The Fluid Dynamics of Tornadoes

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NCAR

Lecture 3: Numerical Simulations & Dynamics of The Two-Celled Vortex

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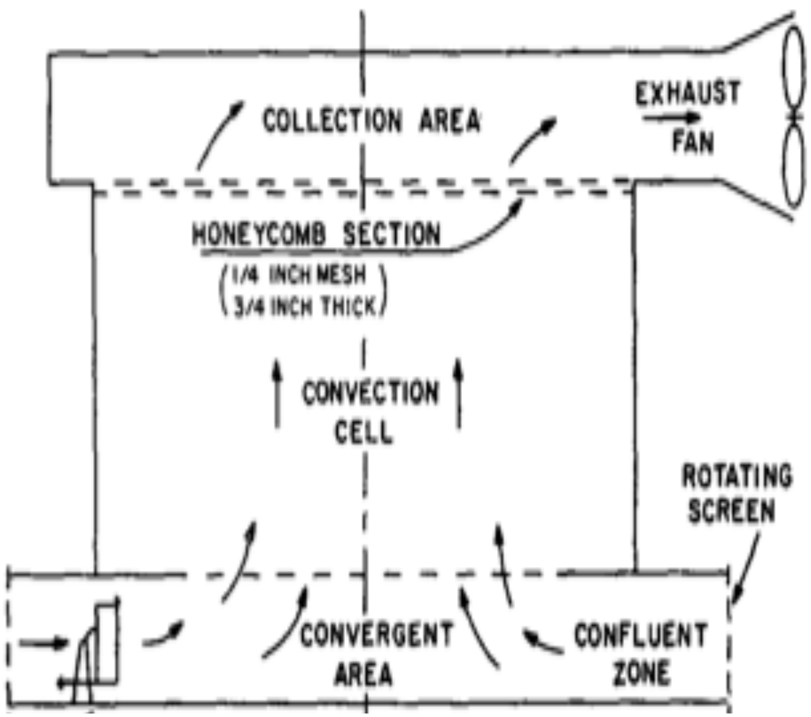
Numerical Model of Ward Vortex Chamber

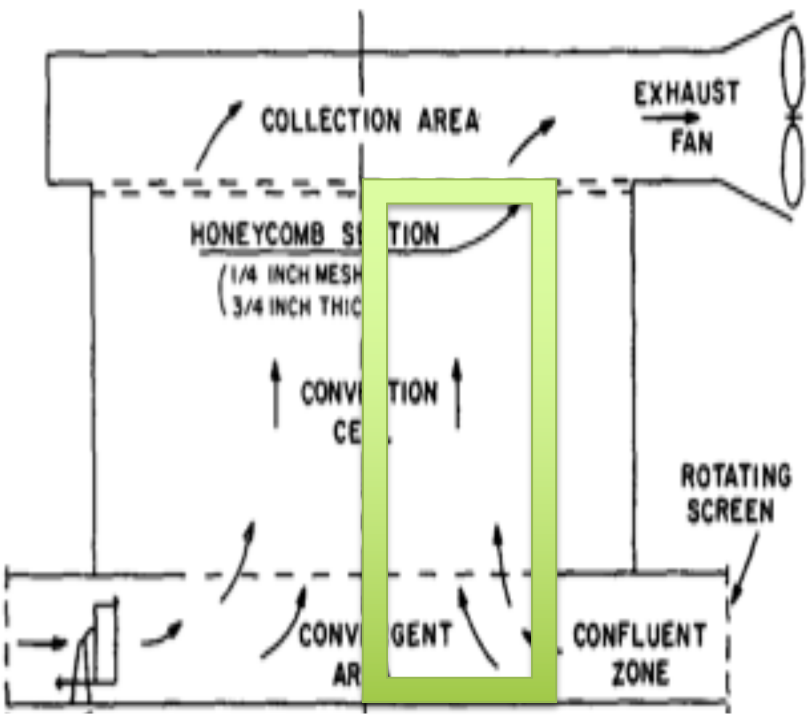
…inadequate top boundary condition…→*…difficult to model top boundary condition…*



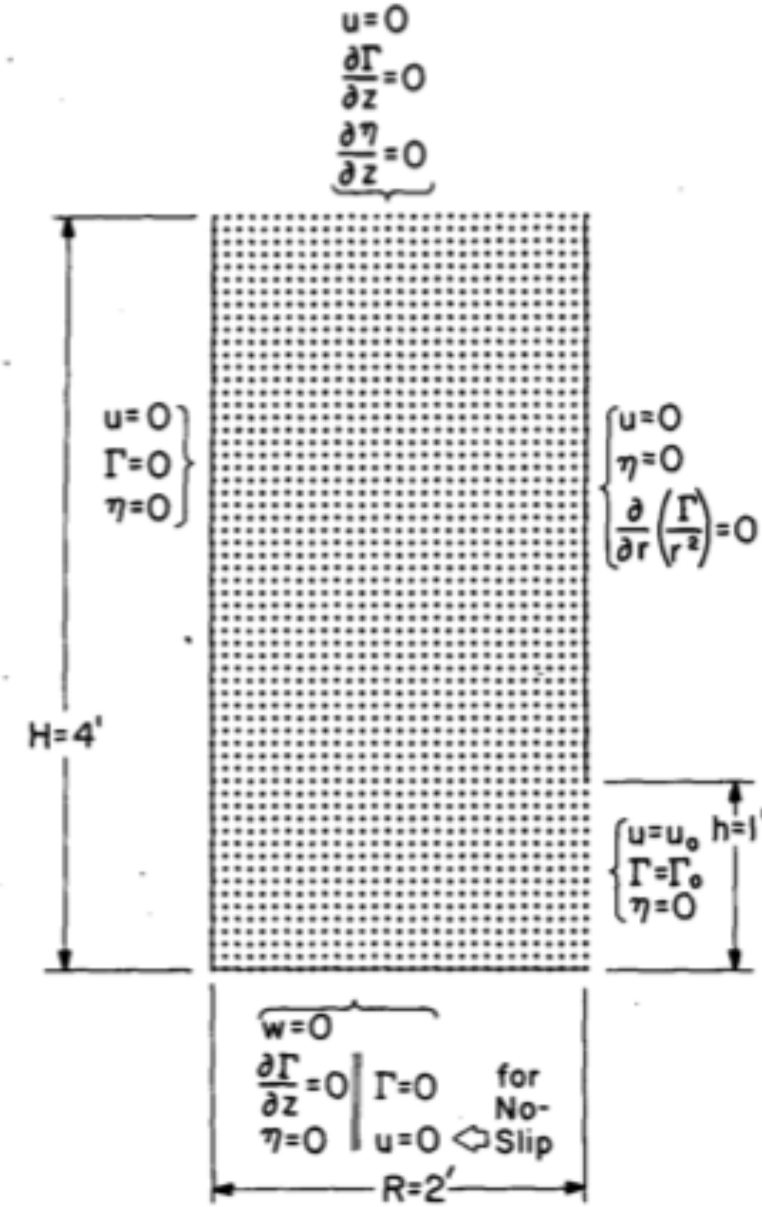
Ward (1972, *JAS*)

Numerical Model of Ward Vortex Chamber

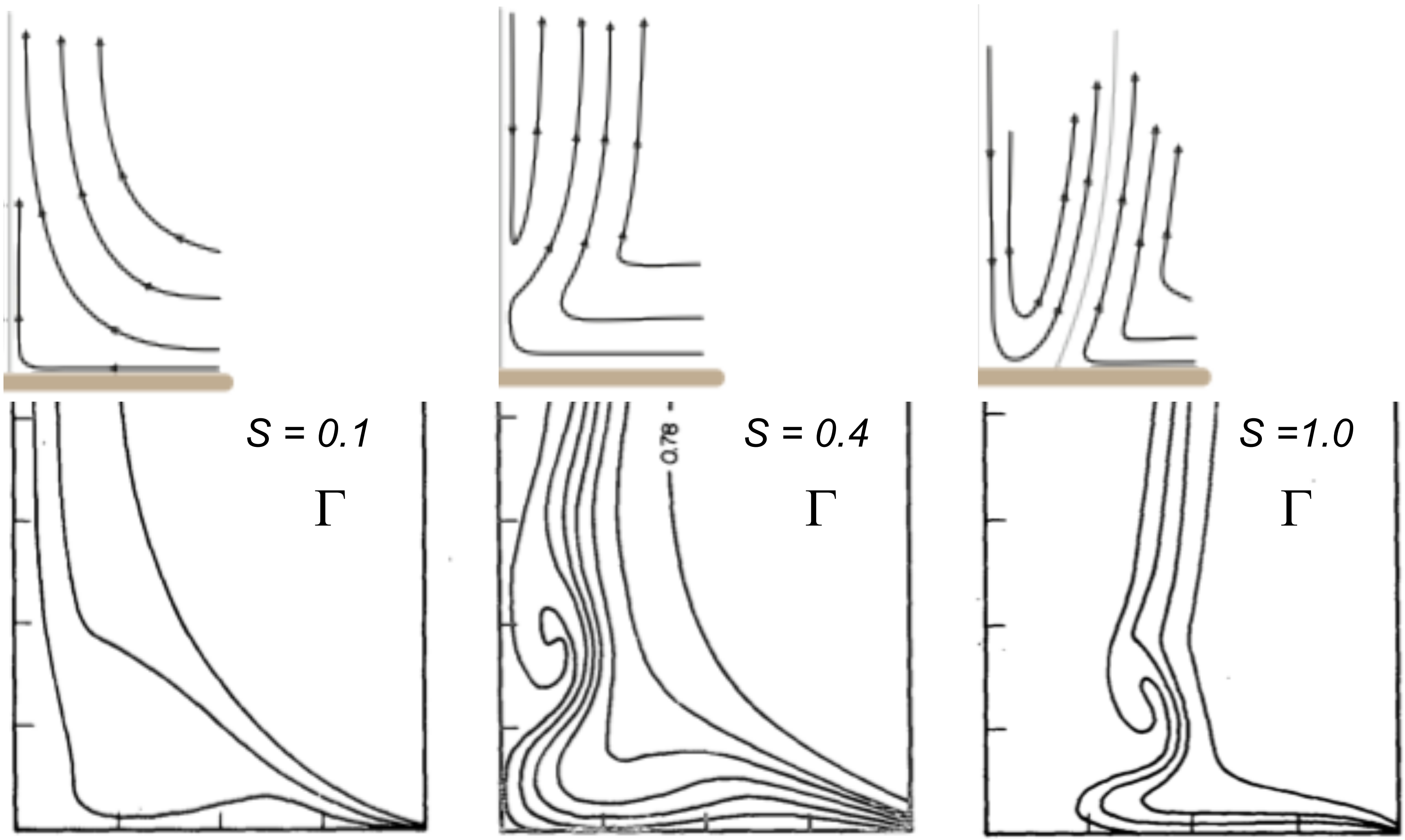
…inadequate top boundary condition…→*…difficult to model top boundary condition…*



Rotunno (1977, *JAS*)



