

Evolution of an Axisymmetric Tropical Cyclone before Reaching Slantwise Moist Neutrality

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(Manuscript received 29 August 2018, in final form 5 April 2019)

ABSTRACT

In a previous study, the authors showed that the intensification process of a numerically simulated axisymmetric tropical cyclone (TC) can be divided into two periods denoted by “phase I” and “phase II.” The intensification process in phase II can be qualitatively described by Emanuel’s intensification theory in which the angular momentum (M) and saturated entropy (s^*) surfaces are congruent in the TC interior. During phase I, however, the M and s^* surfaces evolve from nearly orthogonal to almost congruent, and thus, the intensifying simulated TC has a different physical character as compared to that found in phase II. The present work uses a numerical simulation to investigate the evolution of an axisymmetric TC during phase I. The present results show that sporadic, deep convective annular rings play an important role in the simulated axisymmetric TC evolution in phase I. The convergence in low-level radial (Ekman) inflow in the boundary layer of the TC vortex, together with the increase of near-surface s^* produced by sea surface fluxes, leads to episodes of convective rings around the TC center. These convective rings transport larger values of s^* and M from the lower troposphere upward to the tropopause; the locally large values of M associated with the convective rings cause a radially outward bias in the upper-level radial velocity and an inward bias in the low-level radial velocity. Through a repetition of this process, the pattern (i.e., phase II) gradually emerges. The role of internal gravity waves related to the episodes of convection and the TC intensification process during phase I is also discussed.

1. Introduction

Recently, a theoretical model for axisymmetric tropical cyclone (TC) intensification was developed by Emanuel (2012, hereafter E12); this model improves on an earlier version (Emanuel 1997) by eliminating several ad hoc assumptions. Hydrostatic and gradient wind balance and slantwise moist neutrality are the main assumptions for these models as well as for the steady-state model of Emanuel (1986). Peng et al. (2018, hereafter P18) used a cloud-resolving numerical model, set up to follow closely the physical constraints of the E12 model, in order to assess the latter’s predictions and

assumptions. According to P18, the intensification process of the axisymmetric TC can be divided into two periods, which they called “phase I” and “phase II.” During phase II, the angular momentum (M) and moist entropy (s^*) surfaces in the eyewall and outflow are approximately congruent (i.e., there is slantwise moist neutrality) and the intensification progress can be qualitatively described by Emanuel’s intensification theory. P18 found that the assumption of gradient wind balance in E12 was the reason for the quantitative difference between the E12 and the numerical model results. During phase I, however, the simulated axisymmetric TC intensifies while the M and s^* surfaces evolve from nearly orthogonal to almost congruent. Without a state of slantwise moist neutrality [$s^* = s^*(M)$] the TC

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DOI: 10.1175/JAS-D-18-0264.1

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intensification in phase I cannot be described by the E12 model. The focus of the present paper is the evolution of the axisymmetric TC in phase I, before it reaches a state of slantwise moist neutrality.

A scale analysis for an axisymmetric tropical cyclone-scale vortex shows that, to a first approximation, the flow is mostly in gradient wind and hydrostatic balance, and hence in thermal wind balance in the free atmosphere (Willoughby 1979). Therefore, many theoretical works have focused their attention on balanced dynamics to investigate the TC intensification mechanism, including the E12 theoretical model. TCs are often regarded as quasi-axisymmetric systems composed of the tangential wind and a transverse (radial-vertical) circulation, which are sometimes referred to as the “primary” and “secondary” circulations, respectively.

In the balanced-vortex model (Eliassen 1951; Charney and Eliassen 1964; Ooyama 1969; Emanuel 1986; E12), the secondary circulation acts to exactly cancel the changes caused by eyewall heating and surface friction, in order to maintain thermal wind balance. The assumption of thermal wind balance allows the derivation of a single, linear, diagnostic partial differential equation for the streamfunction of the secondary circulation in the presence of forcing processes such as diabatic heating and tangential friction that, by themselves, would drive the vortex away from such a state of balance. This diagnostic equation is often referred to as the Sawyer–Eliassen (SE) equation. The SE equation diagnostic method is widely used in studies of TC intensification (e.g., Smith 1981; Shapiro and Willoughby 1982; Schubert and Hack 1982; Hack and Schubert 1986; Rozoff et al. 2008; Bui et al. 2009; Pendergrass and Willoughby 2009; Wang and Wang 2014; Abarca and Montgomery 2015). These past studies have found that there are small regions of symmetric instability where solutions of the SE equation cannot be found (Ooyama 1969; Möller and Shapiro 2002; Bui et al. 2009). Kurihara (1975) also found that inertially and/or symmetrically unstable flow occurs in the development of the TC outflow in the upper troposphere. Recently, Cohen et al. (2017) examine the validity of gradient wind balance in the upper levels of 12 TC simulations. They found that gradient wind balance is violated (geostrophic Rossby number $Ro_g < -1/4$) at the top of the simulated storms, especially in the early stages of intensification associated with the contraction of the radius of the maximum wind speed (RMW). Therefore, there might be an important physical process in TC evolution that would not be captured by the balance models.

The development of a warm core is also an important aspect of TC dynamics, as it is associated with the strength of the primary circulation according to thermal wind

balance. The compensating subsidence on either side of convection-associated heating is the generally accepted paradigm for the inner-core warming (Eliassen 1951; Schubert and Hack 1982; Hack and Schubert 1986; Nolan et al. 2007; Pendergrass and Willoughby 2009). Recently, Stern and Zhang (2013) found that for different stages of a simulated TC lifetime, the main factors that contribute to warming of the eye are different. For example, total advection of potential temperature is the only significant term at the start of rapid intensification (RI). However, for a substantial portion of RI, the net advective warming is shown to be a result of eddy radial advection of potential temperature from the region of most rapid warming (ascending area). At a sufficient intensity, mean vertical advective warming becomes concentrated inward of the eyewall. Much uncertainty remains about the detailed mechanisms by which this forced descent occurs, as well as regarding the vertical distribution and magnitude of subsidence and its variability in time.

In this paper, we revisit the dynamics of vortex evolution based on the primitive axisymmetric equations, rather than the approximate balanced equations, with the focus on the evolution of primary and secondary circulation in physical space. In the following section 2 the numerical simulation used here is described to investigate the evolution in phase I. In section 3 the development of the secondary circulation is analyzed. In section 4 we seek to clarify the intensification process of the TC in phase I. Section 5 contains a summary and conclusions.

2. Description of the numerical simulation

Following P18, we use the axisymmetric, nonhydrostatic Cloud Model, version 1 (CM1), as described in Bryan and Rotunno (2009b, hereafter BR09b), to perform an analysis of a numerical experiment investigating the evolution of an axisymmetric TC in phase I. Most of the model settings used in the present simulation are identical to the control experiment in P18. The Coriolis parameter f is set to $0.5 \times 10^{-4} \text{ s}^{-1}$ and the surface exchange coefficient for entropy C_k and momentum C_d are both set to 1×10^{-3} . To exclude the effect of the environmental convective available potential energy (CAPE),¹ the environmental is set to be exactly neutral to moist convection as in Bryan and Rotunno (2009a) and P18. The initial environmental sounding is saturated (Fig. 1a) and was constructed assuming constant pseudoadiabatic equivalent potential temperature. In the present study, in order to reduce

¹ This setting is in the spirit of Rotunno and Emanuel (1987), in which they emphasize that a preexisting conditionally unstable atmosphere is not necessary for the developing stage of TCs.

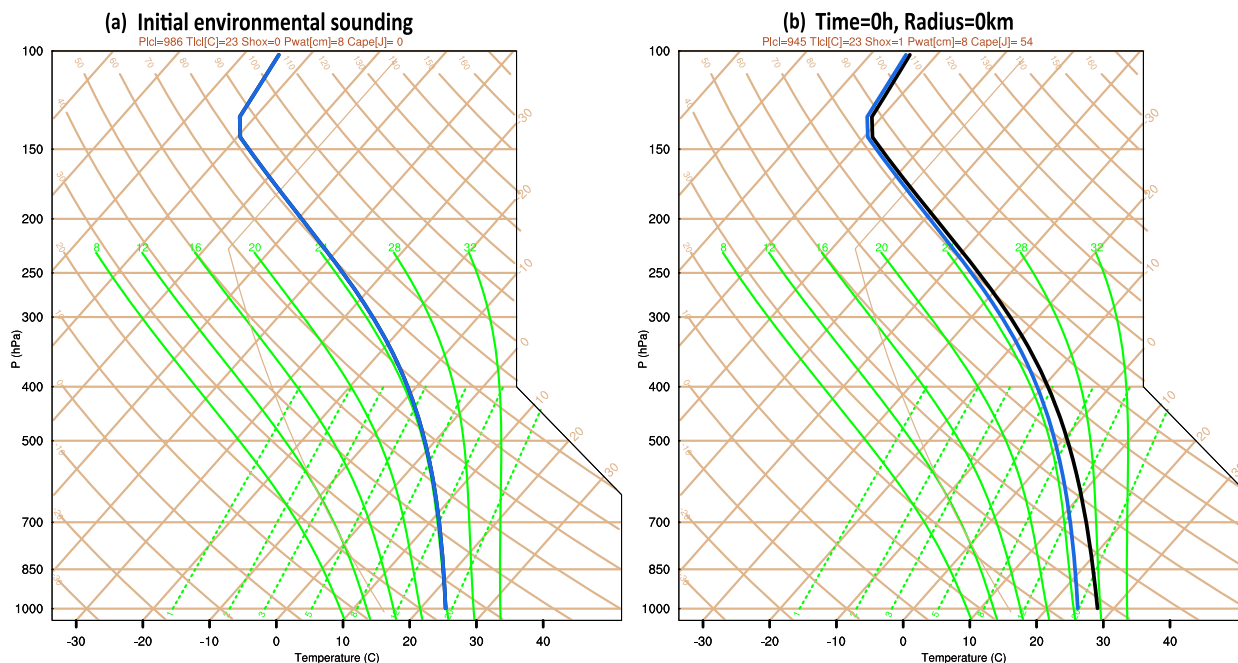


FIG. 1. (a) The initial environmental sounding and (b) the initial sounding at $r = 0$, which reflects the radial temperature gradient needed for thermal wind balance of the initial vortex. The black lines denote the temperature, while the blue lines denote the dewpoint temperature. Note that the initial environment is saturated, so the blue line overlaps the black line entirely in (a).

the effect of any initial CAPE caused by the initial moist vortex in P18, we set the water vapor mixing ratio to be horizontally homogeneous so that the initial vortex is unsaturated; thus, the vortex is conditionally stable at the initial time (Fig. 1b). To make the simulation easier to analyze, the horizontal turbulence length scale l_h here is set to a larger value ($l_h = 1000$ m) than in P18, which produces smoother fields. This value is recommended by Bryan (2012) and is close to the observed value ($l_h = 750$ m) in Zhang and Montgomery (2012). The vertical turbulence length scale l_v is set to 100 m as in P18.

