The Fluid Dynamics of Tornadoes

Richard Rotunno

NCAR

Lecture 4: End-Wall Boundary Layer, Vortex & Corner Flow

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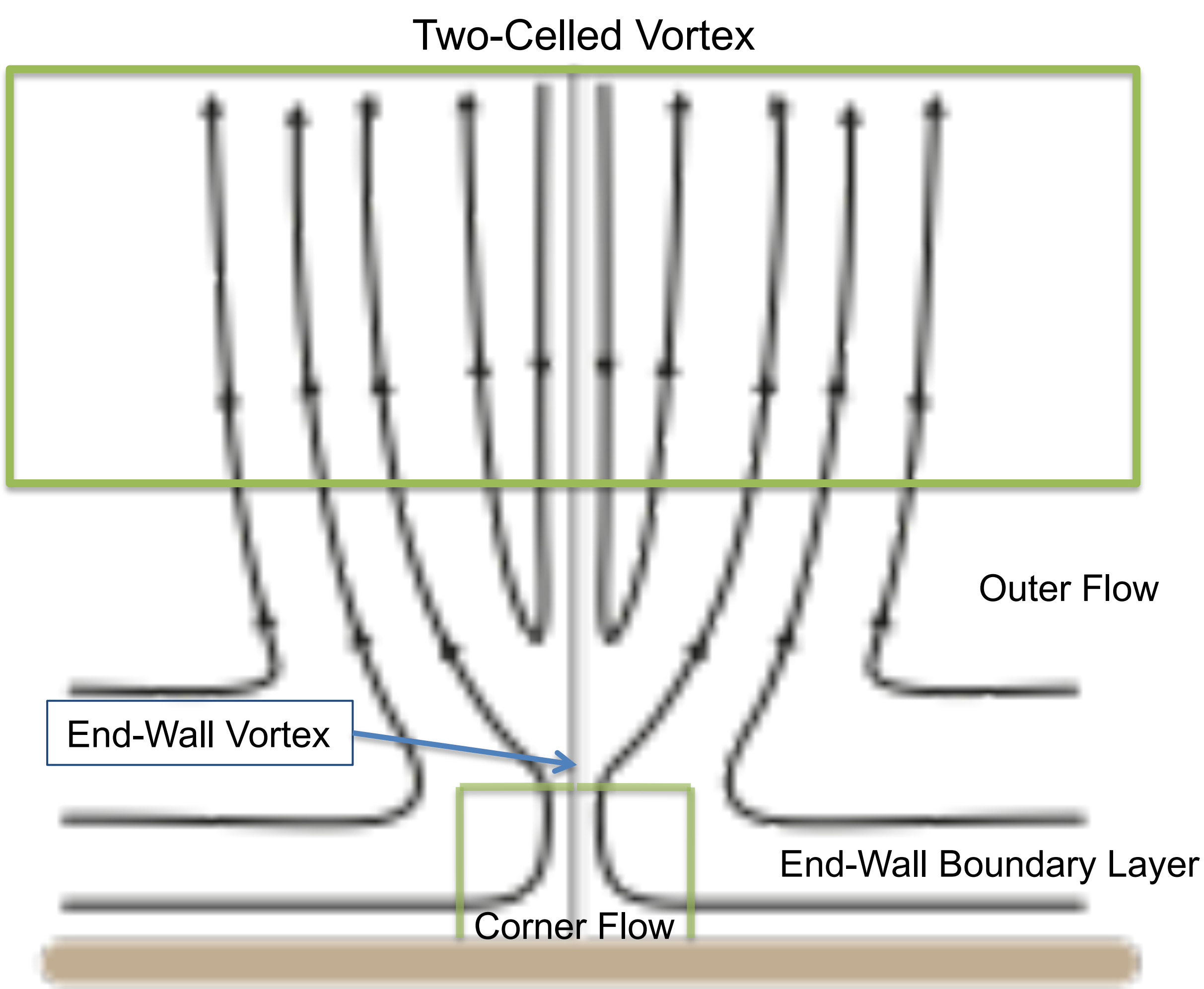
NCAR is funded by the National Science Foundation



Two-Celled Vortex
Outer Flow
End-Wall Vortex
End-Wall Boundary Layer
Corner Flow

Basics of Rotating Boundary-Layer Flow

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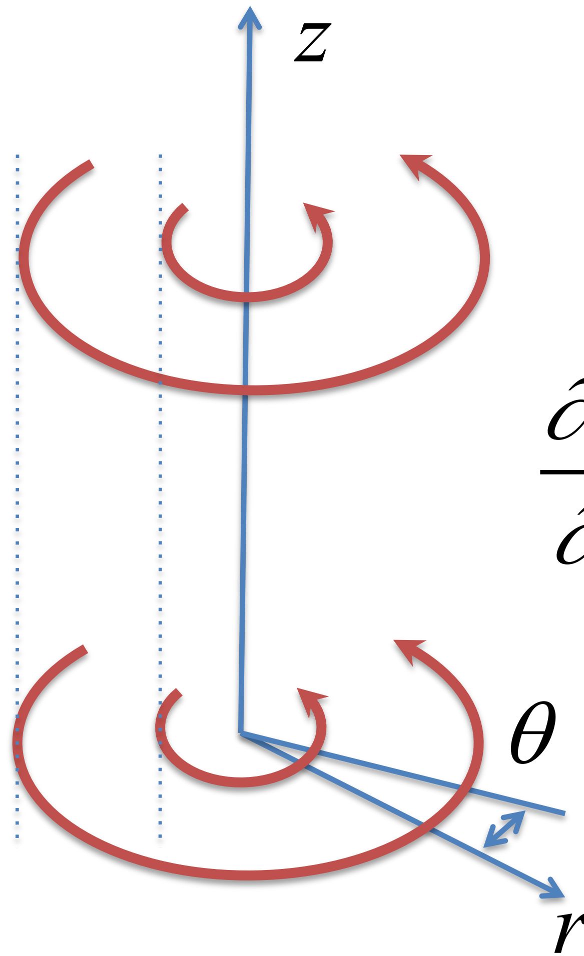
1. Rotating Flow / No Boundary