Numerical Simulations of Ward Chamber

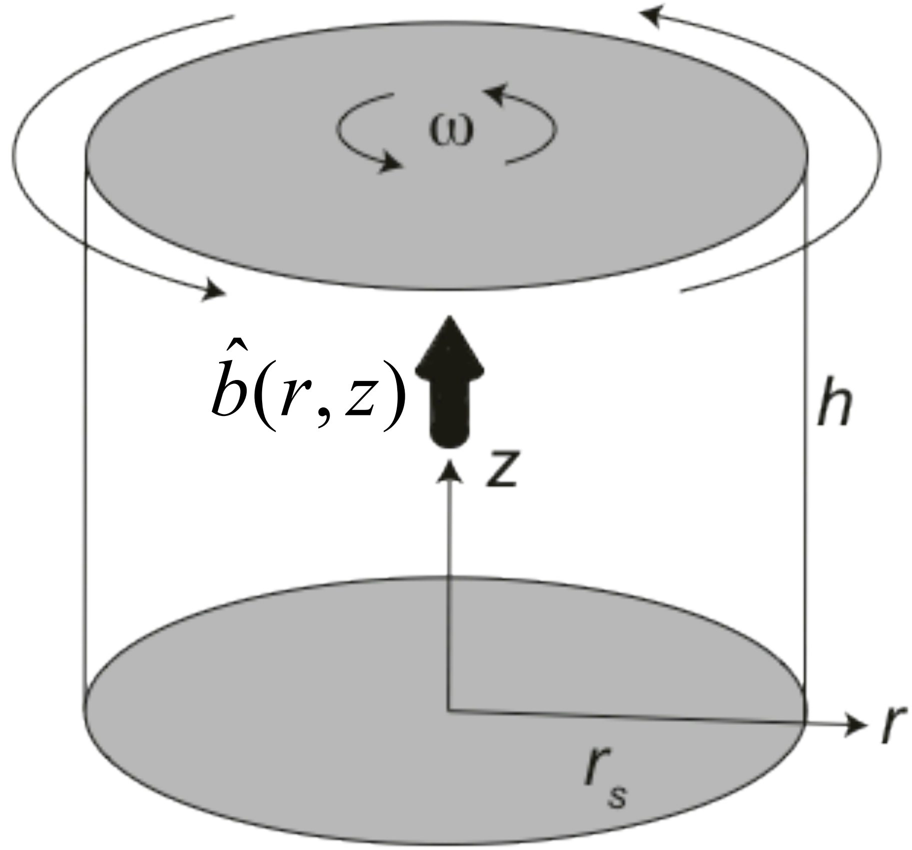


Rotunno (1979, *JAS*)

Fiedler Chamber

Boundary condition

$[u, v, w] = 0$



$$\Omega = \frac{\omega h}{W} \qquad N = \frac{Wh}{\nu} \qquad W \equiv \sqrt{2\int_0^h \hat{b}(0,z)\,dz}$$

Swirl Ratio Reynolds #

Fiedler (1994, *Atmos-Ocn*)

Nondimensional Governing Equations in Cylindrical Polar Coordinates

$$\frac{du}{dt} = -\frac{\partial p}{\partial r} + \frac{v^2}{r} + 2\Omega v + \frac{1}{N}\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)\right) + \frac{\partial^2 u}{\partial z^2}\right)$$

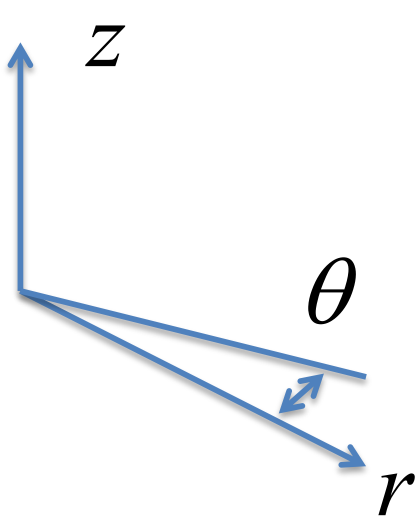
$$\frac{dv}{dt} = -\frac{uv}{r} - 2\Omega u + \frac{1}{N}\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv)\right) + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{dw}{dt} = -\frac{\partial p}{\partial z} + b + \frac{1}{N}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2}\right)$$

$$\frac{1}{r}\frac{\partial ru}{\partial r} + \frac{\partial w}{\partial z} = 0$$

$[u, v, w]$

$[r, \theta, z]$



<u>velocity scale</u>

$$W \equiv \sqrt{2\int_0^h \hat{b}(0,z)\,dz} \cong 50\ \text{m/s}$$

<u>length scale</u>

$$h \approx 10{,}000\text{m}$$

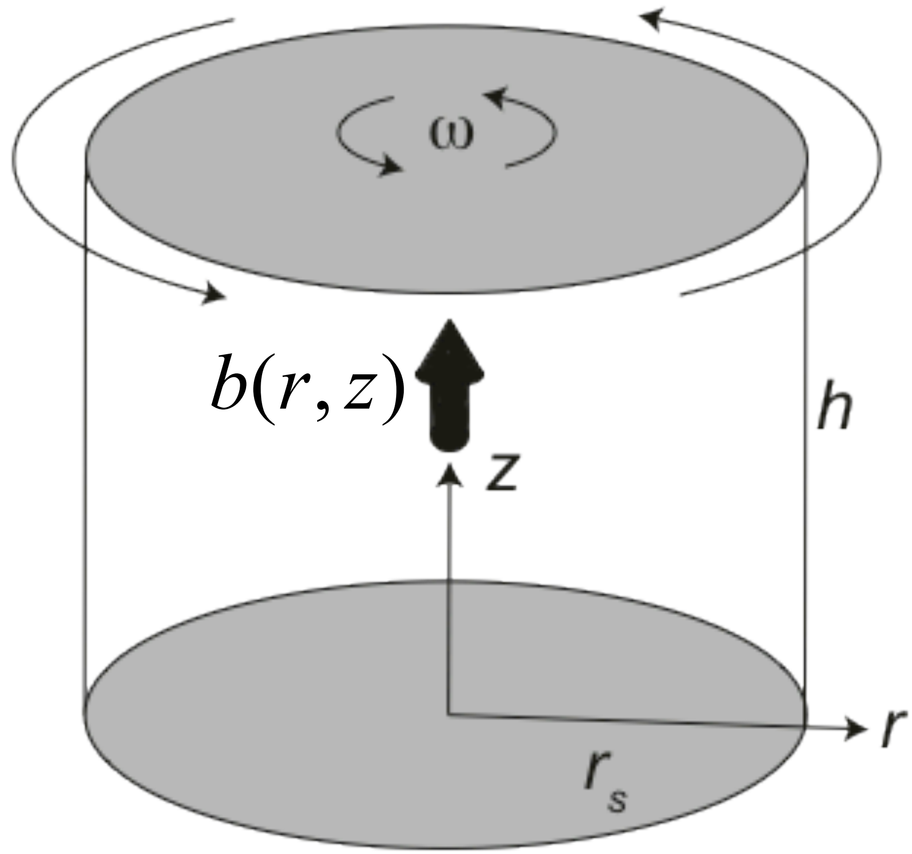
<u>buoyancy scale</u>

$$b_o \approx .125\ \text{m/s}^2 \Leftrightarrow \text{Ave. Cell Temp.} \approx 3.75^\circ$$

Fiedler Chamber

Boundary condition

$$[u, v, w] = 0$$



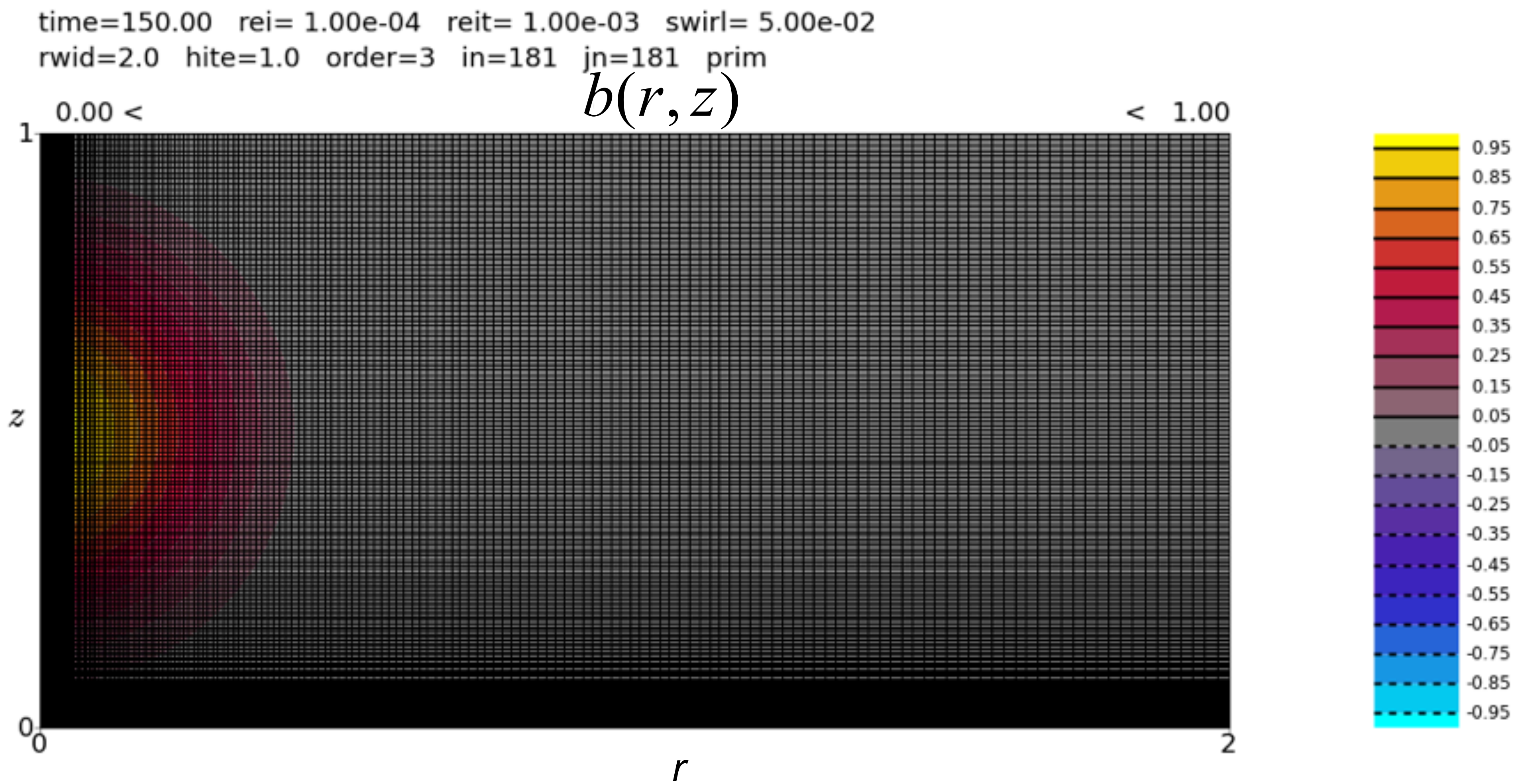
Buoyancy force

$$b(r,z) = 0.5\left[1+\cos(2\pi r_b)\right] \quad \text{for } r_b = \sqrt{r^2+(z-0.5)^2} \le 0.5$$