Figure 2 shows the time series of the maximum tangential velocity V_m and the RMW for the simulation. The RMW is very noisy during the first 38 h; subsequently, there is a transition period (during which the M and s surfaces become nearly congruent; discussed below) that lasts until almost 48 h. After this time, the TC steadily contracts and intensifies. As discussed in P18, the intensification process can be divided into two phases, phase I (0–48 h) and phase II (48–144 h). During phase I, V_m increases from 20 to almost 38 m s^{-1} . During phase II, V_m keeps intensifying to almost 68 m s^{-1} at 144 h, at which time the vortex has not yet reached a steady state (later reached at 163 h). P18 showed that the intensification in phase II can be qualitatively described by the E12 theory, which means it can be considered as a quasi-balanced system (except for the boundary layer). However, the TC intensification in phase I

(by approximately 18 m s^{-1}) cannot be described by the E12 model. In the present study, we seek to clarify the intensification mechanism in phase I. Note that the vortex takes a longer time to reach a steady state than that in P18, which is mainly due to the initially subsaturated vortex (not the larger value of l_h).

The structure of the simulated TC changes drastically during phase I. Initially, the M and s surfaces are nearly orthogonal (Fig. 3a). The entropy s is formulated for pseudoadiabatic processes (Bryan 2008) and is approximately conserved in these simulations. During the

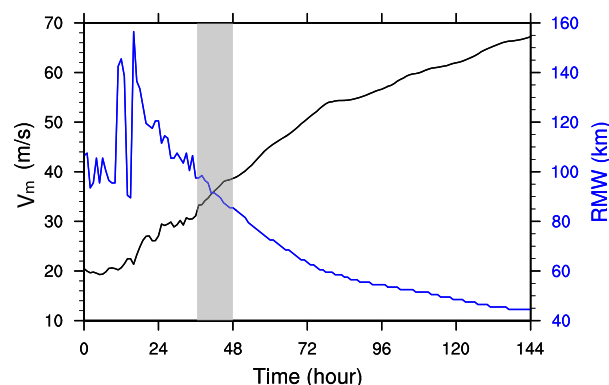


FIG. 2. Time series of the maximum tangential velocity V_m (left axis; black) and the radius of maximum wind speed (RMW; right axis; blue). The gray bar denotes the transition period (during which the M and s surfaces become nearly congruent).

transition period, the M and s surfaces evolve to almost congruent as the vortex approaches slantwise moist neutrality in the eyewall and upper-level outflow (Fig. 3b). During phase II, the M and s surfaces in the eyewall and outflow are congruent as the TC intensifies (as in Fig. 3b). The initial vortex is balanced with no radial or vertical motion (Fig. 3a). By the time phase II is reached, a system-scale overturning (secondary) circulation has developed (Fig. 3b). To understand the evolutionary process that leads to the TC intensification in phase I, we seek to answer three questions in the following sections: 1) How does the secondary circulation build? 2) What is the nature of the transition from phase I to phase II? 3) How does the TC intensify in phase I?

3. Development of the secondary circulation

In the present paper, we study the development of the secondary circulation according to the original equations without the hydrostatic and gradient wind approximations. The azimuthal vorticity, $\eta = \partial u/\partial z - \partial w/\partial r$, together with the anelastic continuity equation, $\partial \rho u/\partial r + \partial \rho w/\partial z = 0$, can be expressed as the Poisson-type equation, $\eta = (\partial/\partial z)(1/\rho r)(\partial \psi/\partial z) + (\partial/\partial r)(1/\rho r)(\partial \psi/\partial r)$, where the streamfunction ψ is defined by the relations $\rho r u = \partial \psi/\partial z$ and $\rho r w = -\partial \psi/\partial r$ and ρ is the fluid density. With the knowledge of $\eta(r, z, t)$ one can deduce ψ from the Poisson-type equation, and therefore, the secondary circulation defined by the radial-vertical velocity (u, w). Qualitatively, the solutions to Poisson equation are such that $\psi \propto -\eta$ so that $\eta < 0$ (> 0) implies a counterclockwise (clockwise) circulation when looking in the positive azimuthal direction (i.e., “into the page”). Accordingly, the evolution of the secondary circulation is investigated through the analysis of the time rate of change of η . The equation for η can be written without approximation as

$$\begin{aligned} \frac{\partial \eta}{\partial t} + \frac{\partial}{\partial r}(u\eta) + \frac{\partial}{\partial z}(w\eta) = & \frac{1}{r^3} \frac{\partial M^2}{\partial z} - \frac{\partial b}{\partial r} \\ & + c_p \left(\frac{\partial \theta_v}{\partial r} \frac{\partial \pi'}{\partial z} - \frac{\partial \theta_v}{\partial z} \frac{\partial \pi'}{\partial r} \right) \\ & + \left(\frac{\partial}{\partial z} D_u - \frac{\partial}{\partial r} D_w \right). \end{aligned} \quad (1)$$

Primes refer to perturbations from a “base state” (indicated by overbars), which is a one-dimensional (z dependent) reference profile in hydrostatic balance [$d\bar{\pi}/dz = -g/(c_p \bar{\theta}_v)$]. The variable π is the nondimensional pressure defined as $\pi \equiv (p/p_0)^{R_d/c_p}$, where p_0 is a reference pressure, R_d is the gas constant for dry air, and c_p is the specific heat of dry air at constant pressure. Potential temperature $\theta \equiv T/\pi$, where T is the absolute temperature. Virtual potential temperature is defined

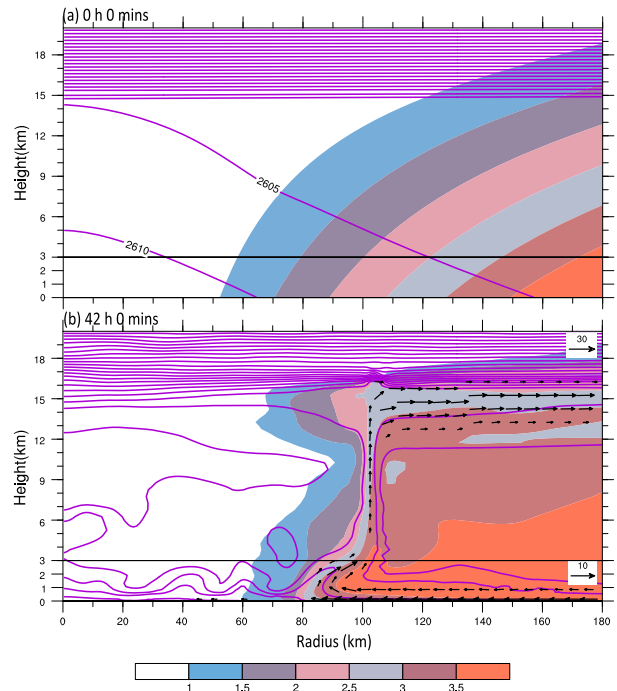


FIG. 3. Entropy s (contour interval = $10 \text{ J kg}^{-1} \text{ K}^{-1}$; purple lines) and angular momentum M ($10^6 \text{ m}^2 \text{ s}^{-1}$; shading) at (a) the initial time and (b) 42 h 0 min. Note that (a) also includes the $2605 \text{ J kg}^{-1} \text{ K}^{-1}$ contour of s . Vectors denote the radial and vertical wind. Vectors are not shown when the wind speed is less than 5 m s^{-1} above 3 km and less than 2 m s^{-1} below 3 km.

as $\theta_v = \theta(1 + q_v R_v/R_d)/(1 + q_v + q_l)$, where R_v is the gas constant for water vapor, q_v is the water-vapor mixing ratio, and q_l is the liquid-water mixing ratio. The buoyancy $b = g(\theta'_v/\bar{\theta}_v)$. The D symbols represent tendencies from turbulent motions (details given in BR09b).

When the first two terms on the rhs of (1) dominate, (1) reduces to the thermal wind equation. Taking the time derivative of the latter and substituting the respective evolution equations for $\partial M/\partial t$ and $\partial b/\partial t$, one arrives at an elliptic² partial difference equation for $\psi(r, z, t)$, forced by friction and diabatic heating, the SE equation. It is generally acknowledged that, while explicit internal inertia-gravity waves are filtered out of the SE equations, they are implicitly responsible for communication between forcing and response in the SE equation. Herein we use the primitive equations in axisymmetric form to examine the explicit behavior of these internal waves.

The form of (1) says that η can be produced by either the first term (M forcing) or the second term (b forcing) on the rhs. The fourth term (friction term) generally acts to diffuse η and the third term represents non-Boussinesq effects and is typically small compared to the first two terms. In the

² Almost everywhere (Ooyama 1969, p. 10).