Simple Vortex

$$[0, V(r), 0]$$



$$\frac{\partial p}{\partial r} = \frac{V^2}{r}$$

2. Rotating Flow/Frictional Boundary layer

National Committee for Fluid Mechanics Films

FILM NOTES

for

SECONDARY FLOW*

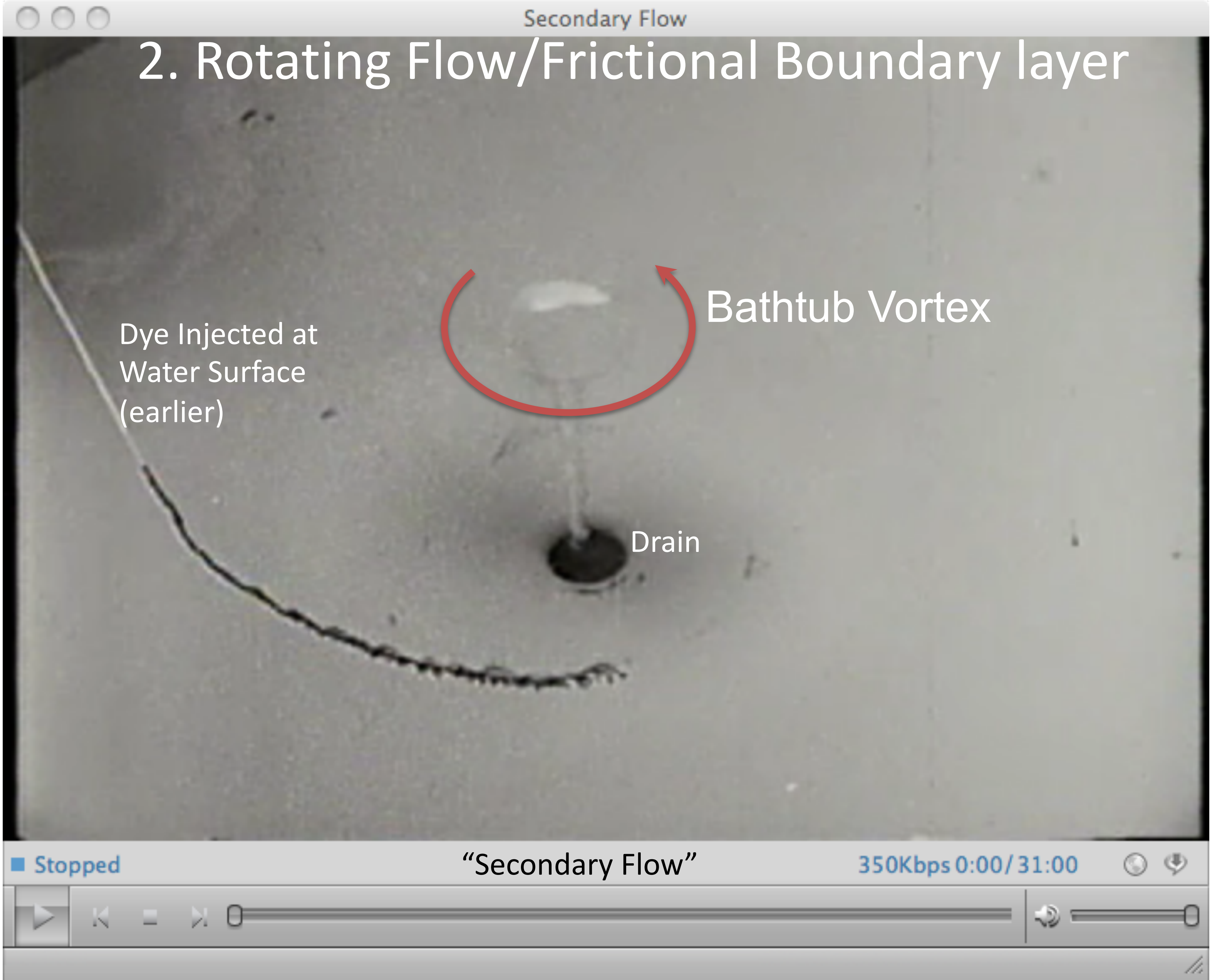
By

EDWARD S. TAYLOR

Massachusetts Institute of Technology

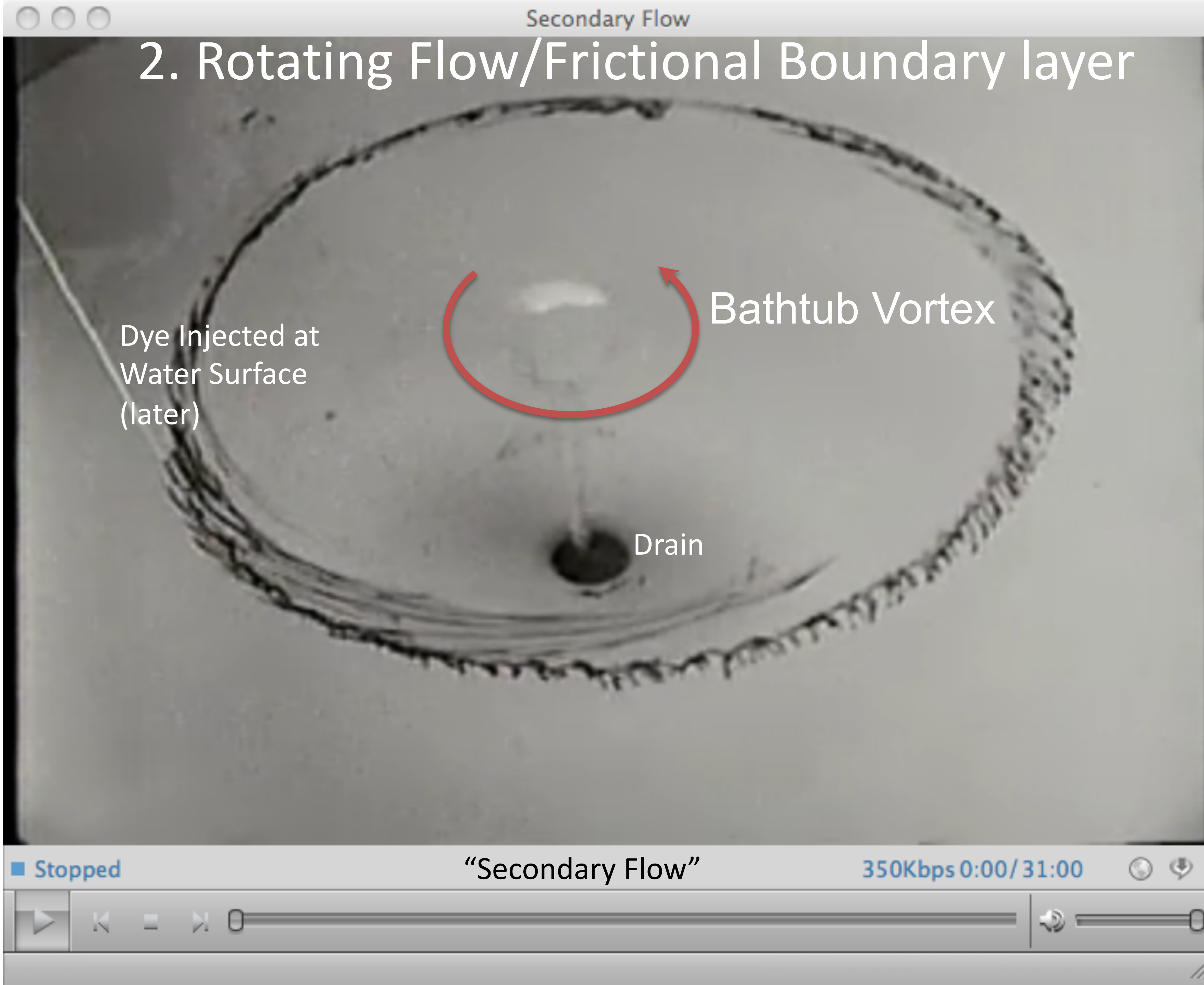
http://web.mit.edu/hml/ncfmf.html

2. Rotating Flow/Frictional Boundary layer

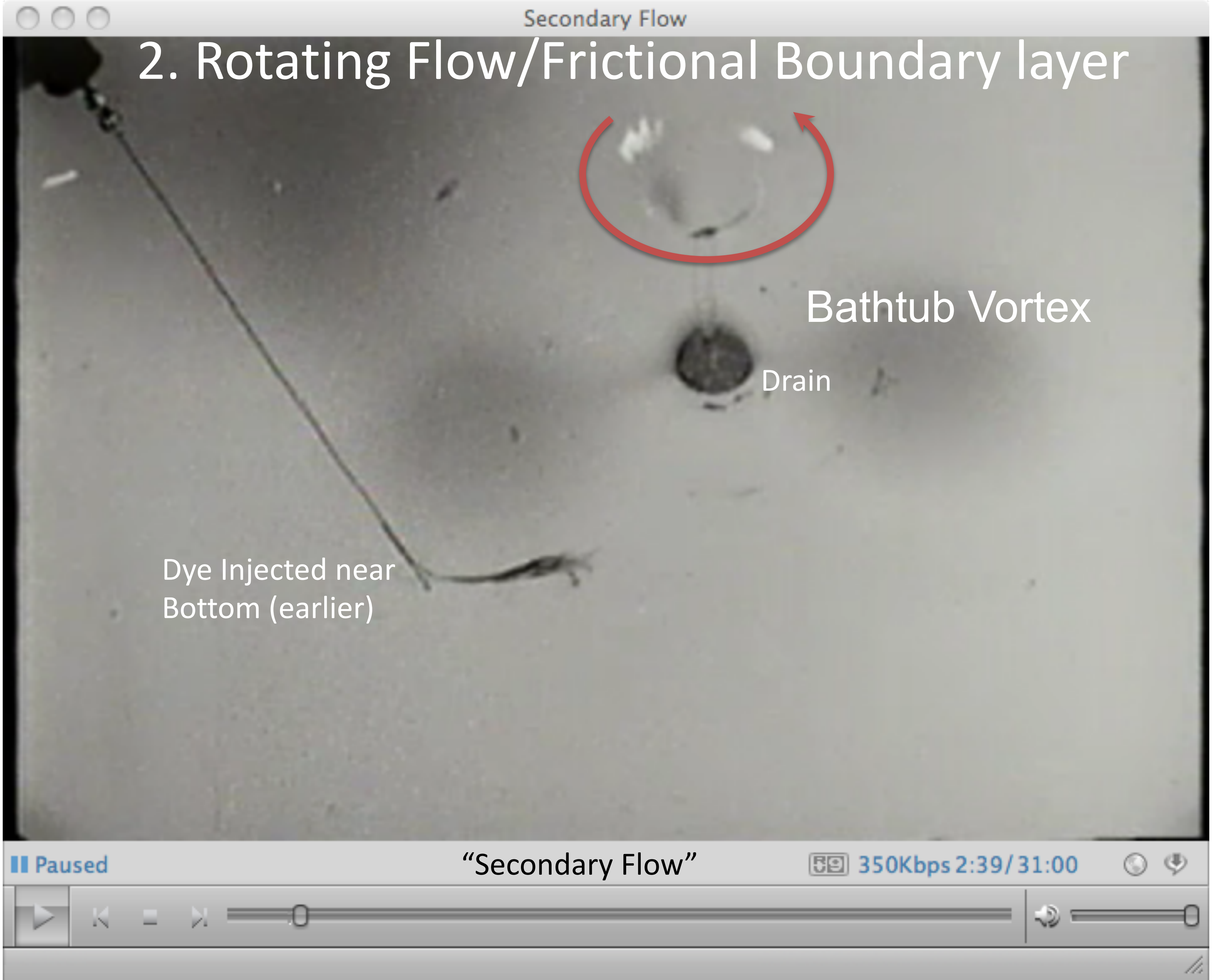


"Secondary Flow"

2. Rotating Flow/Frictional Boundary layer



2. Rotating Flow/Frictional Boundary layer



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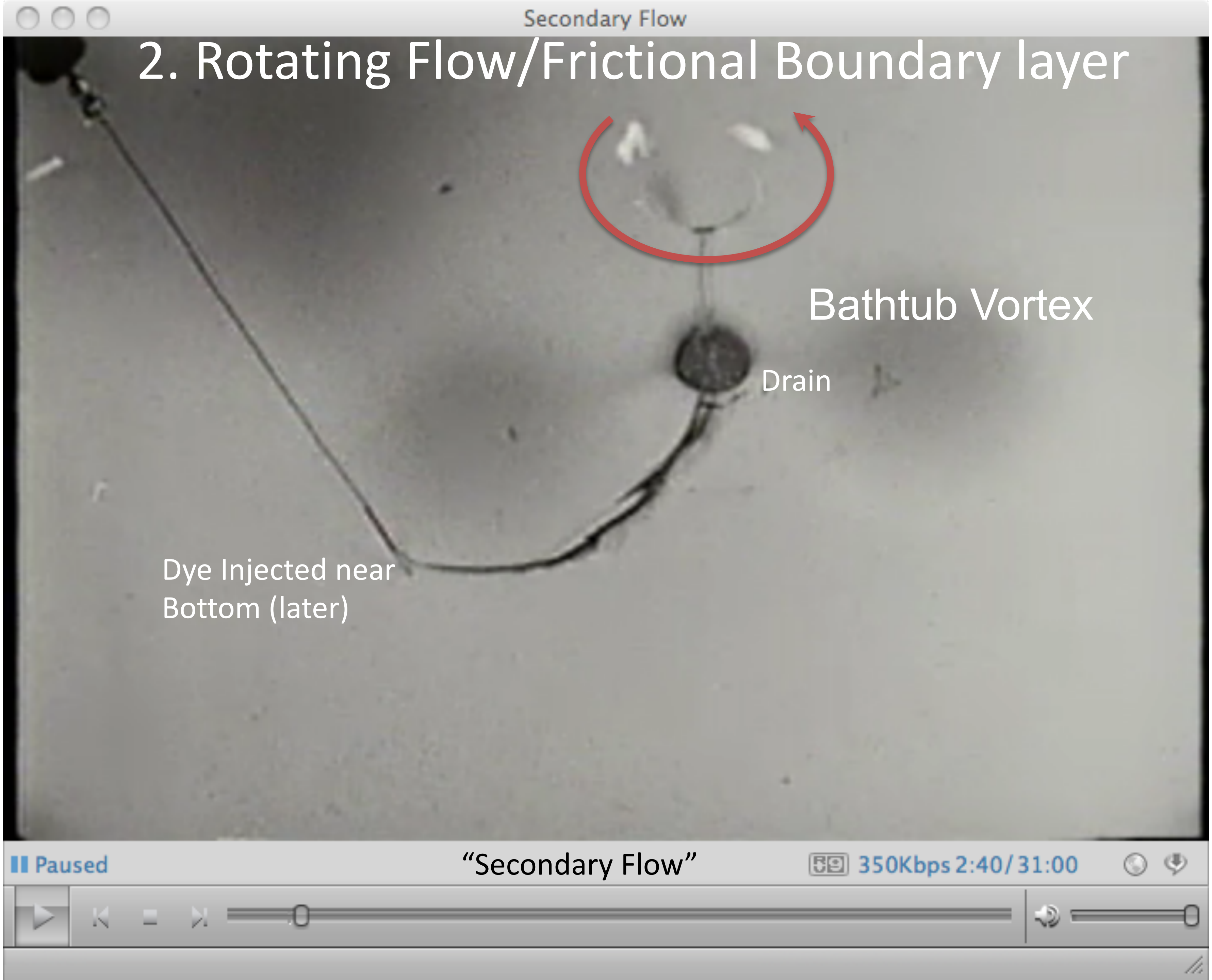