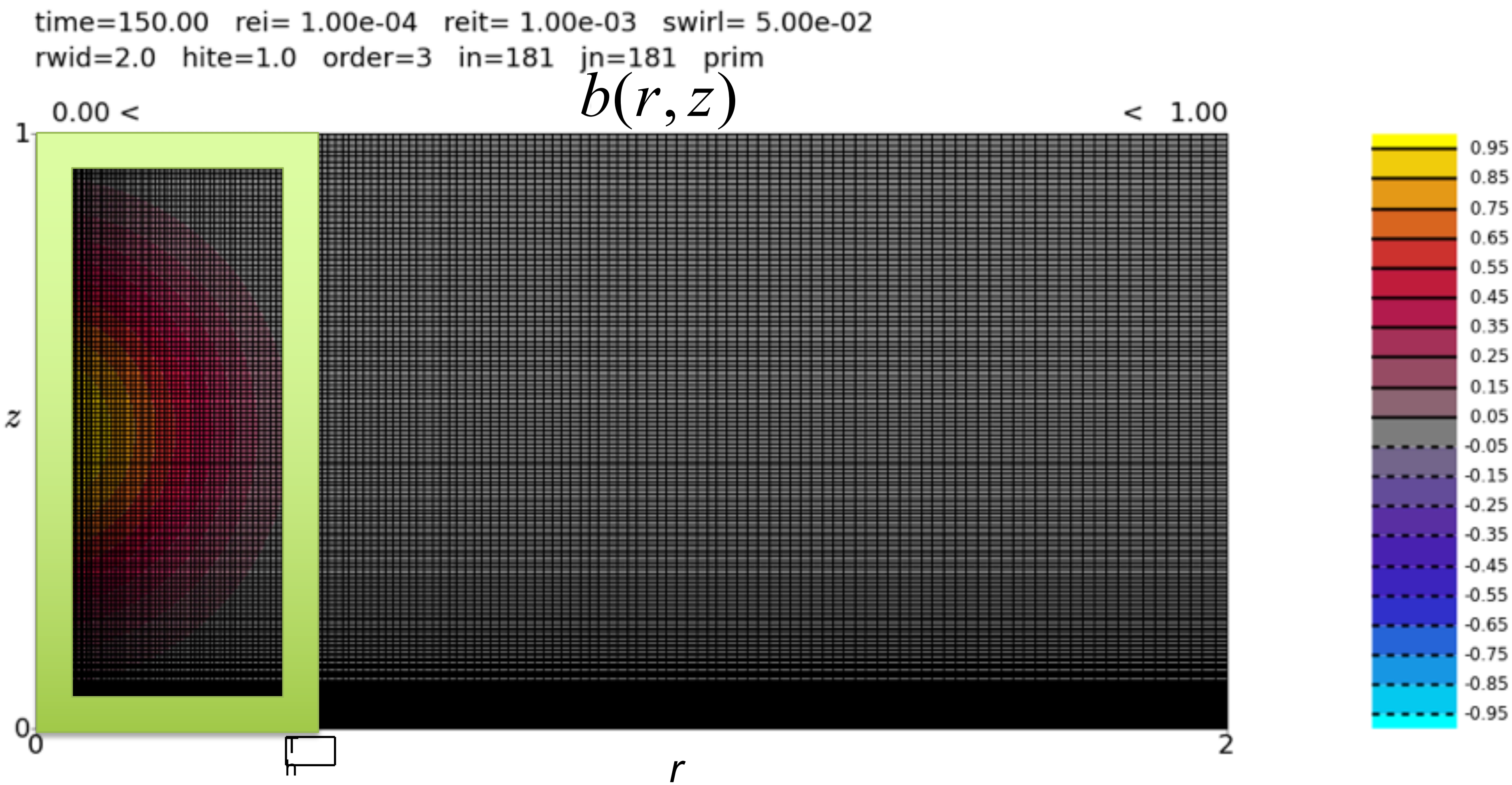
$$b(r,z) = 0 \quad \text{otherwise}$$

Fiedler (1994, *Atmos-Ocn*)

Imposed Buoyancy Force and Grid



Imposed Buoyancy Force and Grid



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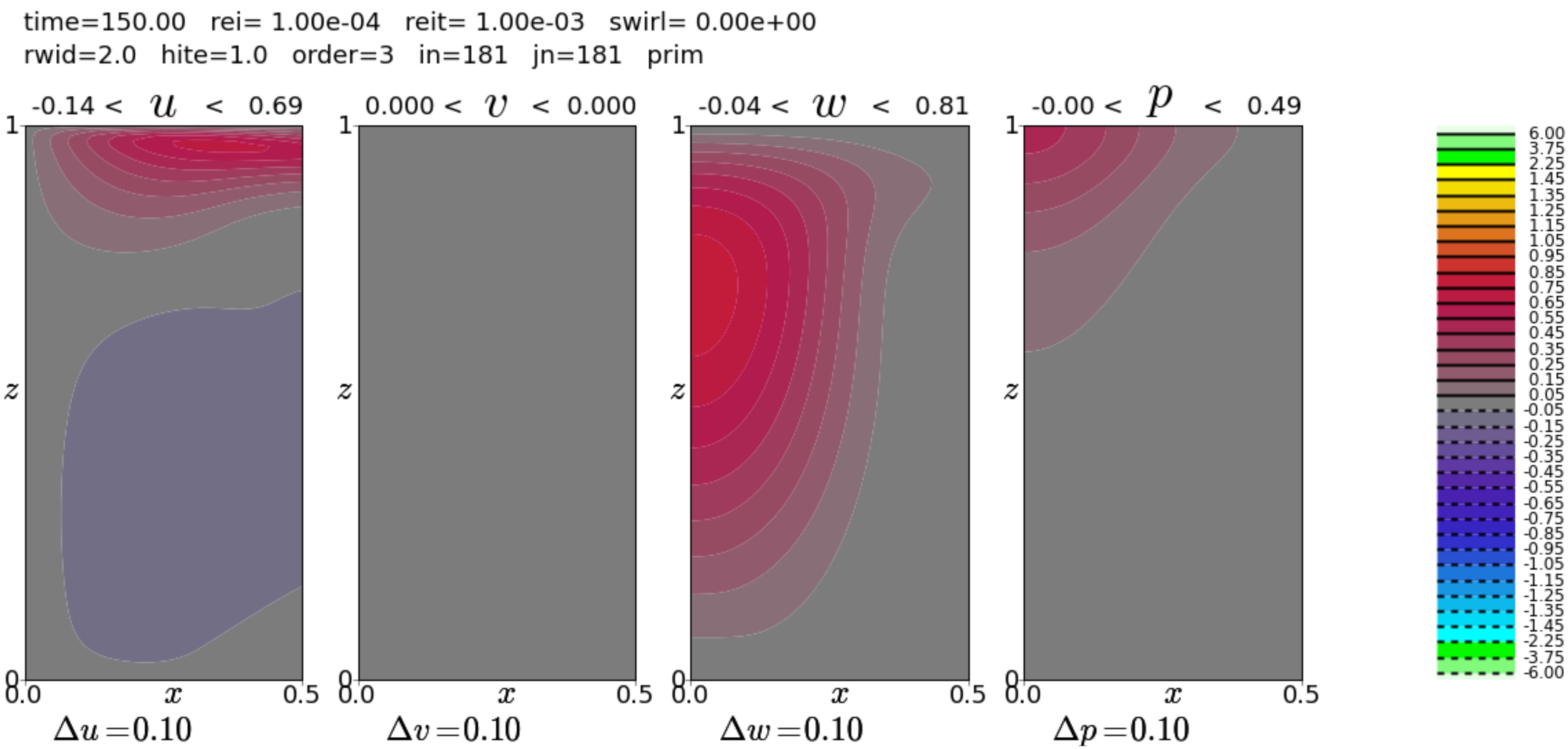
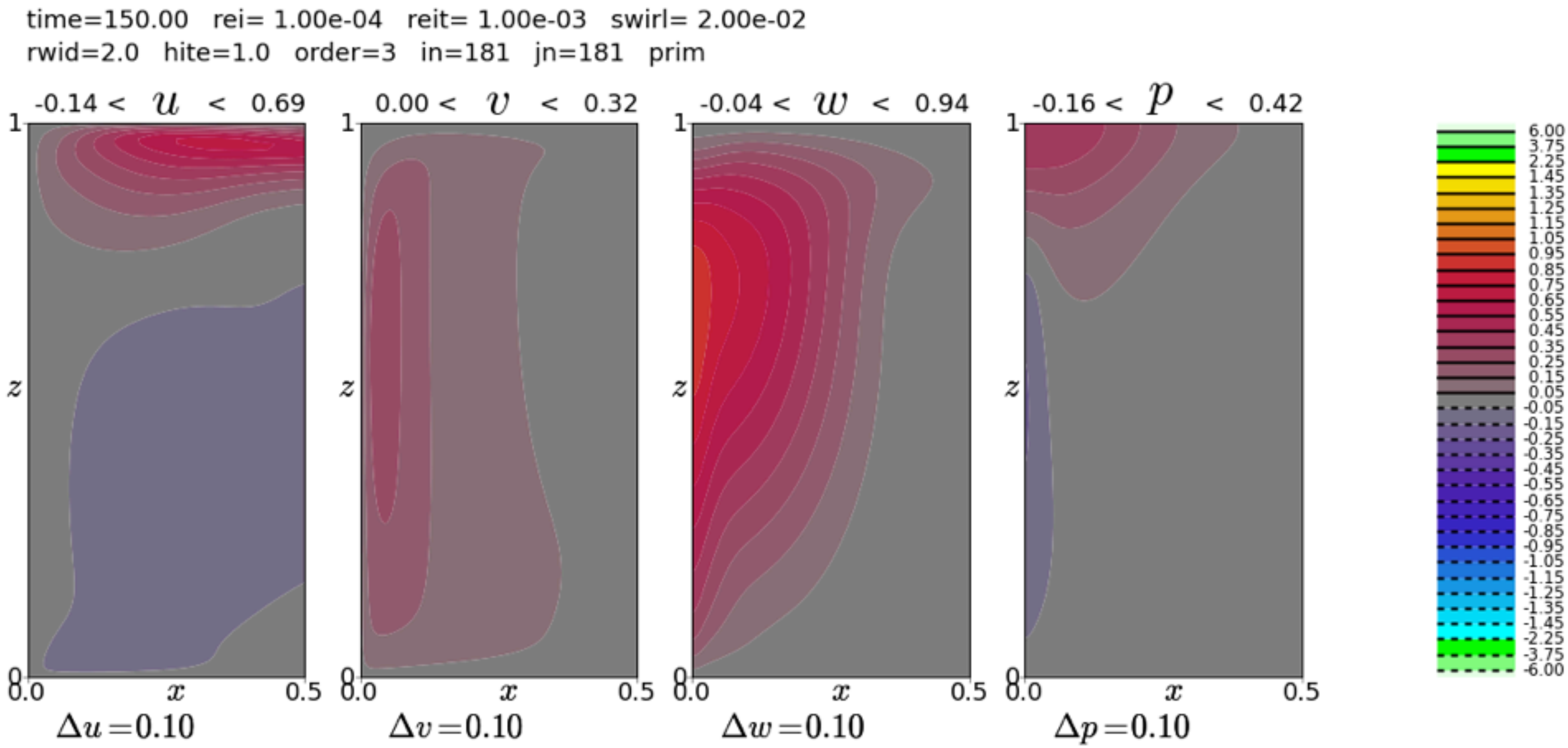
Steady Solution v. Swirl Ratio



