The Fluid Dynamics of Tornadoes

Richard Rotunno

NCAR

Lecture 5:Vortex Breakdown & Generalizations

NCAR is funded by the National Science Foundation

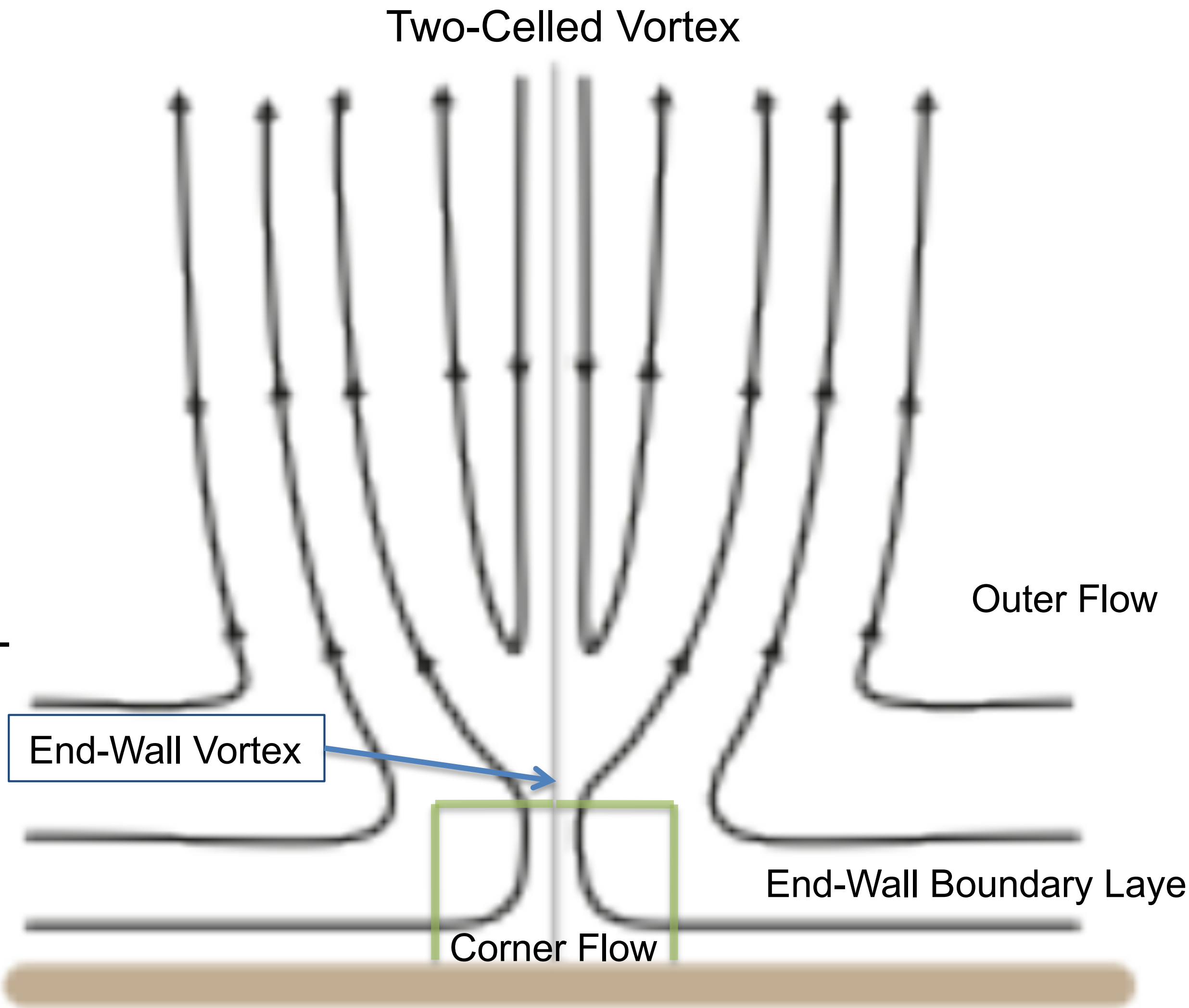
NCAR

$$v_c \approx \frac{1}{\sqrt{2}}$$

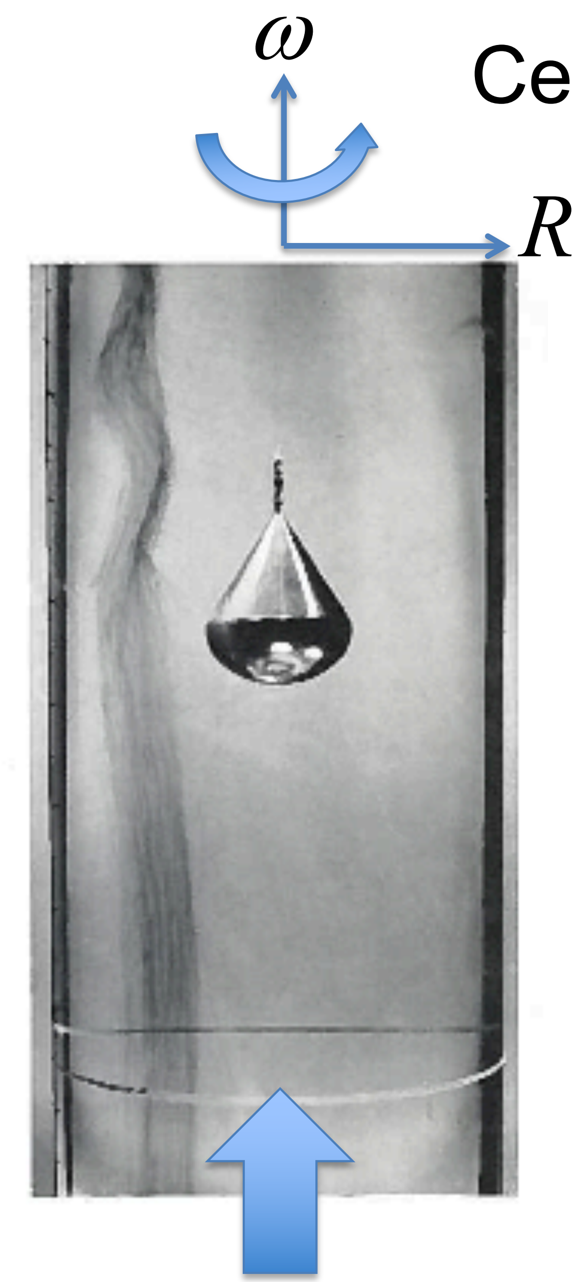
$$r_c \propto \Omega$$

$$v_{jet} \propto \sqrt{N\Omega^3}$$

$$r_{jet} \propto \delta \propto \frac{1}{\sqrt{N\Omega}}$$



Centrifugal (Inertia*) Waves



$$W > c \approx \frac{2\omega R}{3.83}$$

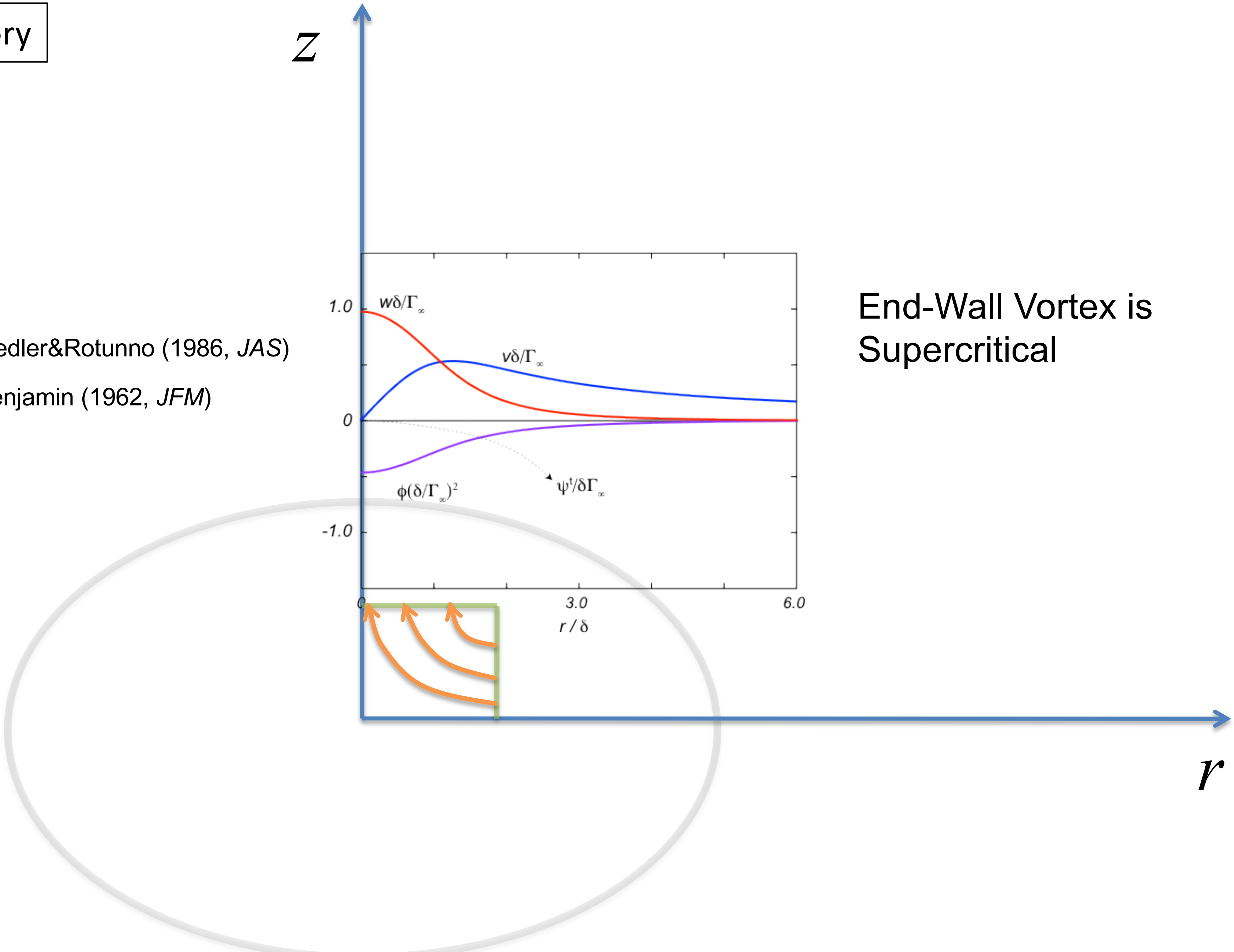
Supercritical