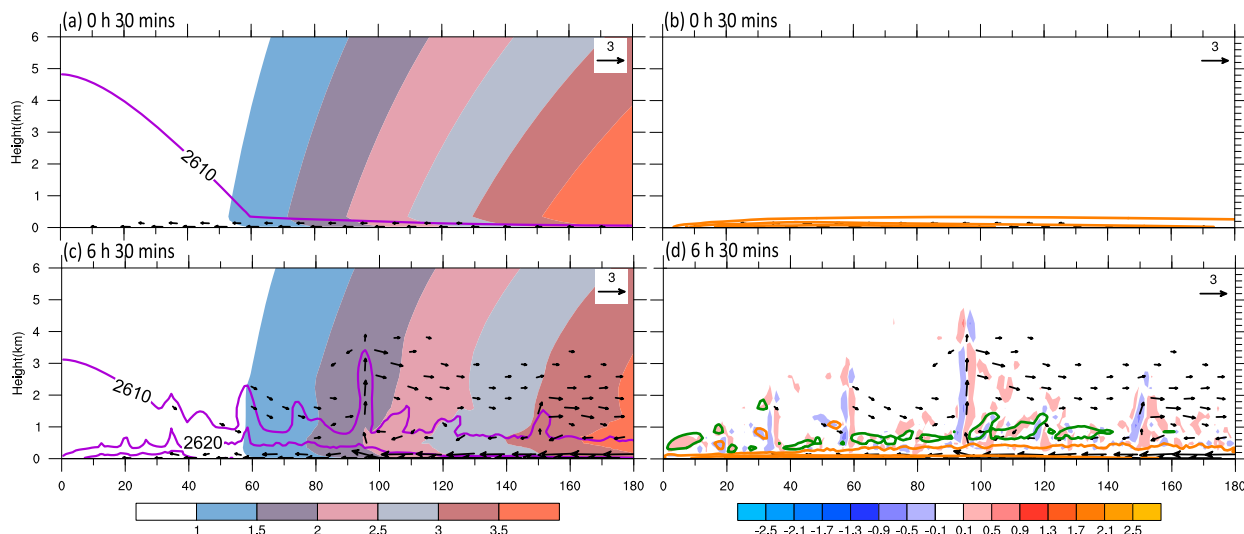


FIG. 4. (left) Entropy s (contour interval = $10 \text{ J kg}^{-1} \text{ K}^{-1}$; purple lines) and angular momentum M ($10^6 \text{ m}^2 \text{ s}^{-1}$; shading) at (a) 30 min and (c) 6 h 30 min and (right) the azimuthal vorticity budget at (b) 30 min and (d) 6 h 30 min, including the M forcing (10^{-5} s^{-2} ; green contours for negative values, orange contours for positive values) and the buoyancy forcing (10^{-5} s^{-2} ; shading). Vectors in all panels denote the radial and vertical wind. Vectors are not shown when the wind speed is less than 0.5 m s^{-1} .

following, we focus on studying the effect of the M and b forcings on the evolution of the secondary circulation. The advection terms on the lhs are not fundamental drivers of the azimuthal vorticity's evolution and are not considered further.

Phase I is considered in three stages: an early stage (0–10 h), a middle stage (10–38 h), and a late stage (38–48 h). In the early stage, an Ekman layer is developing. Subsequently, Ekman-layer convergence leads to episodes of convection that play an important role in developing the radial–vertical circulation during the middle stage. In the late stage (transition period), the M and s surfaces in the eyewall and outflow region approach congruence as a quasi-balanced state.

a. Early stage (0–10 h)

In the present simulation, the initial axisymmetric vortex $v(r, z)$ is in hydrostatic and gradient wind balance with $(u, w) = 0$. Figure 4 shows the s and M surfaces (Figs. 4a,c) and M and b forcings in (1) (Figs. 4b,d) at 30 min and 6 h 30 min, respectively. Radial and vertical velocities larger than certain thresholds are shown by the vectors in Fig. 4. In the earlier period, the angular momentum M near the surface is decreased by vertical diffusion of M to the sea surface, which reshapes the vertical distribution of M (Fig. 4a). The first term on the rhs of (1) is positive and thus induces a shallow layer of positive azimuthal vorticity in the boundary layer (Fig. 4b) and low-level inflow (i.e., an Ekman layer)³ starts to develop

(Fig. 4a). In the boundary layer, the inflow is maximum at the lowest level and decreases with height.

Consistent with previous studies (Emanuel 1986; Rotunno and Emanuel 1987; Bryan and Rotunno 2009a; BR09b; P18), the vortex develops within an environment that is initially moist neutral. As shown in Fig. 4c, the sea surface fluxes transfer larger values of entropy from the ocean surface to the atmosphere and fuel the ensuing convection; at 6 h 30 min, the low-level entropy has increased by more than $10 \text{ J kg}^{-1} \text{ K}^{-1}$. In the meantime, convergence in the low-level inflow induced by the M forcing implies upward vertical motion (Figs. 4c,d). The strongest vertical perturbation occurs at around the radius of 95 km, which is consistent with the theoretical result for Ekman pumping using a drag-law lower boundary condition [Eliassen 1971; Eliassen and Lystad 1977; (28) in Kepert 2001]. Once the vertical perturbation acquires enough moist entropy, the convective elements begin to erupt from the surface layer (Fig. 4d).

b. Middle stage (10–38 h)

At 10 h 50 min, there is a strong convective burst near the radius of 100 km, which brings large M and s to the upper level (Fig. 5a). The convective element impinges upon the tropopause, pushes it higher, and produces a negative buoyancy anomaly in the lower stratosphere. The positive buoyancy anomaly associated with the convection and the negative buoyancy at the tropopause produce the b forcing, which produces two prominent dipoles of η in the mid-to high troposphere and lower stratosphere (Fig. 5b). The b forcing produces vorticity that implies a pair of nearly symmetric radially inward and outward motions at the

³ In the more traditional interpretation, gradient wind imbalance in the boundary layer drives radial inflow toward the central low pressure associated with the initial vortex.

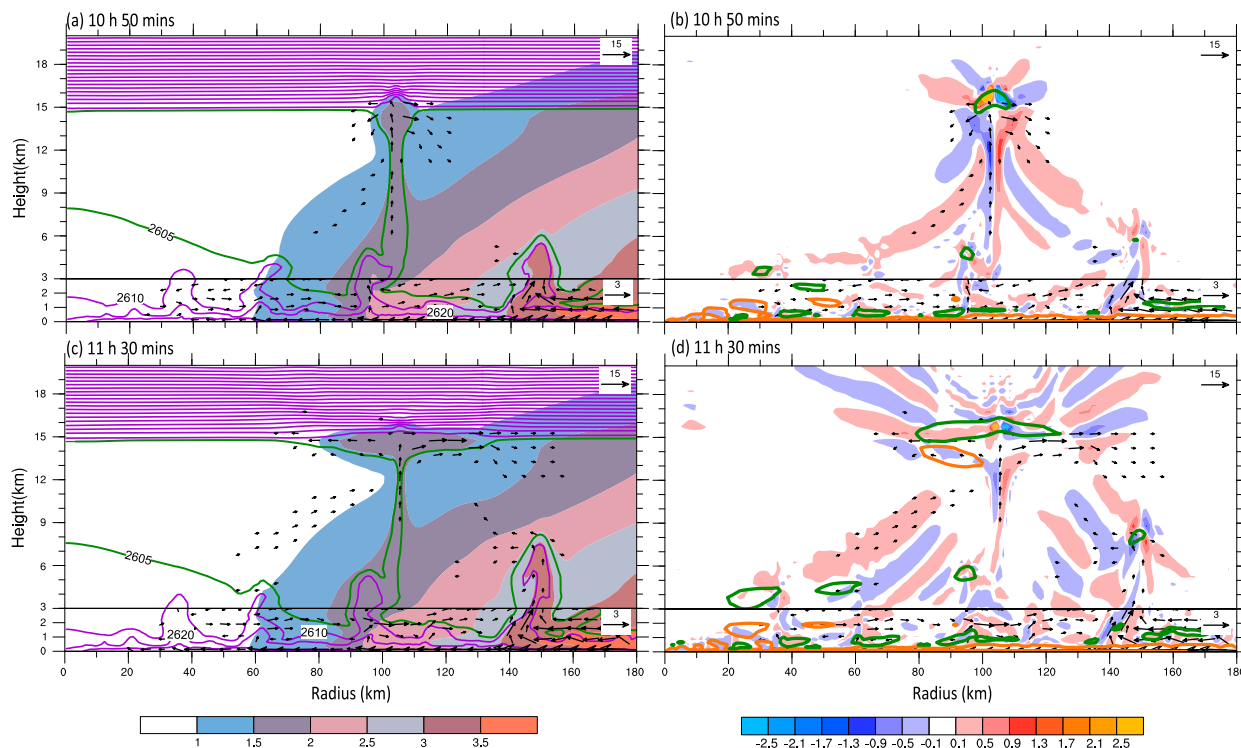


FIG. 5. As in Fig. 4, but at (a),(b) 10 h 50 min and (c),(d) 11 h 30 min. Note that the vertical range is from the height of 0 to 20 km. The green lines in (a) and (c) denote the entropy = $2605 \text{ J kg}^{-1} \text{ K}^{-1}$. Vectors are not shown when the wind speed is less than 2 m s^{-1} above 3 km and less than 0.5 m s^{-1} below 3 km.

top of the convective element (Fig. 5b). At the same time, there is a negative M forcing (i.e., thick green contour) above the top of the convection, which produces outward motion there. However, the value of the M forcing (about $0.1 \times 10^{-5} \text{ s}^{-2}$) at this time is one order of magnitude smaller than the b forcing (absolute value $> 2.5 \times 10^{-5} \text{ s}^{-2}$). Therefore, the b forcing is dominant at this initial stage of the convection.

As the convection develops, the symmetric radial flow at the top of the convective element transports large M and s away from the convection (Fig. 5c), which reduces the b forcing (Fig. 5d). According to Fig. 5d, the M forcing produces negative η above the M perturbation from the radius of 75 to 130 km and positive η below the M perturbation from the radius of 78 to 100 km at 11 h 30 min.⁴ Therefore, the M forcing induces radially outward motion that *resists* the inward motion on the radially inward side of the convection but *enhances* the radially outward motion on the radially outward side of the convection (Fig. 5d). At this time, we can see a clear bias toward radially outward motion at the upper troposphere.

⁴ The M forcing is larger at a smaller radius because of the r^{-3} factor [see (1)].

The initial convection has decayed by 12 h 10 min, but the related radially outward motion remains (Figs. 6a,b). There is now a new convective burst at radius of 150 km, which also induces a pair of radially inward and outward motions at the top of the convection (Figs. 6a,b). The radially inward motion is resisted by the radially outward motion remaining from the previous convection at 100-km radius. Therefore, the sequence of the convection also plays a role in the bias of the outflow in the upper troposphere. As this new convection develops, the evolution proceeds similarly to the previous episode of convection; that is, the M forcing becomes prominent at the top of the convective element and produces radially outward motion, which reduces the radially inward motion and enhances the radially outward motion produced by the subsequent convection (Figs. 6c,d).

Figure 7 shows the convection to be near the radius of 120 km at 30 h; we see that the process is similar to the first episode of convection shown in Fig. 5. The b forcing is dominant at this initial stage of the convection (Fig. 7b) and a radially outward bias in the radial velocity is induced by the M forcing as the convection develops (Fig. 7d).

