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Two Approaches for Studying Cold Pools in Shear

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National Center for Atmospheric Research

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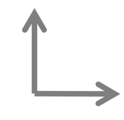
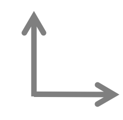
Portland, OR
10 November 2016

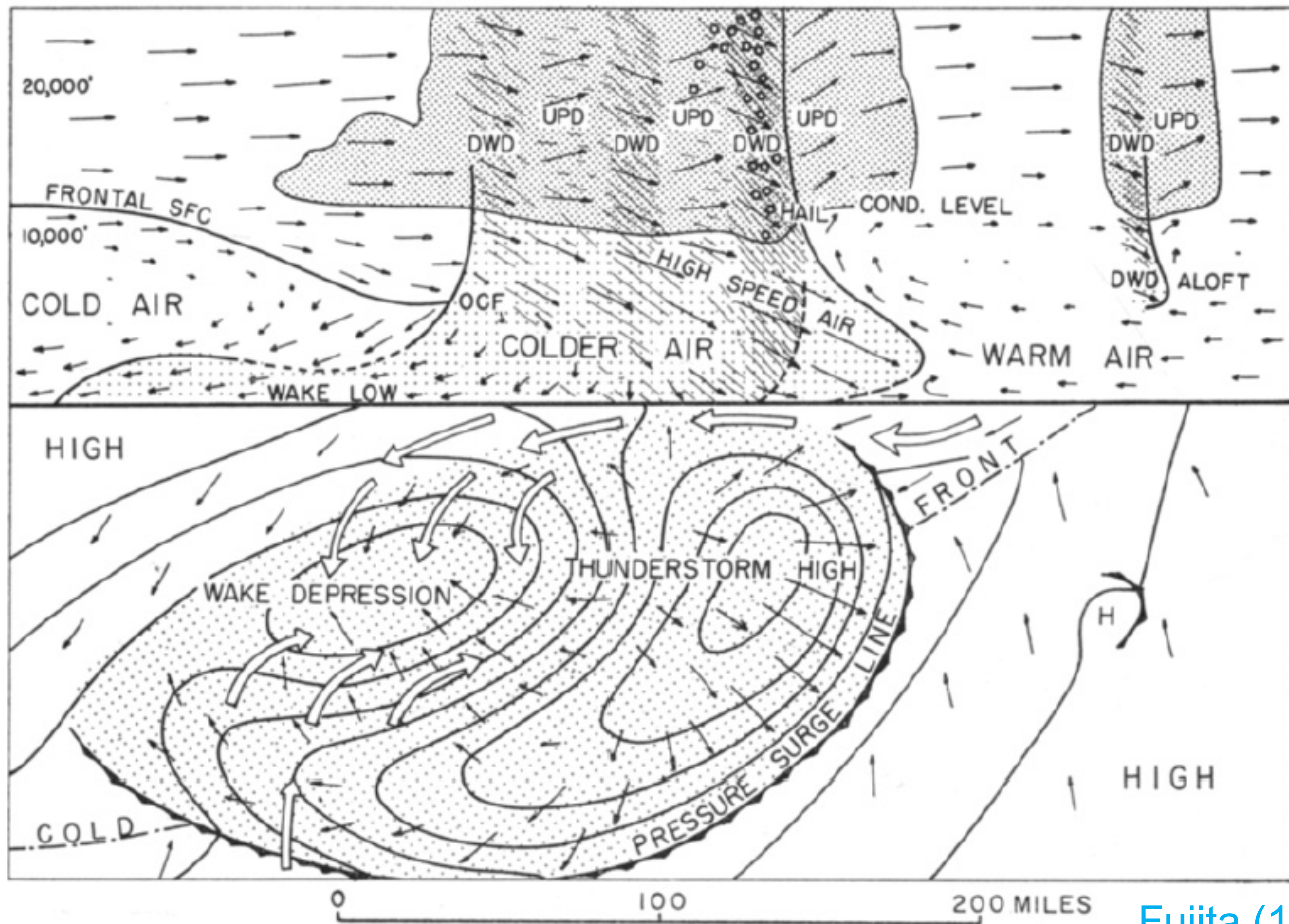
Cold Pools Underneath Thunderstorms

vertical cross section:



surface:

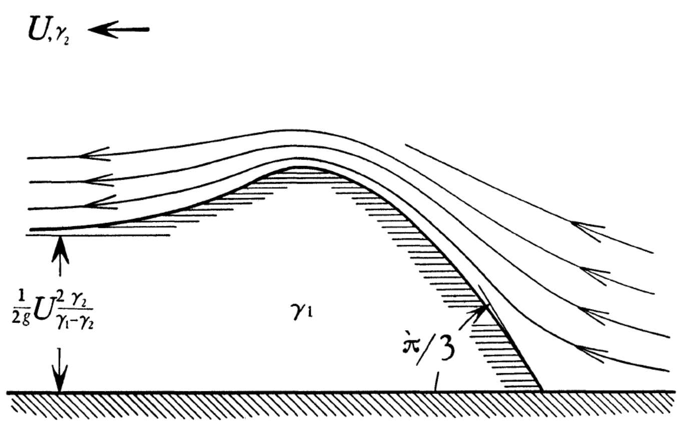




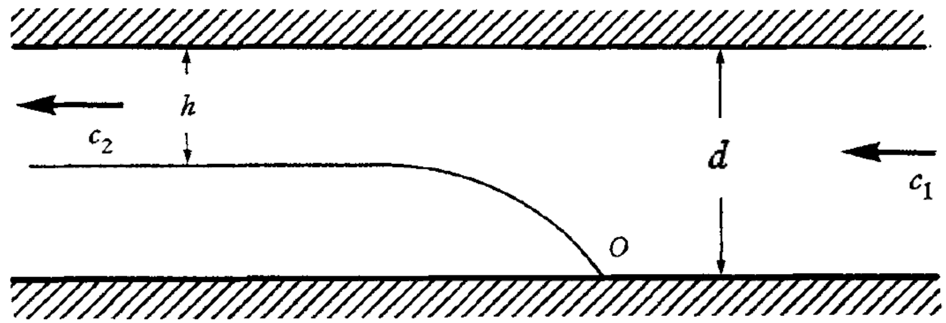
Fujita (1955)

- Cold air plays a role in severe wind production

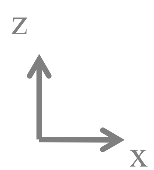
Theoretical Studies of Gravity Currents



von Karman (1940)



Benjamin (1968)



- Clearly quantifies the effects of cold air: $C = k\left(g'h\right)^{1/2}$
- But these classic studies lack environmental shear ….

This Talk:
A Review of Two Theoretical Approaches to Environmental Shear

Vertically confined flow

Bryan and Rotunno (2014b)

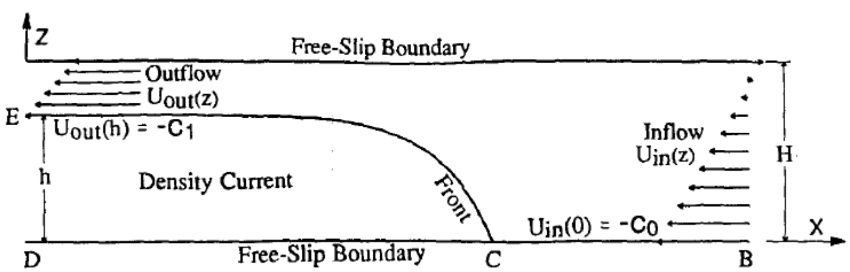
Vertically unconfined flow

Bryan and Rotunno (2014a)

This Talk:
A Review of Two Theoretical Approaches to Environmental Shear

Vertically confined flow

- Originated with Benjamin (1968) (without shear)
- Xu (1992), Xu et al. (1996), Xue (2000), etc, added shear



Xu (1992)

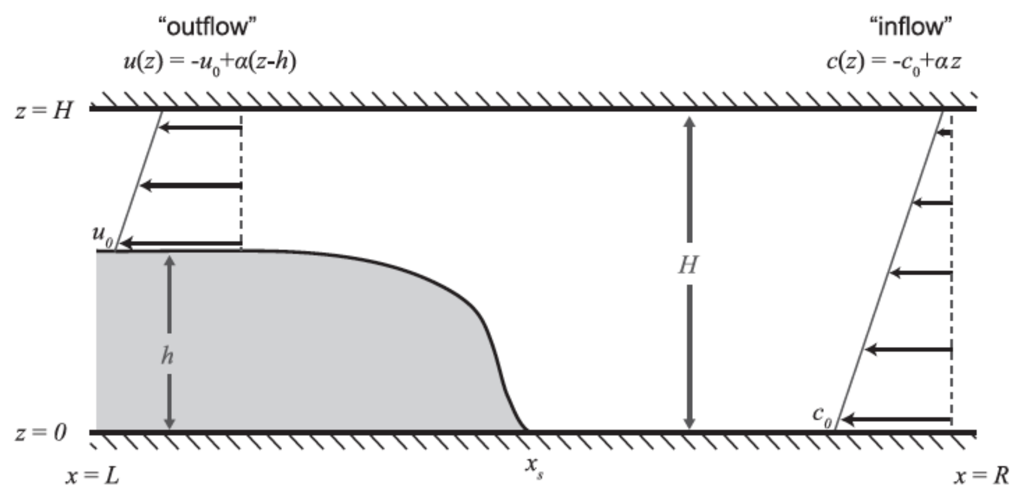
Vertically unconfined flow

(later on)

Gravity Currents with Shear in a Confined Channel

- Step 1. Assume this:

Note: shear is quantified by α (and assumed constant)



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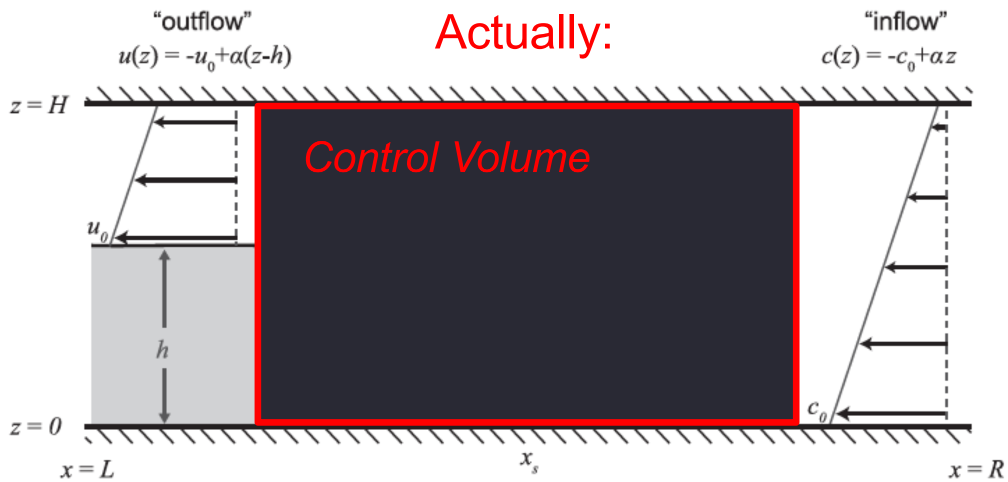