*Batchelor (1967, *In Introduction to Fluid Dynamics, p. 562*)

Theory

Fiedler&Rotunno (1986, *JAS*)

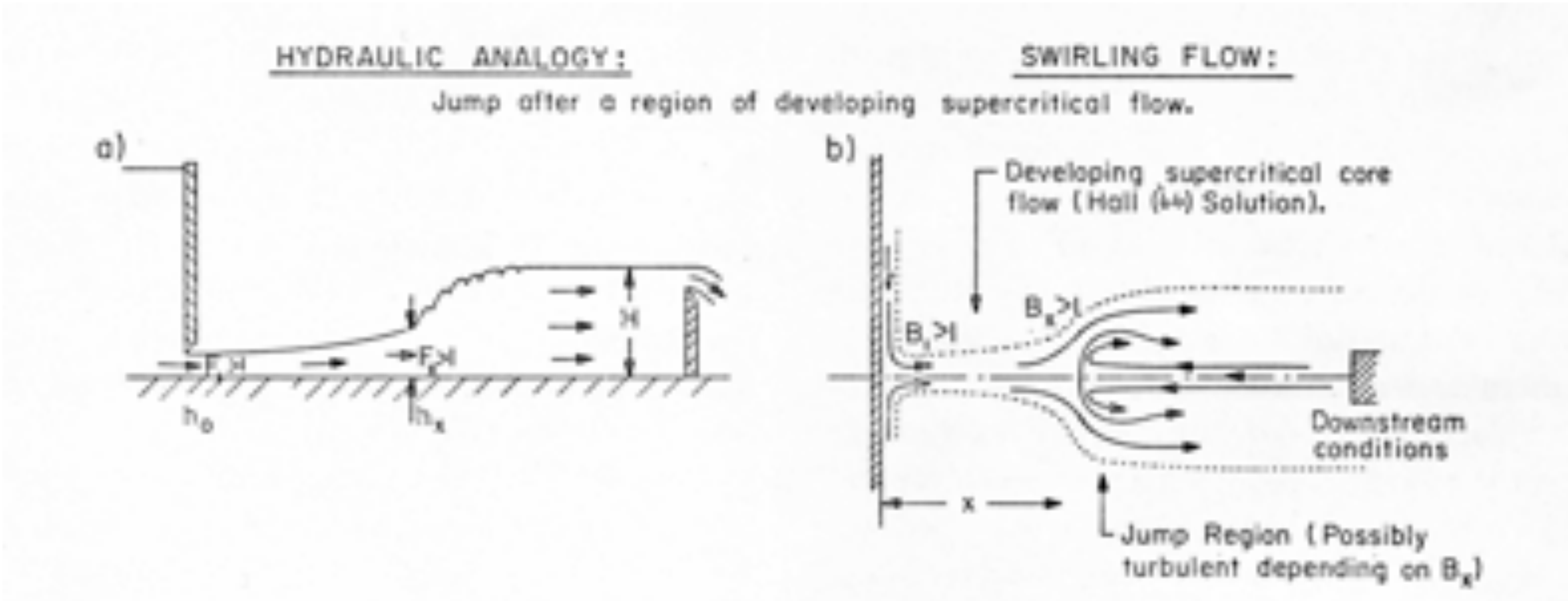
Benjamin (1962, *JFM*)



End-Wall Vortex is Supercritical

Vortex Breakdown

Pauley and Snow (1988, *MWR*)



Maxworthy (1972, *Astro. Acta*)

Compute Subcritical Vortex

1) Assume $w = w_* \text{ for } r \le r_*$

2) Solve

$$\frac{\partial^2 \psi}{\partial z^2} + r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial \psi}{\partial r} = r^2\frac{dH}{d\psi} - \frac{d}{d\psi}\frac{\Gamma^2}{2}$$

