POTENTIAL VORTICITY INVERSION AND MM5

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1. INTRODUCTION

Often in the study of weather phenomena with the aid of a primitive equation model, particularly a mesoscale model such as MM5, it is desirable to separate the so-called slow modes from the fast modes. The former are usually comprised of Rossby waves and coherent structures for which the potential vorticity is the key dynamical variable. The latter consist of inertial waves, gravity waves and deep atmospheric convection. The need to separate the motions is mainly driven by a need for simplification of the often multi-scale, complicated evolution of phenomena produced by a mesoscale model. There are four general areas of concern:

- Static Initialization
- Perturbing the initial state
- Determining the degree of balance
- Dealiasing small-scale motions

The first point relates to the fact that MM5 does not have a static initialization scheme, such as normal mode initialization, which can effectively remove often spurious oscillations near the beginning of a simulation. Traditionally, this has been viewed reasonably acceptable as simulations have typically been integrated for 24 hours or more and the imbalances are typically adjusted on a time scale of a few hours. However, for shorter integrations, erroneous initial imbalances can have a dramatic effect, especially concerning precipitation. Methods such as dynamic initialization and 4D-VAR partially circumvent this problem by using the model itself to perform adjustments during a pre-forecast period. Data assimilation using 3D-VAR also attempts to remove imbalances through model error covariances which contain information about balanced structure or by including penalty terms that help enforce balance. Ideally, proper data assimilation with adequate data should select the correct amount of imbalance in the initial state and filter the remainder. However, there are no methods which reliably do this at present.

The second point refers to sensitivity simulations in which various portions of the balance flow are removed and a new initial state is defined. This has been done for the study of synoptic-scale cyclones by Huo et al. (1999). The present paper shows results for mesoscale flows.

The third point involves the desire to isolate inertia-gravity waves in mesoscale model simulations (Zhang et al. 2000). Here, the separation is done as a simulation proceeds and allows one to monitor the amount of imbalance and diagnose the regions of large imbalance which may be related to the spontaneous emission of inertia-gravity waves.

Finally, there is the desire to understand the influence of the slow evolution of the flow in problems where the fast response is at least as large, or larger than, the slow response. An example is the synoptic-scale forcing of convective motions, where the forcing is weak, and the response is large. There is often a large degree of internal organization in the convection, but there is also a persistent, large scale forcing which must be quantified in order for the overall convective organization to be understood.

2. METHODOLOGY

In this paper, the use of Ertel potential vorticity (PV) and PV inversion using the nonlinear balance constraint and the method of Davis and Emanuel (1991) will be discussed and applied to each of the above mentioned types of problems.

The detailed equations may be found in Davis et al. (1996). Here they will be written symbolically to illustrate the basic concept. Ertel PV can be expressed as a nonlinear function $\mathcal{N}$ of the geopotential Φ (using the hydrostatic approximation) and a horizontally non-divergent stream function Ψ (neglecting the irrotational vertical shear with respect to the vertical shear of the nondivergent wind),

$$q = \mathcal{N}(\Phi, \Psi). \tag{1}$$

This can be combined with an equation relating Φ and Ψ, the nonlinear balance equation, which derives from neglecting terms containing divergence and vertical motion in the divergence equation and can also be obtained by a formal Rossby number expansion (Haltiner and Williams 1980).

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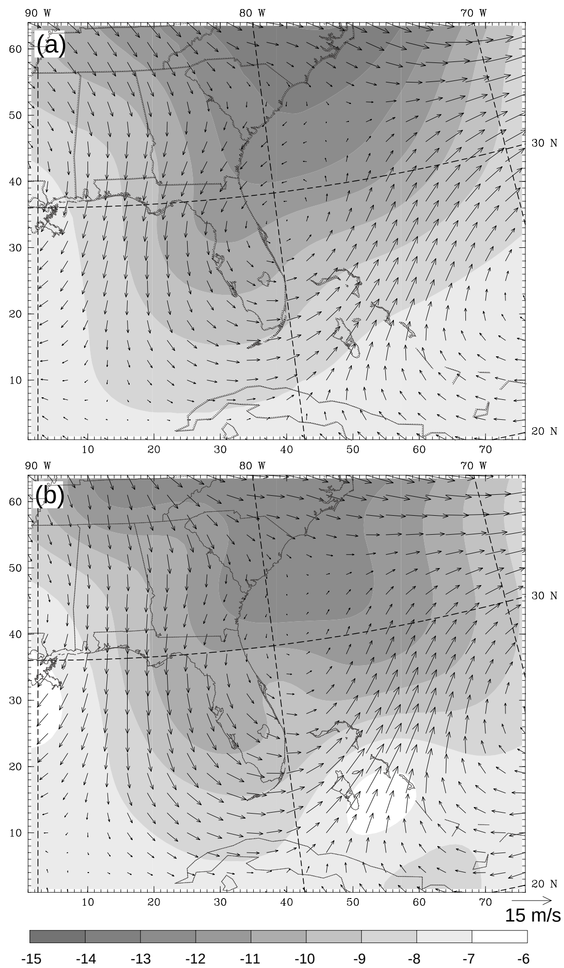