According to the foregoing analysis, we see that the development of the upper-level outflow is strongly related to the sporadic bursts of deep convection during phase I. Note that M in both the interior and the boundary layer

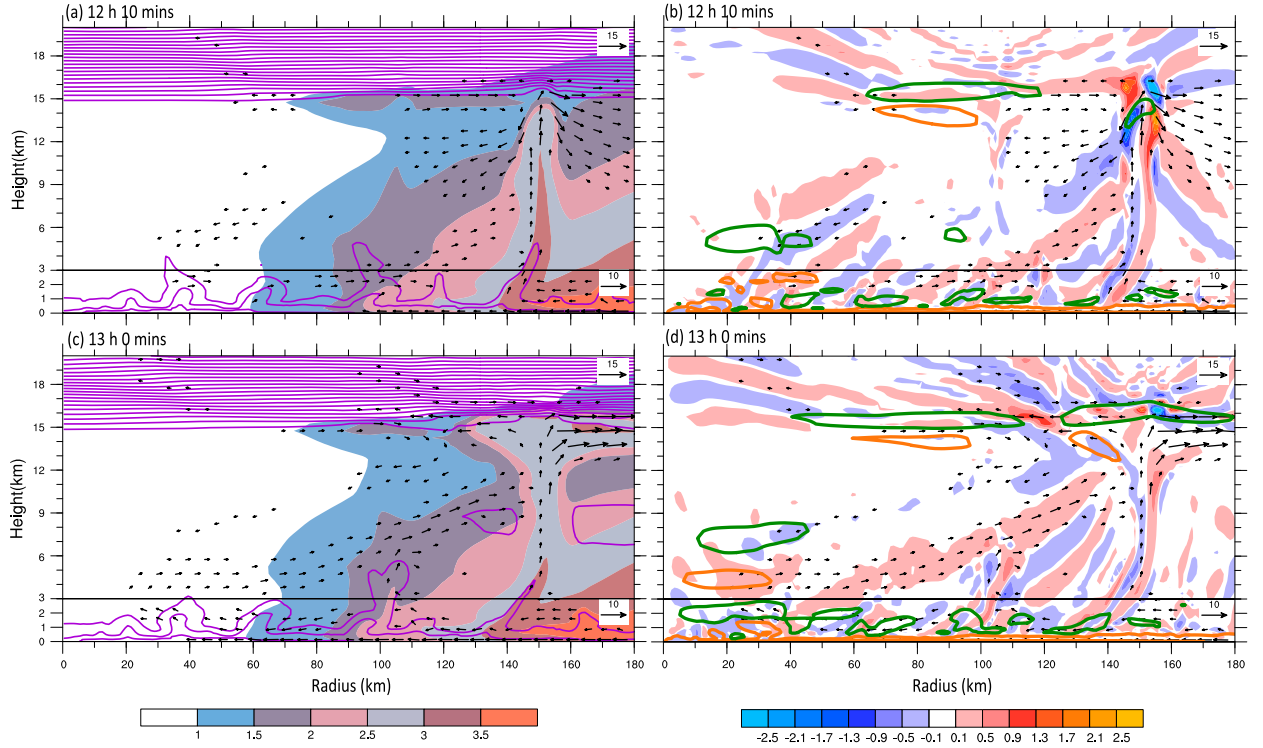


FIG. 6. As in Fig. 5, but at (a),(b) 12 h 10 min and (c),(d) 13 h 0 min. Vectors are not shown when the wind speed is less than 2 m s^{-1} .

increase during phase I (Figs. 6–8). Focusing on the region between the radius of 130 and 150 km, M below 3 km is less than $3 \times 10^6 \text{ m}^2 \text{ s}^{-1}$ at 12 h (Fig. 6a), but increases to $3.5 \times 10^6 \text{ m}^2 \text{ s}^{-1}$ at 38 h (Fig. 8a). According to Fig. 9, which shows the vertical profile of ρu averaged from the radius of 130 to 150 km over the period of 12–36 h, the inflow is only significant below 2 km. Therefore, the progressive increase of M during phase I is mainly contributed by the inflow in the boundary layer, while the increase of M in the interior is owing to the large M brought upward by the convection from the boundary layer. These results imply that the boundary layer spinup mechanism proposed by Smith and Montgomery (2015) plays an important role in TC intensification during phase I.

c. Late stage (38–48 h)

The sporadic deep convective bursts occur randomly during the phase I until 38 h, near which time a persistent eyewall has formed (Figs. 8a,b) and a clear, relatively steady secondary circulation can be seen. By 50 h, the M forcing and b forcing come into equilibrium, except in the eyewall region and boundary layer (Fig. 8d); the TC reaches a quasi-balanced state and phase II stage begins. There is a strong inflow below the height of 1 km related to the positive η produced by M forcing (the Ekman layer).

Figure 10 shows the time-averaged θ_e budget at the lowest level (25 m), which can be written as

$$\frac{\partial \theta_e}{\partial t} = - \left(u \frac{\partial \theta_e}{\partial r} + w \frac{\partial \theta_e}{\partial z} \right) - \left(\frac{1}{r} \frac{\partial r F_r^{\theta_e}}{\partial r} + \frac{\partial F_z^{\theta_e}}{\partial z} \right), \quad (2)$$

where the terms in parentheses on the rhs stand for the advection and turbulence tendency (mostly surface-flux divergence), respectively. From Fig. 10a, it is clear that in the first 12 h, the turbulence tendency is significant while the advection term is relatively small. The large standard deviation in Fig. 10a is an indication that convective cells are forming at different locations in the averaging period. But even with the large standard deviation, the turbulent θ_e flux to the boundary layer is not offset by advection, and thus, there is an increase of θ_e in time (Fig. 10a). The time-average θ_e tendency over the period of 12–24 h is still positive within the radius of 120 km, but the magnitude is smaller (Fig. 10b). During the transition period to the beginning of phase II (38–50 h), as the Emanuel-type vortex develops, the standard deviation for all terms decreases. The gradually enhanced inflow transports low s from the outer region inward to almost neutralize the increase of s provided by the sea surface fluxes (Fig. 10c); this balance is a feature of the E12 model.

Figure 11 shows soundings at 12 and 48 h at the radius of 100 km, which is the position of the first strong convection during phase I (Figs. 4, 5) and the initial location of a consistent eyewall (Fig. 8a) that characterizes phase II. As shown in Figs. 4 and 5, the unstable convective elements

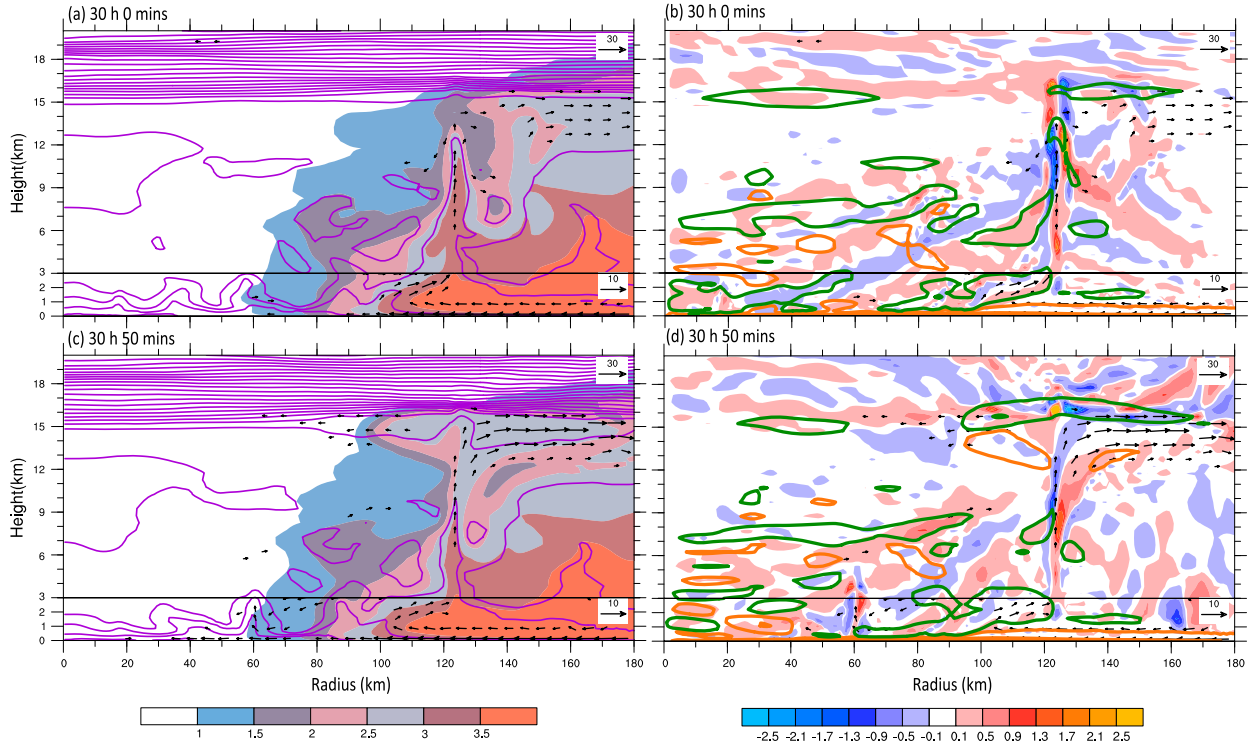


FIG. 7. As in Fig. 5, but at (a),(b) 30 h 10 min and (c),(d) 30 h 50 min. Vectors are not shown when the wind speed is less than 5 m s^{-1} above 3 km and less than 2 m s^{-1} below 3 km.

travel vertically upward and transport larger M and s to higher levels. Consistent with this, the CAPE at the radius of 100 km has increased from essentially zero initially (Fig. 1b) to 1450 J kg^{-1} at 12 h (Fig. 11a) as the boundary layer entropy increases through the air–sea interaction. Thereafter, the upward heat transport heats the column and thus reduces the CAPE during phase I (Fig. 11b). When the boundary layer parcels cannot achieve enough buoyancy, the convection is no longer upright and instead of lifting M surfaces it flows along them.

d. Summary of secondary circulation development

To summarize the evolution of the TC secondary circulation, we integrate the equation of azimuthal vorticity η , (1), over the region $A = [(r, z), 0 \leq r \leq R, 0 \leq z \leq H]$ (with $R = 412.5 \text{ km}^5$ and $H = 20 \text{ km}$, the region of the most intense primary and secondary circulations). The time rate of change of the circulation $C(t) = \int_0^R \int_0^H \eta \, dz \, dr$ around the perimeter of A can be expressed as

$$\begin{aligned} \int_0^R \int_0^H \frac{\partial \eta}{\partial t} \, dz \, dr = & \int_0^R \left[\left(\frac{M^2}{r^3} + D_u \right) \Big|_{z=H} - \left(\frac{M^2}{r^3} + D_u \right) \Big|_{z=0} \right] dr \\ & - \int_0^H (b|_{r=R} - b|_{r=0}) \, dz + \int_0^R \int_0^H c_p \left(\frac{\partial \theta_v}{\partial r} \frac{\partial \pi'}{\partial z} - \frac{\partial \theta_v}{\partial z} \frac{\partial \pi'}{\partial r} \right) dz \, dr. \end{aligned} \quad (3)$$

Since D_w is small, it is neglected to simplify the equation. The first term on the rhs of (3) is called the momentum term, and the second and third terms are called the thermal term. The sum of the momentum term and thermal term is $\partial C / \partial t$.

