Chapter 8: Part I: MM5 Dynamics and Numerics

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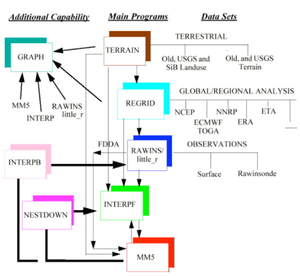
Outline

- Basic Dynamical Equations
- Numerical Methods
- Boundary Conditions
- Nesting
- Data Assimilation

MM5 Modeling System Flow Chart



MM5 Model

- The forecast component of the MM5 System
- Dynamics
  - Compressible, nonhydrostatic with terrain-following coordinate, map-factors, full Coriolis
- Numerics
  - Second order time-split leapfrog time scheme
  - Second-order centered space scheme
- Physics
  - Full physics for NWP applications
  - Many options for each physics component

Basic Equations

- Pressure equation

$$\frac{\partial p'}{\partial t} - \rho_0 g w + \gamma p \nabla \cdot \mathbf{V} = -\mathbf{V} \cdot \nabla p' + \frac{\gamma p}{T}\left(\frac{\dot{Q}}{c_p} + \frac{T_0}{\theta_0} D_\theta\right) \qquad (8.1)$$

- last term is actually neglected in MM5

Basic Equations

- Momentum equation: x component

$$\frac{\partial u}{\partial t} + \frac{m}{\rho}\left(\frac{\partial p'}{\partial x} - \frac{\sigma}{p^*}\frac{\partial p^*}{\partial x}\frac{\partial p'}{\partial \sigma}\right) = -\mathbf{V} \cdot \nabla u + v\left(f + u\frac{\partial m}{\partial y} - v\frac{\partial m}{\partial x}\right) - ew\cos\alpha - \frac{uw}{r_{earth}} + D_u \qquad (8.2)$$

  - the m gradient terms are horizontal curvature terms
  - the r_{earth} term is a vertical curvature term
  - e = $2\Omega\cos\lambda$, the other component of Coriolis
  - $\alpha = \phi - \phi_c$, the rotation angle of the grid

Basic Equations

- Momentum equation: y component

$$\frac{\partial v}{\partial t} + \frac{m}{\rho}\left(\frac{\partial p'}{\partial y} - \frac{\sigma}{p^*}\frac{\partial p^*}{\partial y}\frac{\partial p'}{\partial \sigma}\right) = -\mathbf{V}\cdot\nabla v - u\left(f + u\frac{\partial m}{\partial y} - v\frac{\partial m}{\partial x}\right) + ew\sin\alpha - \frac{vw}{r_{earth}} + D_v \quad (8.3)$$

- Momentum equation: z component

$$\frac{\partial w}{\partial t} + \frac{\rho_0}{\rho}\frac{g}{p^*}\frac{\partial p'}{\partial \sigma} + \frac{g}{\gamma}\frac{p'}{p} = -\mathbf{V}\cdot\nabla w + g\frac{p_0}{p}\frac{T'}{T_0} - \frac{gR_d}{c_p}\frac{p'}{p} + e(u\cos\alpha - v\sin\alpha) + \frac{u^2+v^2}{r_{earth}} + D_w \quad (8.4)$$

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Basic Equations

- Advection terms can be expanded as

$$\mathbf{V}\cdot\nabla A \equiv mu\frac{\partial A}{\partial x} + mv\frac{\partial A}{\partial y} + \dot{\sigma}\frac{\partial A}{\partial \sigma} \quad (8.6)$$

where

$$\dot{\sigma} = -\frac{\rho_0 g}{p^*}w - \frac{m\sigma}{p^*}\frac{\partial p^*}{\partial x}u - \frac{m\sigma}{p^*}\frac{\partial p^*}{\partial y}v \quad (8.7)$$

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Basic Equations

- Thermodynamics

$$\frac{\partial T}{\partial t} = -\mathbf{V}\cdot\nabla T + \frac{1}{\rho c_p}\left(\frac{\partial p'}{\partial t} + \mathbf{V}\cdot\nabla p' - \rho_0 g w\right) + \frac{\dot{Q}}{c_p} + \frac{T_0}{\theta_0}D_\theta \quad (8.5)$$

  - the term in parentheses is the adiabatic warming term
  - the Q term is the diabatic term