Figure 1. Temperature and wind at 500 hPa for (a) the balanced initial state and (b) the initial state obtained after objective analysis. The valid time is 1200 UTC 7 September, 1984.

$$\nabla^2\Phi = \mathcal{M}(\Psi). \tag{2}$$

Here, $\mathcal{M}$ is a nonlinear operator and ∇^2 is the two-dimensional (x, y) Laplacian operator. Equations (1) and (2) may be solved given PV, lateral boundary conditions (either Neumann or Dirichlet conditions on Φ and Ψ) and potential temperature at the upper and lower boundaries of the domain. When inverting PV obtained from MM5 output, fields are first interpolated to pressure surfaces before PV is calculated. At points below ground, the temperature is computed based on the temperature at the lowest level above the ground and the domain average lapse rate at that level (determined from points above ground). In each column, the wind is fixed at its value at the lowest level above the surface.

In the course of solving (1) and (2), negative values of PV are set to a small positive constant. The iterative scheme follows Davis and Emanuel (1991), with the modification that in regions where convergence is hampered, the PV is increased gradually as the iteration proceeds, so that converged fields may have values of PV as much as 0.1 or 0.2 PVU larger than the initial values. This incrementation of the PV during the iteration cycle occurs mainly near the daytime boundary layer where the stratification vanishes.

In order to initialize MM5 with a balanced field, the consistent, balanced vertical velocity is calculated, as well as the "balanced" irrotational (divergent) wind component. One can think of this divergence as slaved to the rotational part of the flow. The vertical motion is obtained using an omega-equation appropriate for the nonlinear balance approximation,

$$\mathcal{L}(\omega) = \mathcal{H}(\Phi, \Psi, \chi, \frac{\partial\Psi}{\partial t}), \tag{3}$$

combined with mass continuity. Note that (3) contains the tendency of the streamfunction on the right-hand-side. In quasi-geostrophic theory, it is possible to eliminate all tendency terms in the derivation of the omega-equation, but here it is not. Thus, the solution for balanced ω requires simultaneous solution of the tendencies of (1) and (2), along with (3) and mass continuity. The tendency of PV is evaluated using the full wind (including the irrotational wind and vertical motion) for advection, and thus must be re-evaluated at each iteration of the system. Effects of condensational heating are included as a reduced static stability where air is ascending and saturated. Because the areas of ascent are not known *a priori*, this also requires iteration.

Solution of both the PV inversion equations and the omega equation results in a complete description of the flow and its tendencies. Currently the tendency information is not used for initializing MM5. It should be noted that the balanced flow is not an accurate depiction of the planetary boundary layer, hence, below roughly the lowest kilometer, the balance solution is modified so that it transitions to the full flow at the surface. This adjustment mainly affects the winds, as the balanced and full temperatures at the 1 km level are nearly identical by definition (via the lower boundary condition for the inversion of PV).

3. RESULTS

3.1 *Balanced Initial Conditions*

An analysis was produced valid for 1200 UTC 7 September , 1984 using the NCEP/NCAR reanalysis data and using RAWINS to analyze the available sounding and surface data. As commonly occurs with the Cressman scheme, the introduction of local observations can produce local extrema. In this case, inclusion of the observation from Nassau results in a temperature maximum at 500 hPa and a strong warm advection region downstream (Fig. 1b). Strong convection occurs almost immediately, not in agreement with satellite observations. The balanced solution