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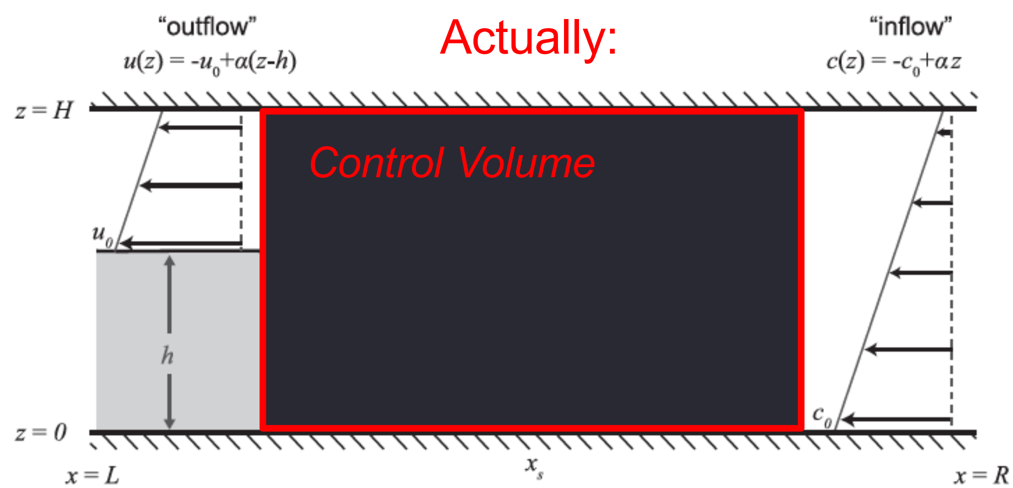

- Step 2. Assume steady flow; integrate the mass-continuity and u-velocity equations over this volume.

Gravity Currents with Shear in a Confined Channel

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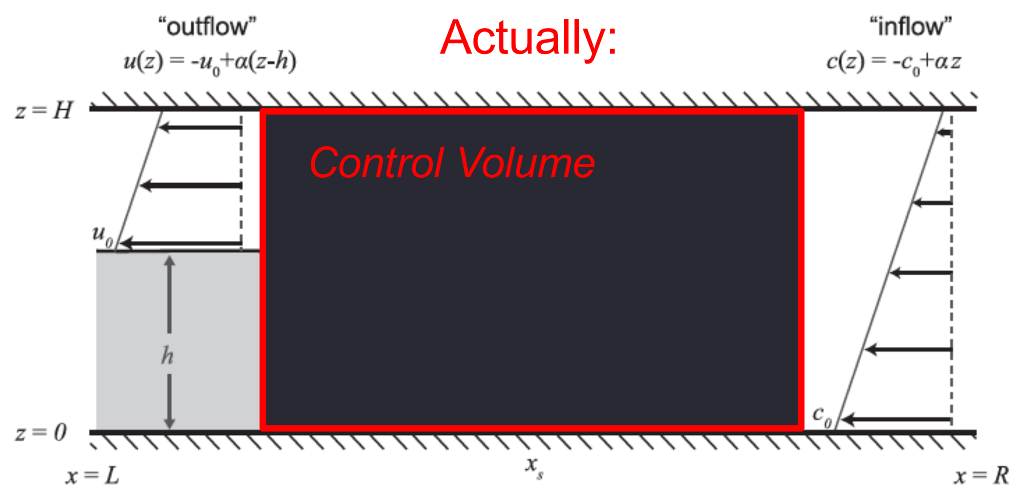

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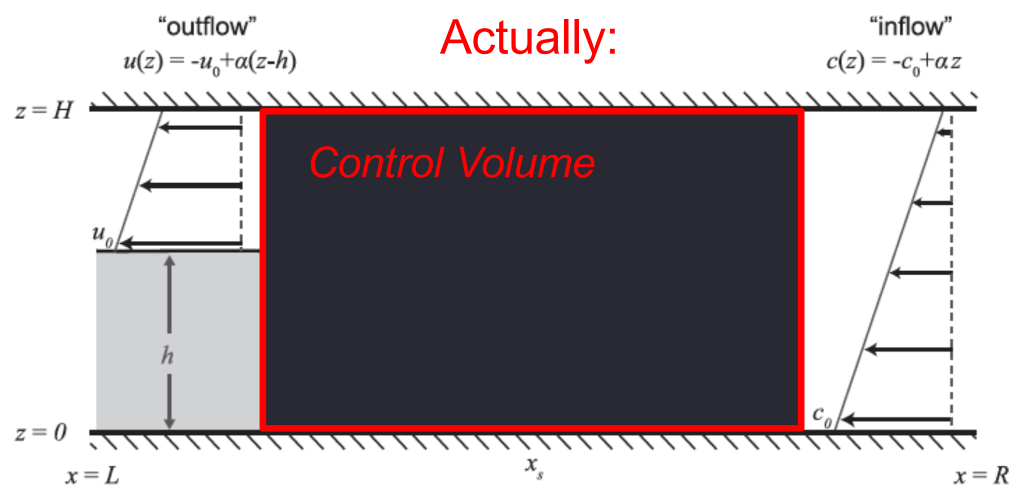

- Step 2. Assume steady flow; integrate the mass-continuity and u-velocity equations over this volume. Do some math….

$$\int (p'_{\text{in}} + u^2_{\text{in}})dz = \int (p'_{\text{out}} + u^2_{\text{out}})dz, \quad (2.8a)$$

or

$$\underset{\text{I}}{\int u^2_{\text{in}} dz} = \underset{\text{II}}{\int u^2_{\text{out}} dz} + \underset{\text{III}}{\int p'_{\text{E}} dz} + \underset{\text{IV}}{\int (p'_{\text{out}} - p'_{\text{E}})dz} \quad (2.8b)$$

$$A_2\alpha^2 + A_1\alpha + A_0 = 0, \quad (2.9a)$$

where

$$A_2 \equiv (1 - h_e{}^2)^2/8 + (1 - h_e{}^3)/3 - (1 - h_e{}^2)/2, \quad (2.9b)$$

$$A_1 \equiv -c_1 h_e h^2/2, \quad (2.9c)$$

$$A_0 \equiv h^2(h - 1/2), \quad (2.9d)$$

Gravity Currents with Shear in a Confined Channel

- Step 1. Assume this:

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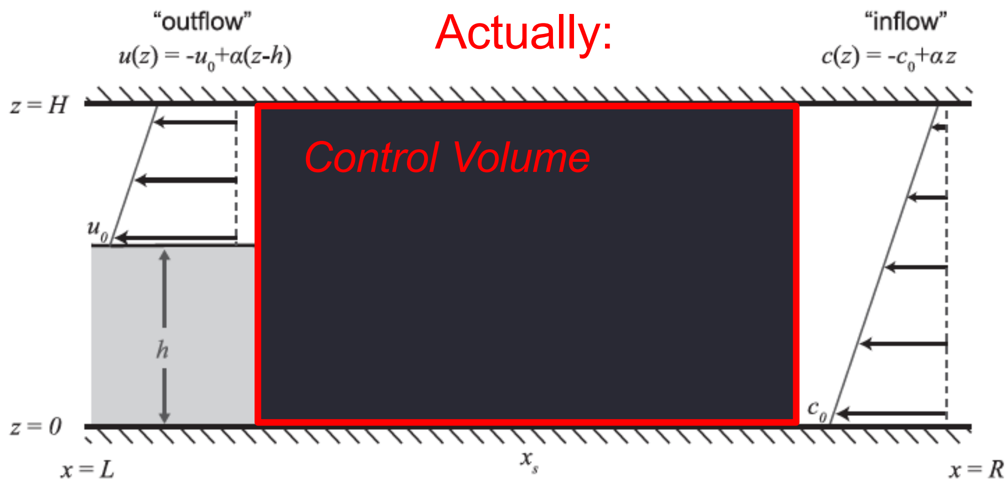


- Step 2. Assume steady flow; integrate the mass-continuity and u-velocity equations over this volume. Do some math….

$$\int (p'_{\mathrm{in}} + u_{\mathrm{in}}^2)\,dz = \int (p'_{\mathrm{out}} + u_{\mathrm{out}}^2)\,dz, \quad (2.8a)$$

or

$$\underbrace{\int u_{\mathrm{in}}^2\,dz}_{\mathrm{I}} = \underbrace{\int u_{\mathrm{out}}^2\,dz}_{\mathrm{II}} + \underbrace{\int p_{\mathrm{E}}\,dz}_{\mathrm{III}} + \underbrace{\int (p'_{\mathrm{out}} - p'_{\mathrm{E}})\,dz}_{\mathrm{IV}} \quad (2.8b)$$

$$A_2\alpha^2 + A_1\alpha + A_0 = 0 \quad (2.9a)$$

where

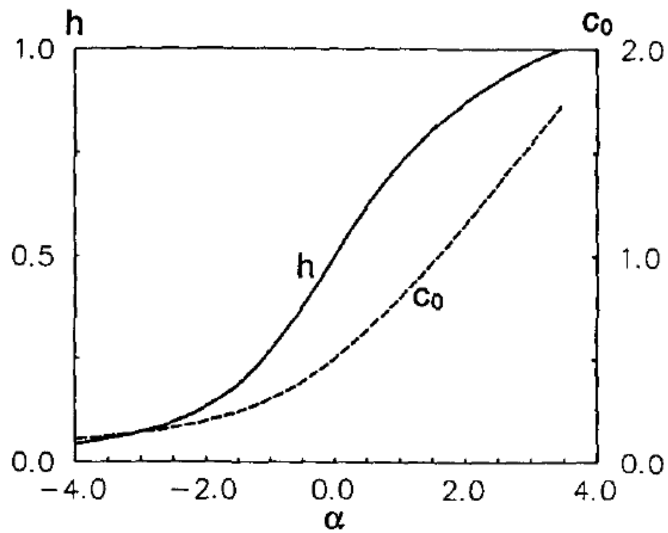
$$A_2 \equiv (1 - h^2)/8 + (1 - h_e^3)/3 - (1 - h_e^2)/2, \quad (2.9b)$$

$$A_1 \equiv -c_0 h_e h/2, \quad (2.9c)$$

$$A_0 \equiv h^2(h - 1/2), \quad (2.9d)$$

Step 3. Plot some results

- As shear (α) increases: cold-pool depth (h) and propagation speed (c_0) must increase