As shown in Fig. 12a, $C(t)$ develops rapidly during the first 12 h, consistent with the tendency due to the thermal term being larger than that of the negative of the momentum term (Fig. 12b). After this time, $C(t)$

becomes noisy, which is related to the convective bursts and internal-wave propagation, as will be illustrated below. The circulation does not change significantly during this time period as the TC primary flow intensifies from

⁵ The radius of 412.5 km is the outer radius r_o , where the initial tangential wind goes to zero.

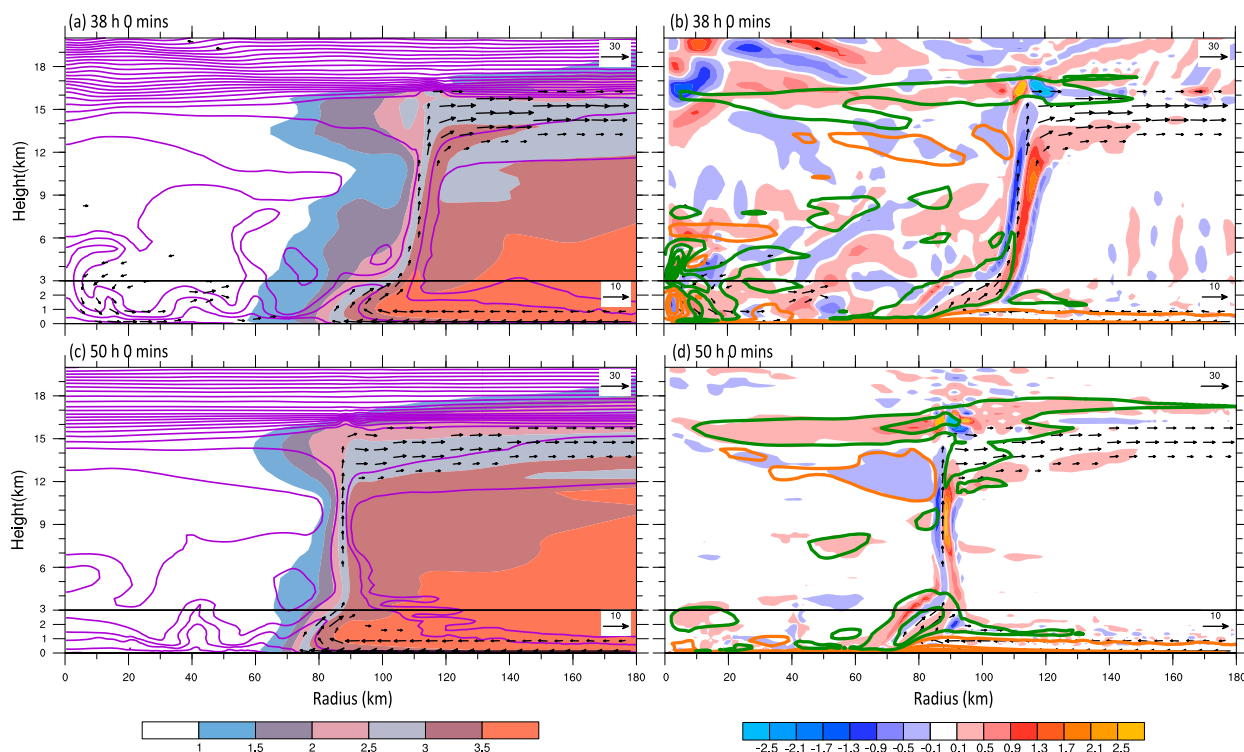


FIG. 8. As in Fig. 5, but at (a),(b) 38 and (c),(d) 50 h. Vectors are not shown when the wind speed is less than 5 m s^{-1} above 3 km and less than 2 m s^{-1} below 3 km.

20 to almost 38 m s^{-1} (Fig. 2). During phase II, the negative of the momentum term and thermal term become almost balanced and increase together (Fig. 12b) as the primary circulation intensifies (Fig. 2) and $C(t)$ is relatively constant (Fig. 12a), although the radial-vertical flow keeps increasing through the entire development of the TC (as shown in the following figures).

In summary, both phase I and phase II ultimately intensify through a balanced response to core heating, but the modalities of the secondary flow are quite different in each phase.

4. TC intensification in phase I

P18 showed that the intensification process can be qualitatively described by the E12 theory in phase II. However, during phase I, the intensification process is characterized by episodes of sporadic convection. The circulation tendency of the momentum term shown in Fig. 12b is mostly contributed by the momentum term at $z = 0$ while the circulation tendency of the thermal term is mostly contributed by the buoyancy term at $r = 0$. In the period from 12 to 46 h, the warm core also keeps developing as the primary circulation intensifies. On the vortex-system scale, the momentum term and the thermal term increase together. Therefore, when integrated

over the region A, the evolution of the TC in phase I approximately satisfies thermal wind balance. In the following, we will illustrate the process of the warm-core enhancement during phase I.

a. Simplified idealized experiment

As mentioned before, the intensification process in phase I is closely related to the episodes of the sporadic

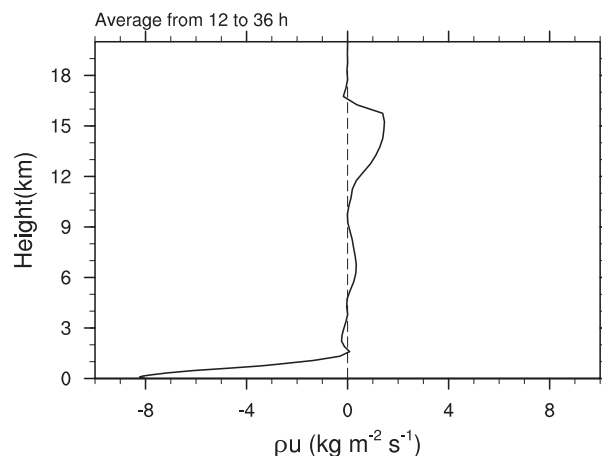


FIG. 9. Vertical distribution of mean ρu averaged from the radii of 130 to 150 km over the period of 12–36 h.

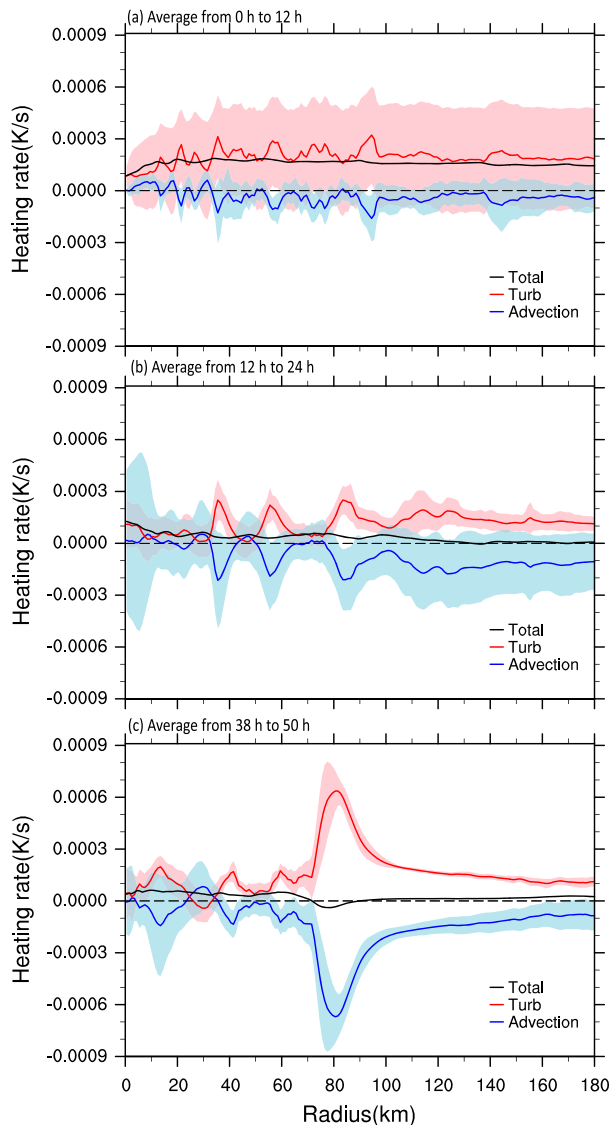


FIG. 10. Terms in time-averaged θ_e budget at the lowest level (25 m): averages are from (a) 0 to 12 h, (b) 12 to 24 h, and (c) 38 to 50 h. The black lines denote the θ_e tendency (which is the sum of all the calculated budget terms). The red lines denote the turbulence tendency. The blue lines denote the advection. The shading denotes the standard deviation for the turbulence term (red) and the advection term (blue).

convection. To help us understand the effect of each episode of convection, a simplified idealized axisymmetric experiment with a warm bubble is conducted. The simplified experiment does not include an initial vortex but retains the Coriolis term with $f = 0.5 \times 10^{-3} \text{ s}^{-1}$ ($f = 0.5 \times 10^{-4} \text{ s}^{-1}$ in the TC simulation). The artificially large f used here is to imitate the large inertial stability in the TC core known to be important in TC evolution (e.g., Schubert and McNoldy 2010). The initial sounding is identical to the control experiment, which

is saturated and moist neutral. The horizontal resolution is 1 km with 300 grid points and the vertical resolution is 500 m with a rigid lid at $z = 15 \text{ km}$. The height of 15 km is approximately the level of the tropopause of the original sounding (Fig. 1). A warm bubble is added at the initial time. The maximum potential temperature perturbation is 10 K, which is located at the position of $(r, z) = (40 \text{ km}, 6 \text{ km})$. Figure 13 shows the buoyancy related to the warm bubble and the sounding at $r = 40 \text{ km}$ at $t = 0$. The other settings are similar to the control simulation.