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Basic Equations

- Divergence term can be expanded as

$$\nabla\cdot\mathbf{V} = m^2\frac{\partial}{\partial x}\left(\frac{u}{m}\right) - \frac{m\sigma}{p^*}\frac{\partial p^*}{\partial x}\frac{\partial u}{\partial \sigma} + m^2\frac{\partial}{\partial y}\left(\frac{v}{m}\right) - \frac{m\sigma}{p^*}\frac{\partial p^*}{\partial y}\frac{\partial v}{\partial \sigma} - \frac{\rho_0 g}{p^*}\frac{\partial w}{\partial \sigma} \quad (8.8)$$

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Notes about the equations

- Appendix A shows derivations of 8.1, 8.4, 8.5 and 8.7
- In (8.1)-(8.4), terms on left of equals sign are treated on a short time-step (sound waves)
- Equations 8.2, 8.3 and 8.8 include terms to account for sloped sigma surfaces
- Prognostic equations also exist for water vapor, cloud, rain, etc.

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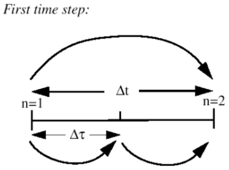
Temporal finite differencing

First time step:



• forward step followed by leapfrog steps with short sub-steps

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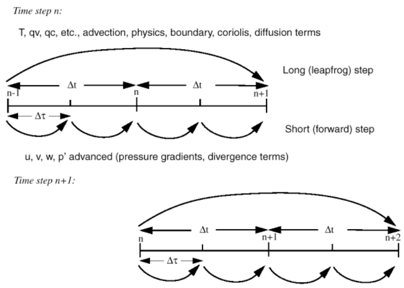
Temporal finite differencing



Semi-implicit short steps

- Horizontally explicit, vertically implicit
- Treats left hand terms in Eqs. (8.1)-(8.4)
- Right hand terms (advection, Coriolis etc.) are kept constant

Semi-implicit short steps

Sequence:

- u and v advanced first
- w and p′ advanced together using implicit scheme where
  - $p'^{\,n+1} = f(w^n, w^{n+1}, p'^{\,n}, u^{n+1}, v^{n+1})$
  - $w^{n+1} = f'(w^n, p'^{\,n}, p'^{\,n+1})$
    - n is current time, n+1 is future time
  - Solved column by column (tri-diagonal matrix)

Implicit schemes in MM5

- Sound waves: allow short time-step to be independent of vertical grid length
- Vertical Diffusion: allow long time-step to be independent of vertical grid length and magnitude of diffusion coefficient

Time-splitting in MM5

CFL condition: $v\Delta t/\Delta x < 1$ for stability

- Sound-waves: allow long time-step to be independent of sound-wave speed
  - $c\Delta\tau/\Delta x < 1$
- Rainfall terms: allow long time-step to be unrestricted by CFL condition for fall speed and vertical grid length
  - $V_f\Delta t'/\Delta z < 1$

Spatial finite differencing

- Second-order centered for horizontal advection, pressure gradients, using B-staggered grid
- Second-order also for vertical advection, pressure gradients and buoyancy, allowing for non-uniform vertical grid spacing and staggered vertical velocity
- First-order upstream for precipitation fall terms
- Fourth-order horizontal diffusive/filter terms

Filters and smoothing

- Leapfrog time scheme uses Asselin filter to control "computational mode"
  - n-1 time level modified by a 1-2-1 smoother in time, e.g.

$T^{n-1} = T^{n-1} + \nu (T^{n-2} - 2 T^{n-1} + T^{n})$ where $\nu = 0.1$

- Horizontal smoothing and diffusion achieved with a fourth-order derivative term
  - Coefficient = constant + horizontal deformation term

Lateral Boundary Conditions

- Fixed
- Time-dependent/Nest
- Relaxation/inflow-outflow

Upper Boundary Condition

- Rigid lid: w = 0 at top
- Radiative: w is a function of p' at top

Lower Boundary Condition

- LOWBDY file (SST, substrate temperature, snow cover, sea-ice)
  - Fixed (ISSTVAR=0)
  - Time-dependent (ISSTVAR=1)
- Terrain-following flow conditions

Nesting

- One-way nesting
  - Run model, then use NESTDOWN to create nest initial and boundary conditions, then run nest
  - Sequential runs
  - No feedback
  - Any integer ratio

Nesting

- Two-way nesting
  - Multiple domains at same time
  - 3:1 grid size ratio
  - 3 nest time-steps per parent step
  - Parent forces nest at its boundaries
  - Nest feeds back to parent in interior
  - Generally more expensive than 1-way