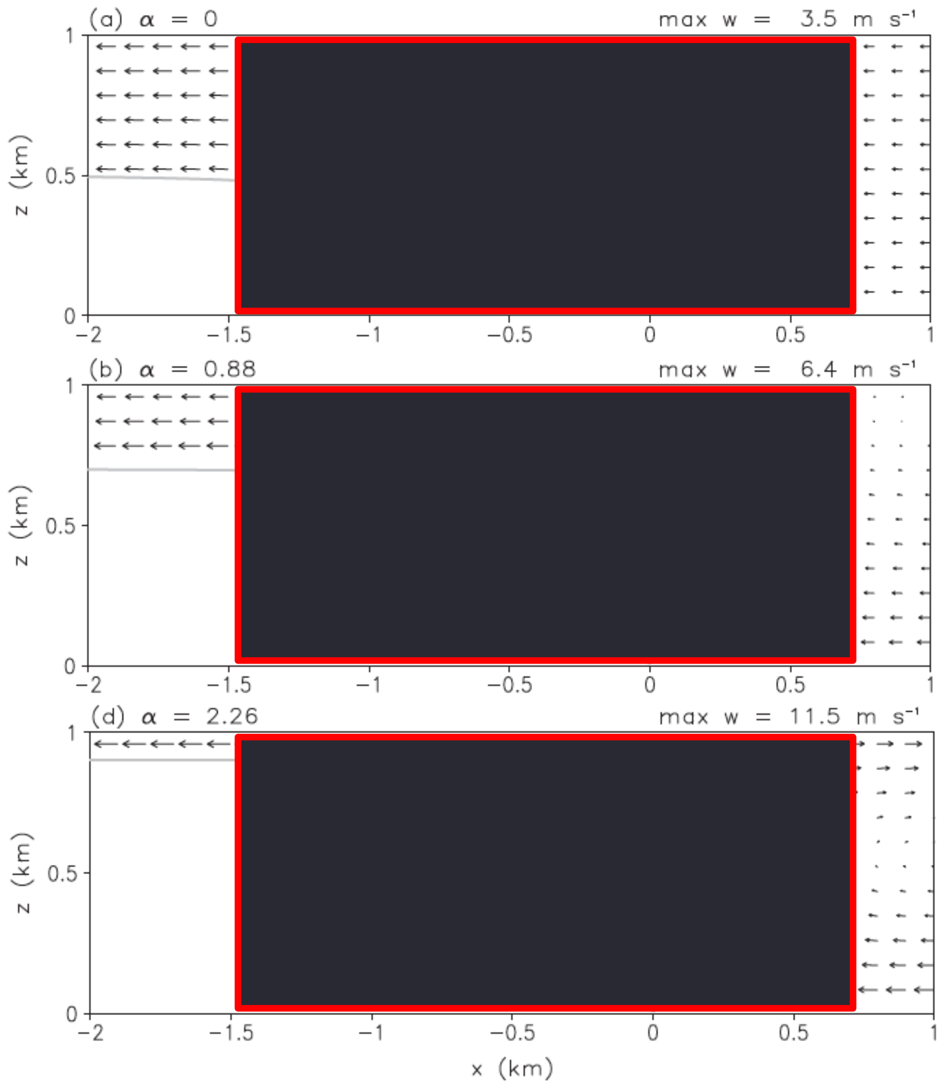
Xu (1992)

Step 3b. Plot some results: "inflow" (right-hand side) and "outflow" (left-hand side)

No shear:
(Benjamin 1968)

Moderate shear:
(Xu et al. 1992)

Strong shear:
(Bryan and Rotunno 2014b)



Step 4. For inviscid flow, solve for interior flow / shape of cold pool

No shear:
(Benjamin 1968)

Moderate shear:
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Strong shear:
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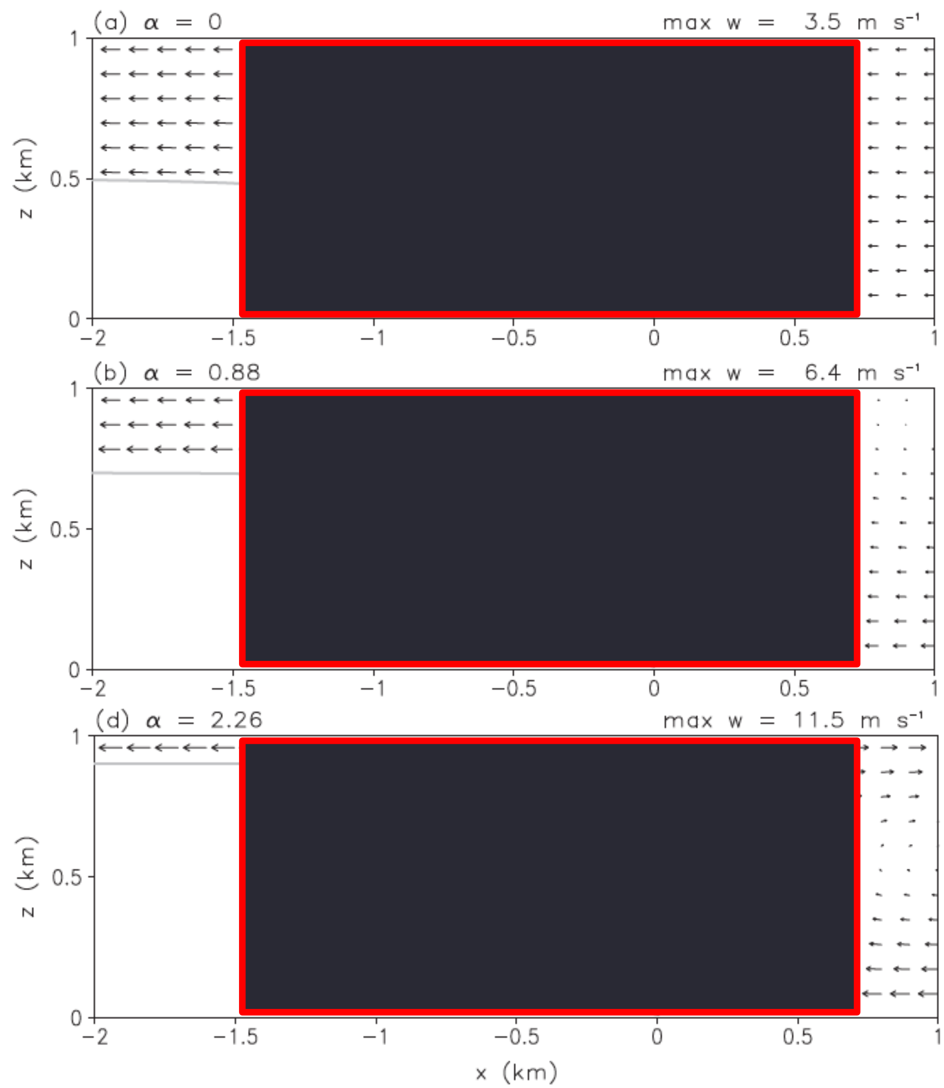

contours:
w (c.i. = 1 m s^{-1})