3) Iterating on $w_*\ ,\ r_*$

until solution is found such that

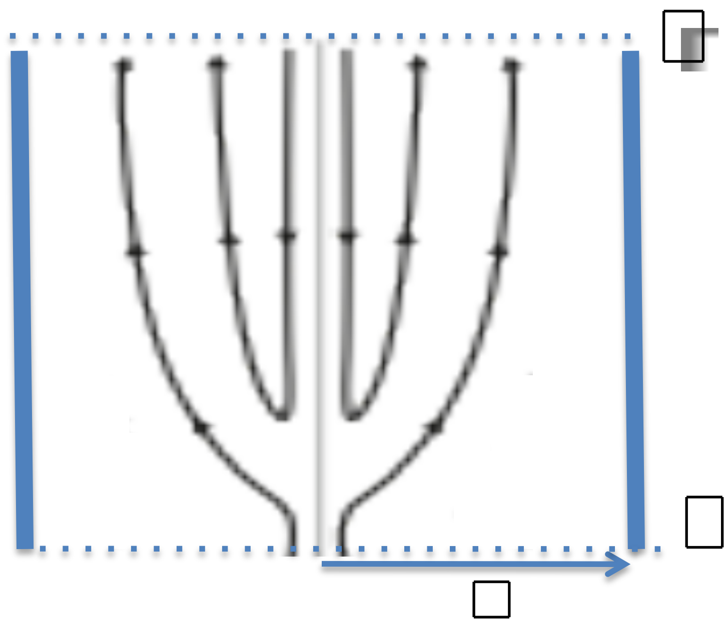
Vertical Momentum Flux = constant

$$2\pi\left(\int_0^R (w^2 + p) r dr\right)_{z_1}^{z_2} = 0$$

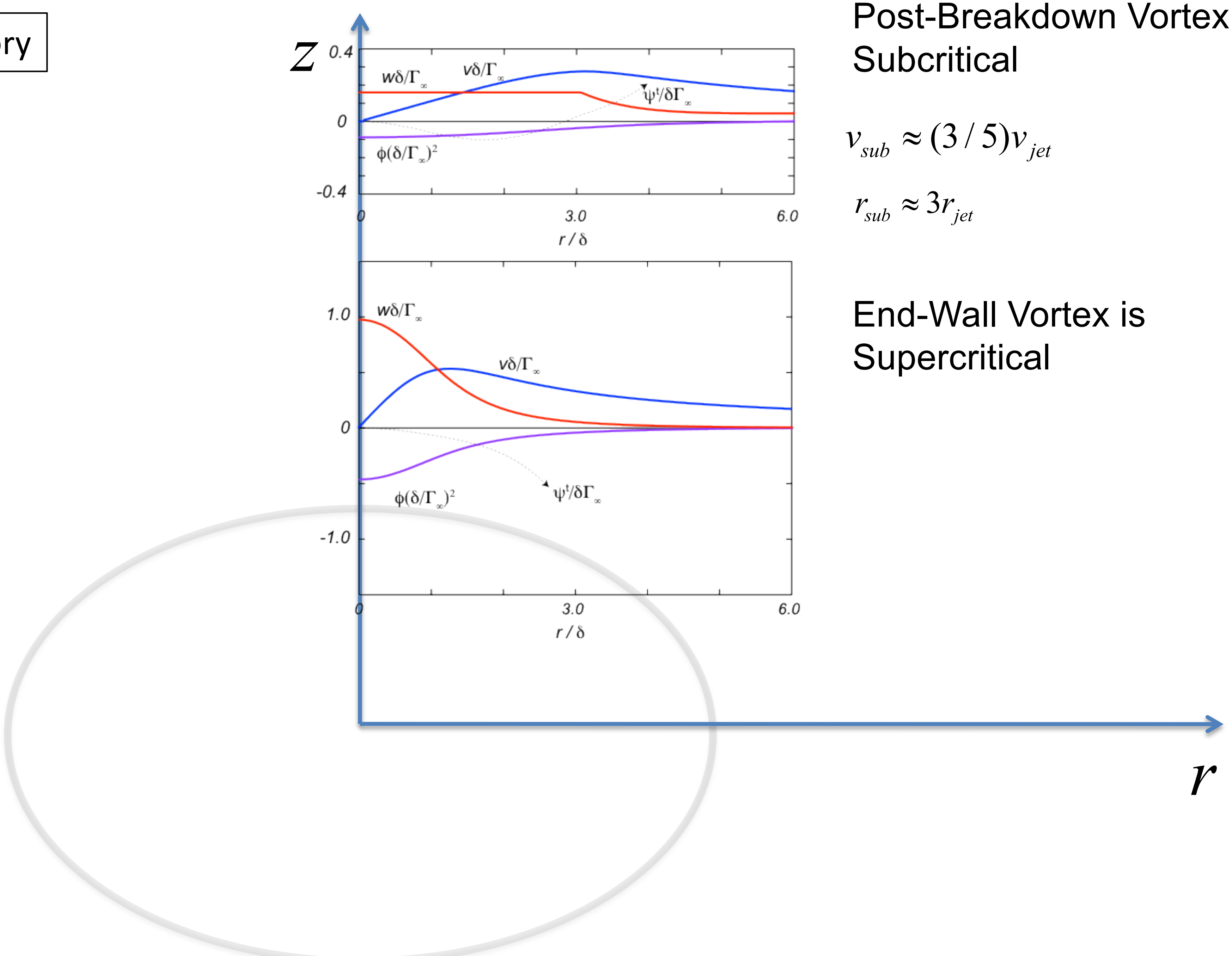
Vertical Mass Flux = constant

$$2\pi\left(\int_0^R w r dr\right)_{z_1}^{z_2} = 0$$

4) Calculate Implied Head Loss



Theory



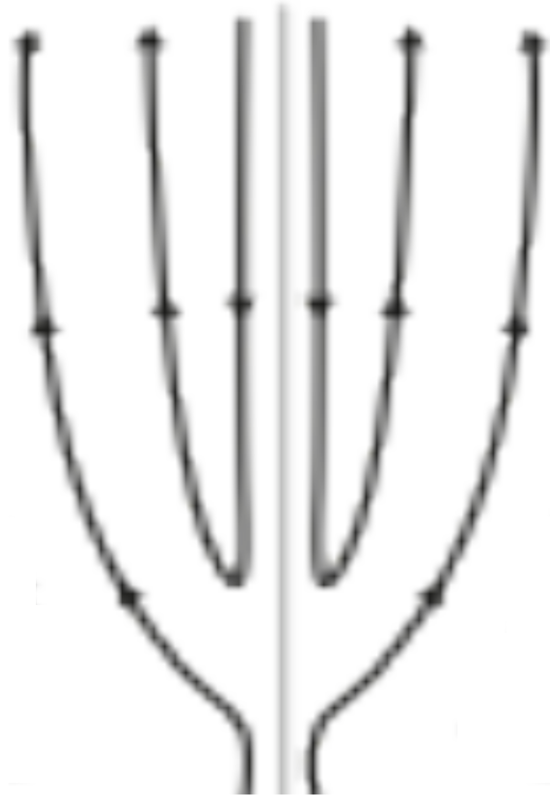
Post-Breakdown Vortex is Subcritical

$$v_{sub} \approx (3/5)v_{jet}$$

$$r_{sub} \approx 3r_{jet}$$

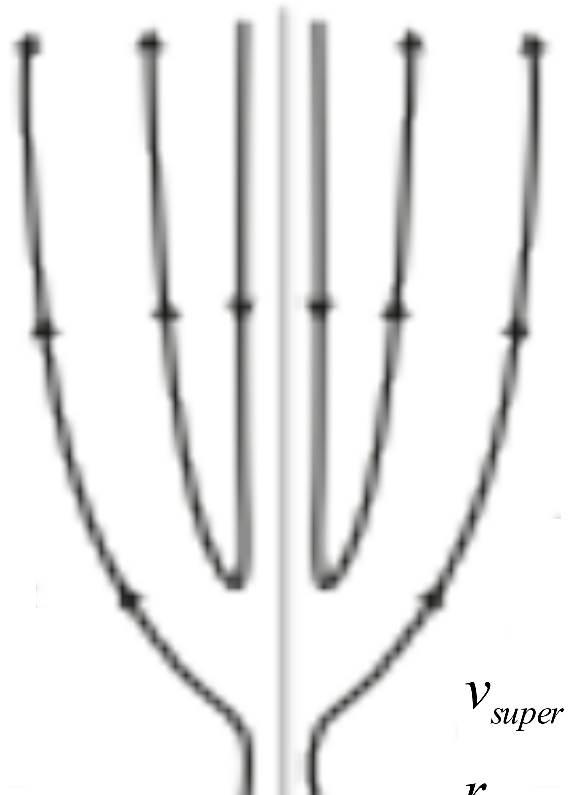
End-Wall Vortex is Supercritical

Condition for Vortex Breakdown to Remain Suspended



$$v_{super} \approx (5/3)v_{sub} \quad r_{super} \approx (1/3)r_{sub}$$

Condition for Vortex Breakdown to Remain Suspended



$$v_{sub} = v_{two-cell} \approx 1$$

$$r_{sub} = r_{two-cell} \propto \Omega$$

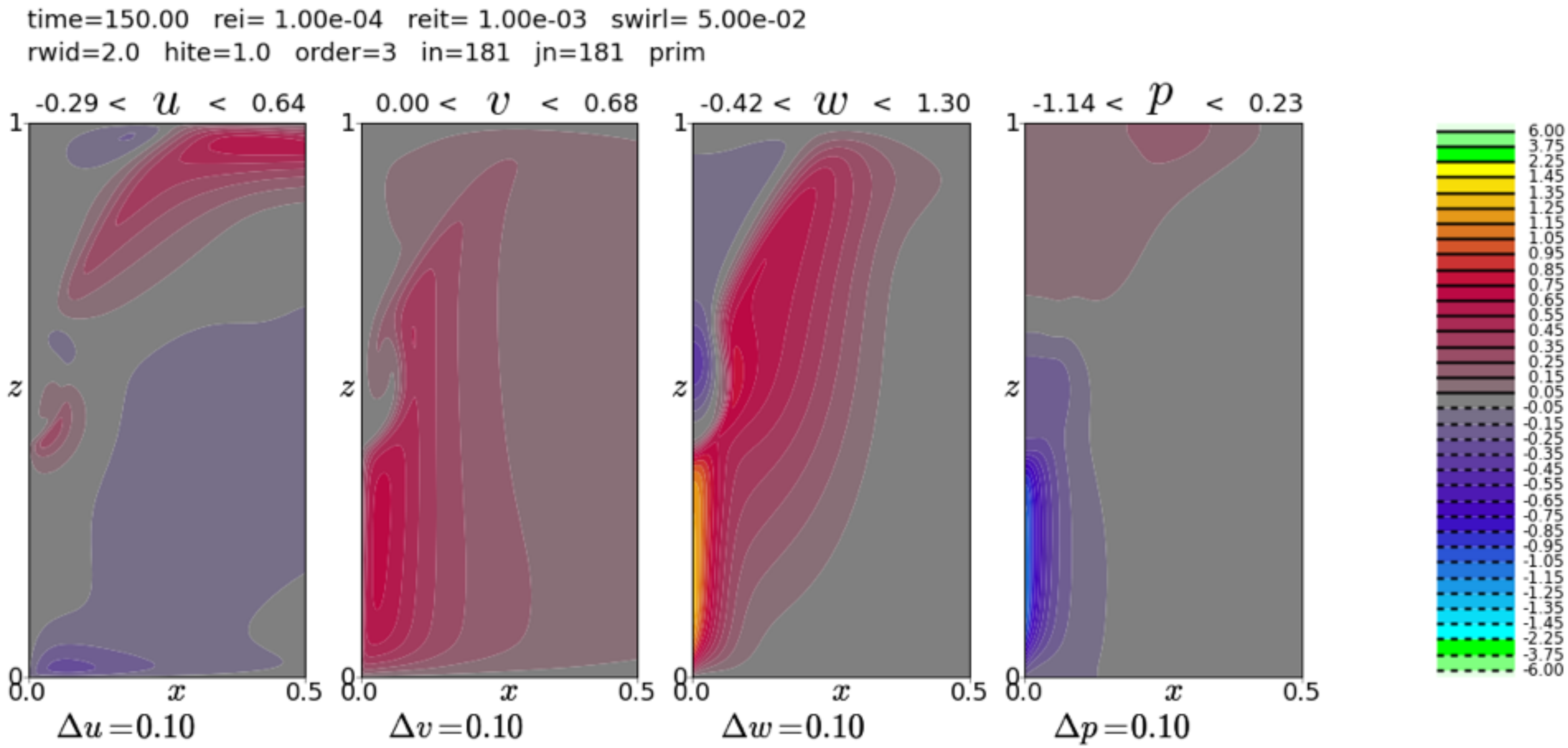
$$v_{super} \approx v_{end-wall} \propto \sqrt{N\Omega^3}$$