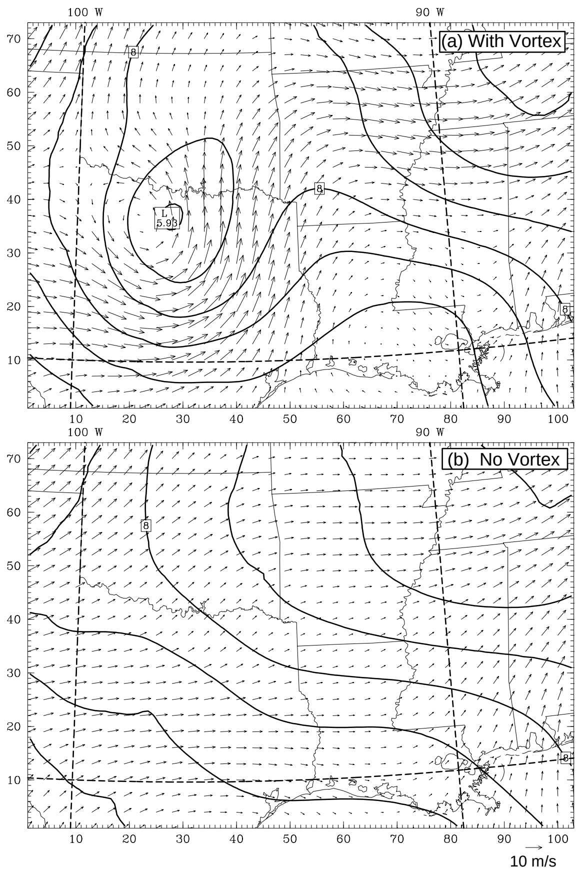

Figure 2. Balanced initial conditions showing wind and temperature at 700 hPa (1 K contour interval) at 1500 UTC 27 May, 1998 for (a) full field and (b) state with mid-tropospheric PV anomalies removed.

produces weaker warm advection and a smoother initial evolution, including a more gradual onset of precipitation. Simulations run with either initial state strongly resemble each other by 12 hours and beyond.

3.2 *Perturbing the Initial State*

Simulations of a mesoscale vortex and its influence on convection were initialized at 1500 UTC 27 May, 1998 using the Rapid Update Cycle (RUC) of NCEP. The vortex is well resolved by the 40 km RUC. We can examine the sensitivity of the convective evolution to the presence of the vortex by removing it (and the downstream ridge) from the initial state. This is done by defining an area (roughly the area shown in Fig. 2) and a pressure interval (850 hPa to 350 hPa) and replacing the PV at each i by its average over j ($\overline{q}$) according to

$$q_{new} = \alpha_1 q|_{j=j_1} + (1-\alpha_1)\overline{q}, \ \left(\alpha_1 = \frac{(j-j_1)}{(J-j_1)}, \ (j_1 < j < J\right);$$

$$q_{new} = \alpha_2 q|_{j=j_2} + (1-\alpha_2)\overline{q}, \ \left(\alpha_2 = \frac{(J-j)}{(j_2-J)}, \ (J < j < j_2\right).$$