Step 4. For inviscid flow, solve for interior flow / shape of cold pool

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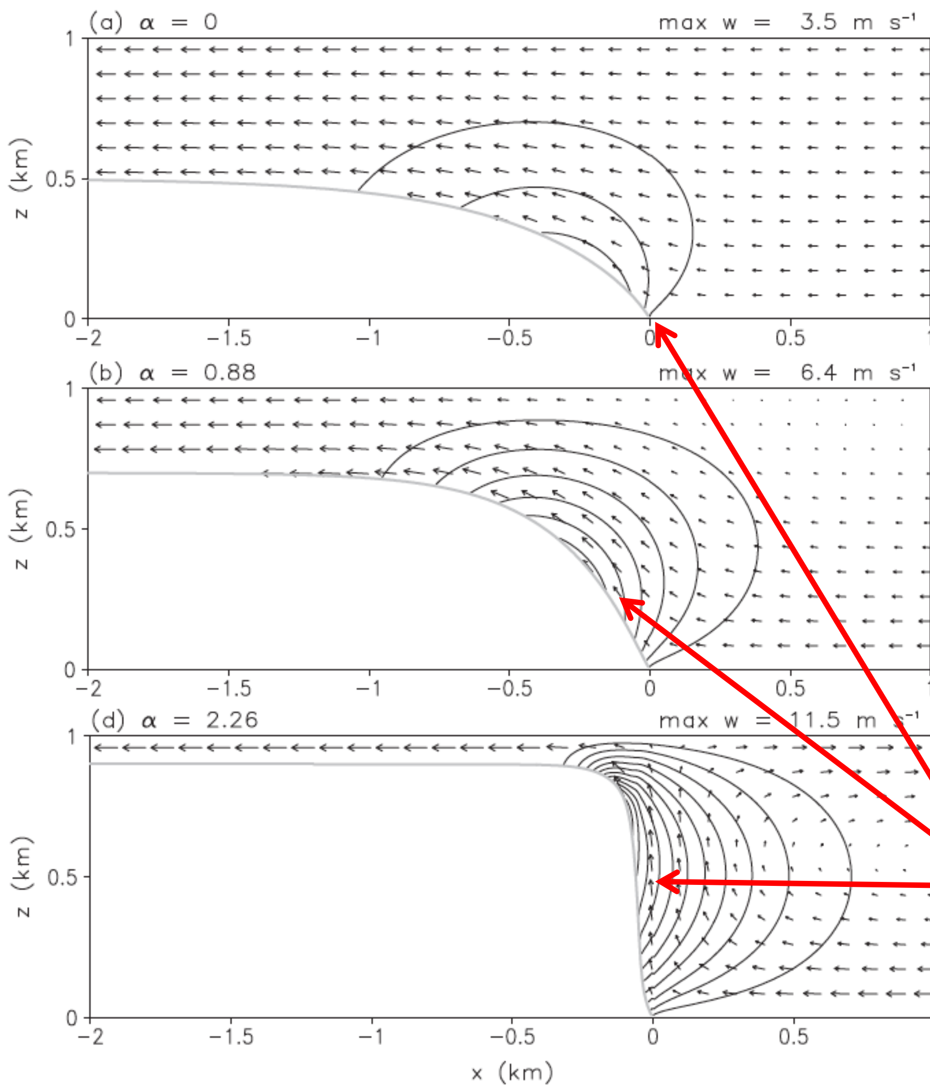
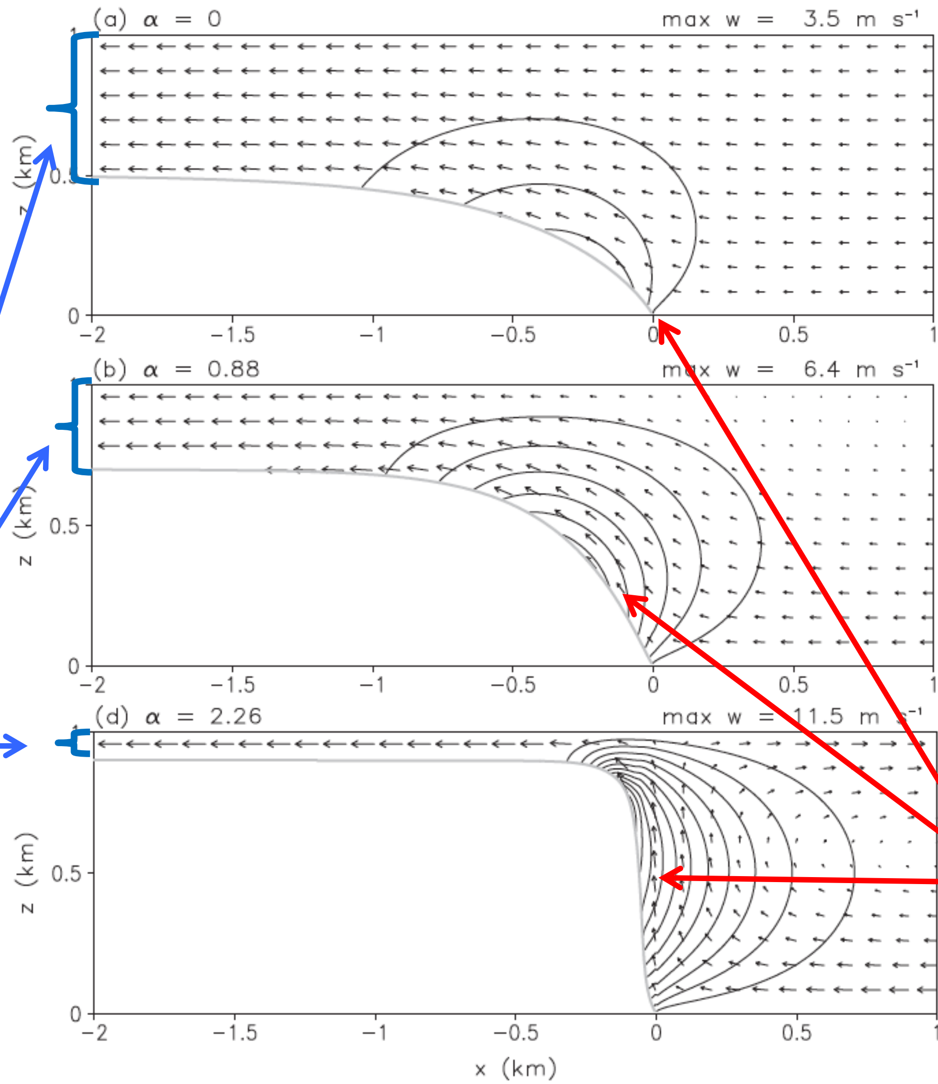


contours:
w (c.i. = 1 m s^{-1})

As shear (α) increases, edge of cold pool becomes steeper

Step 4. For inviscid flow, solve for interior flow / shape of cold pool

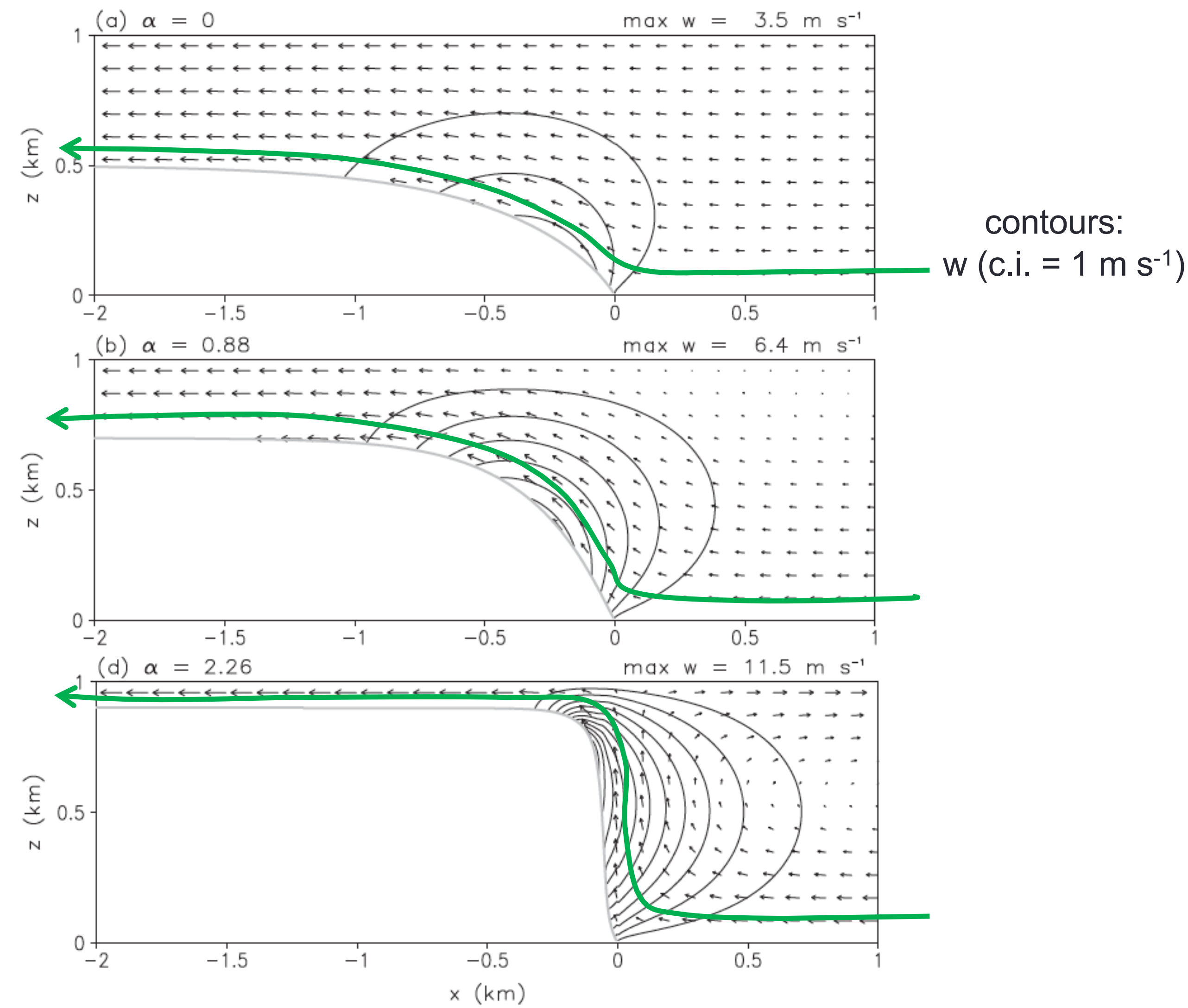


contours:
w (c.i. = 1 m s^{-1})

As shear (α) increases, flow squeezes through a narrower layer

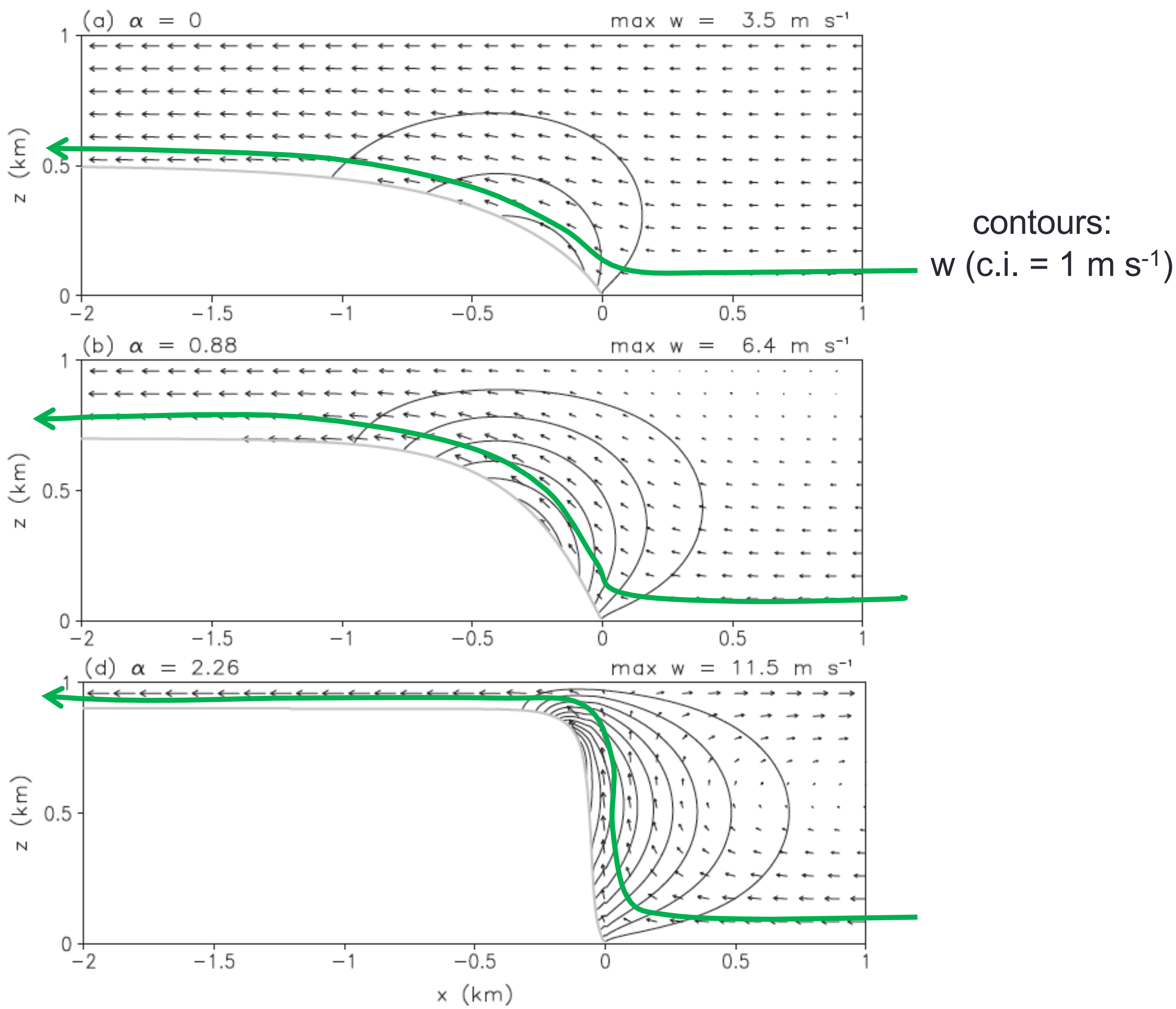
As shear (α) increases, edge of cold pool becomes steeper