Steady Solution v. Swirl Ratio



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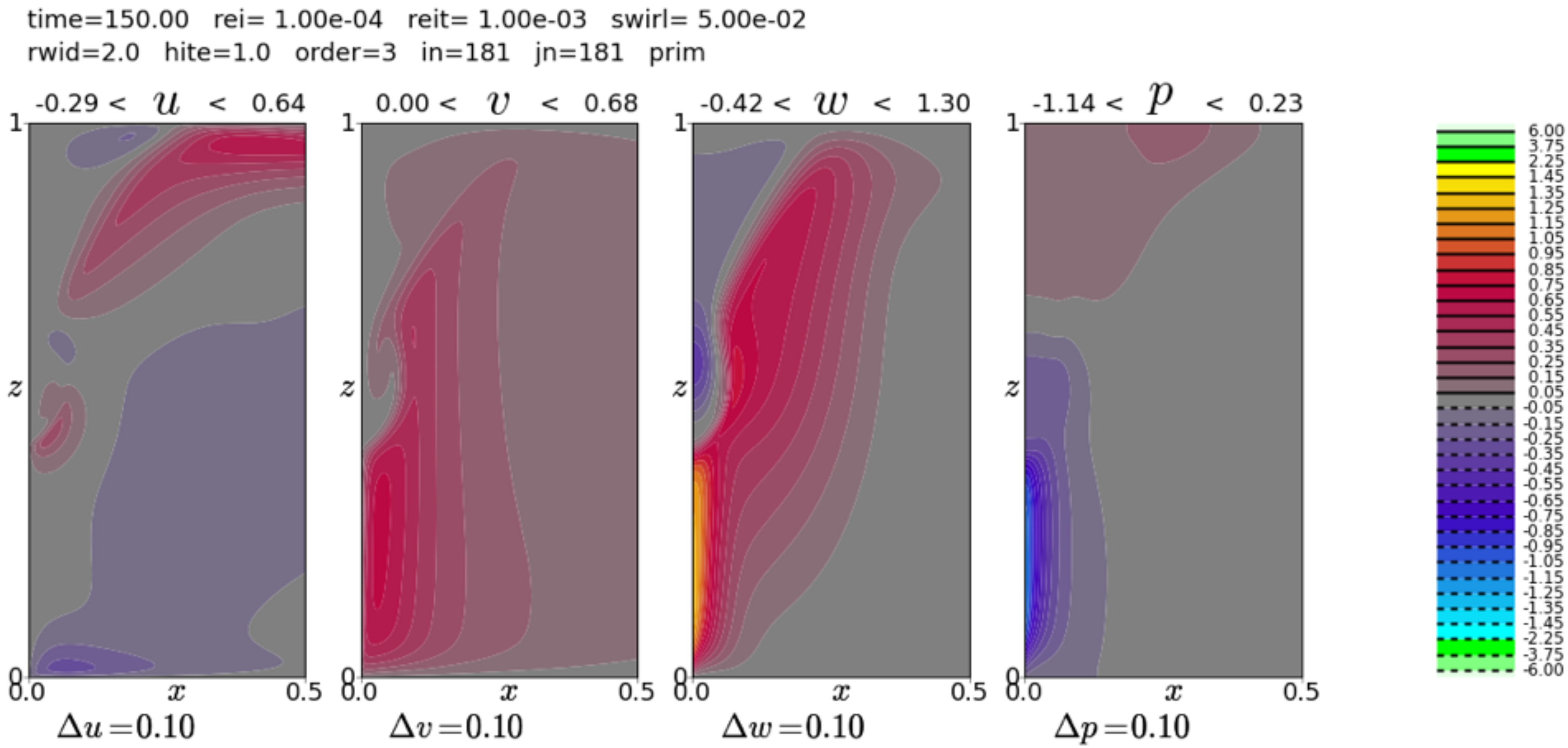

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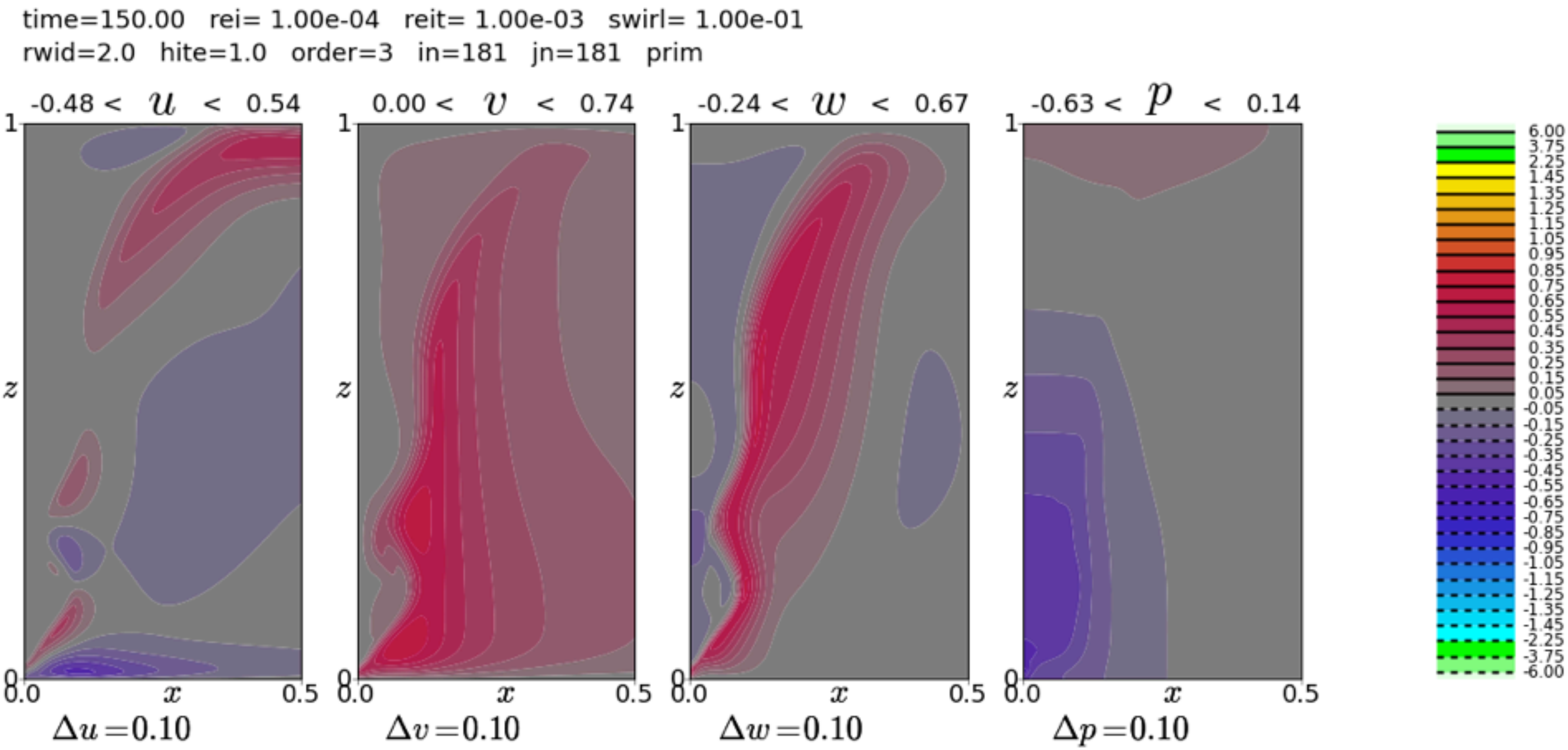
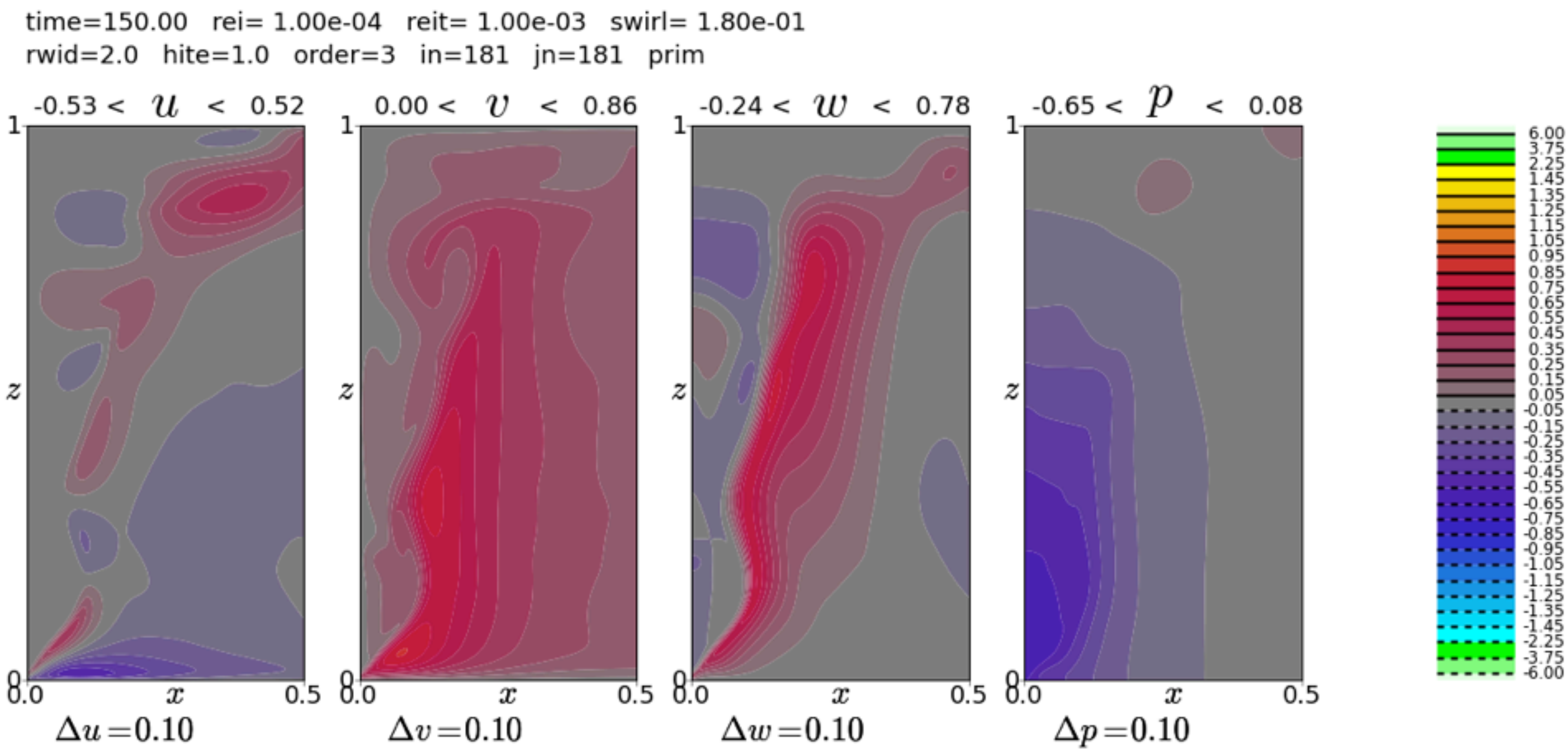
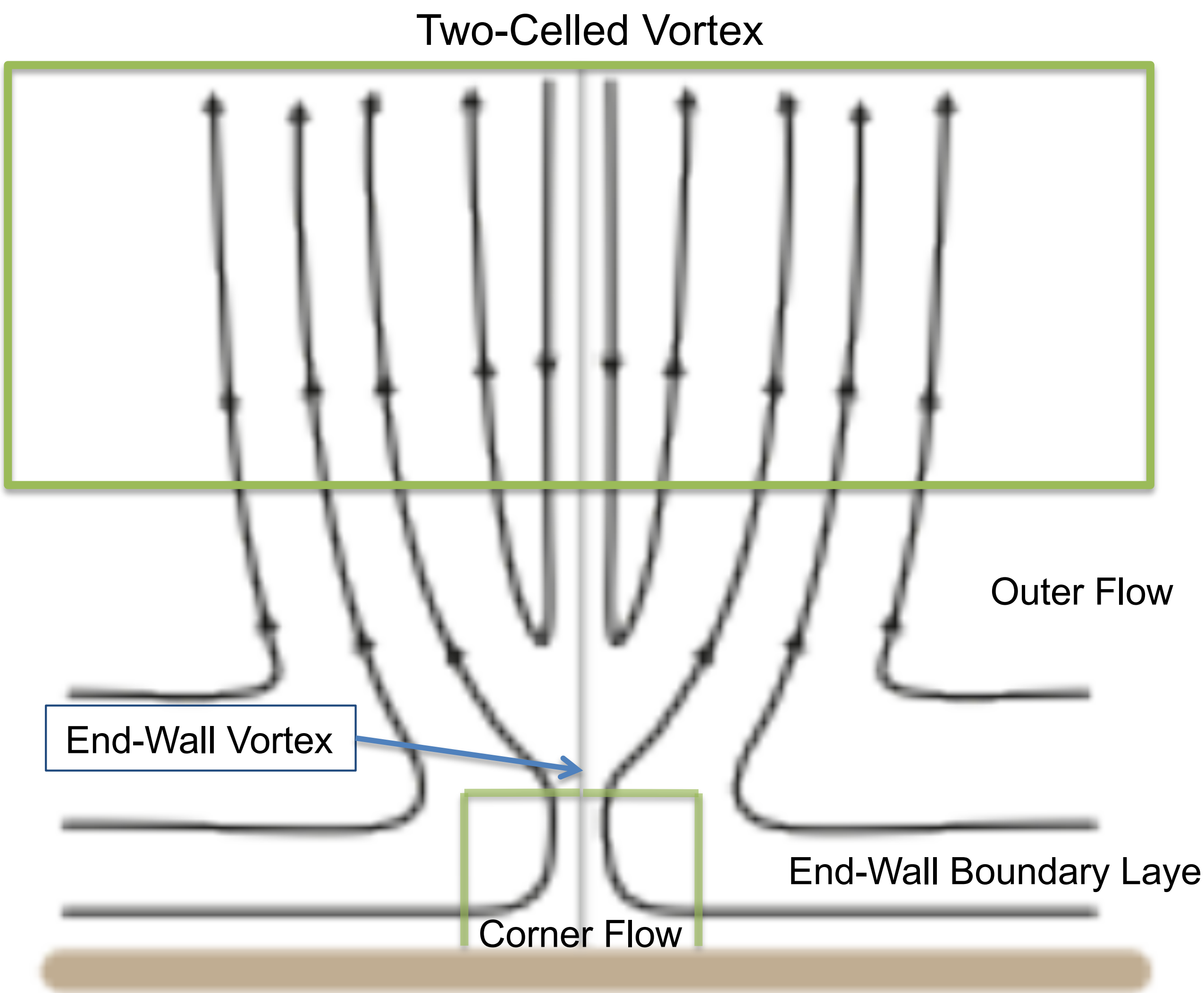


