rate (K m^{-1})
Γ_{dp}	Dewpoint adiabatic lapse rate (K m^{-1})
γ	Ratio of heat capacities (c_p/c_v) for dry air
δ	Solar declination
δM	Supersaturation or undersaturation
Δp	Vertical grid size (Pa)
Δs	Horizontal grid length (m)
Δt	Time step (s)
$\Delta t'$	Short time step for rain fall term (s)
Δx	Horizontal grid length (m)
Δz	Thickness of vertical layer (m)
$\Delta \sigma$	Thickness of model σ levels

$\Delta\sigma_c$	Critical value of convective cloud depth
$\Delta\tau$	Short time step for sound waves (s)
∇_σ^2	Horizontal Laplacian on σ-surfaces
∇_σ^4	Fourth order diffusion operator on σ-surfaces
ϵ	Parameter relating updraft and downdraft mass flux (section 5.3.2.2); also small value; also entrainment coefficient used in high resolution PBL-model (0.2)
ϵ_a	Atmospheric emissivity
ϵ_g	Emissivity of ground
ϵ_u, ϵ_d	Atmospheric longwave emissivity
η_d	Normalized mass flux for downdraft (section 5.3.2.2)
η_u	Normalized mass flux for updraft (section 5.3.2.2)
θ	Potential temperature (K); also angle between y-axis and north for full Coriolis force
θ_a	Potential temperature (K) at lowest layer in model
θ_g	Potential temperature (K) of ground surface
θ_e	Equivalent potential temperature (K)

θ_{es}	Saturation equivalent potential temperature (K)
θ_v	Virtual potential temperature (K)
λ	Longitude; also cloud type (section 5.3.2.2); also thermal conductivity (J m^{-1} s^{-1} K^{-1}); also parameter in raindrop distribution (m^{-1})
μ	Dynamic viscosity of air (kg m^{-1} s^{-1}); also solar zenith angle ; also total net fractional entrainment rate (section 5.3.2.2); also constant in smoother (section 3.3)
μ_u	Total net fractional entrainment rate for updraft (section 5.3.2.2)
μ_{ue}	Gross fractional entrainment rate for updraft (section 5.3.2.2)
μ_{ud}	Gross fractional detrainment rate for updraft (section 5.3.2.2)
ν	Coefficient for Asselin time filter; also for spatial smoother
π	Exner function
ρ	Density of air (kg m^{-3})
ρ_r	Particle density (kg m^{-3})
ρ_u	Density in updraft
ρ_w	Density of water (kg m^{-3})
ρ_0	Reference-state density (kg m^{-3})

ρ'	Perturbation density (kg m^{-3})
σ	Nondimensional vertical coordinate of model
σ'	Dummy variable of integration
$\dot{\sigma}$	Vertical velocity in σ-coordinates (s^{-1})
$\dot{\sigma}_c$	Vertical velocity of convective cloud in σ-coordinates (s^{-1})
σ_{SB}	Stefan-Boltzmann constant (5.67051×10^{-8} J m^{-2} K^{-4} s^{-1})
τ	Half-period of time window of influence of an observation (section 4); also short-wave transmissivity
τ'	Short-wave transmissivity obtained from lookup table
τ_a	Clear air absorption transmissivity
τ_{ac}	Cloudy air absorption transmissivity
τ_s	Clear air scattering transmissivity; also surface stress
τ_{sc}	Cloudy air scattering transmissivity
ϕ	Geopotential; also latitude; also scalar variable in advection equation
ϕ_s	Surface geopotential

Φ	Symbol denoting low-order, monotonic solution to advection equation
χ	Diffusivity of vapor in air (m^2 s^{-1}); also thermal inertia
Ψ	Solar zenith angle; also function of bulk Richardson number
Ψ_m	Nondimensional stability parameter for momentum
Ψ_h	Nondimensional stability parameter for heat and water vapor
ω	Vertical velocity in pressure coordinates (cb s^{-1})
ω_c	Vertical velocity of convective cloud in pressure coordinates (cb s^{-1})
Ω	Angular velocity of earth (7.2722×10^{-5} s^{-1})

Appendix 3. Map Projections

Map projections are constructed by projecting the surface of the earth onto a right circular cone, cutting the cone, and flattening it into a plane surface. Three projections are available for the MM4 system – Polar stereographic, Lambert conformal, and Mercator. Polar stereographic is preferred for high-latitude studies, Lambert conformal for middle-latitude studies, and Mercator for low-latitude studies. This appendix summarizes the map scale factors for each projection and gives the equations for converting from latitude and longitude to the x and y positions on the model grid.

Although the grid size $\Delta x = \Delta y = \Delta s$ is constant on the model's grid, the actual distance represented by Δs on the spherical earth varies with location on the grid because the earth is curved. The map scale factor m is defined as the ratio of the distance on the grid to the corresponding distance on the earth's surface

$$m = \frac{distance\ on\ grid}{actual\ distance\ on\ earth} \qquad A.1$$

a. Lambert Conformal

Conformal means that the scale is equal in all directions about a point, so that shapes of geographic features on the earth are preserved. The Lambert conformal grid is true at latitudes 30° and 60°N so that $m = 1.$ at these latitudes. In general,

$$m = \frac{sin\psi_1}{sin\phi}\left[\frac{tan\phi/2}{tan\psi_1/2}\right]^{0.716}, \qquad A.2$$

where $\psi_1 = 30°$ and ψ is the colatitude ($\psi = 90° - \phi$).