Step 4. For inviscid flow, solve for interior flow / shape of cold pool



- Vertical displacement (δ) of near-surface air increases monotonically with shear

Step 4. For <u>inviscid flow</u>, solve for interior flow / shape of cold pool

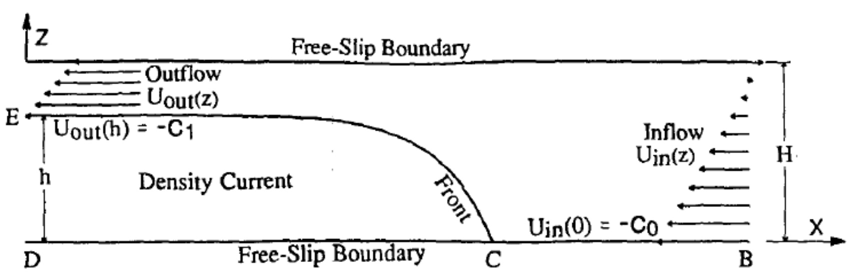


- Vertical displacement (δ) of near-surface air increases monotonically with shear
- Is this what happens in the atmosphere?

Two Approaches to Environmental Shear

Vertically confined flow

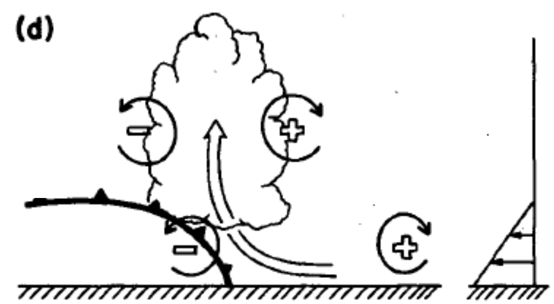
- Originated with Benjamin (1968) (without shear)
- Xu (1992), Xu et al. (1996), Xue (2000), etc, added shear



Xu (1992)

Vertically unconfined flow

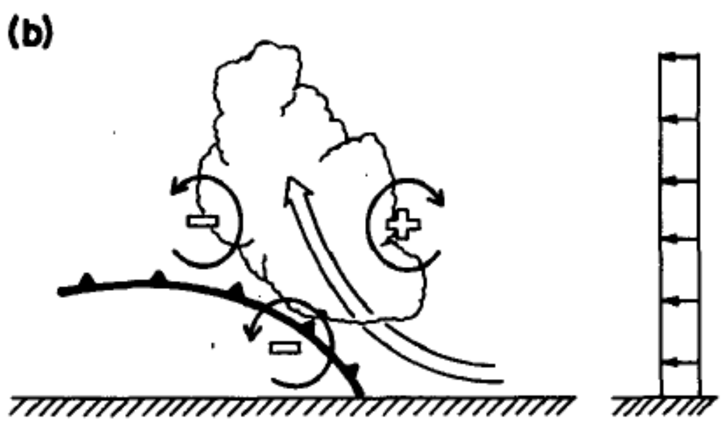
- Originated with Rotunno, Klemp, and Weisman (1988)
- First to include shear analytically



Rotunno et al. (1988)

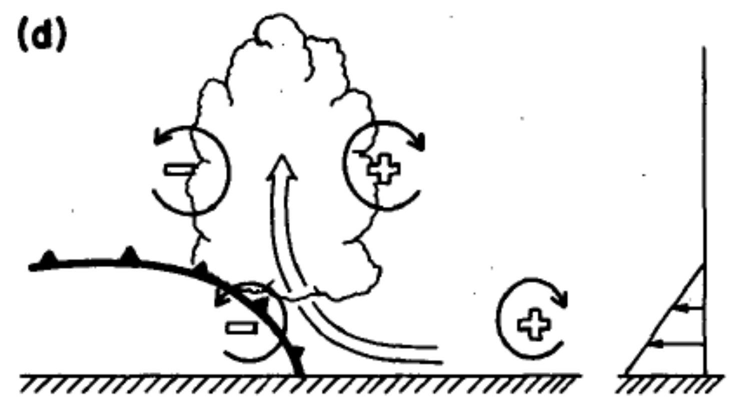
Motivated by Simulations of Squall Lines

Without shear



Air flows over top of cold pool
(as in Benjamin 1968)

With shear

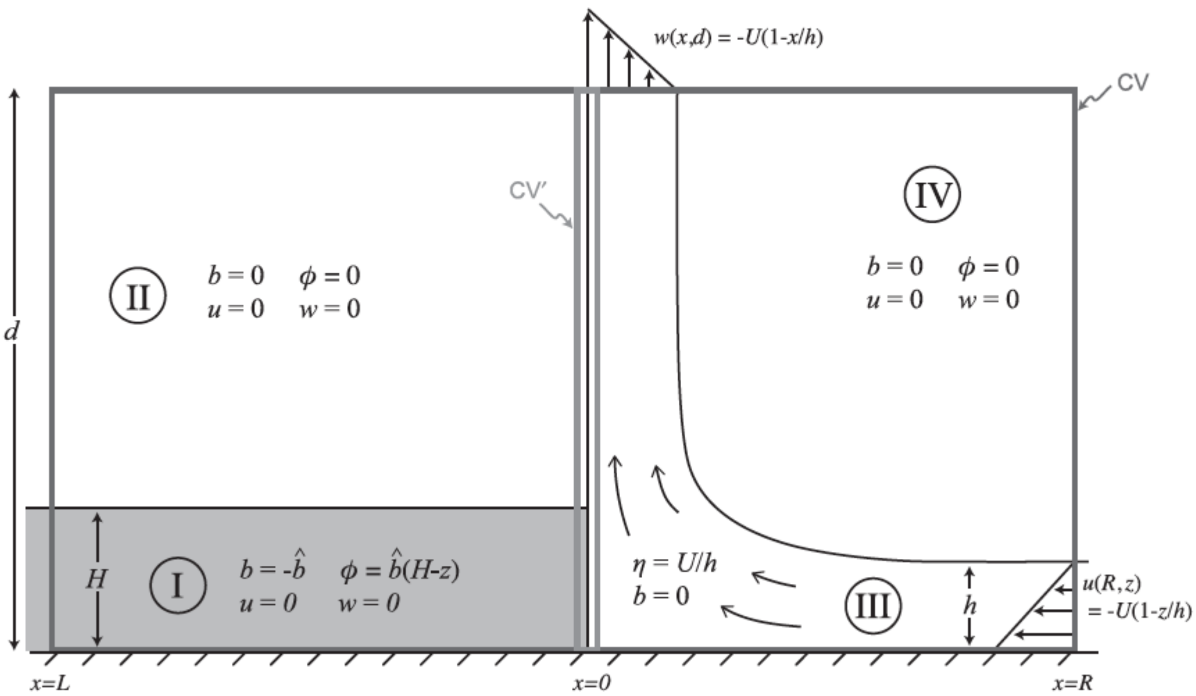


Air separates from cold pool,
goes straight up

Requires a different approach…

Gravity Current with a "Vertically oriented jet"

- Step 1. Assume this:

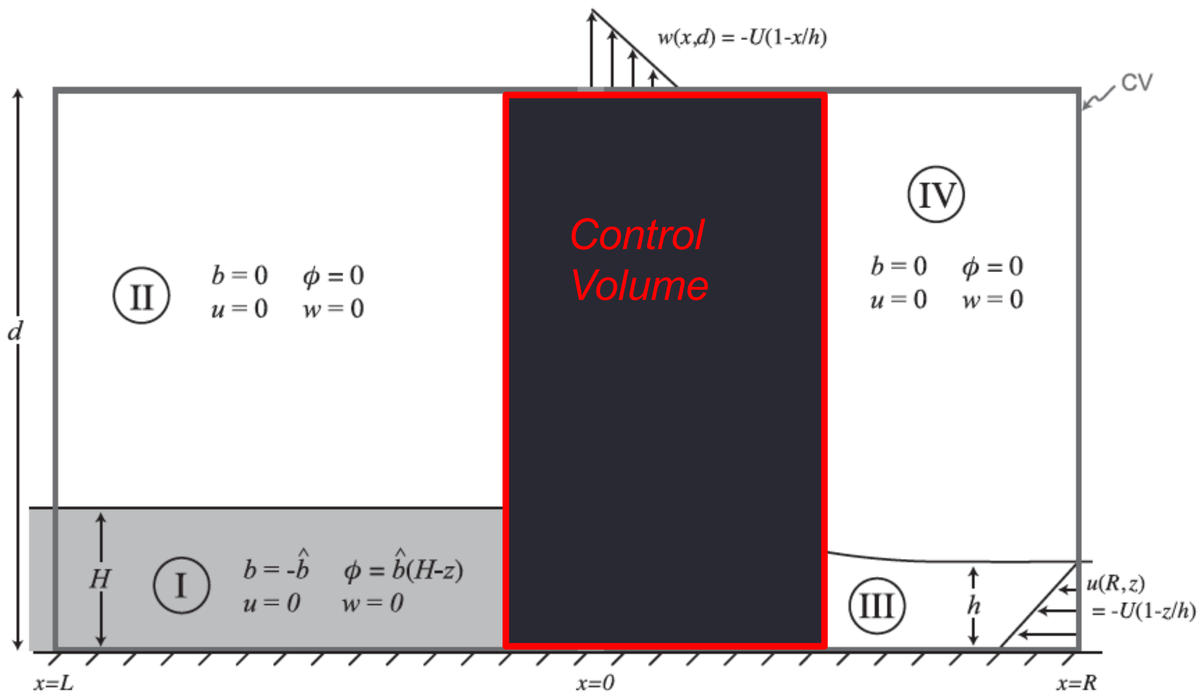


Bryan and Rotunno (2014a)