Steady Solution v. Swirl Ratio





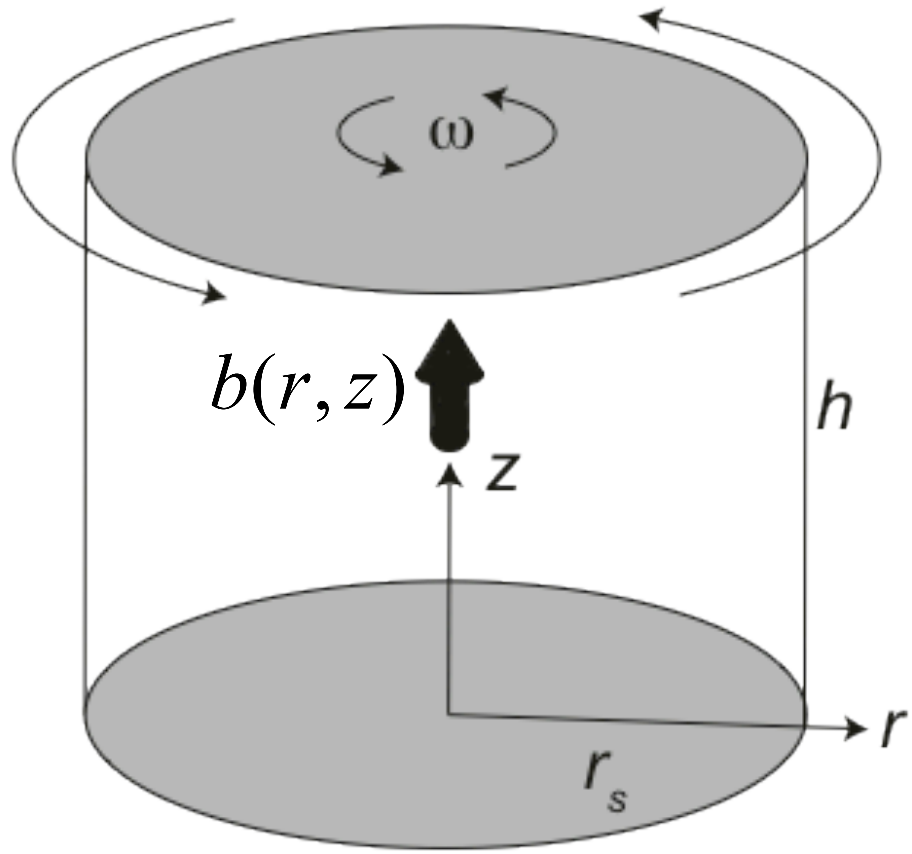
Two-Celled Vortex
Outer Flow
End-Wall Vortex
End-Wall Boundary Layer
Corner Flow

The Two-Celled Vortex

Fiedler Chamber

Boundary condition

$$[u, v, w] = 0$$



Boundary condition @ $z = 0$

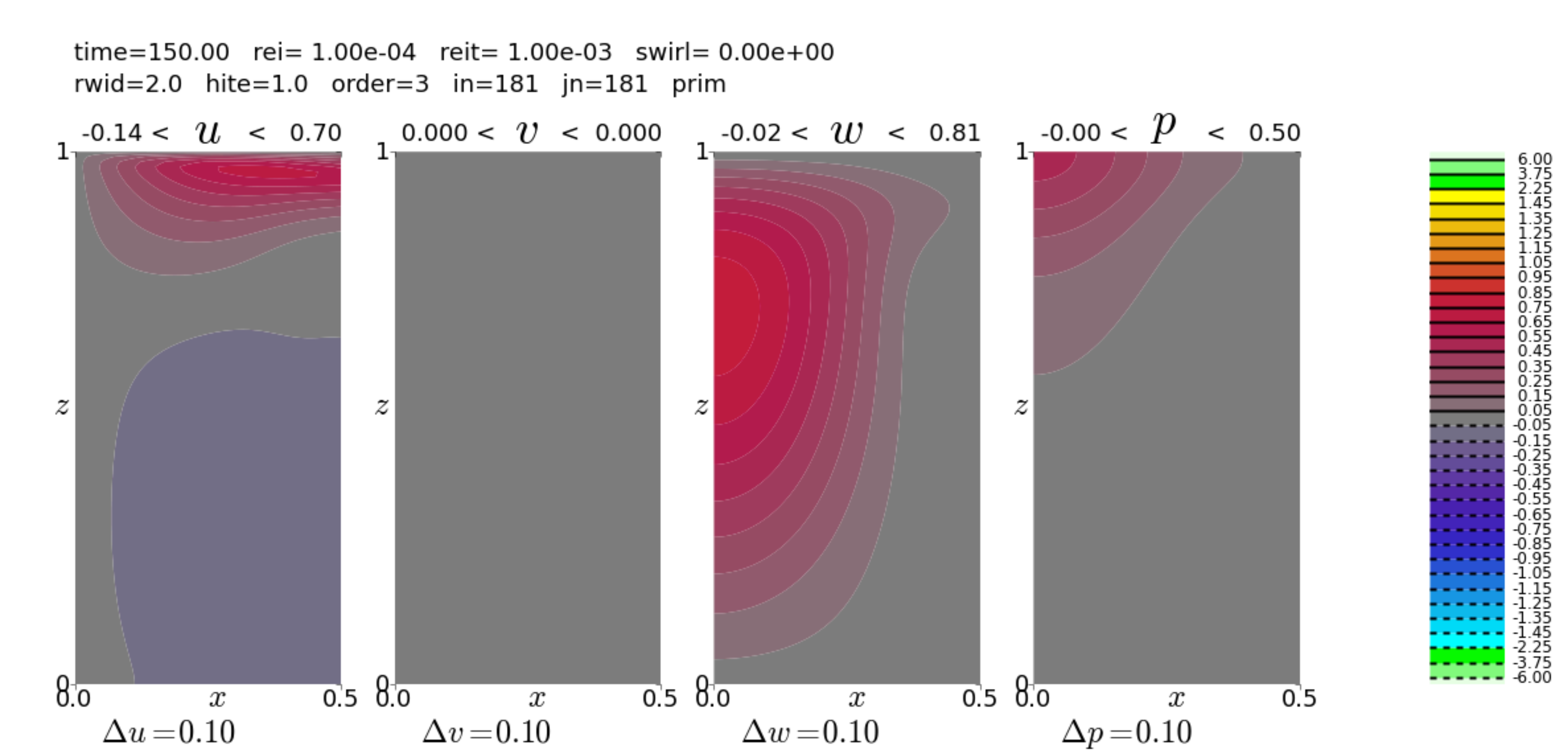
Eliminate lower boundary layer / zero-stress condition →

$$\left[\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z}, w\right] = 0$$

Fiedler (1994, *Atmos-Ocn*)