$$r_{super} \approx r_{end-wall} \propto \frac{1}{\sqrt{N\Omega}}$$

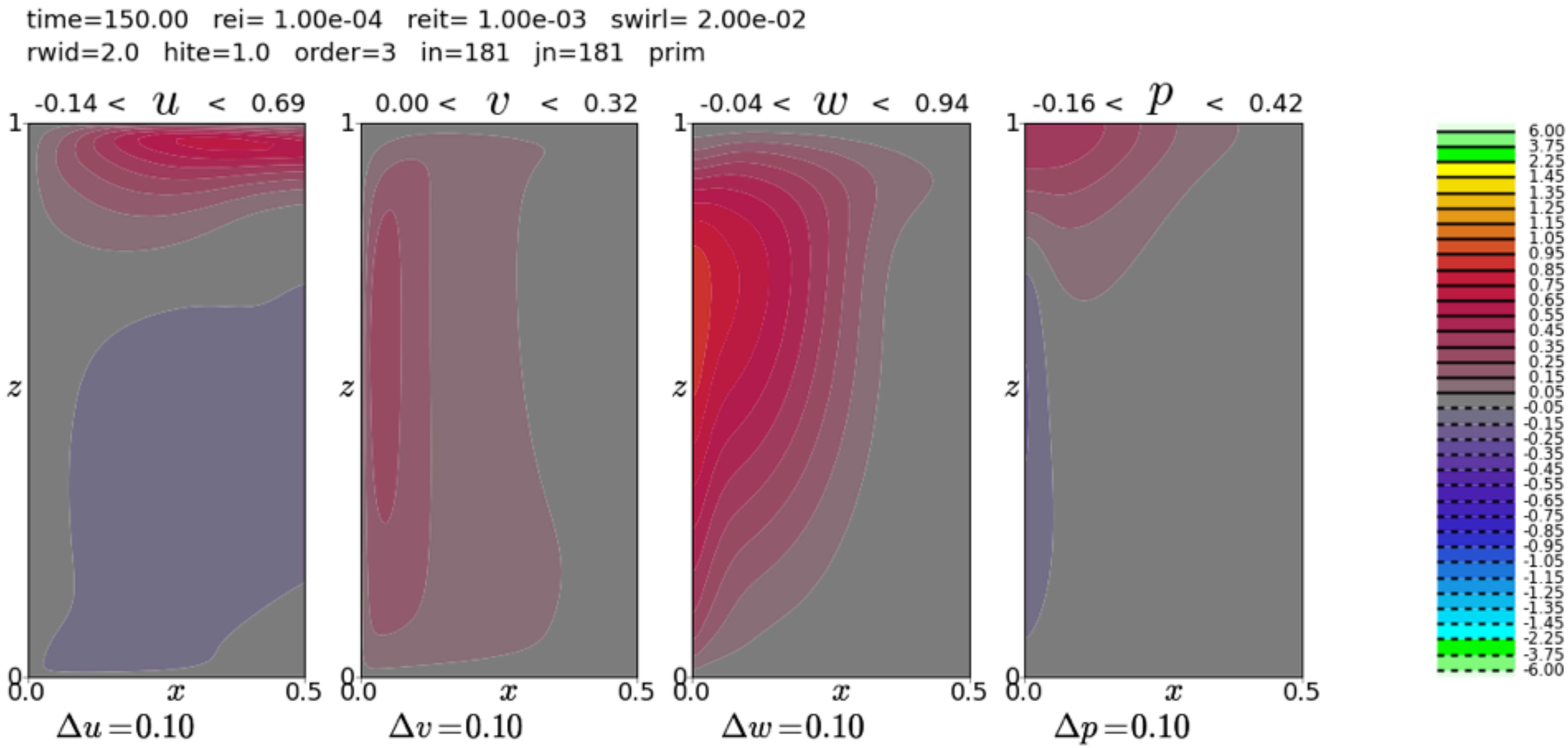
$$v_{super} \approx (5/3)v_{sub} \quad r_{super} \approx (1/3)r_{sub}$$