It is sometimes necessary to compute the position (x, y) on the grid given the latitude and longitude of a point, or vice versa. The following relations pertain to an $X - Y$ grid with center $X = 0,\ Y = 0$ at latitude ϕ_0 and longitude λ_0. Note that the relationship between (x, y) and (X, Y) is

$$x = X + \frac{JMAX - 1}{2}\Delta s, \qquad A.3$$

$$y = Y + \frac{IMAX - 1}{2}\Delta s, \qquad A.4$$

$$\lambda = any\ longitude$$

$$\lambda_0 = \ longitude\ of\ Y\ axis$$

$$\phi = any\ latitude$$

$$\phi_0 = \ latitude\ along\ \lambda_0\ at\ which\ Y = 0$$

$$\psi = 90^\circ - \phi$$

$$n = .716$$

$$\psi_1 = 30^\circ$$

$$\psi_0 = 90^\circ - \phi_0$$

$$a = 6370 km$$

$$r = \frac{a}{n} sin\psi_1 \left[\frac{tan\psi/2}{tan\psi_1/2}\right]^n, \qquad A.5$$

$$C_2 = \frac{a}{n} sin\psi_1 \left[\frac{tan\psi_0/2}{tan\psi_1/2}\right]^n, \qquad A.6$$

$$C_1 = -\lambda_0 - 90^\circ/n, \qquad A.7$$

$$\lambda' = n(\lambda + C_1), \qquad A.8$$

$$X = rcos\lambda', \qquad A.9$$

$$Y = rsin\lambda' + C_2. \qquad A.10$$

The inverse problem to calculate latitude and longitude is done as follows:

$$\lambda' = arctan\left(\frac{Y - C_2}{X}\right), \qquad A.11$$

$$\lambda = \frac{\lambda'}{n} - C_1, \qquad A.12$$

$$r = \frac{X}{cos\lambda'} or \frac{Y - C_2}{sin\lambda'} \qquad A.13$$

$$\psi = 2arctan\left[tan\psi_1/2\left(\frac{nr}{asin\psi_1}\right)^{1/n}\right], \qquad A.14$$

$$\phi = 90^\circ - \psi. \qquad A.15$$

b. Polar Stereographic

For the polar stereographic projection, true at latitude $\phi_1 = 60°N$, the map scale factor is

$$m = \frac{1 + sin\phi_1}{1 + sin\phi} \qquad A.16$$

The relationships between latitude and longitude and X and Y on the polar stereographic grid are calculated as before on the Lambert conformal grid, but now $n = 1$.

$$r = amcos\phi, \qquad A.17$$

$$C_2 = asin\psi_1 \left[\frac{tan\psi_0/2}{tan\psi_1/2}\right], \qquad A.18$$

$$C_1 = -\lambda_0 - 90°, \qquad A.19$$

$$\lambda' = \lambda + C_1, \qquad A.20$$

$$X = rcos\lambda', \qquad A.21$$

$$Y = rsin\lambda' + C_2. \qquad A.22$$

and for the inverse problem

$$\lambda' = arctan\left(\frac{Y - C_2}{X}\right), \qquad A.23$$

$$\lambda = \lambda' - C_1, \qquad A.24$$

$$r = \frac{X}{cos\lambda'} or \frac{Y - C_2}{sin\lambda'} \qquad A.25$$

$$\psi = 2arctan\left[tan\psi_1/2\left(\frac{r}{asin\psi_1}\right)\right], \qquad A.26$$

$$\phi = 90° - \psi. \qquad A.27$$

Note that the signs of $Y - C_2$ and X in (A.23) must be considered to obtain the correct quadrant for λ'.

c. Mercator

For the Mercator grid, $\phi_0(Y = 0)$ corresponds to the equator and the relationships between X and Y and ϕ and λ are relatively simple

$$X = (acos\phi_1)(\lambda - \lambda_0), \qquad A.28$$

$$Y = (acos\phi_1)ln\left[\frac{1 + sin\phi}{cos\phi}\right], \qquad A.29a$$

$$Y = (acos\phi_1)ln[tan(45^\circ + \phi/2)]. \qquad A.29b$$

Note that $(\lambda - \lambda_0)$ in (A.28) must be expressed in radians. The latitude ϕ_1 at which the Mercator projection is true is often taken to be 30°.

The reverse problem, to obtain X and Y from ϕ and λ, is also simple

$$\lambda = \lambda_0 + \frac{X}{acos\phi_1}. \qquad A.30$$

To solve for ϕ, use (A.29b)

$$\phi = -90^\circ + 2arctan\left[exp\left(\frac{Y}{acos\phi_1}\right)\right]. \qquad A.31$$