Two-Celled Vortex

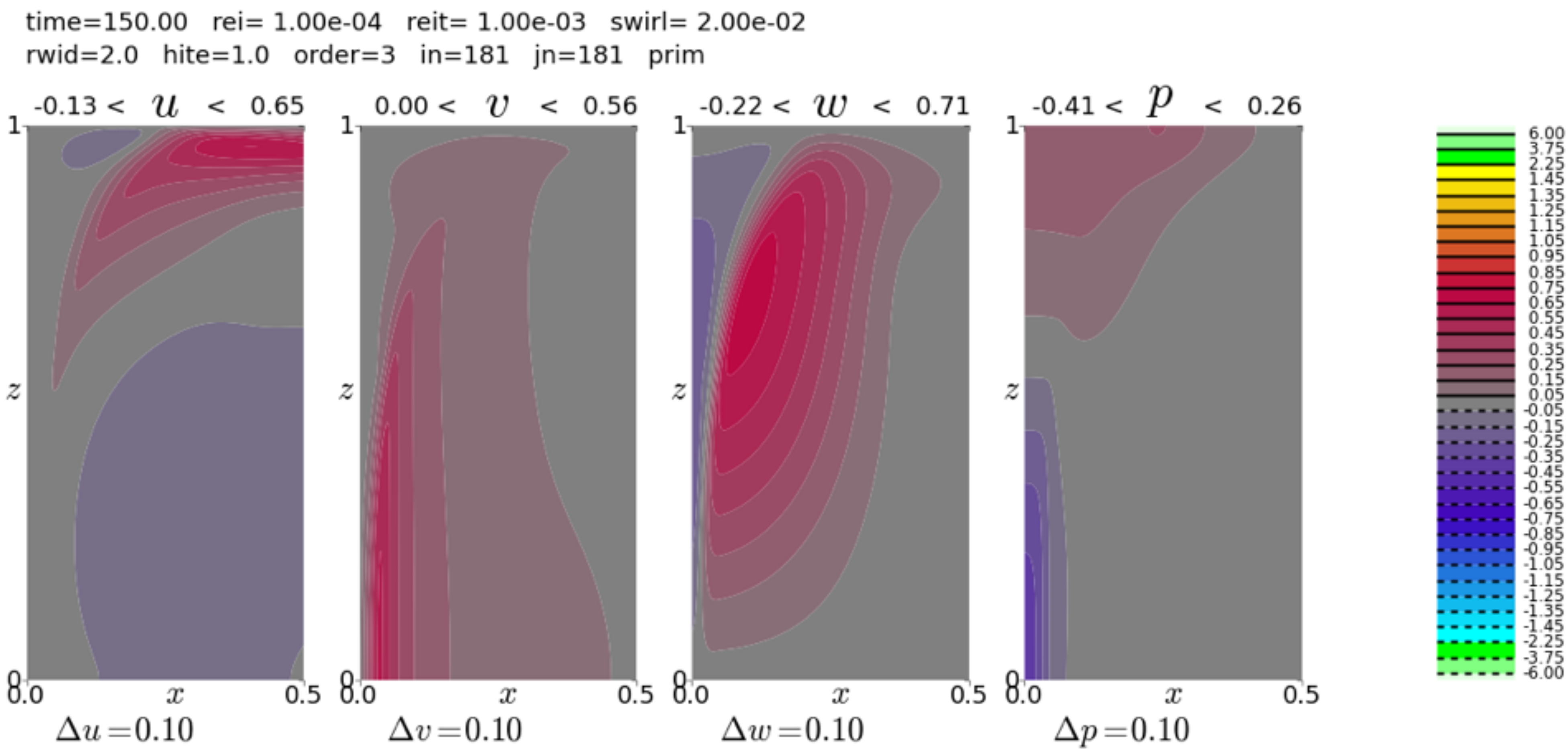
Steady Solution v. Swirl Ratio



Free-Slip @ z=0

Two-Celled Vortex

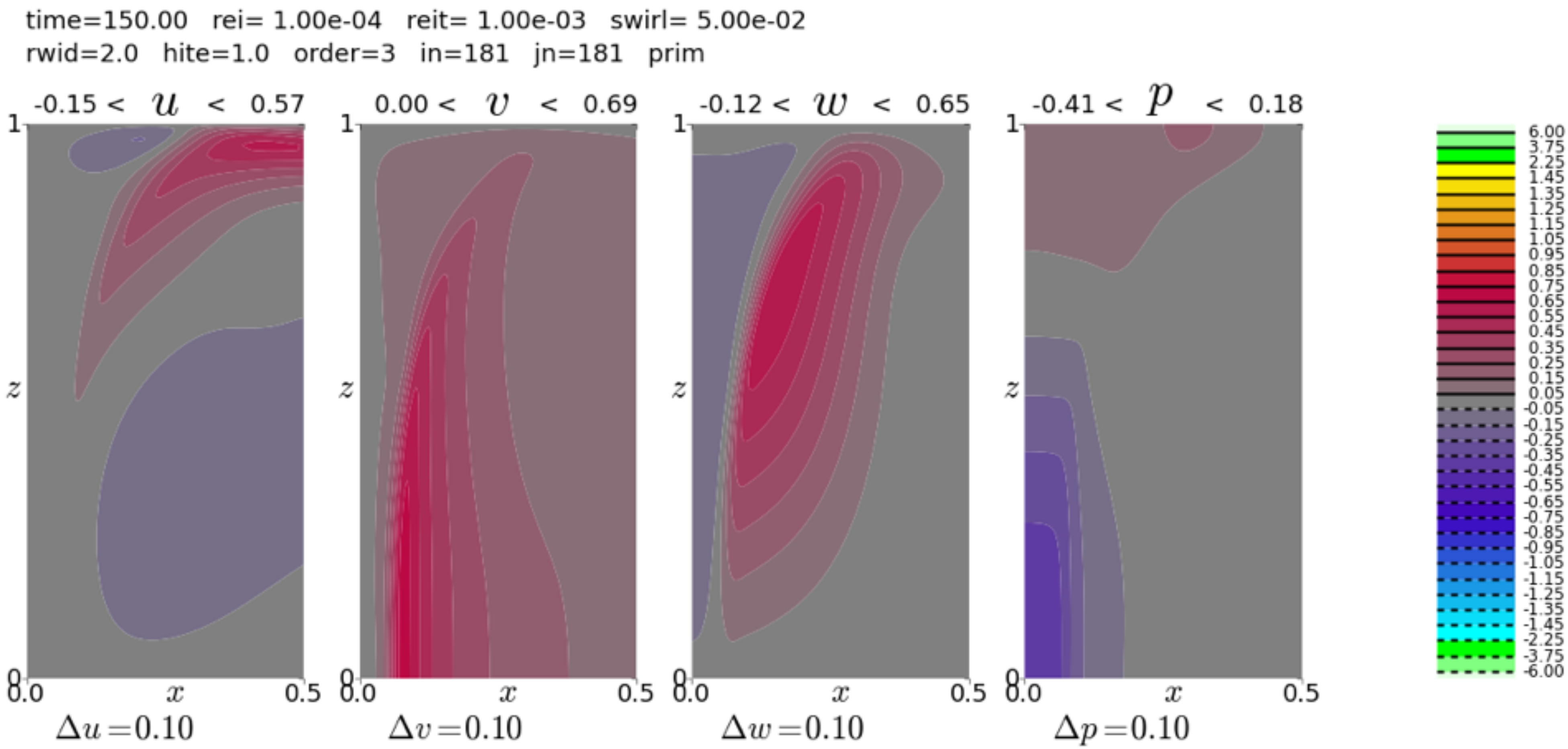
Steady Solution v. Swirl Ratio



Free-Slip @ z=0

Two-Celled Vortex

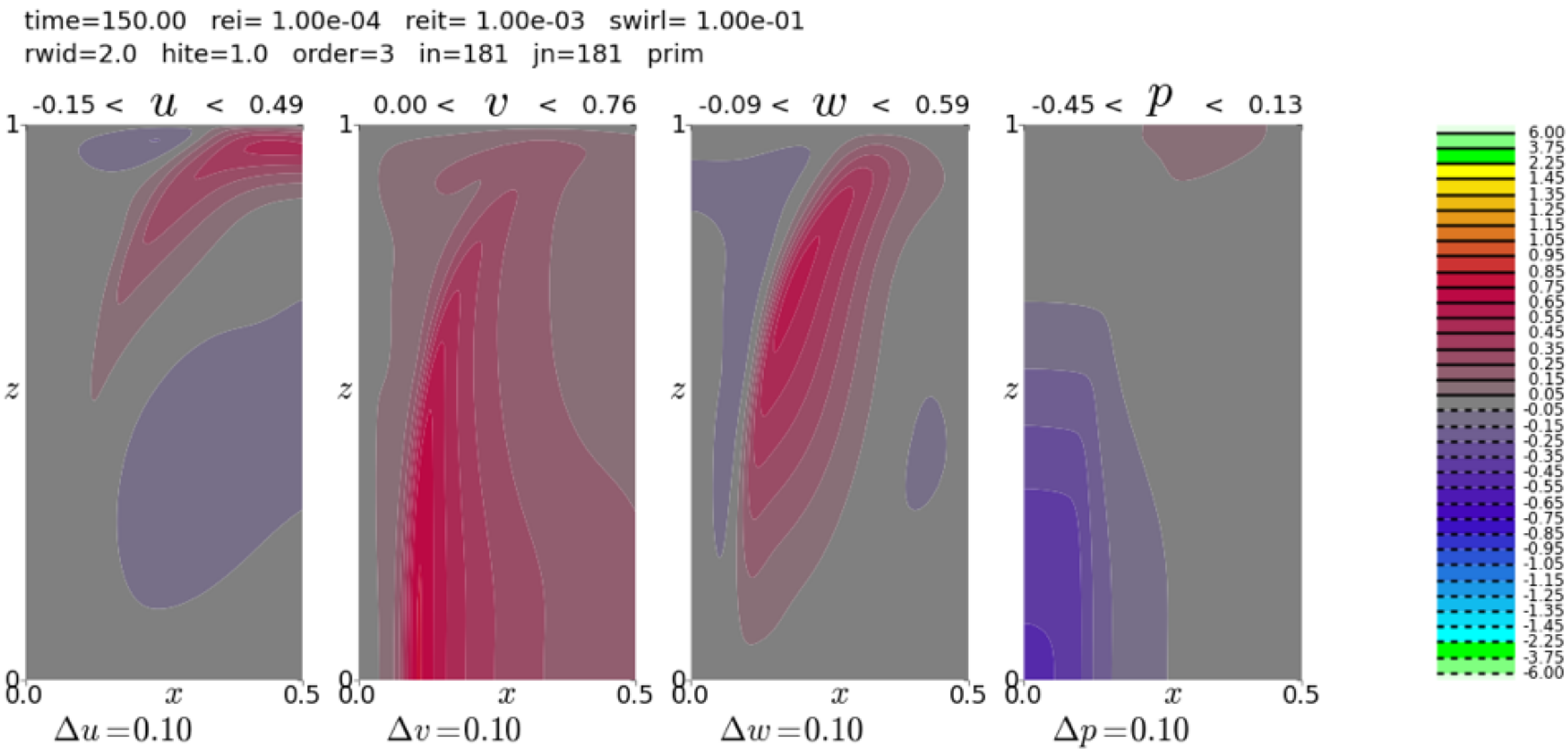
Steady Solution v. Swirl Ratio



Free-Slip @ z=0

Two-Celled Vortex

Steady Solution v. Swirl Ratio



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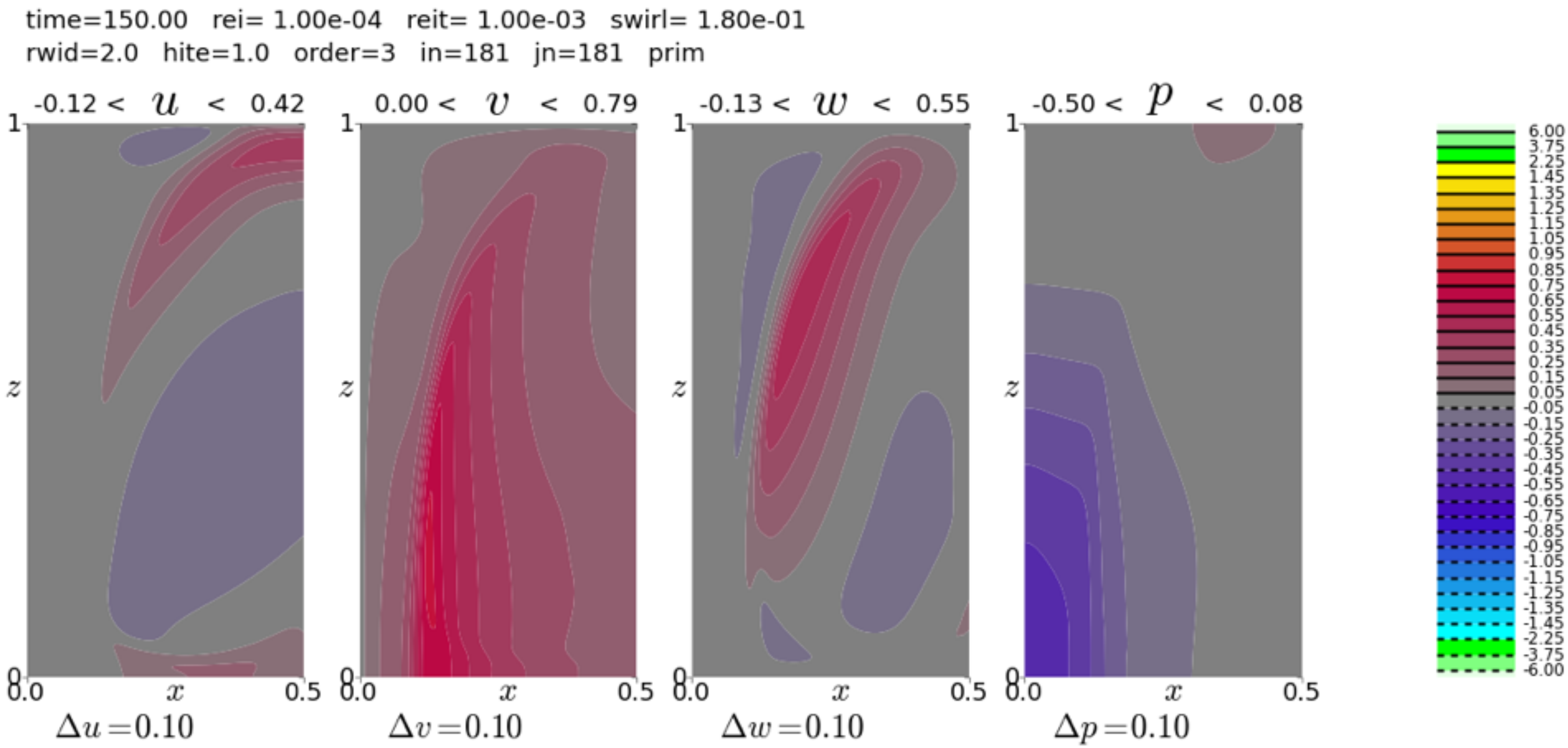
Two-Celled Vortex

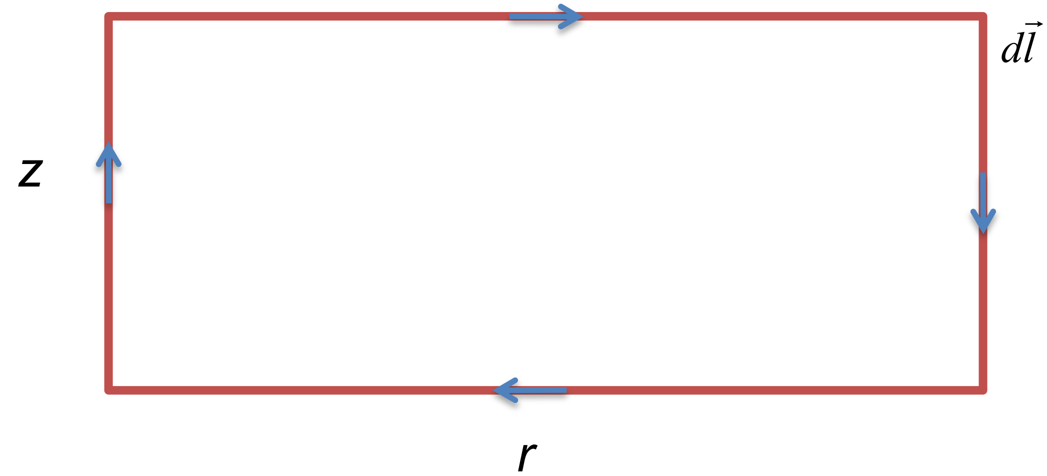
Steady Solution v. Swirl Ratio



Free-Slip @ z=0

Two-Celled Vortex

Circulation Around the Domain