As the bubble rises, it induces compensating downdrafts, which desaturate the surroundings (Fig. 14a) and thus the response is in the form of internal gravity waves. The internal-wave response to the rising warm bubble takes the form of rays emanating outward and downward from it (Fig. 14b). This pattern is analogous to internal gravity waves generated by steady vertical motion of a sphere through a uniformly stratified, nonrotating fluid, which has been shown and explained by Lighthill (1967, their section 7). As the present experiments are in cylindrical coordinates, and axisymmetry requires $u(r = 0, z, t) = 0$, the downward motion on the inner side of the bubble is stronger than the outer side (Figs. 14a,b) because of the effect of the “image” vortex for $r < 0$. Therefore, the region and magnitude of the warming induced by the downward motion on the inner side is larger than the outer side. As the internal waves propagate outward from the bubble, the “inner core” keeps warming because of the stronger downward motion (Figs. 14c,d) and a significant warming can be seen at $z = 4\text{--}6 \text{ km}$ at 16 min (Fig. 14d). At the upper level, the bubble hits the rigid lid at 15 km and spreads outward (Figs. 14c,d). Figure 14 also shows that the warm bubble induces convergence (divergence) below (above) the center of the warm bubble, which transports larger absolute angular momentum inward and smaller absolute angular momentum M outward, since the basic distribution of $M = fr^2/2$. Therefore, the warm bubble produces the positive tangential wind in the lower inflow region and negative tangential wind in the upper outflow region.

Figure 14 illustrates how internal gravity waves communicate the transient effects of convection to the inner core through adiabatic warming and enhancement of the tangential wind. Over the short period of the foregoing experiment, however, the further effects of rotation on the buoyancy are not established since a simulation with $f = 0$ (not shown) is nearly identical to the present simulation. In the following we will see that it is the continuing episodes of convection that lead to eventual thermal wind balance.

b. Warm-core enhancement in control experiment

Figure 15 shows the M surfaces and buoyancy field of the control experiment at the beginning of the middle stage of phase I. It can be seen that the TC eye has a

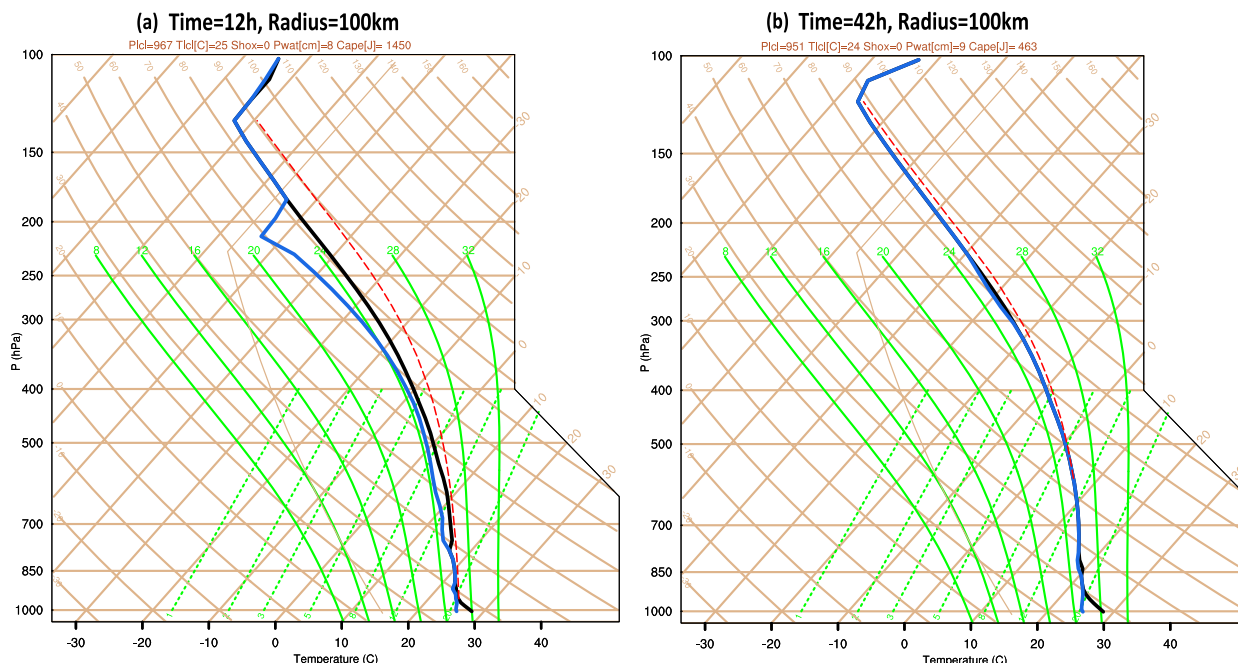


FIG. 11. Soundings at the radius of 100 km (the position of the eyewall) for (a) 12 and (b) 48 h. The red dashed line denotes the parcel path from the surface upward.

double warm-core structure, in which there is a mid- to low-level warm core below $z = 10$ km and a higher-level warm core located at $z = 14$ – 16 km. At 10 h 50 min, a new convective cell starts at the radius of 145 km (Fig. 15a). The positive buoyancy that drives the convection is derived from the high entropy in the boundary layer. The rising parcel also induces compensating downdrafts, which desaturate the surroundings as in Fig. 14a. Similar to Fig. 14, the internal-wave response to a rising disturbance takes the form of rays emanating outward and downward from it (Fig. 15). Figure 15b shows how the subsidence warming is produced by the rising convective element on both sides at 12 h. Figure 15c shows that the internal gravity waves propagate downward to the mid- to low level of the center of TC. A maximum warming can be seen at the height of 2 to 4 km in the TC center. These features are similar to the simplified experiment results shown in Fig. 14.

Figure 16 shows the M surfaces and buoyancy field at the end of the middle stage of phase I. Compared to Fig. 15, the mid- to low-level and upper-level warm core are both enhanced. The enhancement of the upper-level warm core is likely caused by the subsidence related to the convection-induced radial inflow (Fig. 15 and Fig. 16). The convection at the radius of 118 km also produces internal gravity waves, which enhance the warm core under $z = 6$ km (Fig. 16). One of the observational studies of Hurricane Bonnie (Heymsfield et al. 2001) also found that there was a

broad current of strong (several meters per second) descent associated with an intense eyewall convective cell. They proposed that the midlevel eye warming observed in Bonnie may arise from one or more of

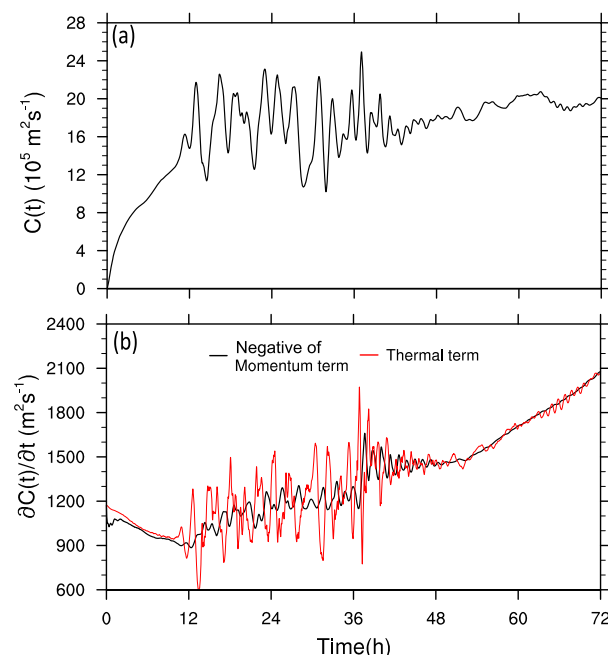


FIG. 12. Time dependence of (a) circulation $C(t)$ and (b) circulation tendency of the negative of the momentum term ($\text{m}^2 \text{s}^{-2}$) and the circulation tendency of the thermal term ($\text{m}^2 \text{s}^{-2}$).

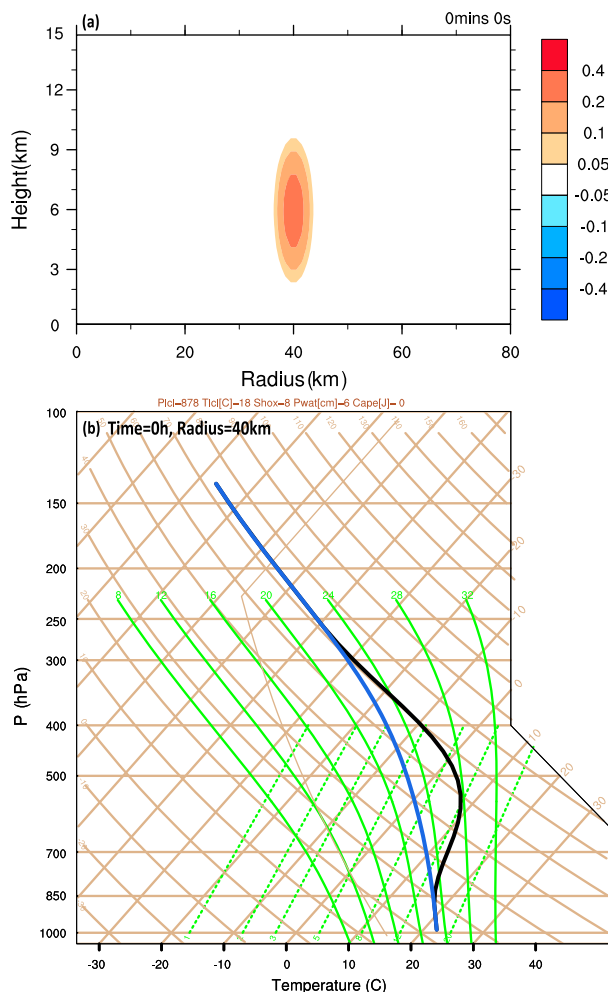


FIG. 13. (a) Buoyancy (m s^{-2} ; color) and (b) the sounding at the radius of 40 km (the position of initial warm bubble) at $t = 0$ for the simple idealized experiment.

these convectively induced episodes rather than as a result of a gradual sinking motion applied uniformly throughout the eye, which is consistent with our results.