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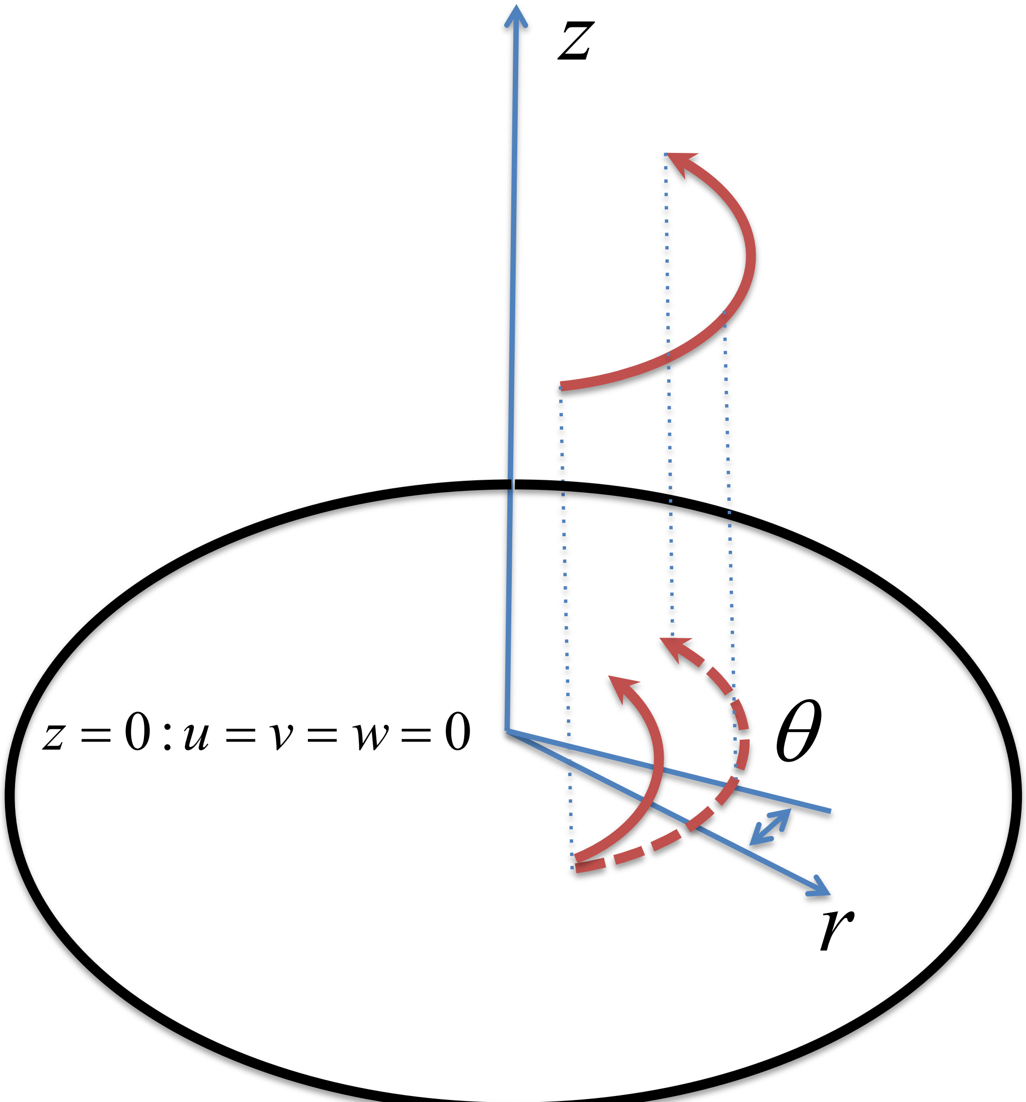
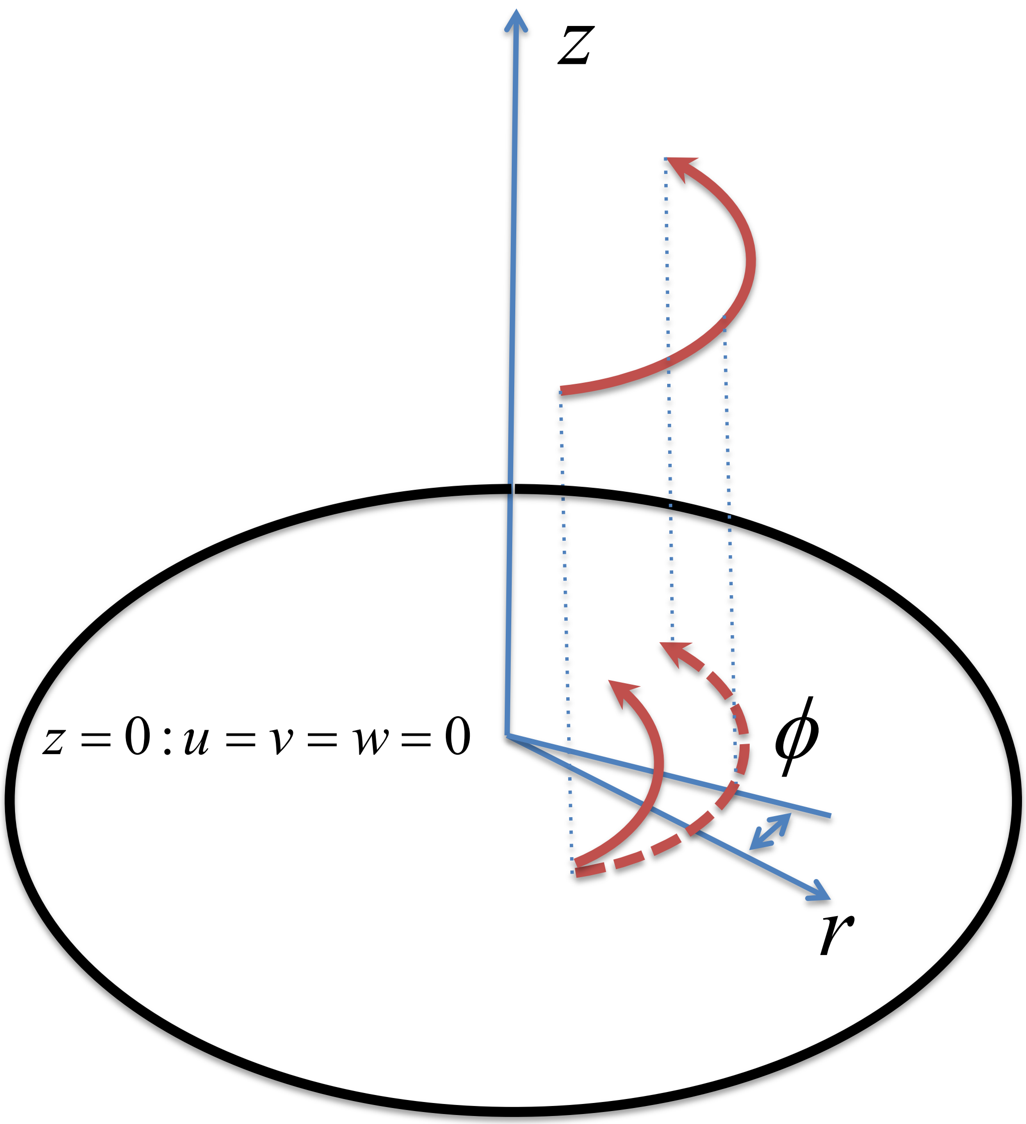


$$0 = -\frac{\partial p}{\partial r} + \frac{V^2}{r}$$

$$0 = -\frac{\partial p}{\partial r} + \nu \frac{\partial^2 u}{\partial z^2}$$

2. Rotating Flow/Frictional Boundary layer



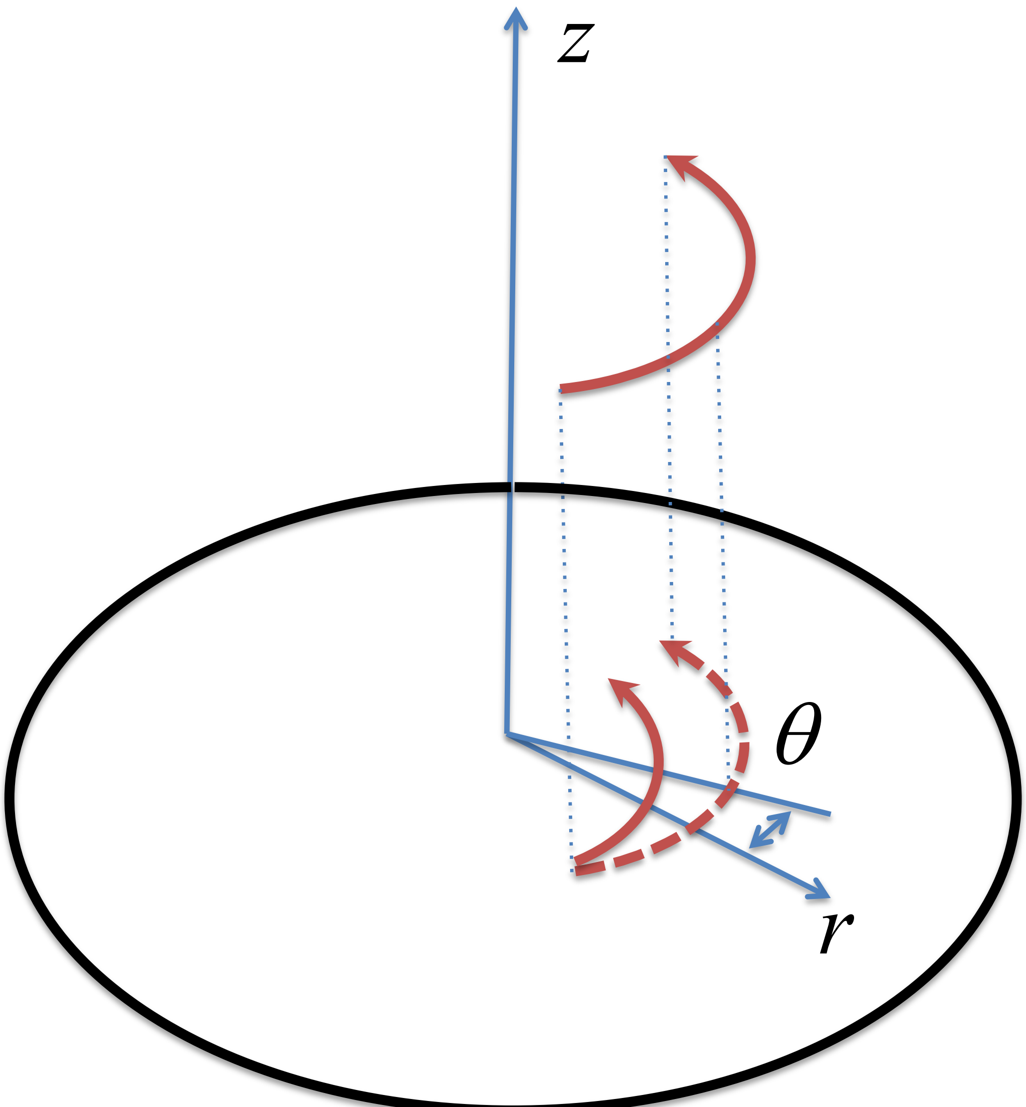
$$0 = -\frac{\partial p}{\partial r} + \frac{V^2}{r}$$

Thought experiment:
Apply lower b.c. at t=0→

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = -\frac{\partial p}{\partial r}$$