$$\Omega^* \propto N^{-1/3}$$

$$\Omega \approx \Omega^*$$

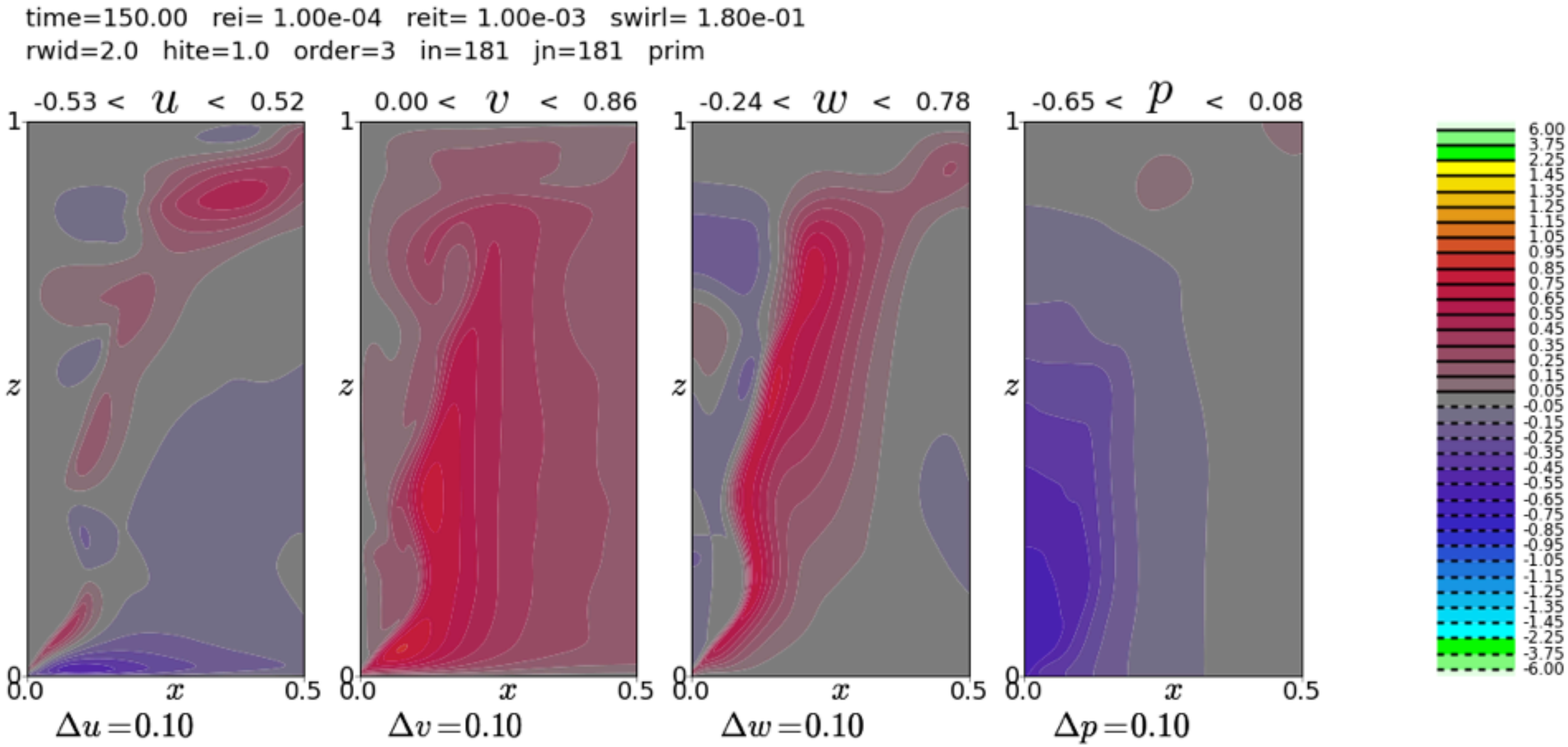


$\Omega < \Omega^*$

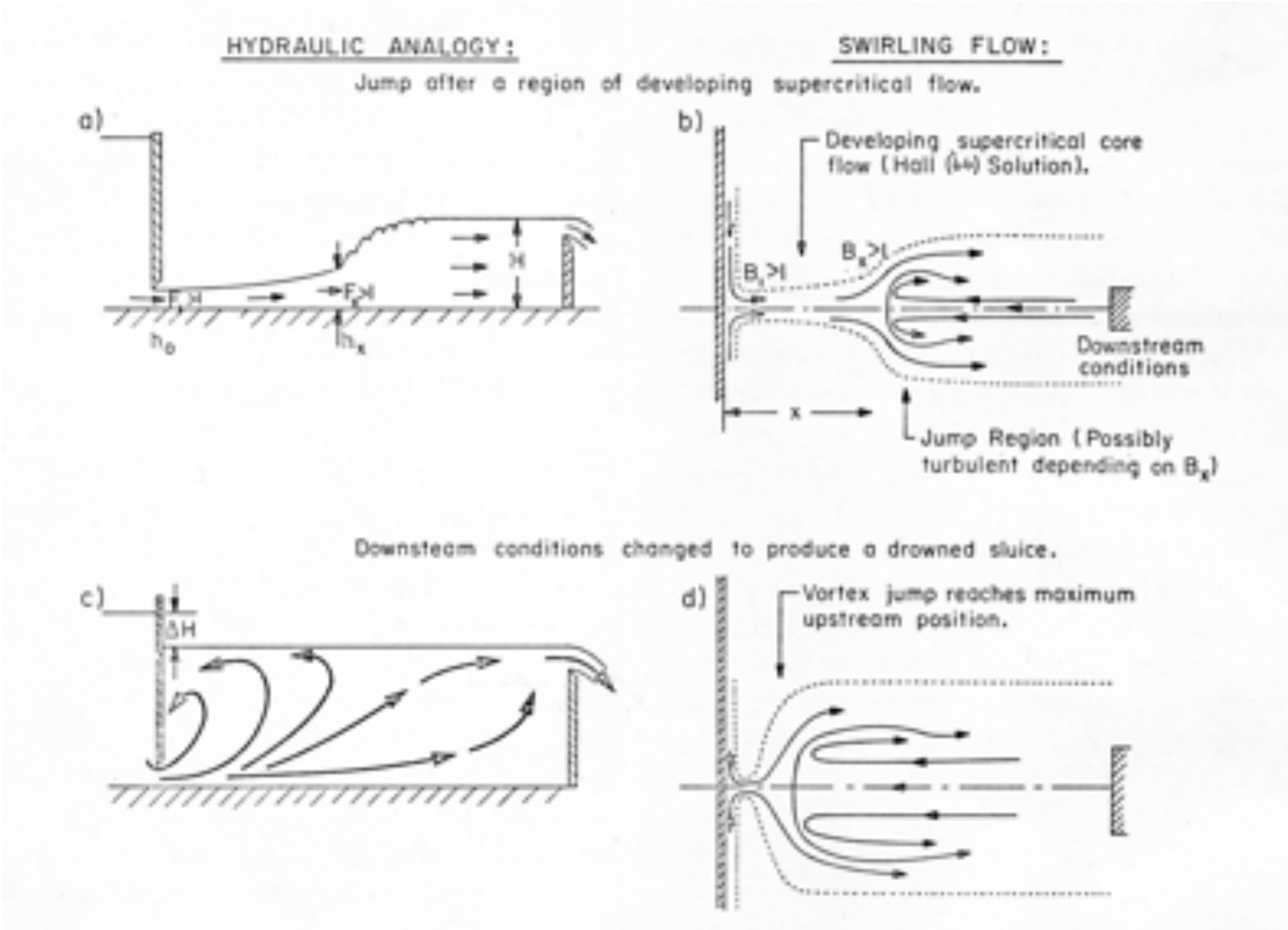


time=150.00 rei= 1.00e-04 reit= 1.00e-03 swirl= 2.00e-02
rwid=2.0 hite=1.0 order=3 in=181 jn=181 prim
-0.14 < u < 0.69
0.00 < v < 0.32
-0.04 < w < 0.94
-0.16 < p < 0.42
z
x
0.0
0.5
1
0
Δu =0.10
Δv =0.10
Δw =0.10
Δp =0.10
6.00
3.75
2.25
1.45
1.35
1.25
1.15
1.05
0.95
0.85
0.75
0.65
0.55
0.45
0.35
0.25
0.15
0.05
-0.05
-0.15
-0.25
-0.35
-0.45
-0.55
-0.65
-0.75
-0.85
-0.95
-1.05
-1.15
-1.25
-1.35
-1.45
-2.25
-3.75
-6.00