Nesting

- Two-way nesting options
  - Interpolation (IOVERW=0)
    - Requires no additional files
    - Nest can start any time and move
  - Nest input file (IOVERW=1)
    - Requires MMINPUT_DOMAINn file
    - Nest starts at time zero of coarsest mesh
  - Terrain input file (IOVERW=2)
    - Requires TERRAIN_DOMAINn file
    - Nest can start at any time

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Nesting

- Two-way nesting feedback options
  - No feedback
  - 9-point weighted average
  - 1-point feedback with no smoothing
  - 1-point feedback with smoother-desmoother (recommended)
  - 1-point feedback with heavy smoothing

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FDDA

- Method of nudging model towards observations or analysis
- May be used for
  - Dynamical initialization (pre-forecast period)
  - Creating 4D meteorological datasets (e.g. for air quality model)
  - Boundary conditions (outer domain nudged towards analysis)

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FDDA (cont)

- Methods
  - Grid or analysis nudging (suitable for coarse resolution)
  - Observation or station nudging (suitable for fine-scale or asynoptic obs)
- Nudging can be applied to winds, temp, and water vapor

Note: nudging terms are fake sources, so avoid FDDA use in dynamics or budget studies

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FDDA (cont)

- Analysis nudging
- Each grid-point is nudged towards a value that is time-interpolated from analyses

$$\frac{\partial p^* \alpha}{\partial t} = F(\alpha, \mathbf{x}, t) + G_\alpha \cdot W_\alpha \cdot \epsilon_\alpha(\mathbf{x}) \cdot p^*(\hat{\alpha}_0 - \alpha)$$

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FDDA (cont)

$$\frac{\partial p^* \alpha}{\partial t} = F(\alpha, \mathbf{x}, t) + G_\alpha \cdot W_\alpha \cdot \epsilon_\alpha(\mathbf{x}) \cdot p^*(\hat{\alpha}_0 - \alpha)$$

- G is nudging inverse time scale
- W is vertical weight (upper air and surface)
- $\varepsilon$is a horizontal weight for obs density

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FDDA (cont)

- Upper-air analysis nudging uses the MMINPUT fields
  - Only used above boundary layer
- Surface analysis nudging uses SFCFDDA file from RAWINS/Little_R (may have higher frequency than upper-air analysis)
  - Only used at lowest layer and within boundary layer (PBL depth comes from PBL parameterization)

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FDDA (cont)

- Observation nudging
- Each grid point is nudged using a weighted average of differences from observations within a radius of influence and time window

$$\frac{\partial p^* \alpha}{\partial t} = F(\alpha, \mathbf{x}, t) + G_\alpha \cdot p^* \frac{\sum_{i=1}^{N} W_i^2(\mathbf{x}, t) \cdot \gamma_i \cdot (\alpha_o - \hat{\alpha})_i}{\sum_{i=1}^{N} W_i(\mathbf{x}, t)}$$

$$W(\mathbf{x}, t) = w_{xy} \cdot w_\sigma \cdot w_t$$

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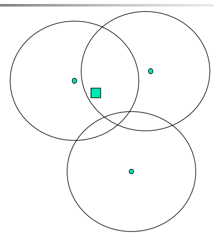
Obs nudging (cont)



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Obs nudging (cont)

$$w_t = 1 \qquad |t - t_0| < \tau/2$$

$$w_t = \frac{\tau - |t - t_0|}{\tau/2} \qquad \tau/2 \le |t - t_0| \le \tau$$

- τ is the specified time window for the obs
- This is a function that ramps up and down

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Obs nudging (cont)

$$w_{xy} = \frac{R^2 - D^2}{R^2 + D^2} \qquad 0 \le D \le R$$

$$w_{xy} = 0 \qquad D > R,$$

- R is radius of influence
- D is distance from ob modified by elevation difference

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Obs nudging (cont)

- w_σ is the vertical weighting – usually the vertical influence is set small (0.005 sigma) so that data is only assimilated on its own sigma level
- obs input file is a special ascii file with obs sorted in chronological order
  - each record is the obs (u, v, T, Q) at a given model position and time

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FDDA Summary

- FDDA grid nudging is suitable for coarser grid sizes where analysis can be better than model-produced fields
- Obs nudging can be used to assimilate asynoptic or high-frequency observations
- FDDA has fake sources and sinks and so should not be used on the domain of interest and in the time period of interest for scientific studies and simulations

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