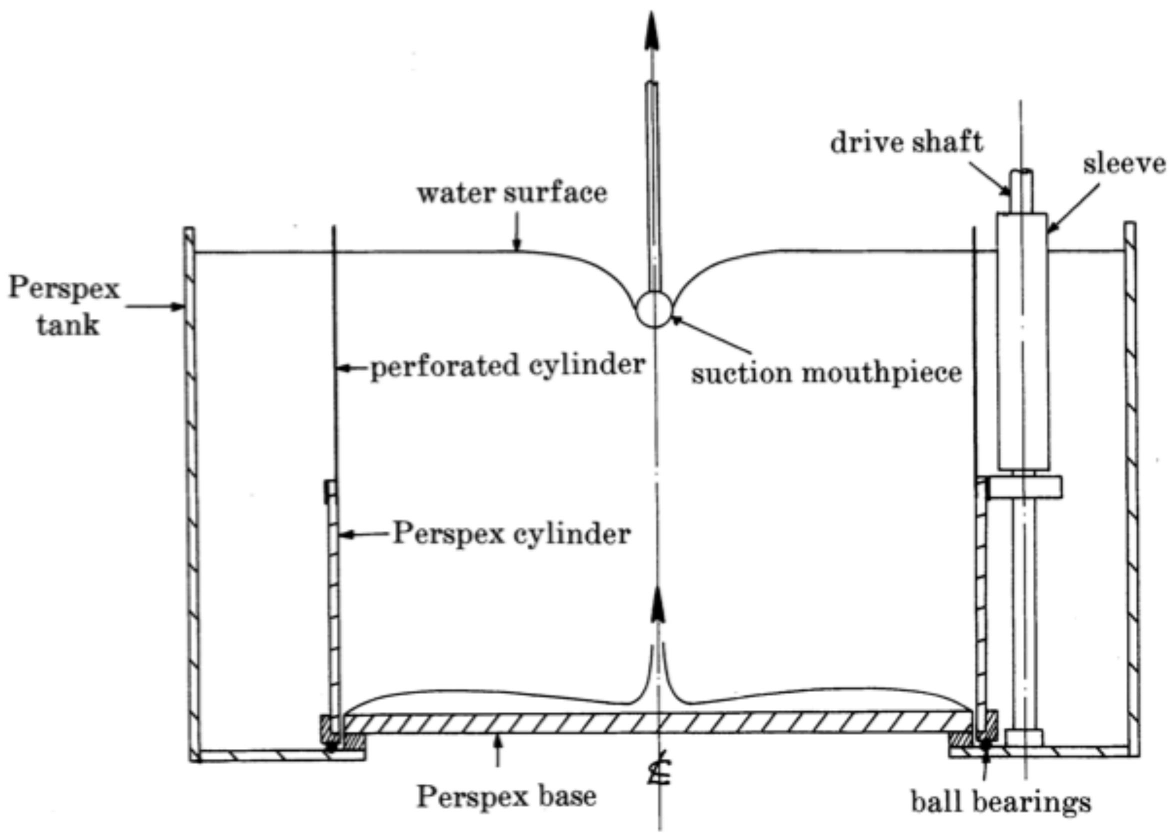
2. Rotating Flow/Frictional Boundary layer

For any $V(r)$, $u < 0$ (radial inflow) near frictional boundary



3. Potential Vortex Above a Stationary Disk: Experiment

$$V \propto r^{-1}$$



Phillips and Khoo (1987, *Proc. Roy. Soc. London*)

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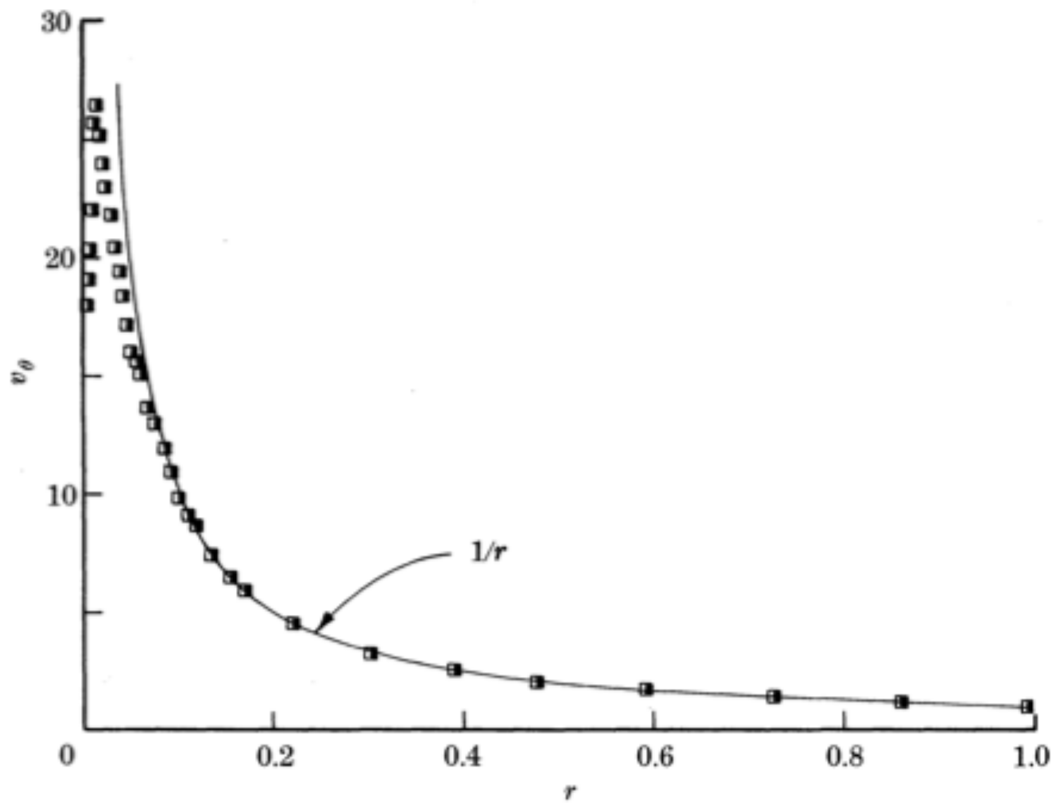
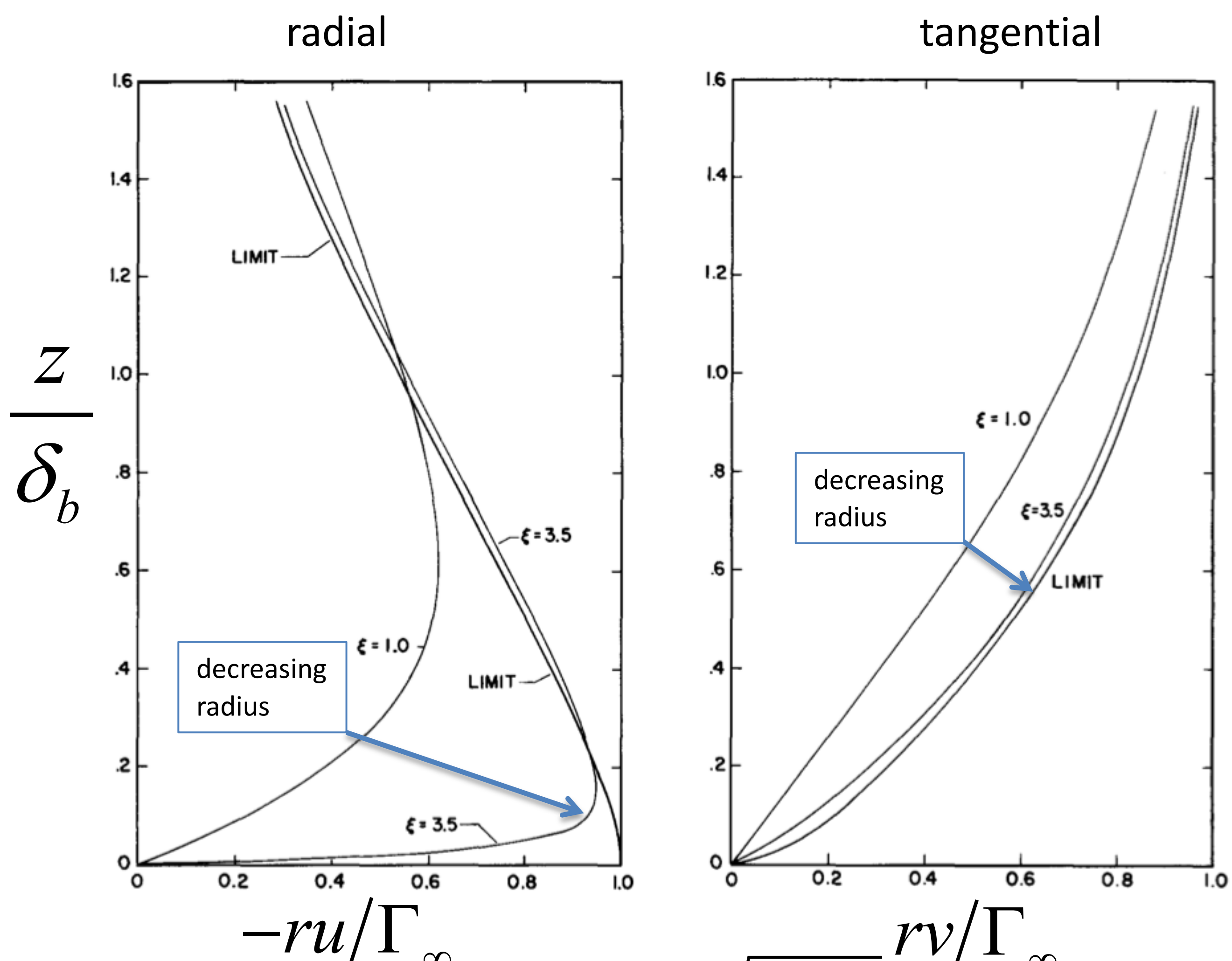


FIGURE 2. Circumferential velocity field in outer flow. $Re = 10000$, $z = 0.127$.