Previous studies suggested that the mid- to low-level warm core is an integral part of the TC-balanced dynamics (e.g., Stern and Nolan 2012). In the present study, we illustrate the detailed inner-core warming process by individual episodes of convection. The episodes of convection transport large M inward in the boundary layer and upward to the interior and, at the same time, they produce gravity waves propagating in a ray-like manner to warm the mid- to low levels of the center of the TC. Figure 17 shows a time–height diagram of buoyancy at the center of the TC, which can be compared with the circulation tendency from the momentum term (black curve) and the thermal term (red curve) in (3) replotted in Fig. 17. Figure 17 shows that

after the column of the TC center is warmed by an episode of convection, the primary vortex starts to adjust to a new state of thermal wind balance. For example, a maximum warming at the low levels in the TC center can be seen in Fig. 17 at around 12 h 40 min, which is caused by the downward-propagated internal gravity waves from the convection that occurs at the radius of 150 km (Fig. 15). At the same time, we see a local maximum value of the thermal term and a local minimum value of the magnitude of the momentum term at 12 h 40 min in Fig. 17. Subsequently, the circulation tendency from the momentum term, which is proportional to the overall TC intensity, gradually increases as the buoyancy at low levels gradually moves upward through the column (Fig. 17). These episodes of convection and adjustment repeat and produce an oscillation in TC intensity in phase I.

Figure 18 shows a time-averaged θ budget in phase I (from 6 to 30 h) at the center of TC, which can be written as

$$\frac{\partial \theta}{\partial t} = -u \frac{\partial \theta}{\partial r} - w \frac{\partial \theta}{\partial z} + \dot{q} + D_{\theta} + R + N_{\theta}, \quad (4)$$

where $\dot{q}$ represents the tendency from microphysical scheme, D_{θ} is the turbulence tendency, R represents the tendency from radiation scheme, and N_{θ} is the tendency from Rayleigh damping (see BR09b for details). It can be seen that during the phase I, the increase of θ in the midlevel (~ 3 – 9 km) and upper level (above 14 km) in the troposphere are mainly contributed to by the vertical advection term. The effects of turbulence and microphysics are very small above a height of 5 km. The tendency from radiation is negative everywhere. The values of horizontal advection and N_{θ} are close to zero (not shown). We notice that the vertical advection is negative below a height of 4 km, which is associated with shallow convection occurring at the center of the TC. However, the negative effect is cancelled by the microphysical process of latent heating. Therefore, after several episodes of convection and adjustment, a net increase of buoyancy in the center is left behind and the vortex gradually intensifies.

By the time the simulated TC approaches phase II, the mid- to low-level inner core has been filled with warm air. The sounding in the eyewall region becomes essentially moist neutral (Fig. 11b). The buoyancy associated with the boundary layer air in the eyewall region is comparable to the interior air (Fig. 19), and thus the wavelike enhancement of the mid- to low-level warm core stops. We also show the “gradient nonbalance” region, defined by Cohen et al. (2017), in Figs. 15c, 16c, and 19c. During phase I, the nonbalance region exists in the areas that have convection and gravity waves

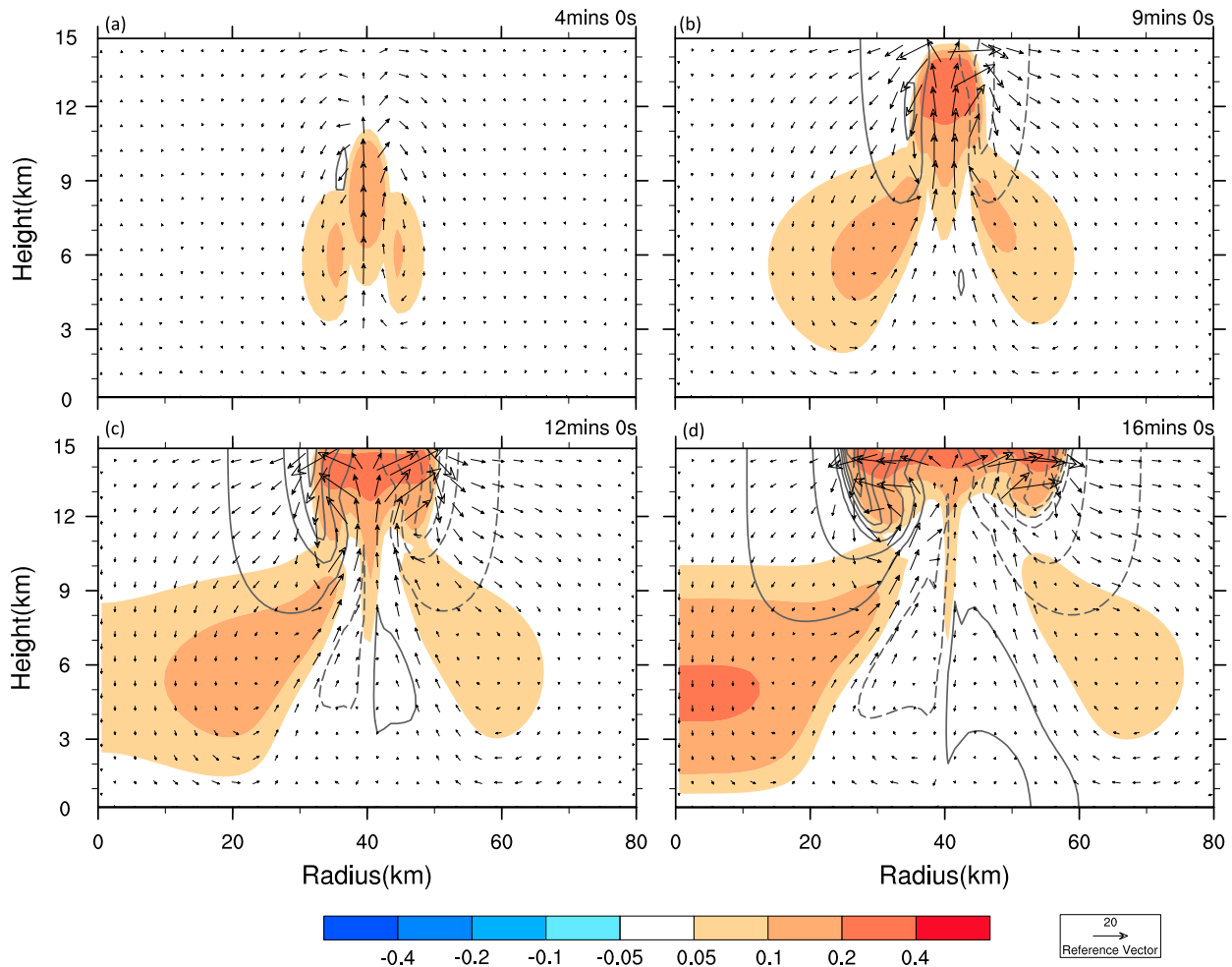


FIG. 14. Buoyancy (m s^{-2} , color), tangential wind (gray lines; contour interval: 1 m s^{-1} ; the positive azimuthal direction, dashed for negative azimuthal direction) and wind vectors (u, w) for the simple idealized experiment at (a) 4, (b) 9, (c) 12, and (d) 16 min.

(Figs. 15c, 16c). As the TC is approaching phase II (Fig. 19c), the nonbalance area shrinks significantly. Therefore, the vortex adjusts to near balance in phase II as discussed in P18 and shown in Fig. 8.

5. Summary and conclusions

According to our previous numerical results (P18), the intensification process of a simulated axisymmetric tropical cyclone can be divided into two periods, phase I and phase II. During phase I, the angular momentum (M) and saturated entropy (s^*) surfaces evolve from nearly orthogonal to almost congruent. During phase II, the M and s^* surfaces in the eyewall and outflow are approximately congruent as the TC intensifies. In this paper, we investigate the evolutionary process of the TC in phase I through a focus on the development of the secondary circulation, the nature of the end of

phase I, and the TC intensification mechanism in phase I.

The equation for the azimuthal vorticity is used to investigate the development of the secondary circulation. At the initial stage, the M near the sea surface is decreased by vertical diffusion in the boundary layer, which reshapes the vertical distribution of M . The positive M forcing induces a shallow layer of positive azimuthal vorticity and low-level inflow (i.e., an Ekman layer) starts to develop. In the first few hours, the sounding within the vortex is almost moist neutral and CAPE is very small (Fig. 20c). As the sea surface fluxes transfer more entropy from the sea to the air, the environment becomes unstable. The perturbation triggered in the Ekman layer tends to develop and convection begins.

We find that the formation of the TC outflow is strongly related to the episodes of convection in phase I