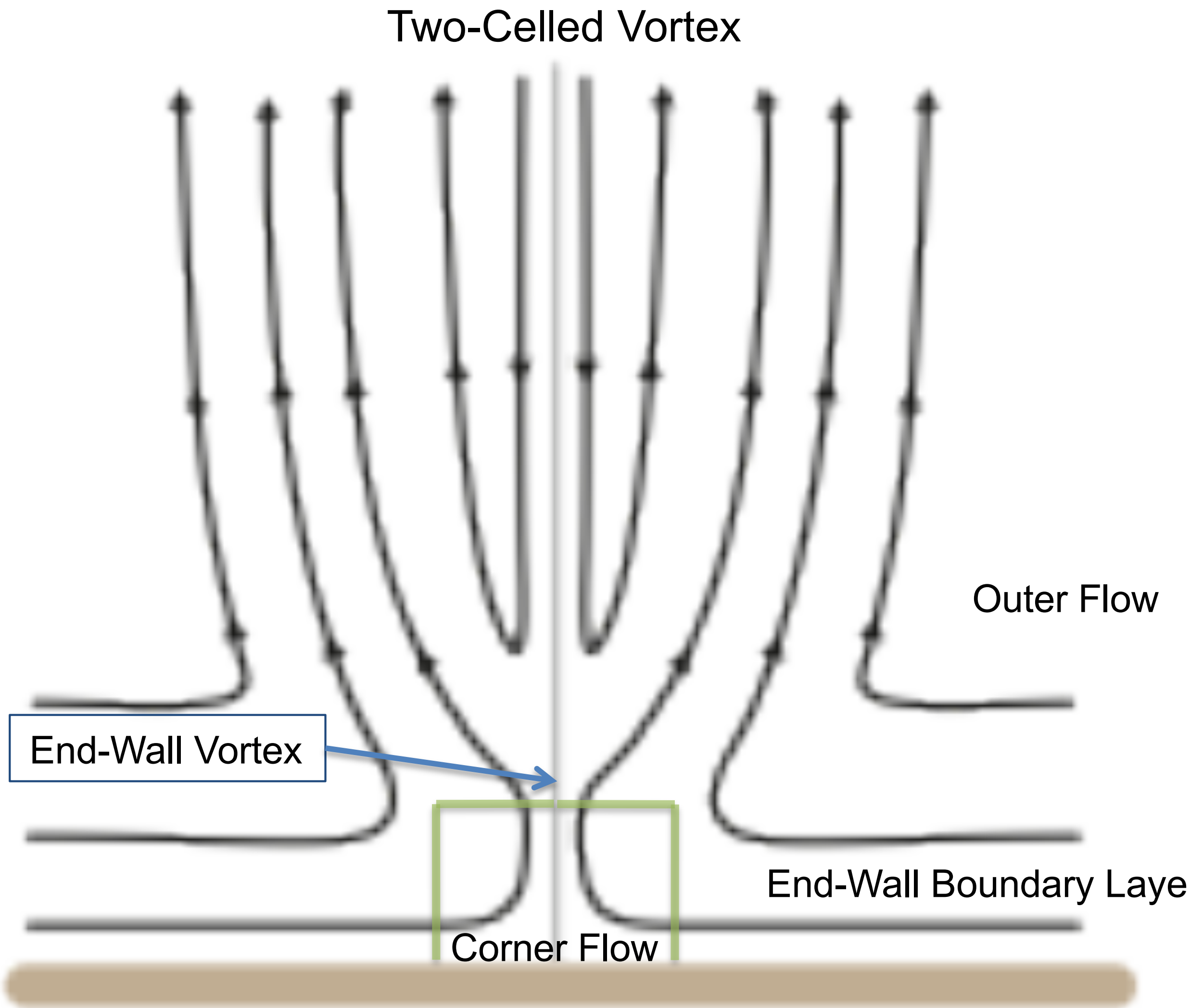
$$\Omega > \Omega^*$$



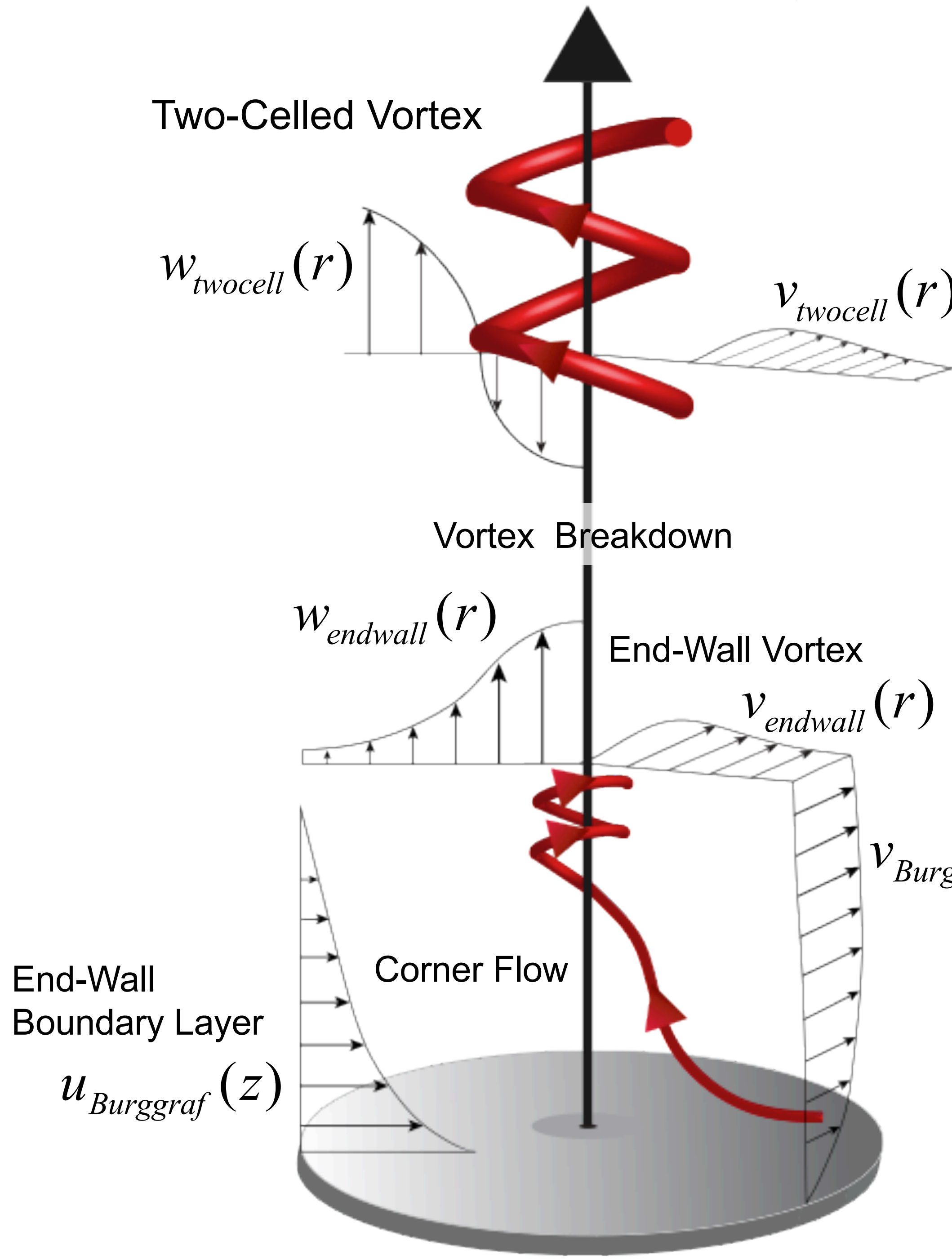
time=150.00 rei= 1.00e-04 reit= 1.00e-03 swirl= 1.80e-01
rwid=2.0 hite=1.0 order=3 in=181 jn=181 prim
-0.53 < u < 0.52
0.00 < v < 0.86
-0.24 < w < 0.78
-0.65 < p < 0.08
z
x
0.0
0.5
1
0
Δu=0.10
Δv=0.10
Δw=0.10
Δp=0.10
6.00
3.75
2.25
1.45
1.35
1.25
1.15
1.05
0.95
0.85
0.75
0.65
0.55
0.45
0.35
0.25
0.15
0.05
-0.05
-0.15
-0.25
-0.35
-0.45
-0.55
-0.65
-0.75
-0.85
-0.95
-1.05
-1.15
-1.25
-1.35
-1.45
-2.25
-3.75
-6.00



Maxworthy (1972, *Astro. Acta*)


Two-Celled Vortex
Outer Flow
End-Wall Vortex
End-Wall Boundary Layer
Corner Flow

Vortex Lines and Velocity Fields



$$\nabla H = \vec{u} \times \vec{\omega} = 0$$

vortex lines = streamlines
Burggraf et al. '71
End-Wall Boundary Layer &,
consequently, also true for
Corner Flow & End-Wall Vortex

Generalizations

$$N = \frac{Wh}{\nu}$$

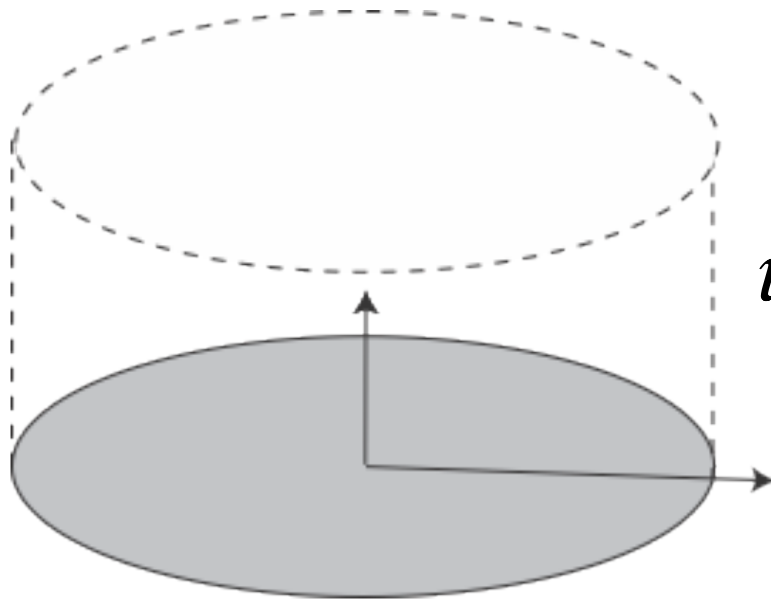
$$W \equiv \sqrt{2\int_0^h b(0,z)dz} \approx 50m/s \qquad h \approx 10^4 m \qquad \nu \approx 10^{-5} m^2/s$$

$$N \approx O(10^{10})$$

Effects of Turbulence?