Phillips and Khoo (1987, *Proc. Roy. Soc. London*)

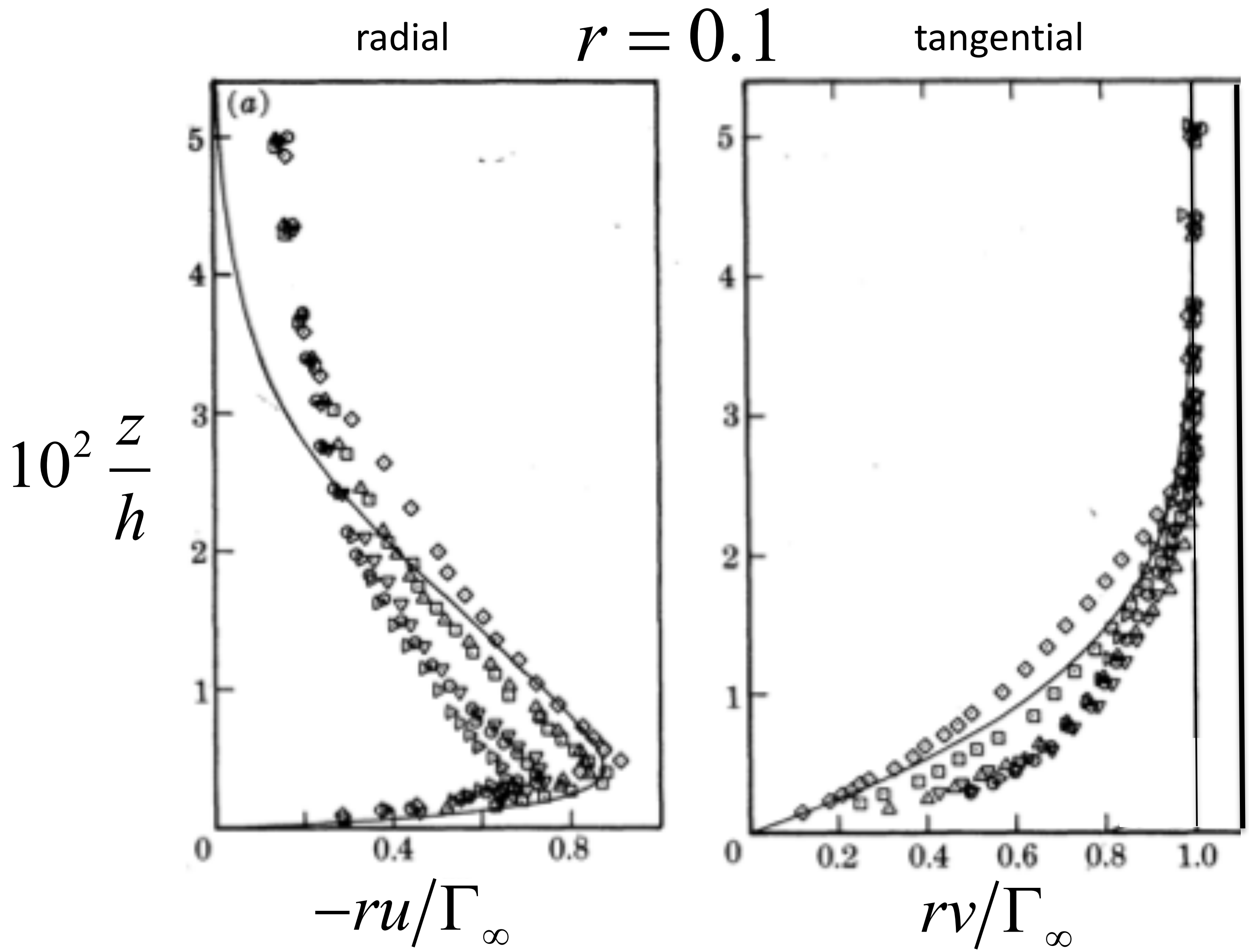
3. Potential Vortex Above a Stationary Disk: Theory



$$\delta_b = r_{disc}\sqrt{\nu/\Gamma_\infty}$$

Burggraf, Stewartson and Belcher (1971, *Phys. Fluids*)

3. Potential Vortex Above a Stationary Disk: Theory & Experiment



Phillips and Khoo (1987, *Proc. Roy. Soc. London*)

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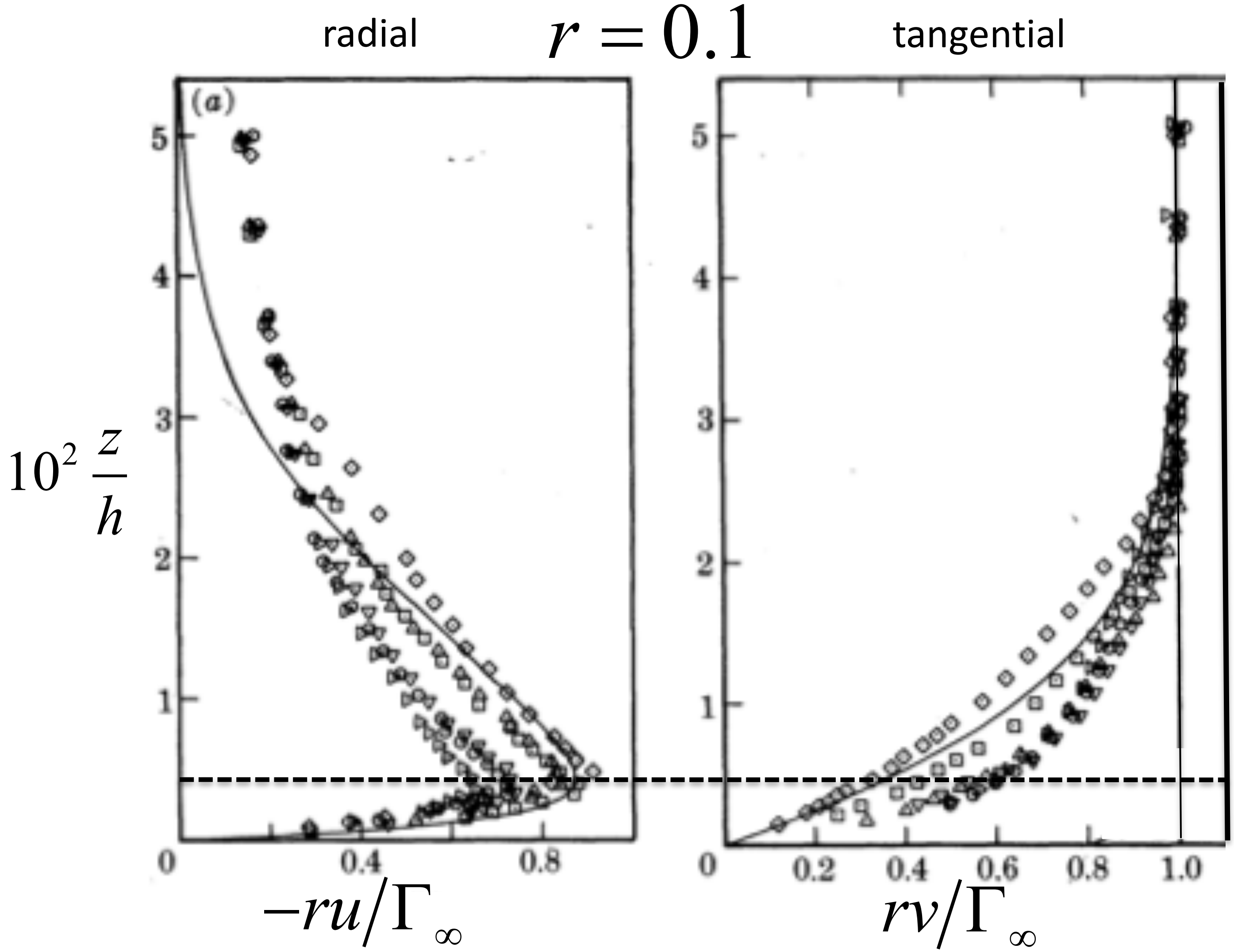


Phillips and Khoo (1987, *Proc. Roy. Soc. London*)

Inertial layer:

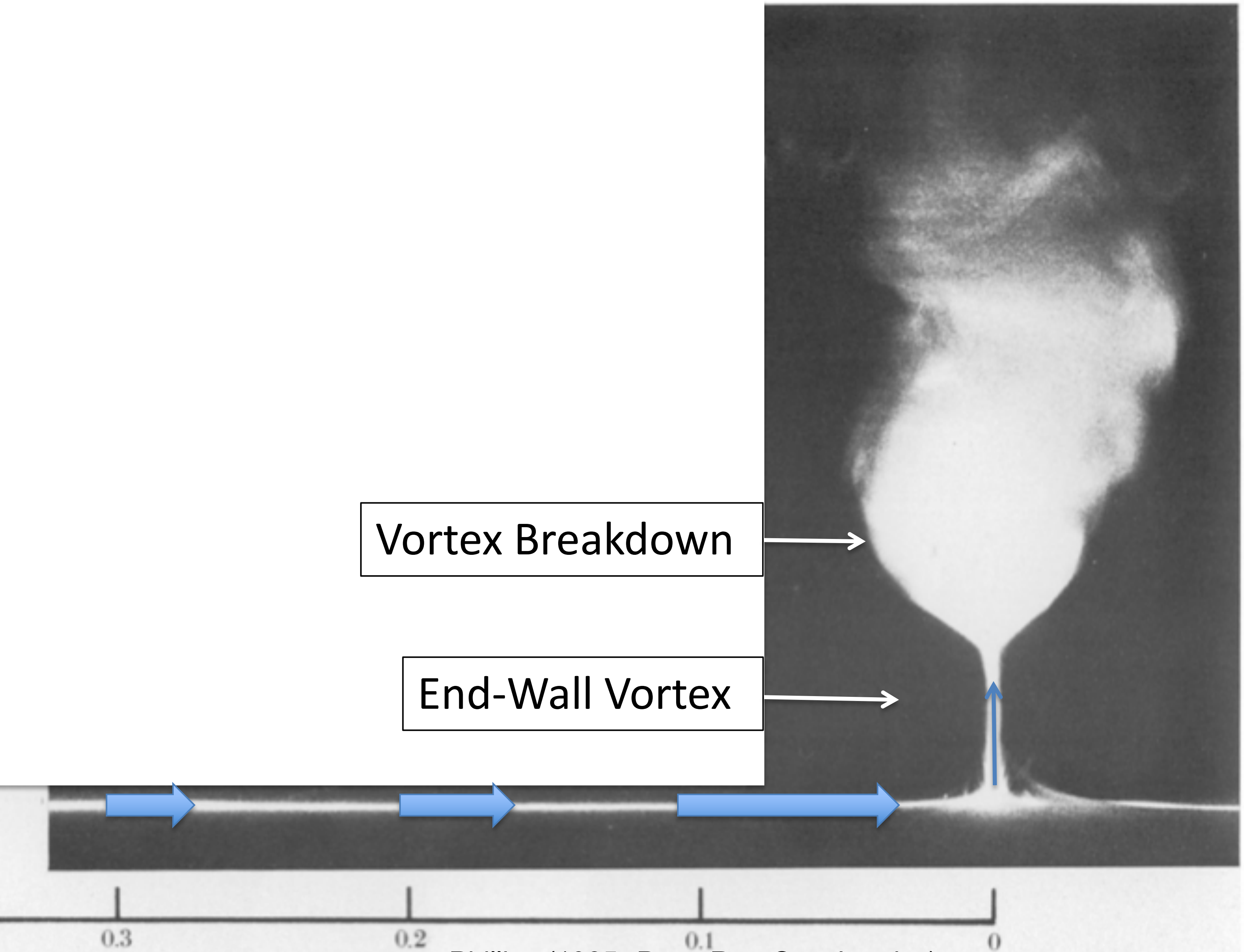
$$\frac{du}{dt} \approx \frac{v^2 - V^2}{r} < 0$$