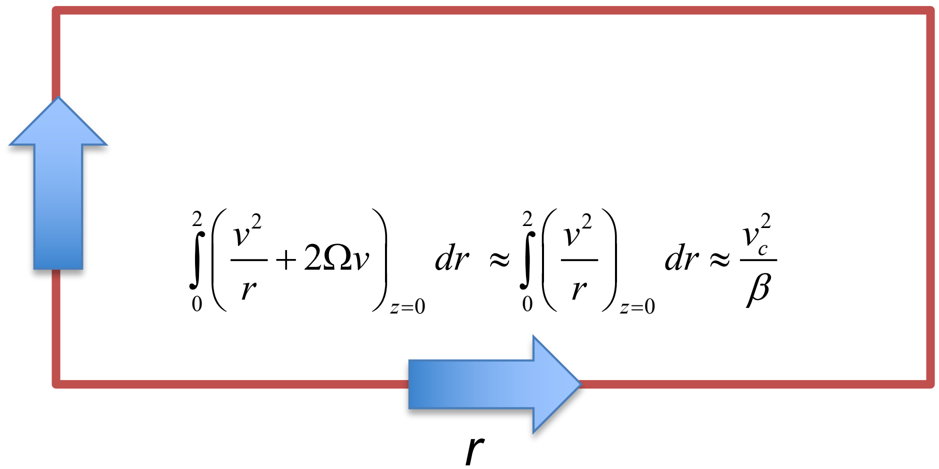
$$\frac{\partial}{\partial t}\oint \vec{u} \bullet d\vec{l} = \oint \vec{F} \bullet d\vec{l}$$

At steady state and applying boundary conditions →

$$0 = \int_0^1 b\,|_{r=0}\, dz - \int_0^2 \left(\frac{v^2}{r} + 2\Omega v \right)_{z=0} dr + \oint D \bullet dl$$

Two-Celled Vortex: Balance of Buoyancy and Centrifugal Tendencies

$$\int_0^1 b\,|_{r=0}\; dz = \frac{1}{2}$$



$$\int_0^2 \left(\frac{v^2}{r} + 2\Omega v\right)_{z=0} dr \approx \int_0^2 \left(\frac{v^2}{r}\right)_{z=0} dr \approx \frac{v_c^2}{\beta}$$

$$v_c \approx \sqrt{\frac{\beta}{2}}$$

Thermodynamic Speed

$\beta = 1$ (Rankine Vortex)