LES Nested in an Idealized Supercell

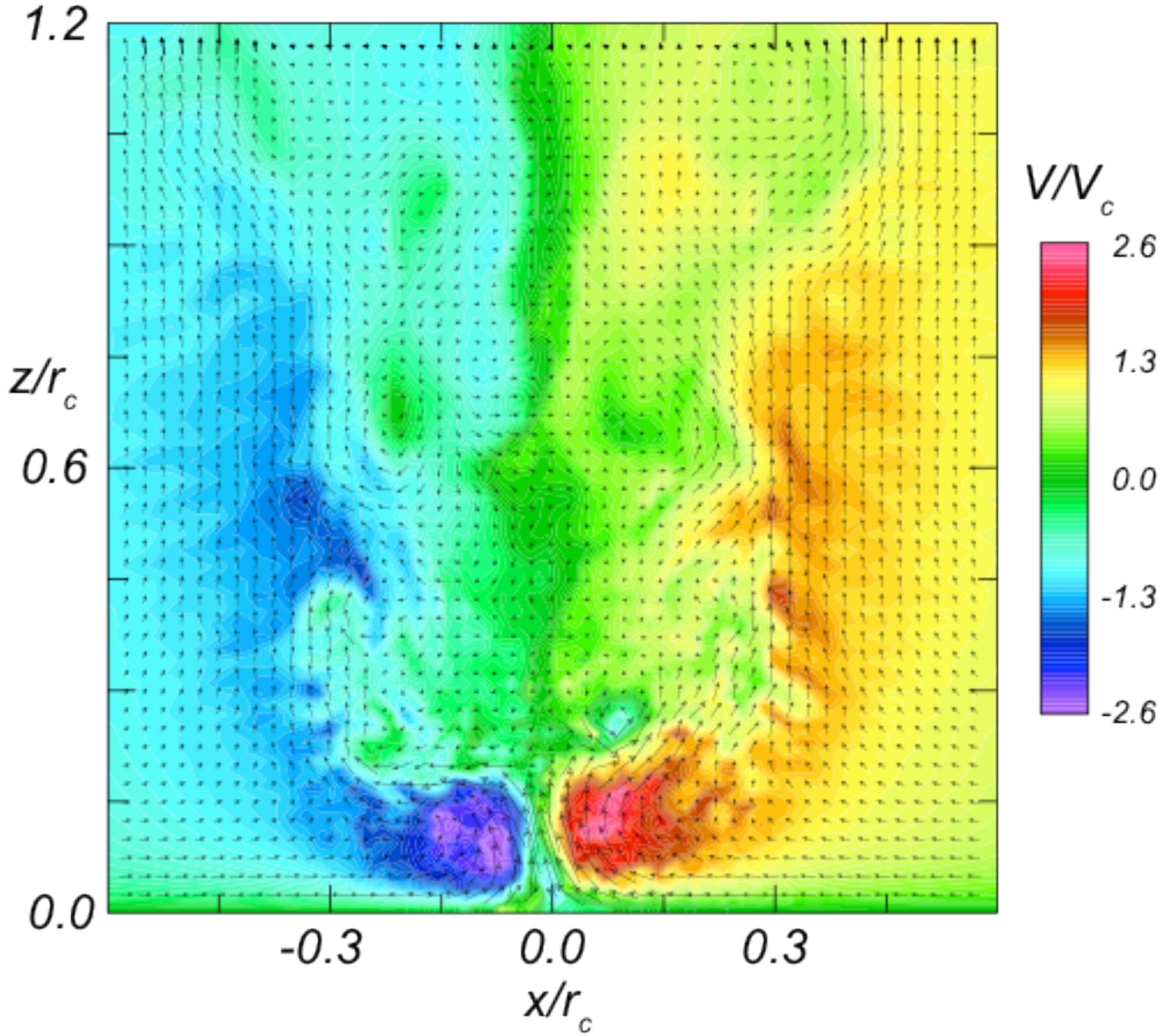
$$w = w(r_s, \theta, z, t)$$



$$u = u(r_s, \theta, z, t)$$

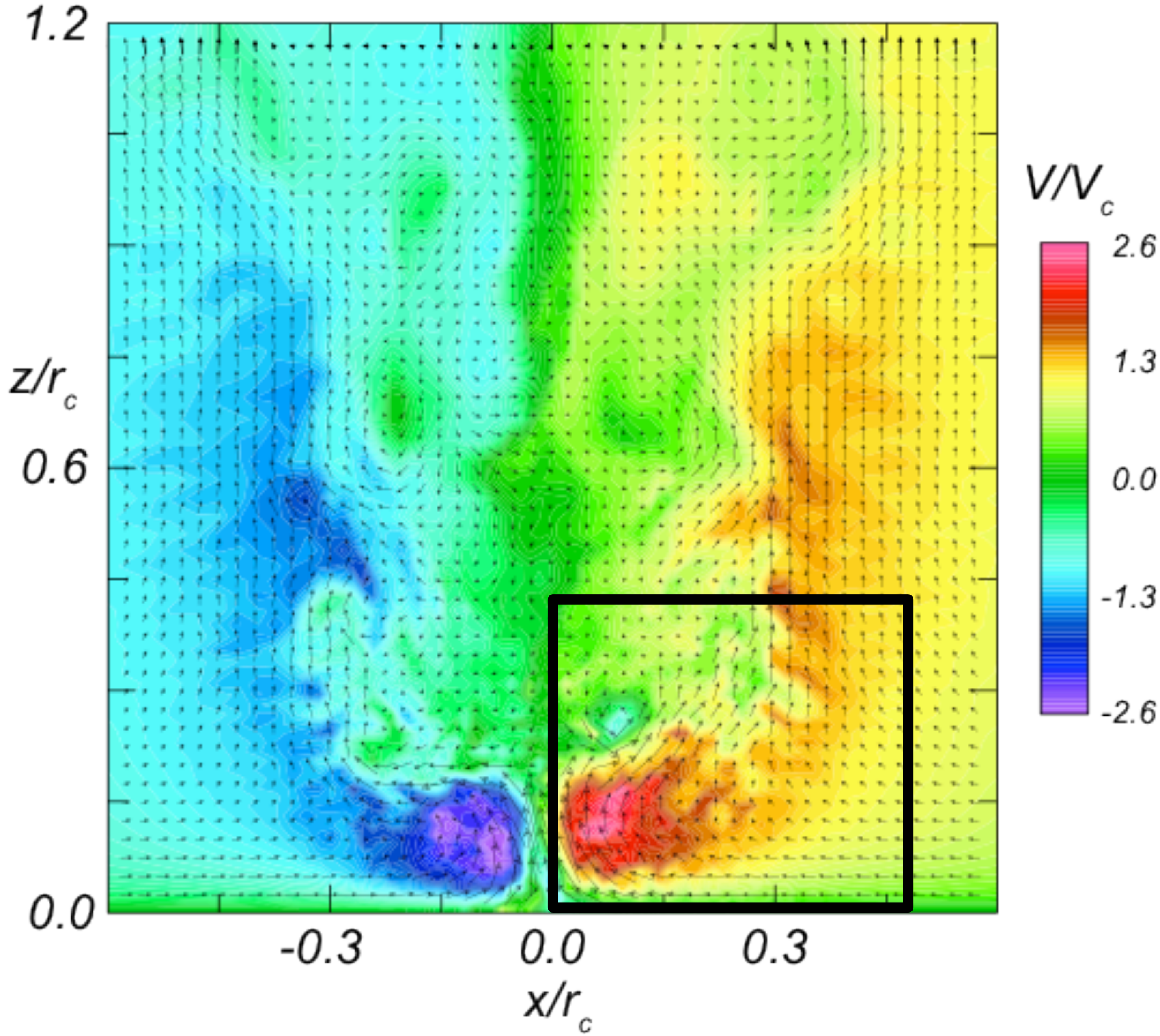
$$v = v(r_s, \theta, z, t)$$

Large Eddy Simulation



Lewellen et al (2000, *JAS*)

Large Eddy Simulation



Lewellen et al (2000, *JAS*)

Corner-Flow Swirl Ratio

$$S_c = \frac{r_c \Gamma_\infty^2}{\Upsilon}$$

$$\Upsilon = -2\pi \int_0^{z_1} \left\langle u(r_1, z)\left[\Gamma_\infty(r_1, z_1) - \Gamma(r_1, z)\right]\right\rangle r_1 dz$$

$$S_c = 0.7 - 1.7$$

→ most intense vortices in LES experiments

Lewellen et al (2000, *JAS*)

$$S_c = \frac{r_c}{\Upsilon / \Gamma_\infty^2} = \frac{r_c}{\delta_\Gamma} \approx \frac{3\delta}{3\delta} \approx 1$$ → end-wall boundary layer to subcritical vortex

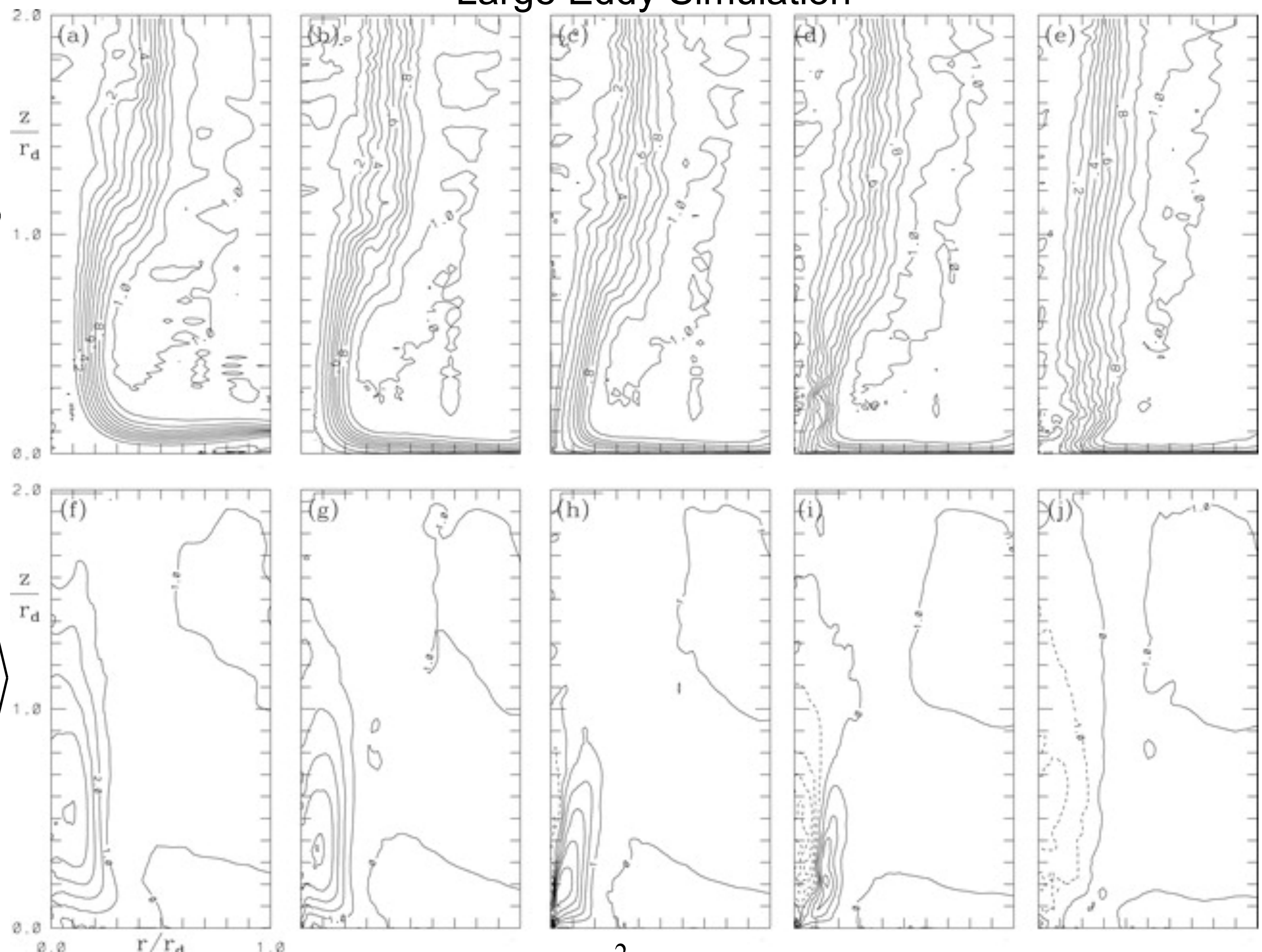
Fiedler (2009, *Atmos. Sci. Let.*)