Friction layer:

$$0 \approx -\frac{V^2}{r} + \nu \frac{\partial^2 u}{\partial z^2}$$

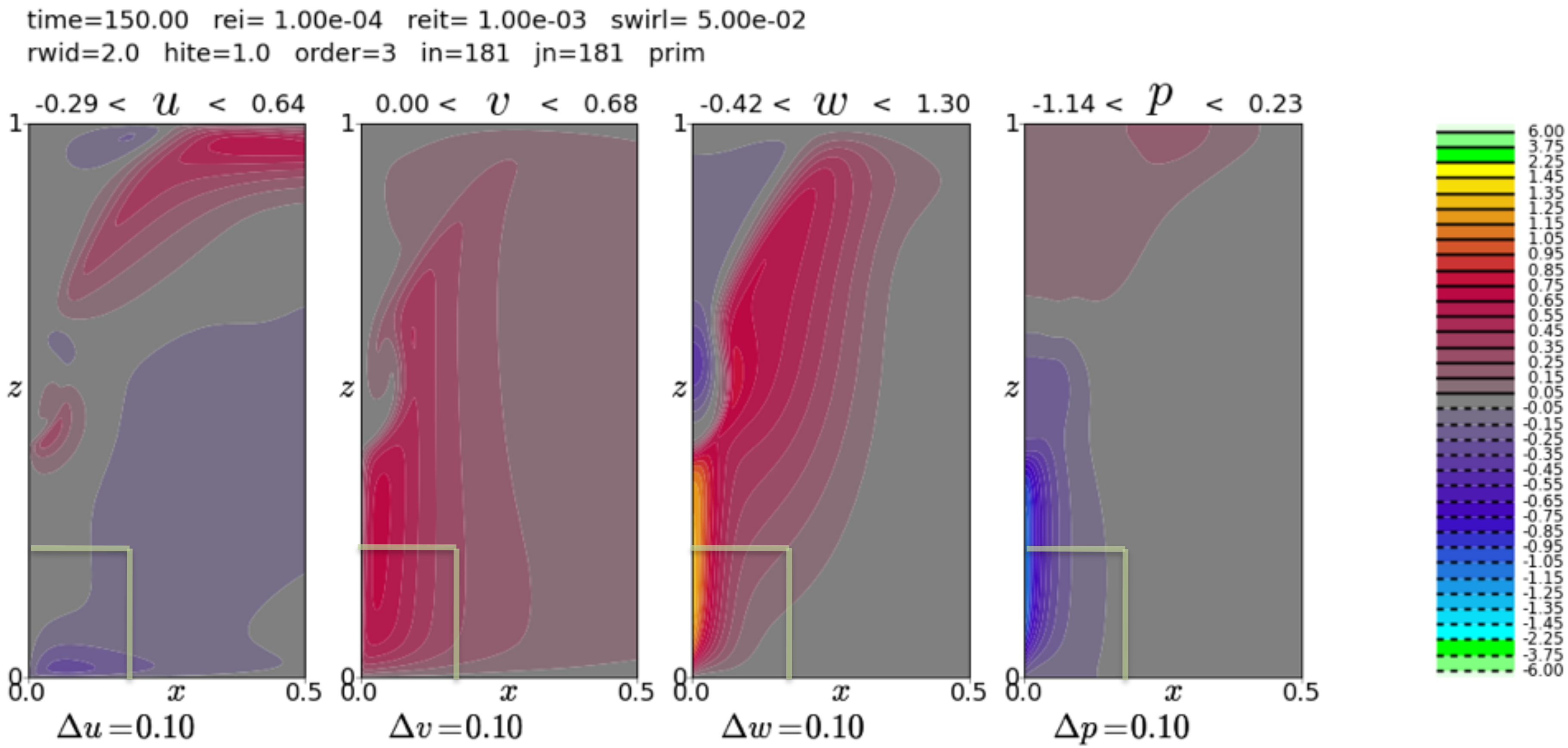
Wilson and Rotunno (1986, *Phys. Fluids*)

3. Potential Vortex Above a Stationary Disk: Experiment



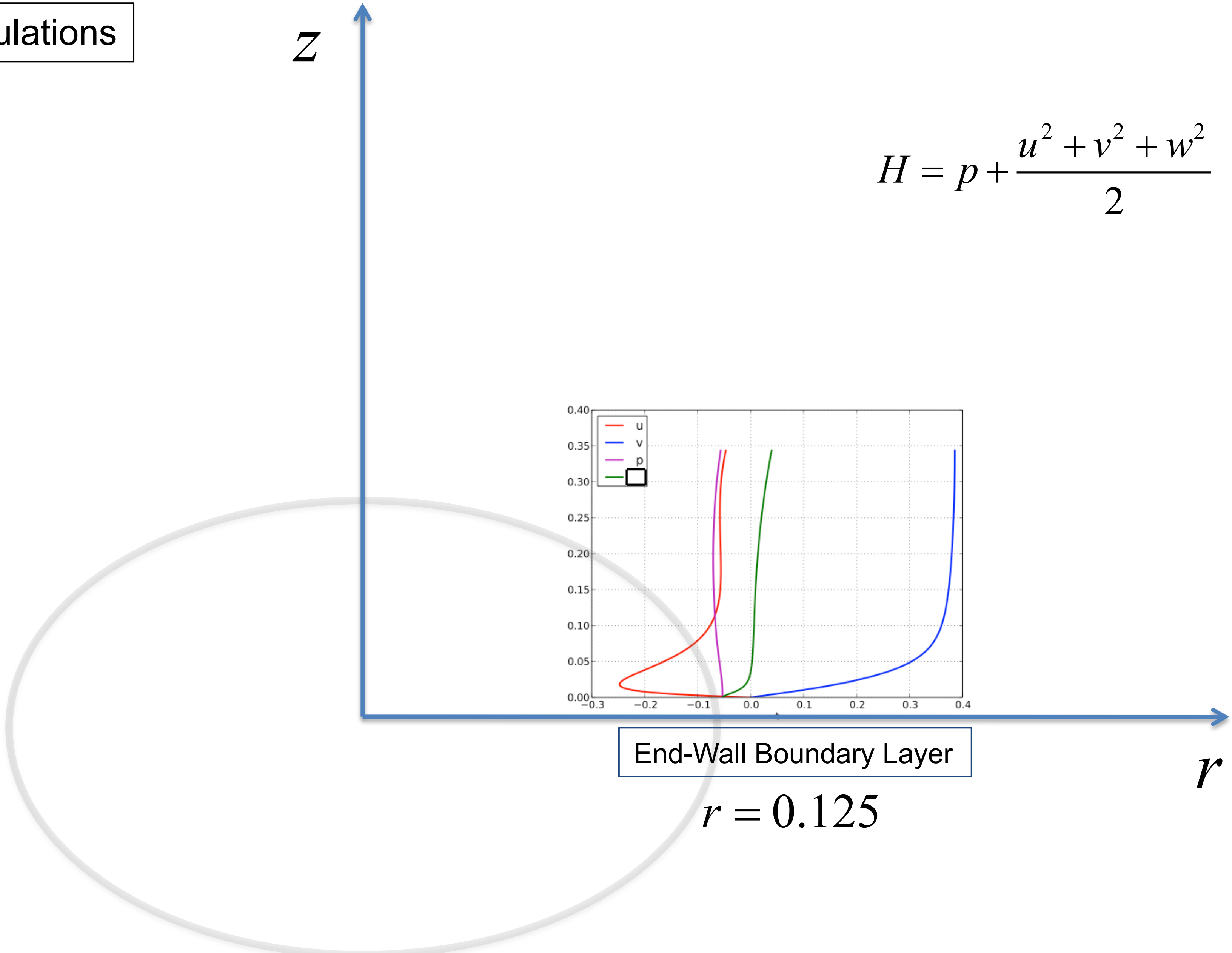
Phillips (1985, *Proc. Roy. Soc. London*)

