Table 1. *Properties of the various simulations, where w_{max} is the maximum updraft velocity, and $\theta_{e\text{-}max}$ and is the maximum θ_e below 300 mb. Only the MM5 experiments labeled "new turbulence" can be compared to the ARPS experiments, since they contain similar physics.*

Experiment	w_{max} (m s^{-1})	$\theta_{e\text{-}max}$ (K)
Control	58.83	363.88
Iterative condensation	48.15	358.09
Original condensation, after "sound"	59.41	354.79
Iterative condensation, after "sound"	47.69	350.33
No drag	77.83	365.77
New turbulence code	57.56	354.22
New turbulence code, iterative condensation	44.95	349.64
ARPS, control	58.00	352.81
ARPS, forward-in-time	57.23	349.43
ARPS, iterative condensation	42.49	348.31
ARPS, forward-in-time, iterative condensation	42.31	347.71

ARPS and MM5. The two models were configured to be as similar as possible. All ARPS simulations use a three-dimensional turbulent mixing parameterization similar to the scheme designed for MM5 by Bryan and Fritsch (2000); therefore, the MM5 simulations noted by "new turbulence" in Table 1 are most comparable to the simulations from ARPS. Notice how ARPS also produces the θ_e problem, and that the magnitudes of w_{max} and $\theta_{e\text{-}max}$ are similar. The next two sections elaborate on the two mechanisms that we suspect are the causes of the θ_e problem.

3.1 *Condensation Closure*

The manner in which condensation is treated in MM5 was found to play a major role in the intensity of the θ_e problem. All condensation schemes in MM5 follow the methodology of Soong and Ogura (1973). That is, temperature and moisture are stepped forward in time without consideration of condensation. Then, if the resulting state at a grid point is supersaturated, an "adjustment step" is applied that converts water vapor to liquid water and warms the air. A final relative humidity of 100% is assumed. Typically, modelers apply an equation that uses a single step to approximately reach 100% relative humidity (see Soong and Ogura 1973 for the derivation of this equation). A Taylor series expansion is used to derive this equation, where only the first-order terms are retained. Therefore, this equation only *approximates* the amount of condensation necessary to obtain 100% relative humidity.

By examining the amount of condensation in the warm rain scheme during every time step, we have found that this technique can artificially increase θ_e at a grid point. It seems that the approximations used to derive the one-step condensation equation tend to overestimate the final value of mixing ratio and/or temperature such that the resulting θ_e is too high. The artificial increase in θ_e during condensation is usually small (about 0.01 K), but when applied *every time step* the θ_e problem tends to magnify over time – this is especially true for the very small time steps and grid lengths used at cloud-scale resolution.

Based on plots of $\theta_{e\text{-}max}$ and maximum updraft for different model configurations, and after analysis of tendency terms during model integration, it appears that the θ_e problem is actually a positive feedback loop:

- θ_e inside the cloud increases,
- Higher θ_e results in larger positive buoyancy in the vertical velocity equation,
- The thunderstorm updraft increases,
- More moisture is advected upward, which condenses and releases more latent heat,
- θ_e inside the cloud increases

We have designed an iterative condensation closure that retains the value of liquid-water potential temperature (Tripoli and Cotton 1982). The results using this scheme are compared to the warm rain scheme in Table 1. Notice that the θ_e problem is reduced.

Another problem was found with the manner in which condensation is treated in MM5. The Soong-Ogura condensation procedure should (technically) be applied *after* the model equations are stepped forward. However, in MM5, the relative humidity is set to 100% using the value of pressure at time t-Δt, then pressure is stepped forward in time (in the "sound" subroutine), and the resulting relative humidity from the new temperature *and* the new pressure is greater than (less than) 100% when pressure increases (decreases). We have applied the condensation adjustment step after the "sound" subroutine using both the original warm rain scheme and with our iterative condensation closure. The results are summarized in Table 1; note how this change further decreases the value of $\theta_{e\text{-}max}$.

3.2 *Time Integration*

MM5 (and many other models) use centered-in-time (leapfrog) time integration for advection:

$$\alpha^{t+\Delta t} = \hat{\alpha}^{t-\Delta t} + 2\Delta t \frac{\partial \alpha}{\partial t} ,$$