l is a mixing length scale, R_i is the Richardson number, and R_{ic} is a critical Richardson number. Notice that (5)-(7) do not consider the effects of horizontal shear on the generation of turbulence. Other PBL schemes in MM5 also fail to use a three-dimensional shear production term for the parameterization of turbulent activity.

2.2 *Turbulence Closure for a Cloud Model*

We have designed a turbulence scheme for MM5 that has been used in several other cloud models (e.g., Klemp and Wilhelmson 1978, Redelsperger and Sommeria 1984, Xue et al. 1995). The turbulent fluxes for potential temperature, water vapor, and liquid water constituents are expressed in the form:

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x_i}\left(K_h \frac{\partial \phi}{\partial x_i}\right), \qquad (8)$$

where K_h is the eddy mixing coefficient for heat and moisture. The turbulent fluxes for momentum are given by:

$$\frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x_j}\left(K_m\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right) \qquad (9)$$

where K_m is the eddy mixing coefficient for momentum. A key difference between this formulation and codes available in MM5 is the turbulent mixing of vertical momentum ($u_3 = w$), which is currently ignored in MM5.

The eddy mixing coefficients are determined by the subgrid-scale turbulence kinetic energy (E):

$$K_m = C_m E^{\frac{1}{2}} l, \qquad (10)$$

$$K_h = \frac{K_m}{P_r}, \qquad (11)$$

where C_m is a constant, $l = (\Delta x \Delta y \Delta z)^{\frac{1}{3}}$, and P_r is the Prandtl number (determined in this code following the methodology of Deardorff 1980). The following prognostic equation (Klemp and Wilhelmson 1978) is used to determine E:

$$\frac{\partial E}{\partial t} = -\frac{\partial u_i E}{\partial x_i} + E\frac{\partial u_i}{\partial x_i} + \frac{\partial u_i}{\partial x_j} K_m \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + B + \frac{\partial}{\partial x_i}\left(K_m \frac{\partial E}{\partial x_i}\right) - \frac{C_\varepsilon}{l} E^{\frac{3}{2}} + D_E \qquad (12)$$

where B is the buoyancy term, C_ε is a constant, and D_E represents the fourth-order computational diffusion scheme in MM5 (used to control small-wavelength noise in the model). The first two terms on the right of (12) represent the flux-form advection of E, the third term is the shear production term, the fourth term is buoyancy, the fifth term is the turbulent flux divergence of E, and the sixth term is the dissipation term.

3. IDEALIZED THUNDERSTORM SIMULATIONS

The first set of simulations presented in this paper is performed with 1 km horizontal and 250 m vertical grid spacing. The constant vertical grid spacing ensures that l remains independent of height. The warm rain microphysics scheme is used. Surface fluxes of heat, moisture, and momentum, the Coriolis terms, curvature terms in the momentum equations, radiation, and map scale factors are ignored.

The analytic temperature and moisture profiles from Weisman and Klemp (1982) are used to define a horizontally homogeneous initial state. The initial wind profile is unidirectional, with wind speed increasing linearly from –5 m s^{-1} at the surface to +5 m s^{-1} at 10 km. Clouds are initiated with a warm bubble that has a temperature perturbation of 1 K at the center.

The turbulence closure denoted by equations (5)-(7) will hereafter be referred to as the Richardson number (R_i) based turbulence closure. This scheme is used above the planetary boundary layer (PBL) whenever the user selects the Blackadar PBL option, or the MRF PBL option. The new code (section 2.2) will be referred to as the turbulence kinetic energy (TKE) based turbulence closure. Note that, since radiation and surface fluxes of heat are ignored, there is no well-mixed boundary layer in these simulations, i.e., the vertical mixing is handled by the R_i scheme or the TKE scheme over the entire depth of the model.

Figure 1 shows K_m fields produced by the two turbulence schemes. The K_m field in Fig. 1a was never actually used in a simulation. Rather, in order to compare it to a K_m field generated by using the TKE scheme, the K_m field from the R_i based scheme (Fig. 1a) was computed from the three-dimensional fields that were output from the simulation that used the TKE scheme. Note how the TKE scheme generates more turbulence (i.e., more mixing) on the sides of the cloud. Furthermore, K_m from the R_i scheme tends to exist in thin vertical layers; thus, mixing from the R_i scheme is confined to small depths in the vertical direction.

Figures 2 and 3 show the effects of the two different methods of computing K_m when each scheme is applied in a cloud-scale simulation.