Corner Flow, End-Wall Vortex and Boundary layer



Simulations

$$H = p + \frac{u^2 + v^2 + w^2}{2}$$



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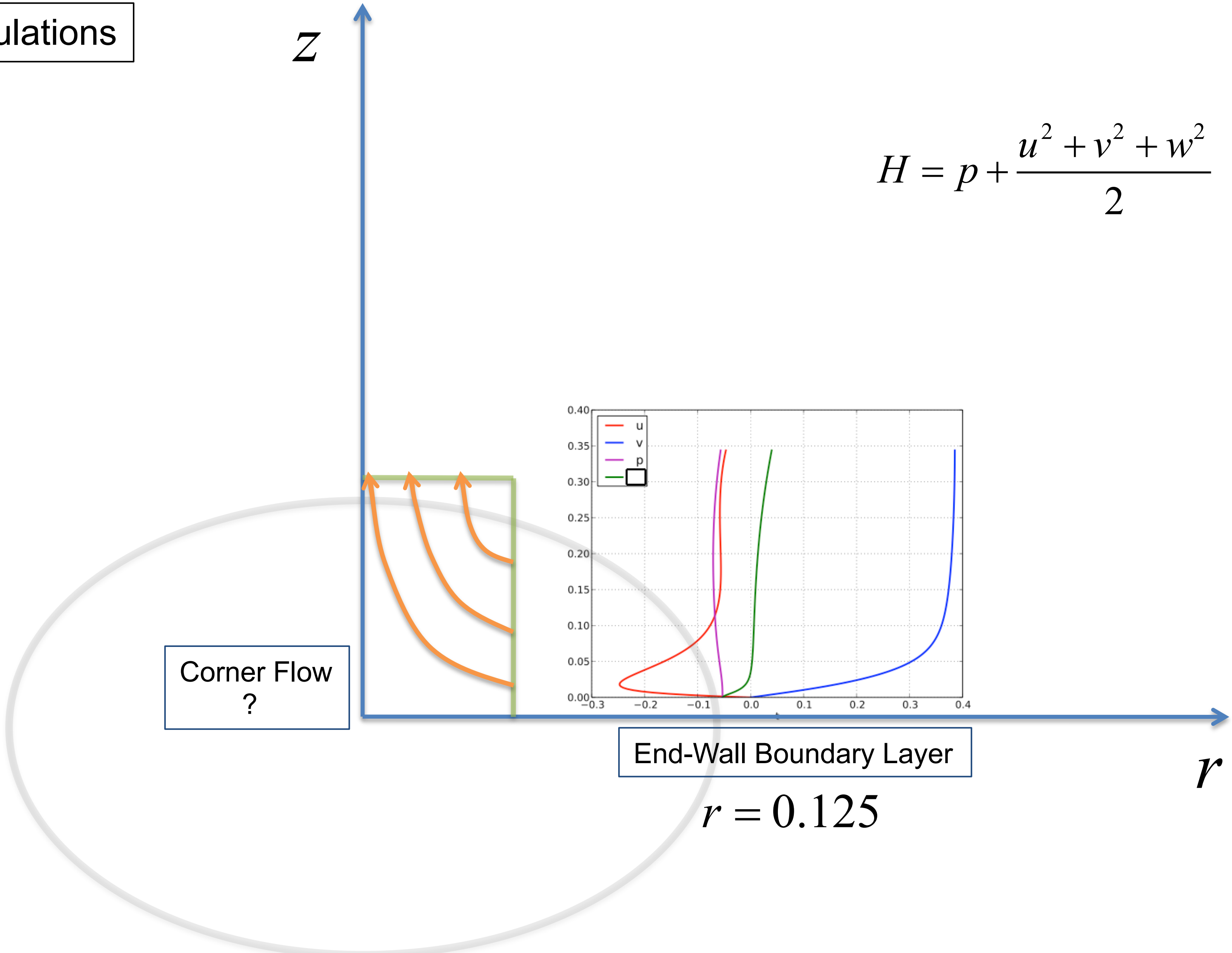
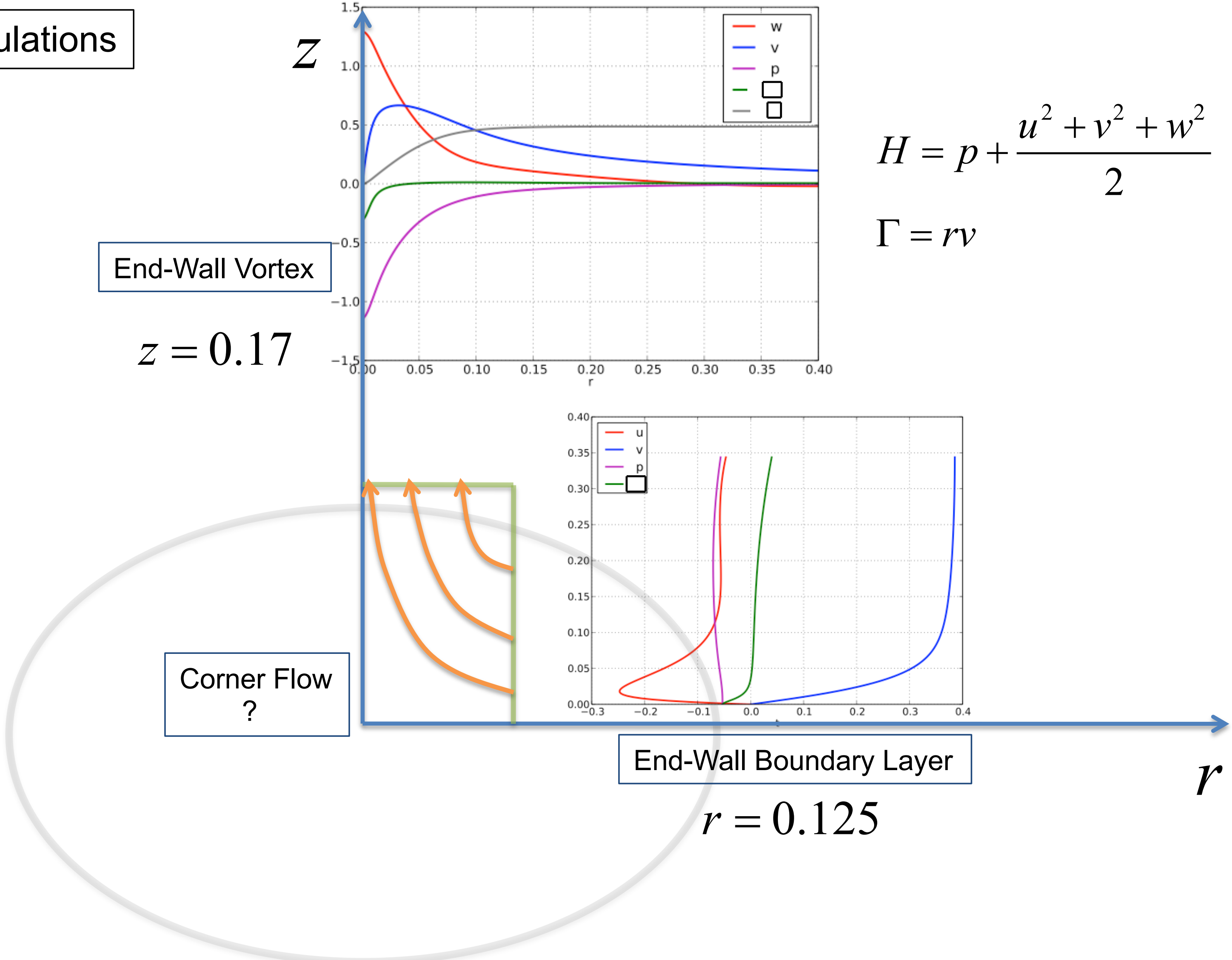


Simulations



$$H = p + \frac{u^2 + v^2 + w^2}{2}$$

$$\Gamma = rv$$

Steady, Inviscid, Nondimensional Momentum Equations

$$r:\ w\eta - v\varsigma = -\frac{\partial H}{\partial r}$$

$$\theta:\ u\varsigma - w\xi = 0$$

$$z:\ v\xi - u\eta = -\frac{\partial H}{\partial z}$$

Vorticity

$$\vec{\omega} = (\xi, \eta, \zeta) = \left(-\frac{1}{r}\frac{\partial \Gamma}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}, \frac{1}{r}\frac{\partial \Gamma}{\partial r} \right)$$

$$H = p + \frac{u^2 + v^2 + w^2}{2}$$

$$\Gamma = rv$$

Batchelor (1967*, An Introduction to Fluid Dynamics*, Chap. 7.5)

Steady, Inviscid, Nondimensional Momentum Equations

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$$H = p + \frac{u^2 + v^2 + w^2}{2}$$

$$\Gamma = rv$$

$$\frac{1}{r}\frac{\partial ru}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad \rightarrow \quad u = \frac{1}{r}\frac{\partial\psi}{\partial z}\ ;\ w = -\frac{1}{r}\frac{\partial\psi}{\partial z}$$

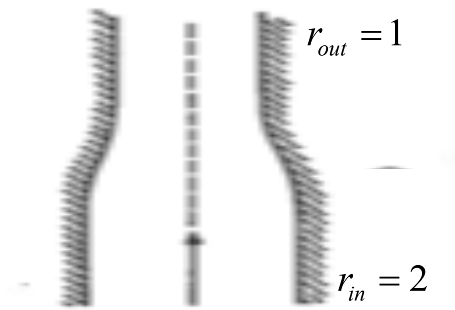
$$\boxed{\frac{\partial^2\psi}{\partial z^2} + r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial\psi}{\partial r} = r^2\frac{dH}{d\psi} - \frac{d}{d\psi}\frac{\Gamma^2}{2}}$$

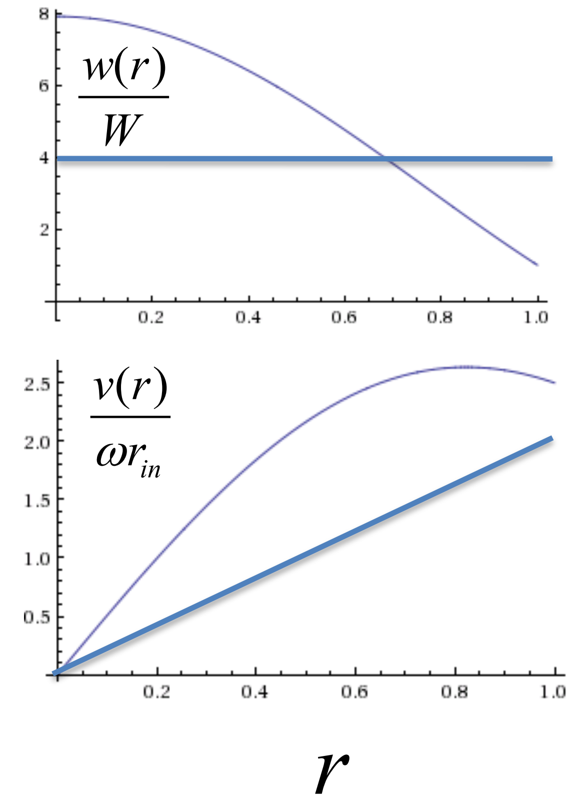
Batchelor (1967*, An Introduction to Fluid Dynamics*, Chap. 7.5)

Example

Outflow → $$r\frac{\partial}{dr}\frac{1}{r}\frac{d\psi}{\partial r} = \frac{2\omega^2}{W}r^2 - \frac{4\omega^2}{W^2}\psi$$