Dynamic Corner-Flow Collapse

Large Eddy Simulation

$$\left\langle \frac{\Gamma}{\Gamma_\infty} \right\rangle$$

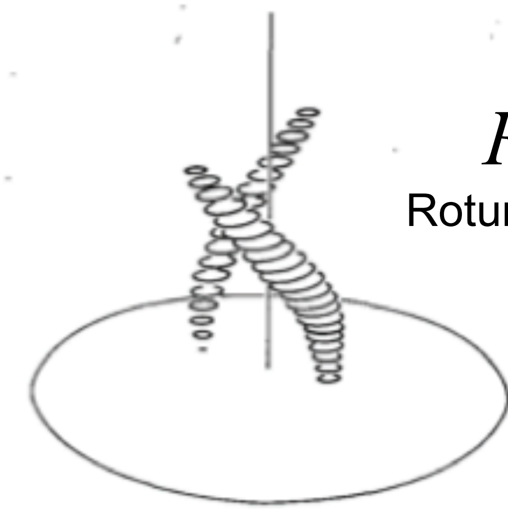
$$\left\langle \frac{w r_d}{\Gamma_\infty} \right\rangle$$



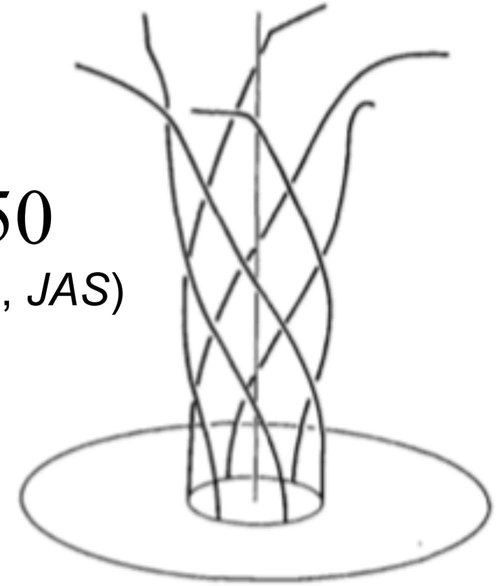
$$\frac{t r_d^2}{\Gamma_\infty} \rightarrow$$

Lewellen&Lewellen (2007, *JAS*)

Pressure Isosurface



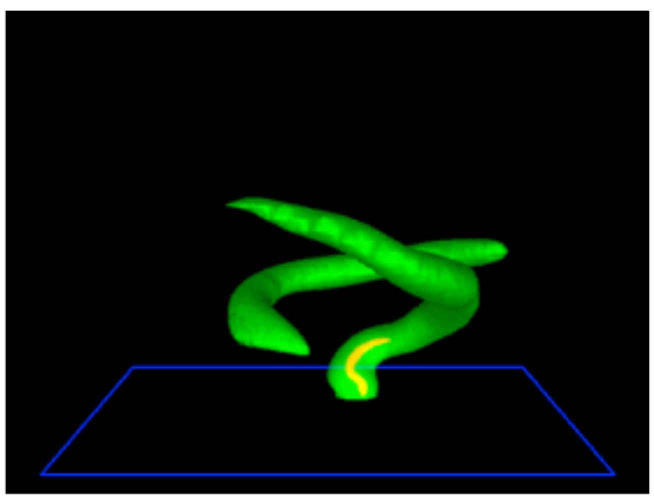
Vortex Lines in Two-Celled Vortex



$Re = 150$

Rotunno (1984, *JAS*)

$Re = 2500$

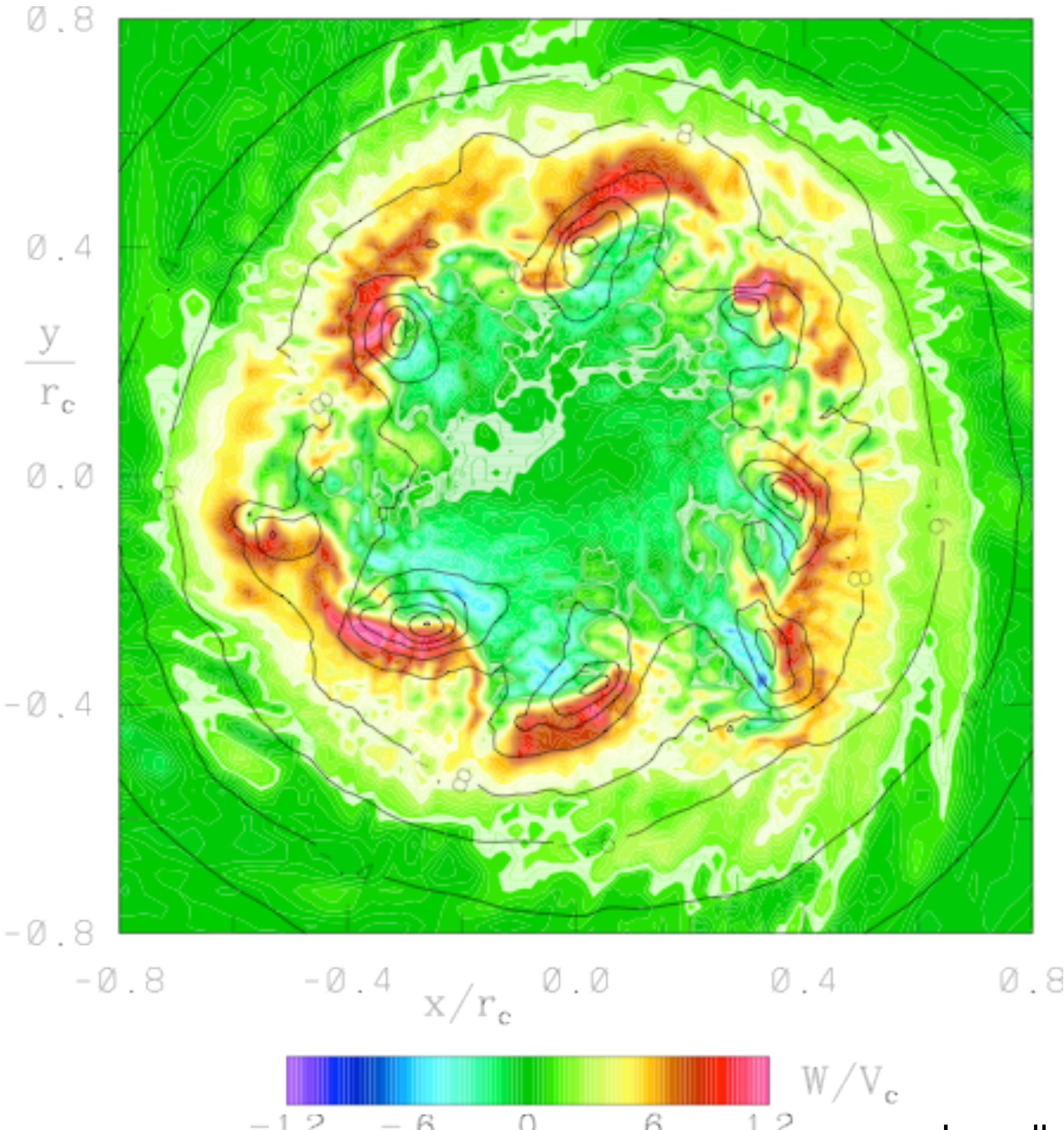


Fiedler (2009, *Atmos. Sci. Let.*)

LES

Lewellen et al (1997, *JAS*)

Large Eddy Simulation



Each satellite vortex has locally amplified velocity in the boundary layer

Lewellen et al (2000, *JAS*)

Conclusions

- Steady-State Theory of Axisymmetric Tornado-Like Vortices Well Developed and Tested in Laboratory and Numerical Models; 3D Effects Less So…
- Knowledge of Tornadogenesis Primitive
- Intersection of Storm-Scale and Vortex-Scale Dynamics Next Frontier