Gravity Current with a "Vertically oriented jet"

- Step 1. Assume this:

Actually:



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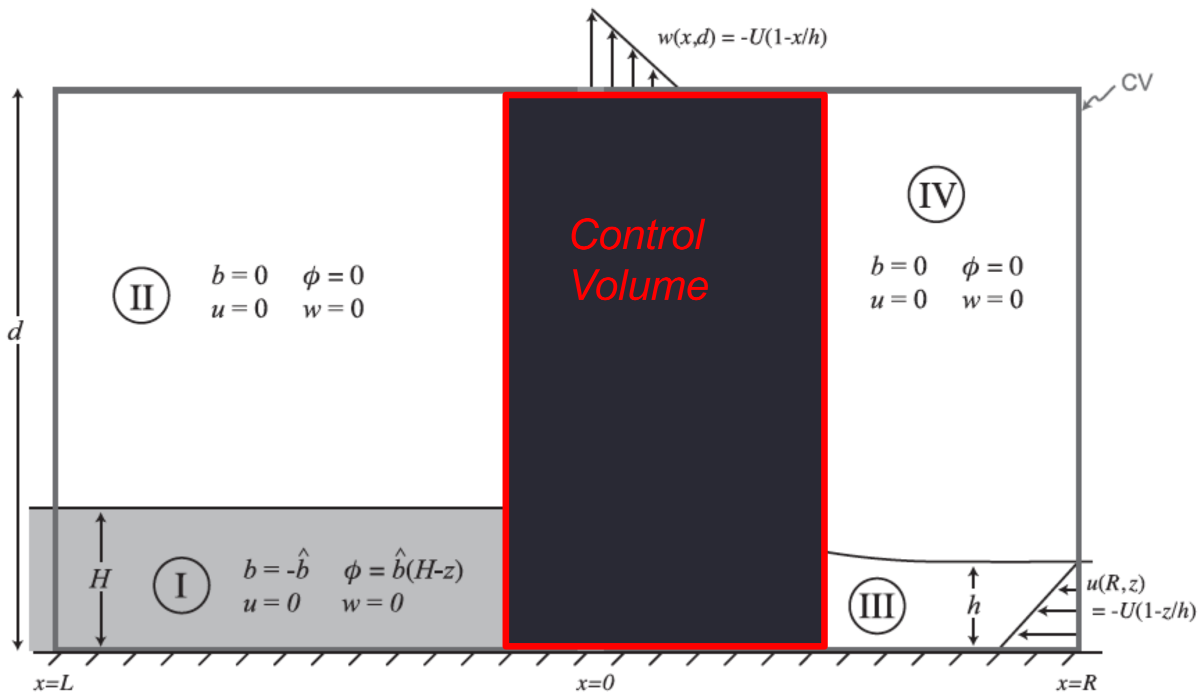
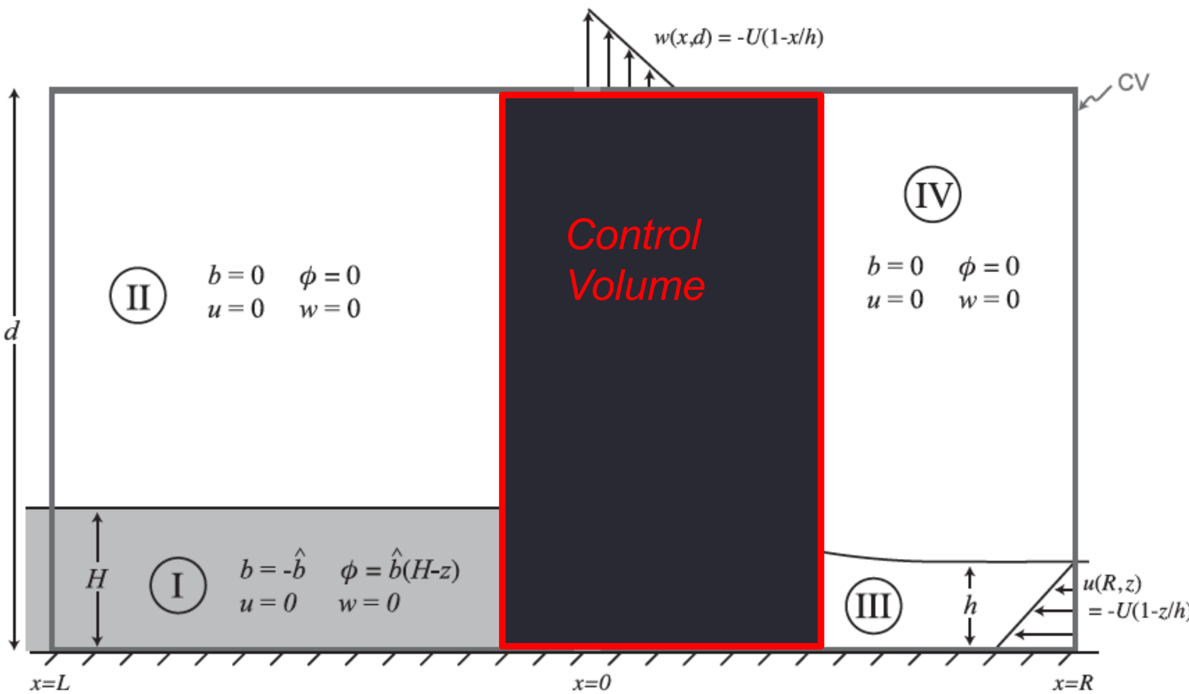


- Step 2. Integrate vorticity equation over the control volume:

$$U = (2\hat{b}H)^{1/2}$$

Gravity Current with a "Vertically oriented jet"

- Step 1. Assume this: Actually:



- Step 2. Integrate vorticity equation over the control volume:

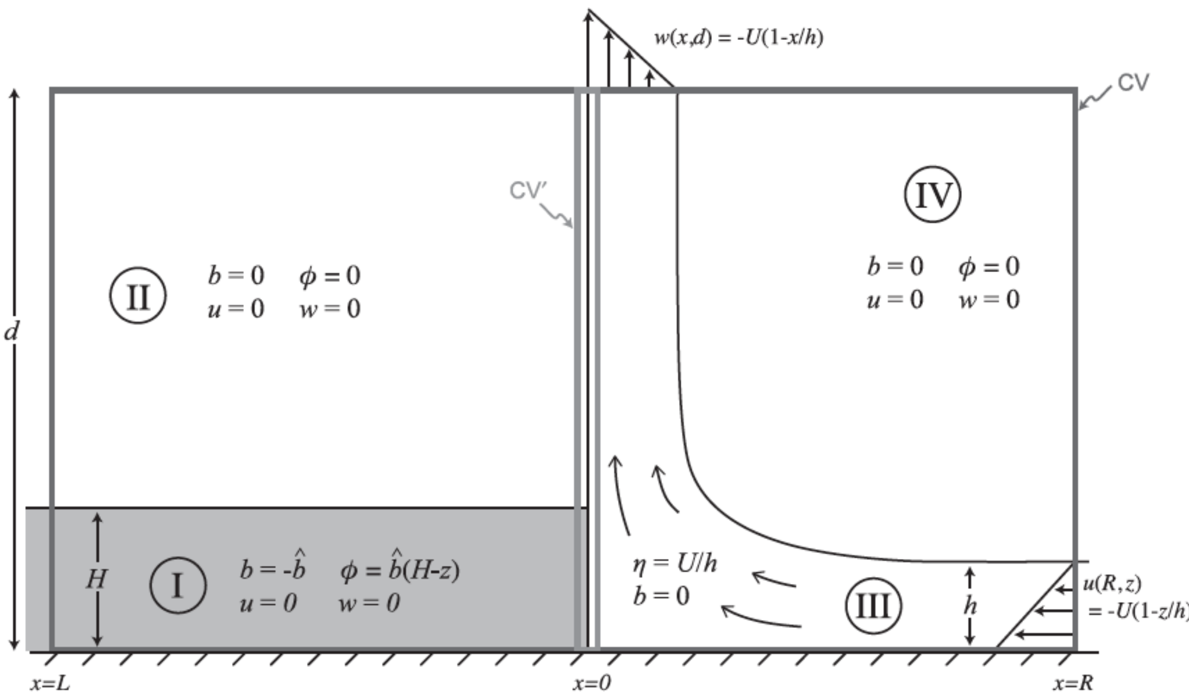
$$U = (2\hat{b}H)^{1/2}$$

- Step 3. Integrate mass-continuity and u-velocity equations:

$$h/H = 3/4$$ * For linear u(z) profile

Gravity Current with a "Vertically oriented jet"

- Step 1. Assume this:



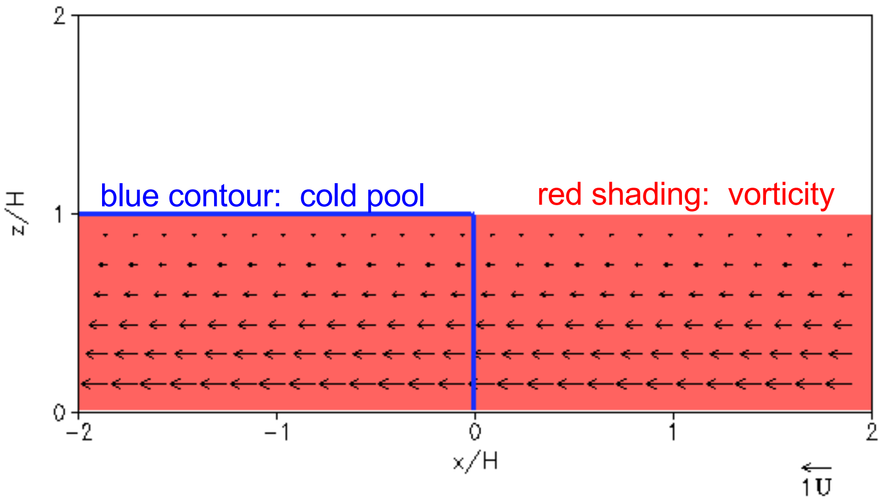
Bryan and Rotunno (2014a)

Two criteria:

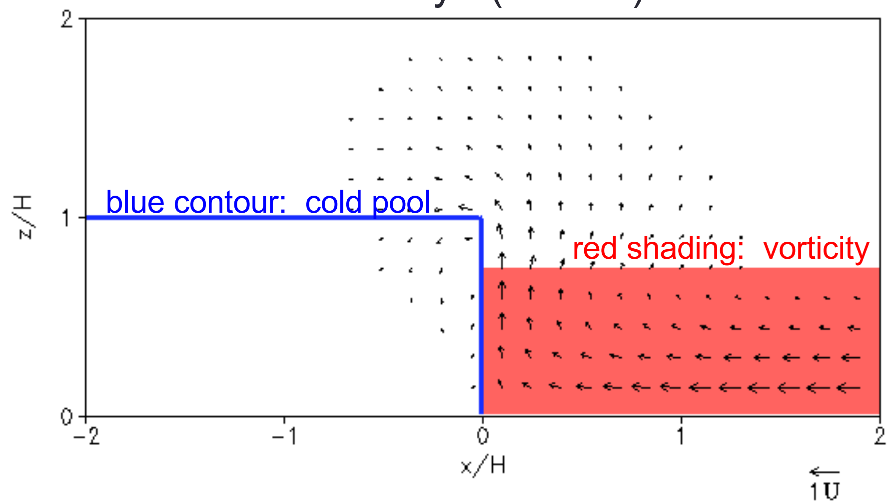
$$U = (2\hat{b}H)^{1/2} \qquad h/H = 3/4$$

Initial Conditions for a Numerical Simulation

Old way: (RKW88, WR04)



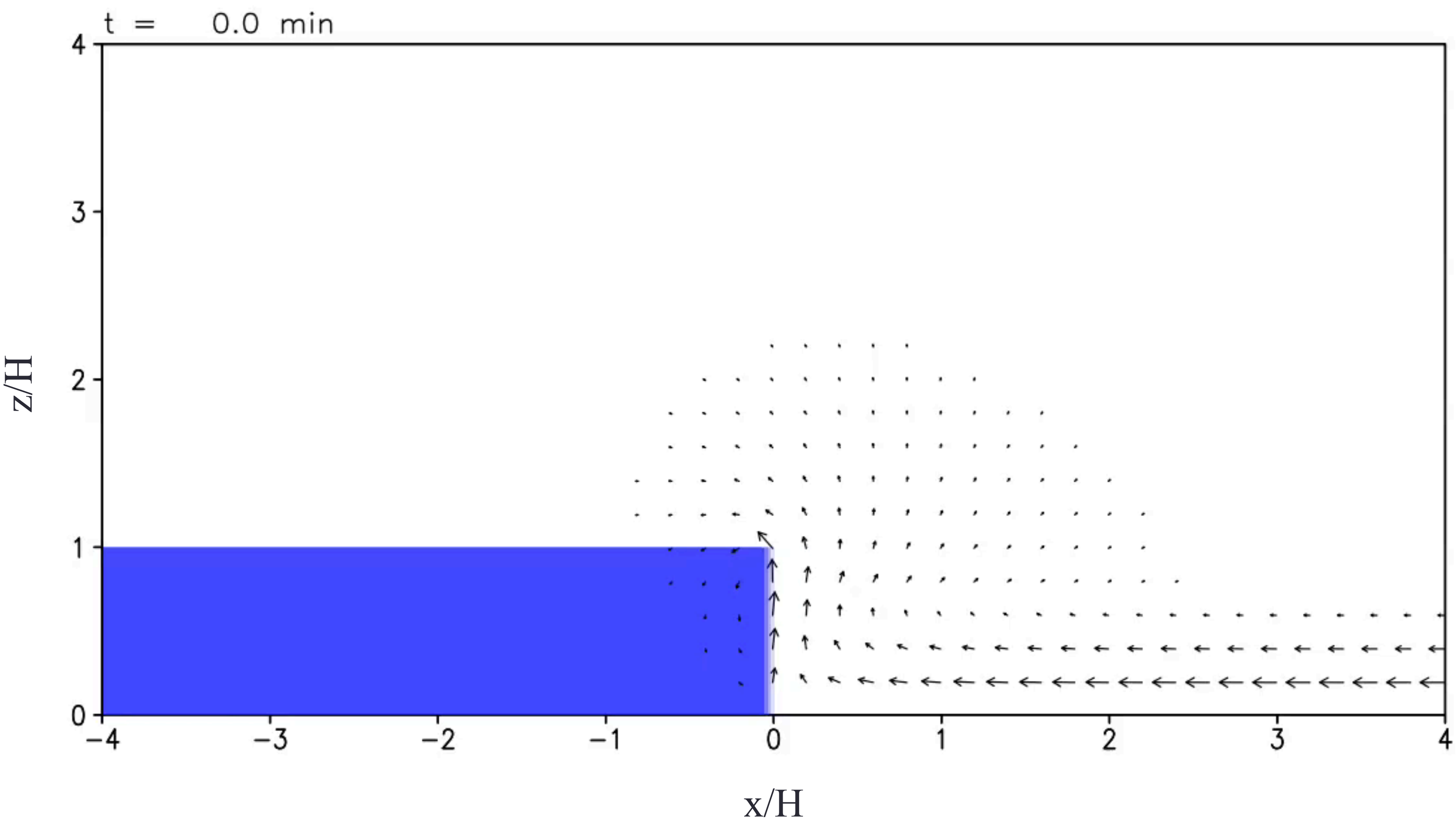
New way: (BR14a)



- Simulations using CM1:
  - 2D, $\Delta x = \Delta z = 31$ m; 50 km x 20 km domain
  - No moisture
  - Direct numerical simulations, Re = 10,000

Simulation with necessary conditions for vertically oriented jet

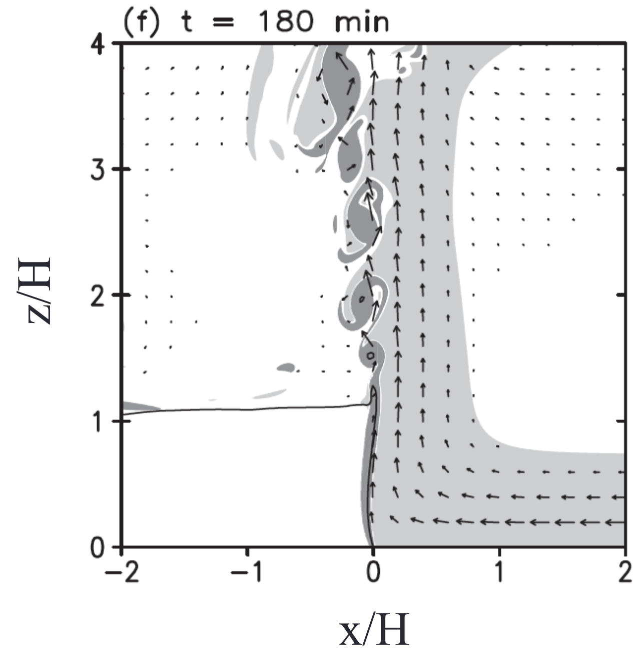
$$U = (2\hat{b}H)^{1/2} \qquad h/H = 3/4$$



Simulation with necessary conditions for vertically oriented jet

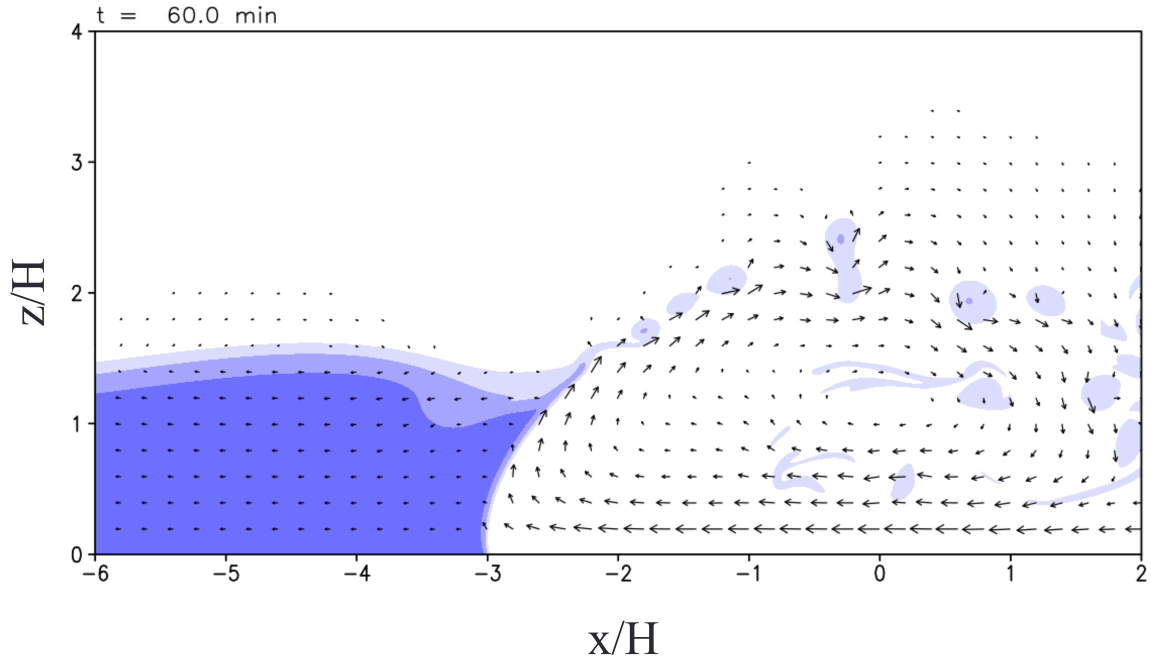
$$U = (2\hat{b}H)^{1/2} \qquad h/H = 3/4$$

2 hours later:

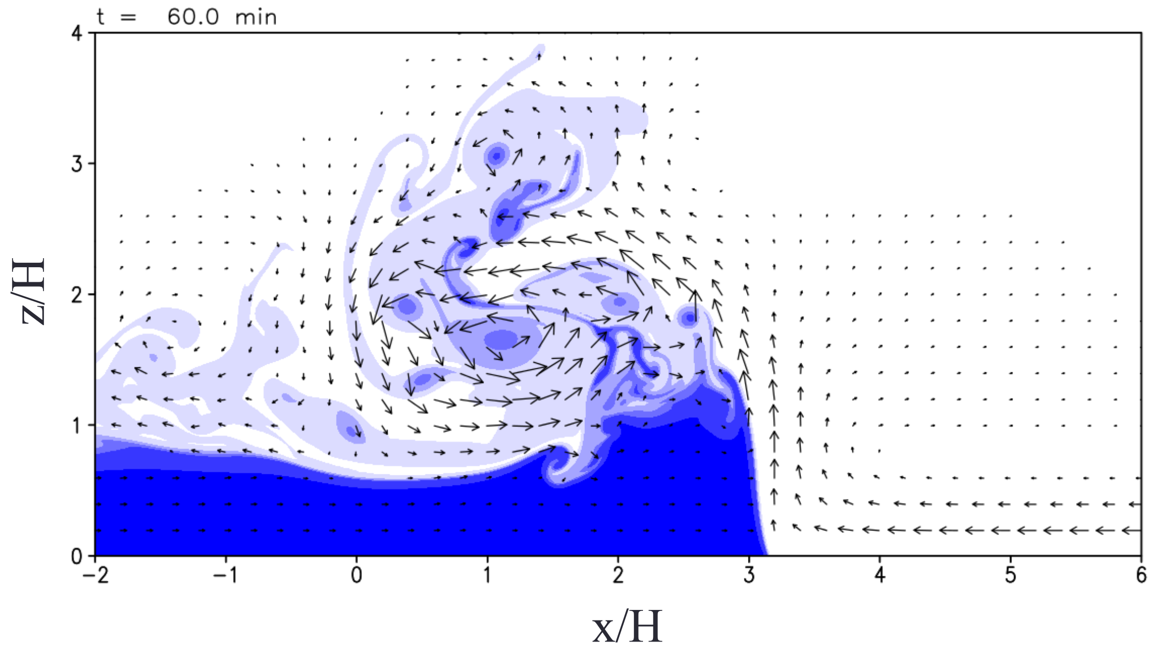


Exactly the same, except:

warmer cold pool
(C < ΔU)

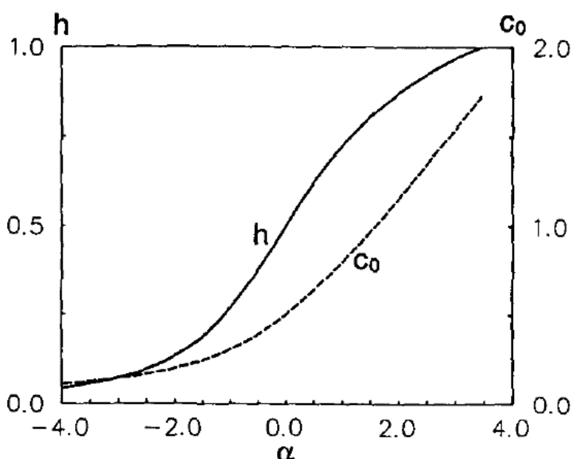


colder cold pool
(C > ΔU)



Vertically confined flow

- As shear increases, vertical displacements (δ) increases

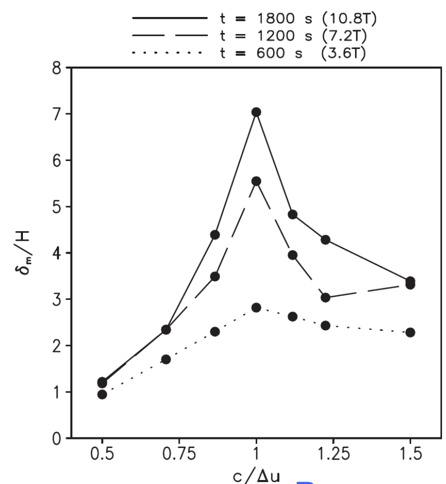


Xu (1992)

- Maximum vertical displacement: depth of cold pool (h)

Vertically unconfined flow

- Vertical displacement (δ) is maximized for a certain set of conditions (C = ΔU)



Bryan and Rotunno (2014a)

- Maximum vertical displacement: unbounded/infinite

Vertically confined flow

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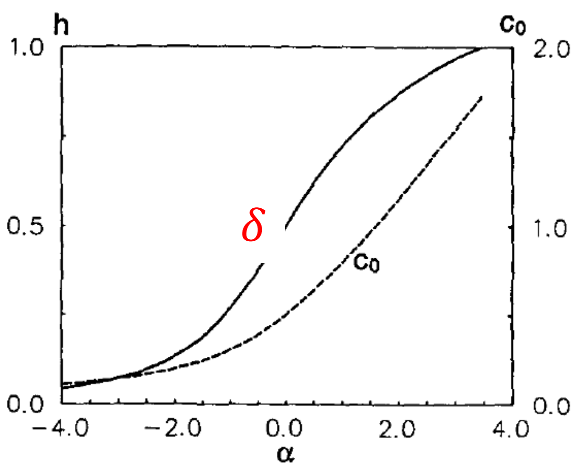

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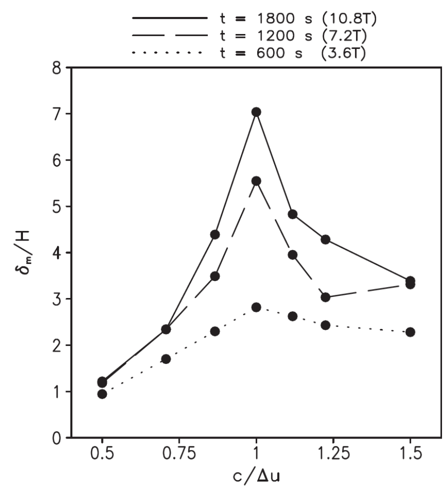
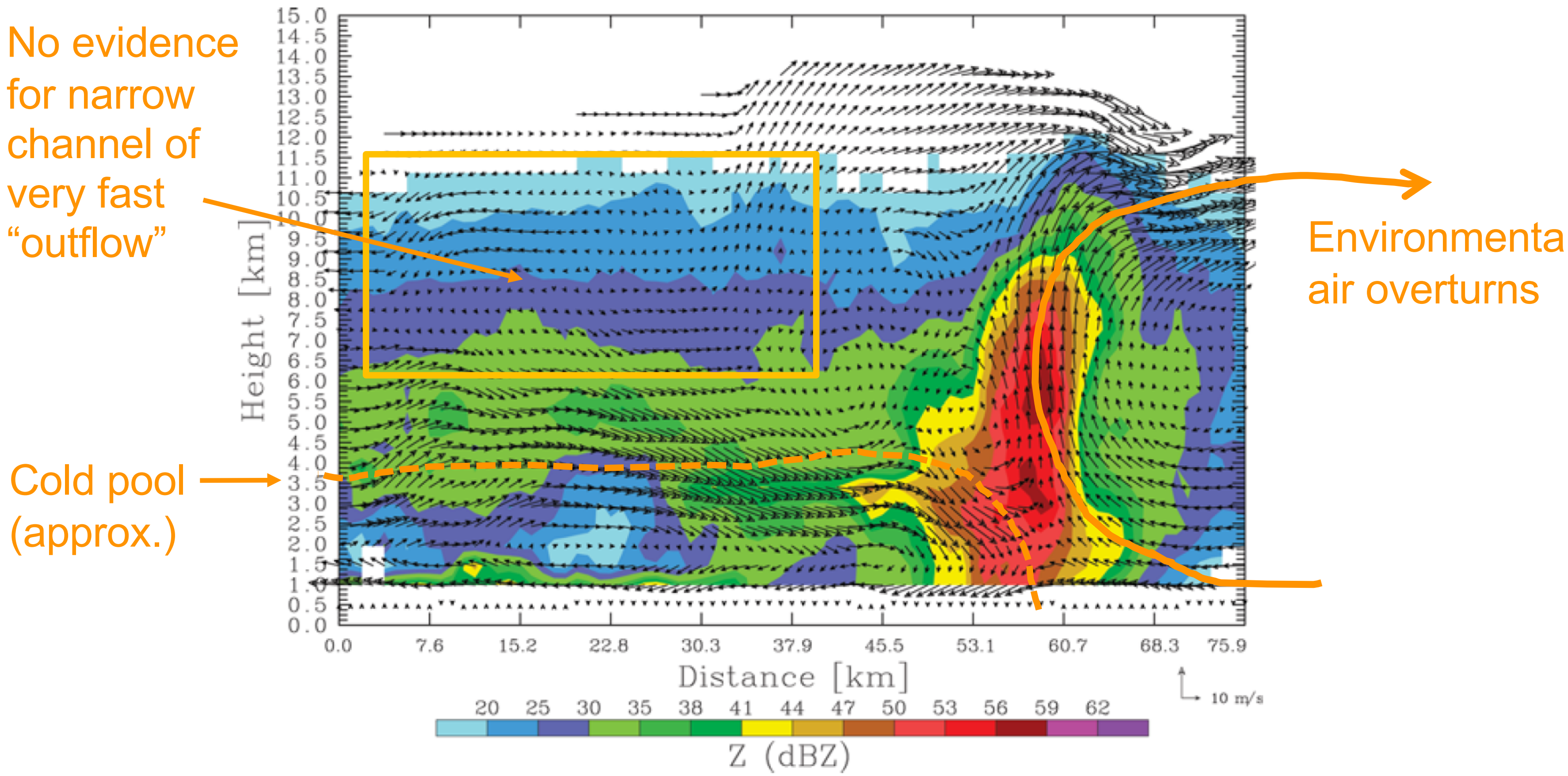


Bryan and Rotunno (2014a)

- Maximum vertical displacement: unbounded/infinite

Back to the atmosphere: a BAMEX bow echo

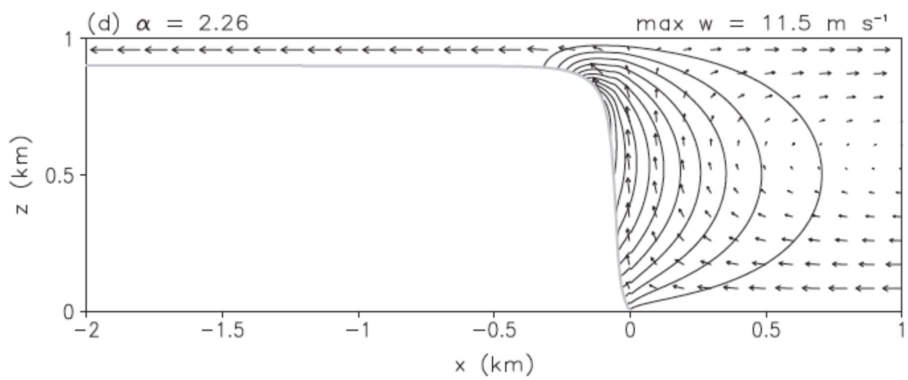


Davis et al. (2004)

For more info:

- Bryan and Rotunno (2014a, 2014b, JAS)

Vertically confined flow
with large shear



Vertically unconfined flow
with "optimal" shear