$$\partial / \partial z = 0$$

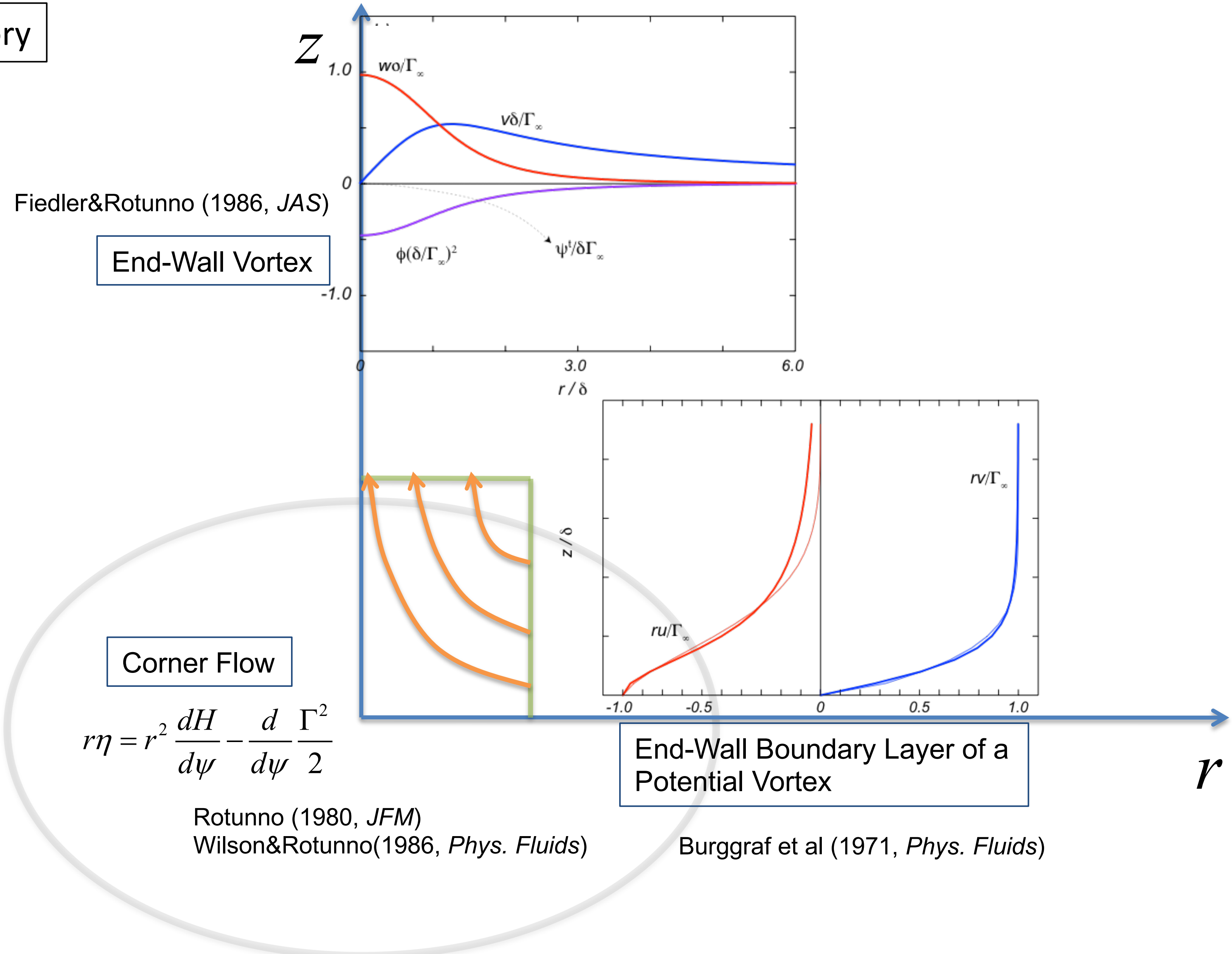




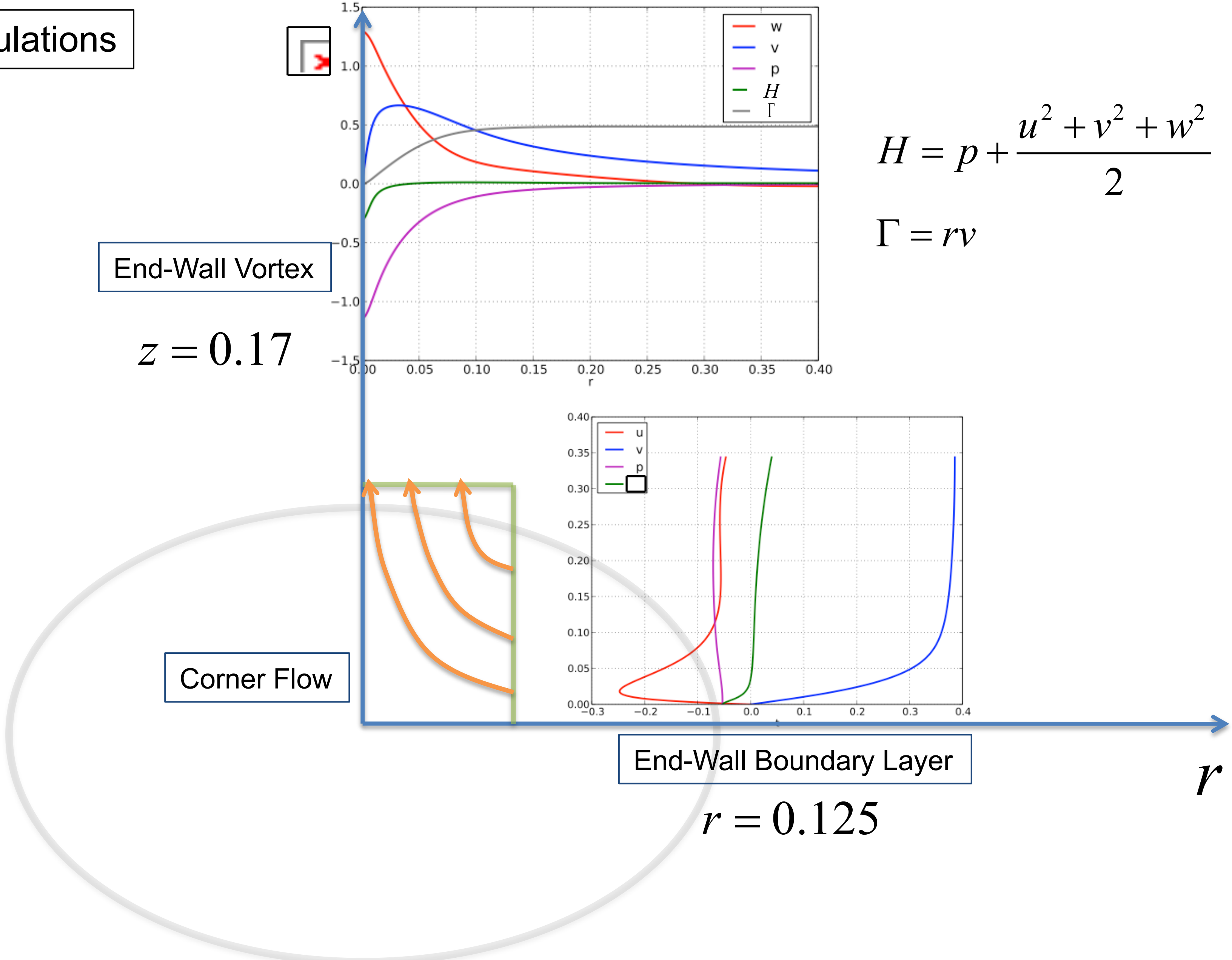
Inflow → $$w = W\ ,\ v = \omega r$$

Batchelor (1967*, An Introduction to Fluid Dynamics*, Chap. 7.5)

Theory


z
r
wo/Γ∞
vδ/Γ∞
φ(δ/Γ∞)²
ψ'/δΓ∞
1.0
0
-1.0
0
3.0
6.0
r / δ
Fiedler&Rotunno (1986, *JAS*)
End-Wall Vortex
Corner Flow
$r\eta = r^2 \frac{dH}{d\psi} - \frac{d}{d\psi}\frac{\Gamma^2}{2}$
Rotunno (1980, *JFM*)
Wilson&Rotunno(1986, *Phys. Fluids*)
z / δ
ru/Γ∞
rv/Γ∞
-1.0
-0.5
0
0.5
1.0
End-Wall Boundary Layer of a Potential Vortex
Burggraf et al (1971, *Phys. Fluids*)

Simulations



End-Wall Boundary Layer

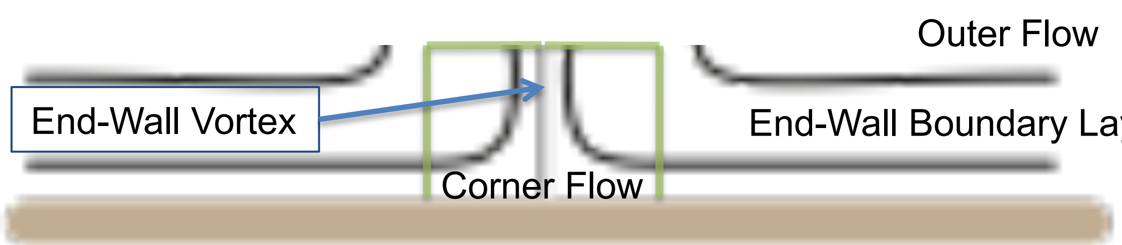
$$\delta_b = r_{disc}\sqrt{\nu/\Gamma_\infty} = \sqrt{\nu / \omega}$$

$$\delta = \delta_b / h = \sqrt{\nu / \omega h^2} \times \sqrt{W / W} = 1 / \sqrt{N\Omega}$$

End-Wall Vortex

$$r_{jet} \propto \delta \propto \frac{1}{\sqrt{N\Omega}}$$

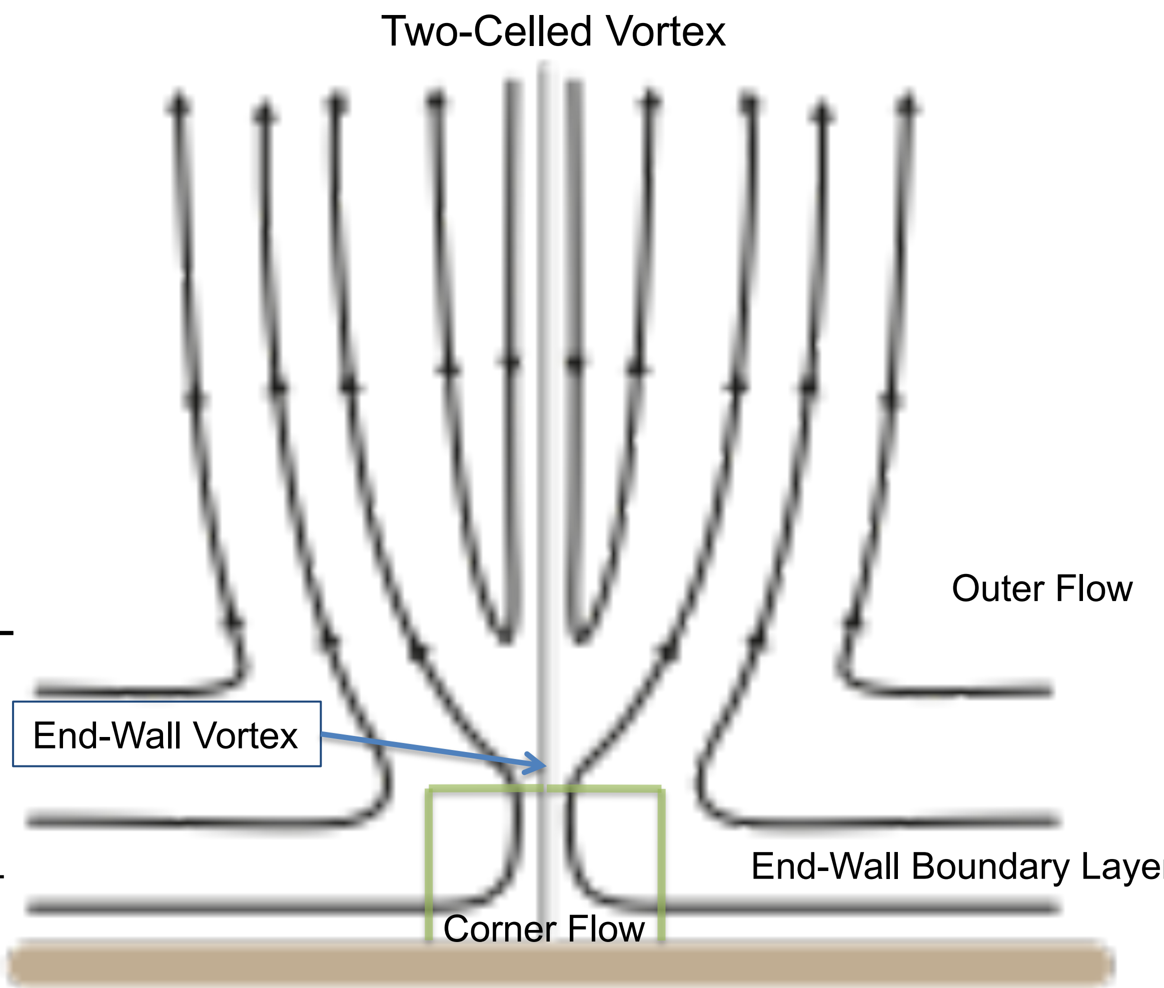
$$v_{jet} \propto \frac{\Gamma}{r_{jet}} \propto \sqrt{N\Omega^3}$$



$$\delta \propto \frac{1}{\sqrt{N\Omega}}$$

$$v_c \approx \frac{1}{\sqrt{2}}$$

$$r_c \propto \Omega$$



$$v_{jet} \propto \sqrt{N\Omega^3}$$

$$r_{jet} \propto \delta \propto \frac{1}{\sqrt{N\Omega}}$$

Vortex Breakdown

Pauley and Snow (1988, *MWR*)

Summary

1. Boundary Layer of a Potential Vortex → $ru \rightarrow \text{constant as } r \rightarrow 0$

2. Boundary Layer of a Potential Vortex → Corner Flow and End-Wall Vortex

3. Generally Giving a Mismatch Between End-Wall and Two-Celled Vortices