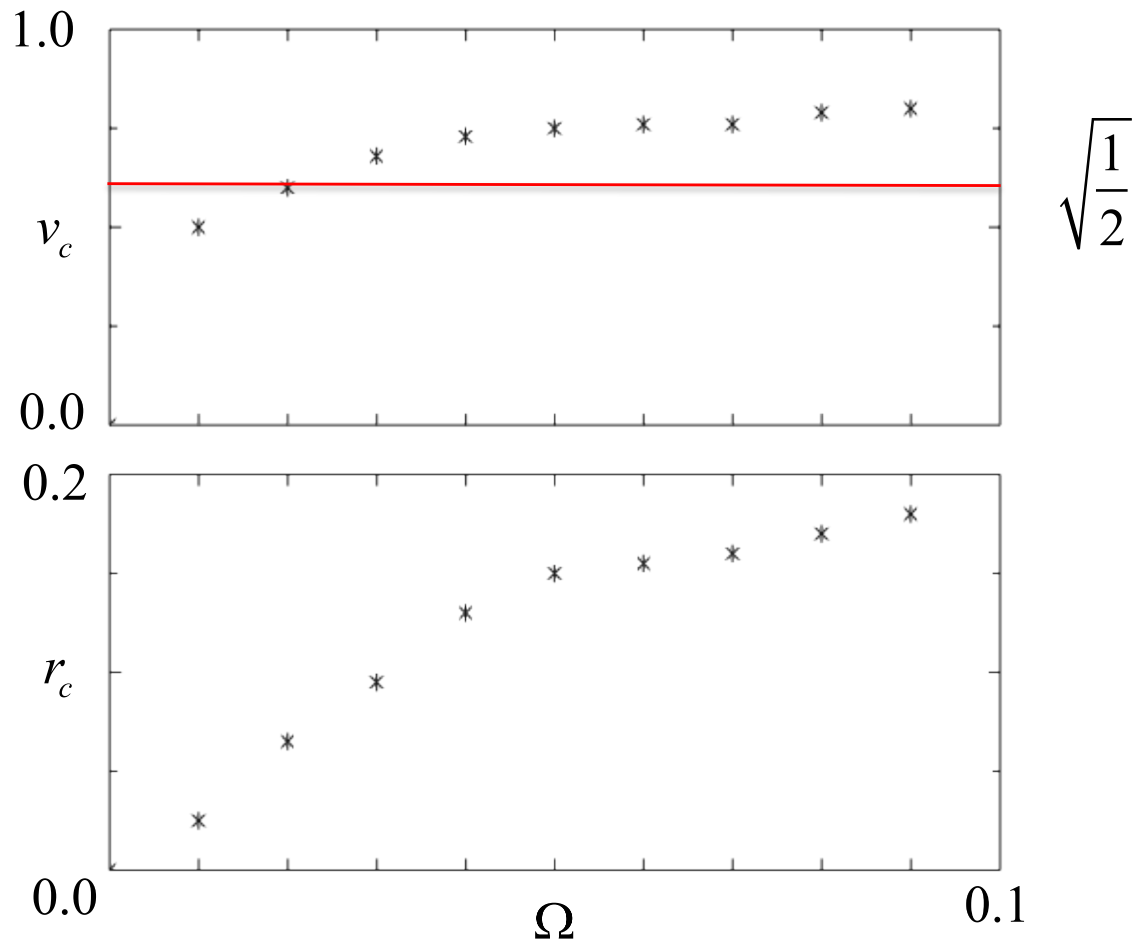
Conservation of Angular Momentum

$$r_c v_c + \Omega r_c^2 \approx r_e v_e + \Omega r_e^2$$

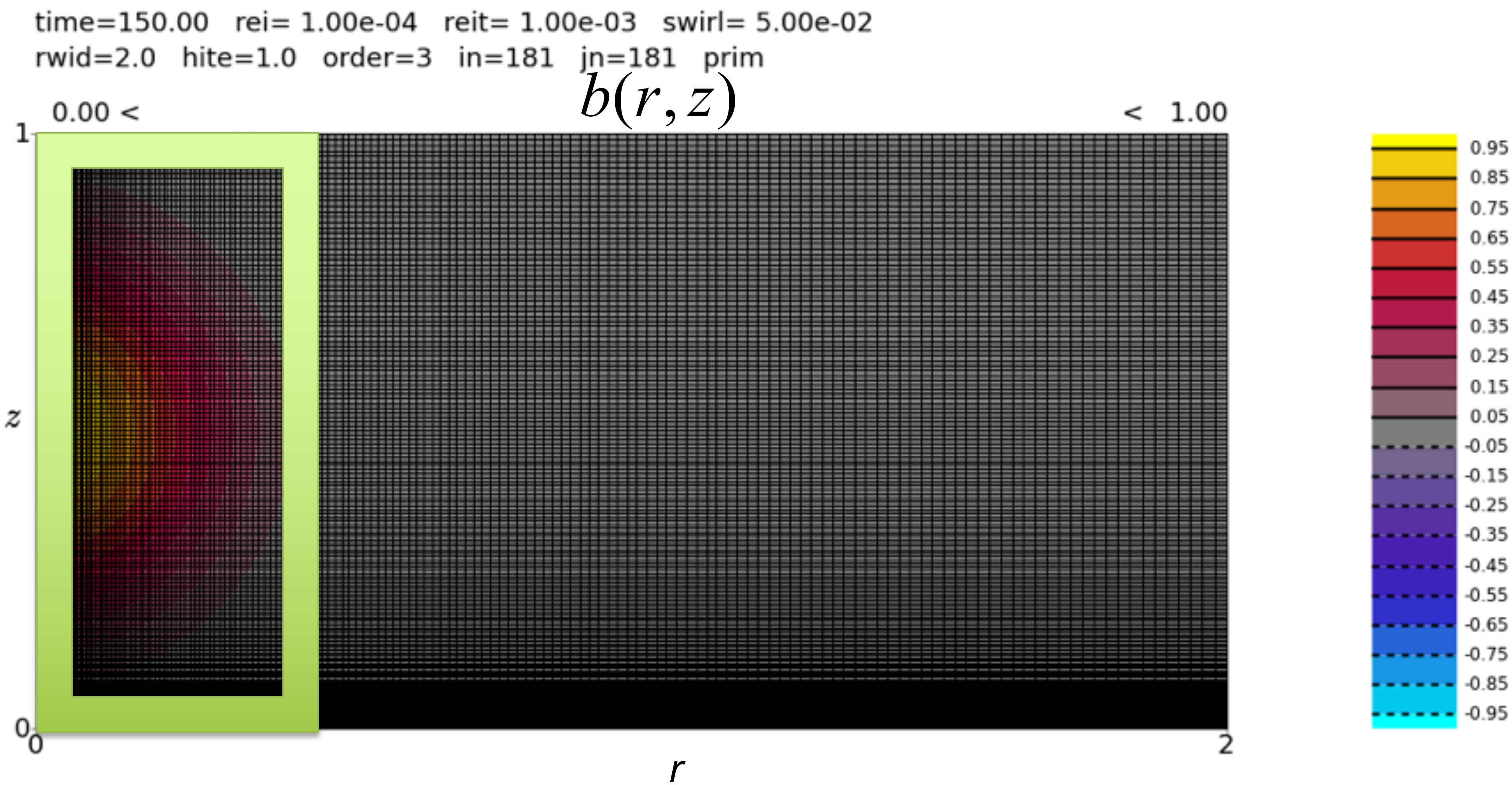
$$r_c \propto \Omega$$

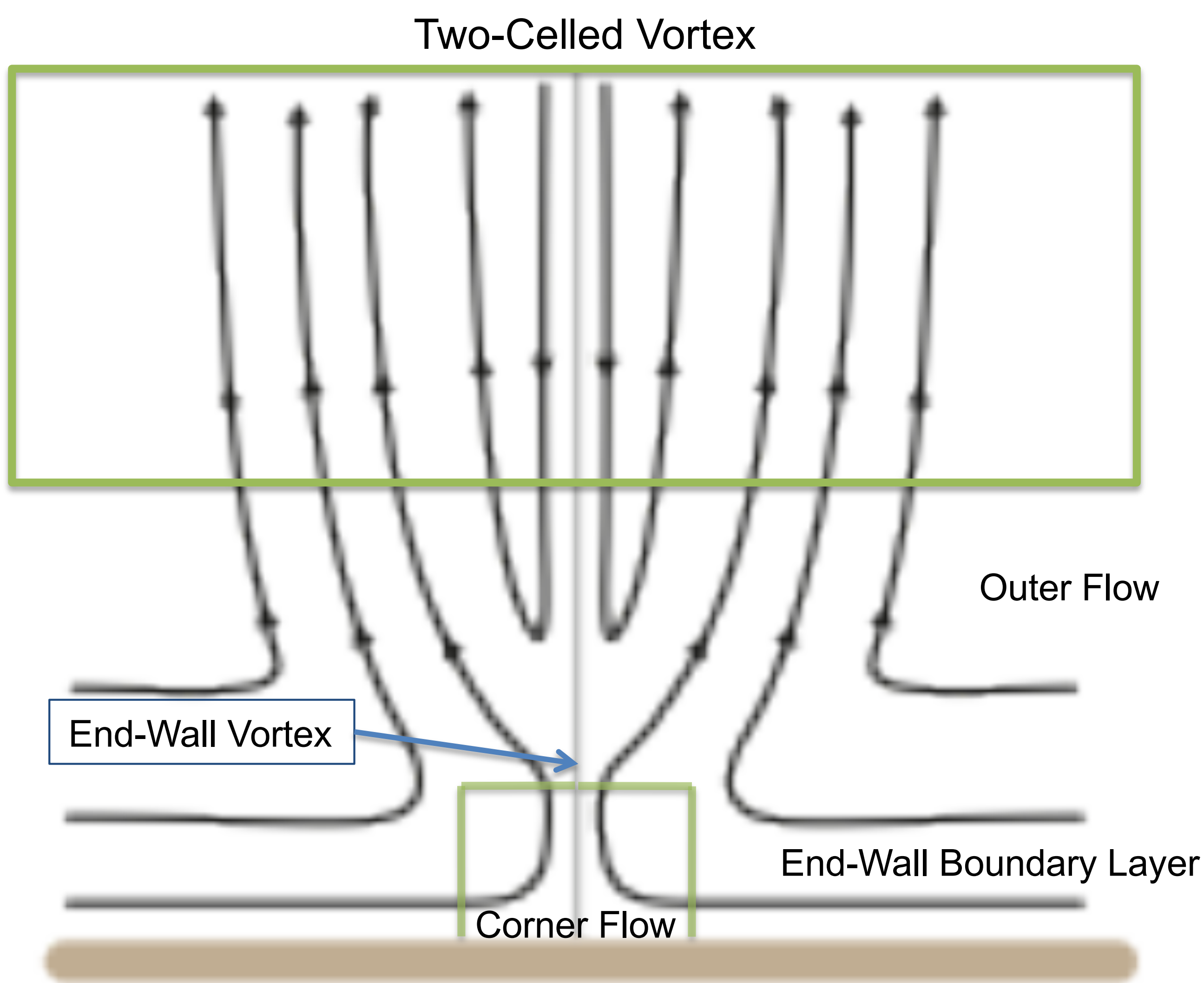
Core size Increases with Swirl Ratio

Core Size and Max Velocity from
Free-Slip Two-Celled Vortex Experiments



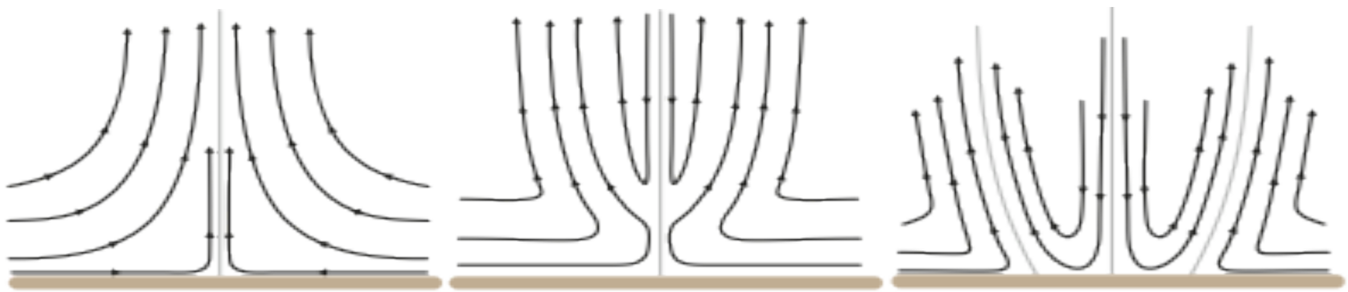
Imposed Buoyancy Force and Grid




Two-Celled Vortex
Outer Flow
End-Wall Vortex
End-Wall Boundary Layer
Corner Flow

Summary

1. Fiedler Chamber also produces a realistic range of vortex structures



2. Free-slip conditions eliminate the surface boundary layer and corner flow →

3. Two-celled vortex has its own distinct dynamics, essentially the classic mechanism of conservation of angular momentum