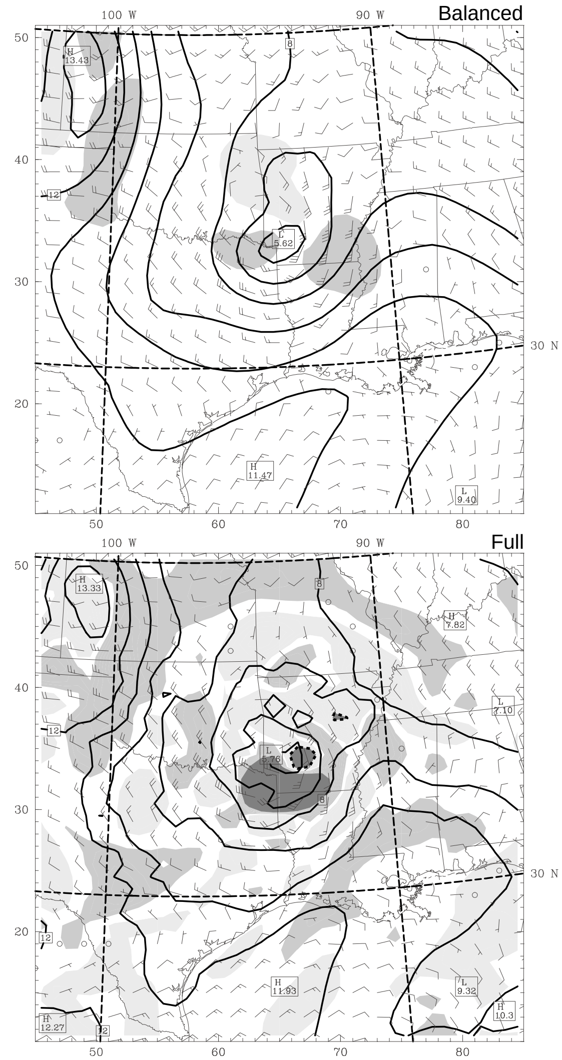

Figure 3. Vertical motion, horizontal wind and temperature at 700 hPa for 0900 UTC 28 may, 1998 (18 h forecast) for (a) balanced flow and (b) total model flow. Both show fields on Domain 1 (40 km) of a 3 domain simulation. Light gray shading is subsidence weaker than 10 cm s^{-}1, medium gray shading indicates ascent weaker than 10 cm s^{-}1. Dark shading in (b) indicates strong vertical motion greater than 10 cm s^{-}1 in magnitude with subsidence denoted by a surrounding dashed contour. Contour interval for temperature is 1 K.

here $J = (j_1 + j_2)/2$. Figure 2 shows a comparison of two balanced initial states, one with the PV anomalies removed. This method differs somewhat from Huo et al. (1999) because here, the full nonlinear inversion problem is solved with the modified PV. Huo et al. (1999) invert just the anomalous part of the flow using a linearized inversion operator, then subtract this perturbation from the full fields. Because of nonlinearity, this does not result in a balanced flow. For synoptic-scale problems where the dynamics are adiabatic to first order, this may be acceptable, but for mesoscale problems where convection triggering is sensitive to the detailed thermal structure, this distinction can be important.

3.3 *Determining the Degree of Imbalance*

For a quantitative determination of the importance of inertia gravity waves in initiating precipitation, or just to isolate these motions, it is necessary to have a framework in which one can separate balance and unbalanced motions. The PV inversion concepts, subject to the constraint of nonlinear balance have been applied by Zhang et al. (2000) for this purpose. In their study, Zhang et al. compared vertical motions, geopotential height and streamfunction derived from PV inversion with the full model output fields to help understand the processes and location of the genesis of large-amplitude inertia-gravity waves.

3.4 *Balanced Motions near Convection*

A profoundly difficult problem is the attempt to understand the role of large-scale weak ascending motion in the modulation of deep convection in a flow in which deep convection is widespread. Any attempt to examine the model vertical motion for weak signals is all but impossible due to the contamination from convection and gravity waves and their aliasing onto larger scales. The approach described herein provides a way to do this, because the PV is largely independent of the convective motions, especially if we adopt the coarse-graining approach to PV inversion, in which we recognize that the balanced part of the flow will be relatively insensitive to small-scale variations in the PV.

In Fig. 3 we show the balanced and full model fields for a mesoscale convective system developing in the presence of a mesoscale vortex. The dark shading in Fig. 3b indicates areas of strong upward and downward motion that are clear signals of the convection that is occurring. Note that this is a 3 domain simulation (40 km, 13.3 km and 4.5 km resolution domains). The balanced fields (calculated on domain 1, 40 km) show weak ascending motion on the southeast and southwest flanks of the vortex, the latter involving isentropic ascent directly into the convective region, suggesting that balanced lifting is playing a role in forcing parcels to their level of free convection.

4. CONCLUSIONS

The techniques described herein are believed to be useful tools for helping to understand numerical simulations produced by mesoscale numerical weather prediction models by attempting to separate slow and fast elements of the flow. The former are obtained through PV inversion and hence retain the conceptual apparatus that has been developed over the years to understand balanced flows. The method makes it possible to perturb the initial state of the model without introducing large imbalances. It also allows one to understand the action of the slow part of the flow in situations where the amplitude of the fast modes (i.e. convection) is of the same order as the balanced part. The PV inversion code can be run with MM5 version 3 model output or input.

Shortcomings of the present method include (a) occasional slow or marginal convergence of the iterations; (b) no compensating adjustment of the moisture field to go with adjustments in the PV and (c) a rather ad-hoc treatment of areas near high topography, where the proper boundary condition at the ground is not incorporated. Improvement in each of these areas is a potentially major endeavor, especially (a) and (c) as little is known about the mathematical properties of the balance equations and how to solve them in the presence of topologically complicated boundary conditions.

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