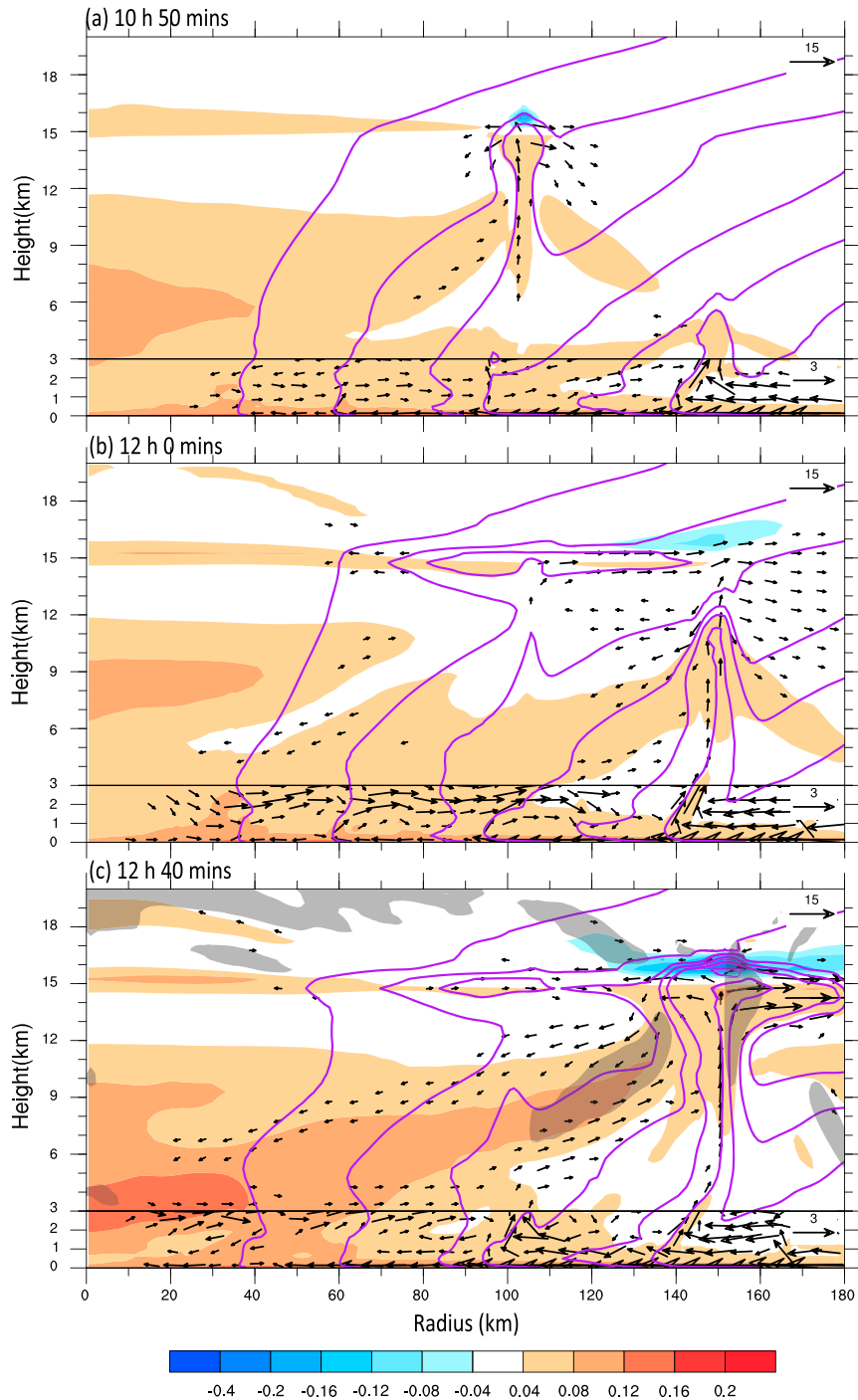


FIG. 15. Buoyancy (m s^{-2} ; color), M ($10^6 \text{ m}^2 \text{ s}^{-1}$; contours) and wind vectors at (a) 10 h 50 min, (b) 12 h 0 min, and (c) 12 h 40 min for the control TC experiment. Vectors are not shown when the wind speed is less than 2 m s^{-1} above 3 km and less than 0.5 m s^{-1} below 3 km. The gray shadow in (c) represents the area that is in a state of “nonbalance” (geostrophic Rossby number $\text{Ro}_g < -1/4$).

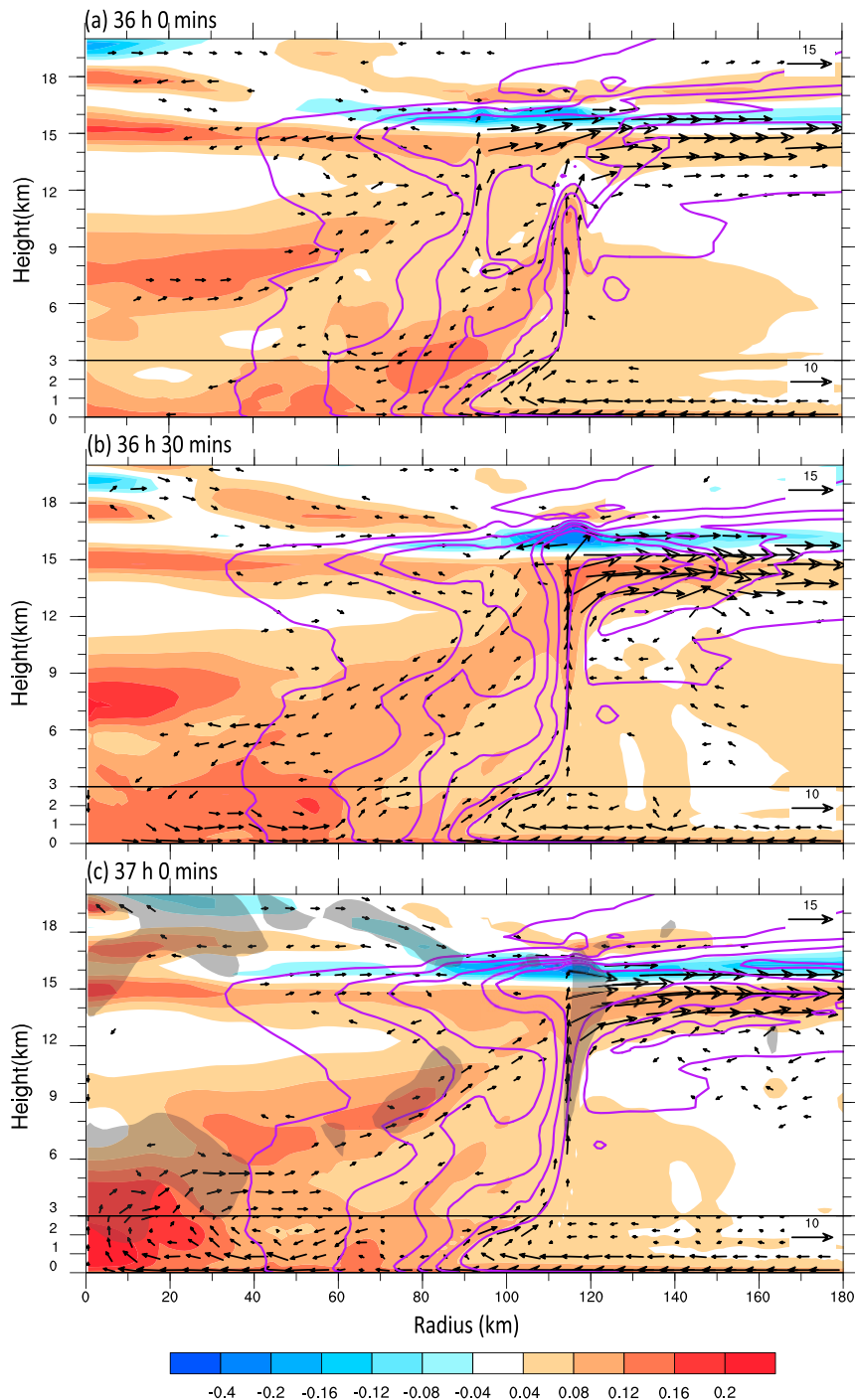


FIG. 16. As in Fig. 15, but at (a) 36 h 0 min, (b) 36 h 30 min, and (c) 37 h 0 min. Vectors are not shown when the wind speed is less than 2 m s^{-1} .

(Fig. 20a). Each episode of convection results in radially inward and outward flow (relative to the TC) at its top. According to the budget analysis of η (Fig. 5), this inward–outward motion is first produced by the b forcing. The M forcing contributes to producing a

radially outward bias in the radial velocity, which can be interpreted as the outward centrifugal and Coriolis forces overcoming the inward pressure gradient force. The sequence of continuing episodes of convection also plays a role in the continued outward

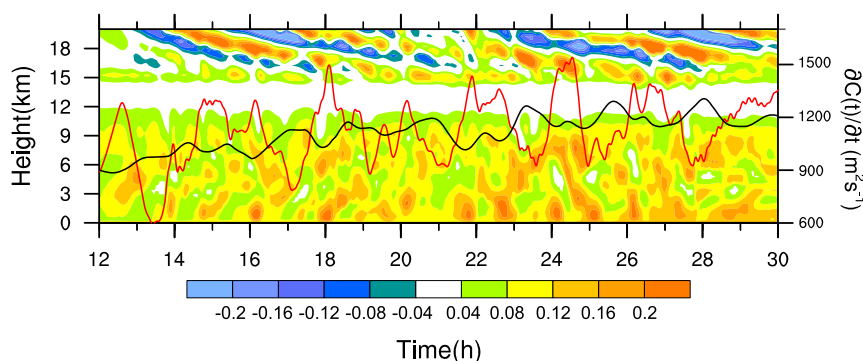


FIG. 17. Time–height plot of buoyancy ($\text{m}^2 \text{s}^{-2}$; color) at the center of the TC, the circulation tendency of the negative of the momentum term ($\text{m}^2 \text{s}^{-2}$; black line), and the circulation tendency of the thermal term ($\text{m}^2 \text{s}^{-2}$; red line).

radial bias of the outflow at the upper troposphere (Fig. 6).

The enhancement of the warm core is consistent with the intensification of the primary TC circulation during phase I. The enhancement of the warm core is also related to the convection in phase I. The rising buoyant elements of convection produce compensating subsidence in the form of internal gravity waves (Lighthill 1967), which emanate outward and downward and produce warming on both sides. The internal gravity waves propagate to mid- to low levels near the center of the TC and enhance the mid- to low-level warm core. In Fig. 20b, it is clear that the magnitude of vertical integral of buoyancy expands outward with every episode of convection in phase I. As the warming caused by the downward propagated internal gravity wave reaches the TC center, the vortex starts to adjust to a new state of thermal wind balance. The overall TC intensity is gradually increasing as the buoyancy at low levels gradually moves upward through the column (Fig. 17). After several episodes of convection and adjustment, a net increase of buoyancy in the center is left behind and the vortex gradually intensifies.

After almost 20 h from the onset of first convection, the sporadic episodes of convection keep reducing CAPE by warming the TC interior during phase I. The increase of s through the surface fluxes is neutralized by the low s transported by the boundary layer inflow (Fig. 10c). Thus the boundary layer air is no longer unstable to convect in an upright manner and lift the M surfaces; instead the upward motion starts to follow the M and s surfaces. At this time, except for the eyewall region and the boundary layer, the vortex adjusts to near balance everywhere—that is, phase I ends and phase II begins. It should be noted that there is no convection at very small radii, even though the CAPE there is very large (Fig. 20c). This is mainly because of

the systematically downward motion and a level of free convection above 700 hPa (not shown), which suppresses convection (Fig. 20d).

Previous studies have also found that the evolution of the TC (either axisymmetric or three-dimensional) can

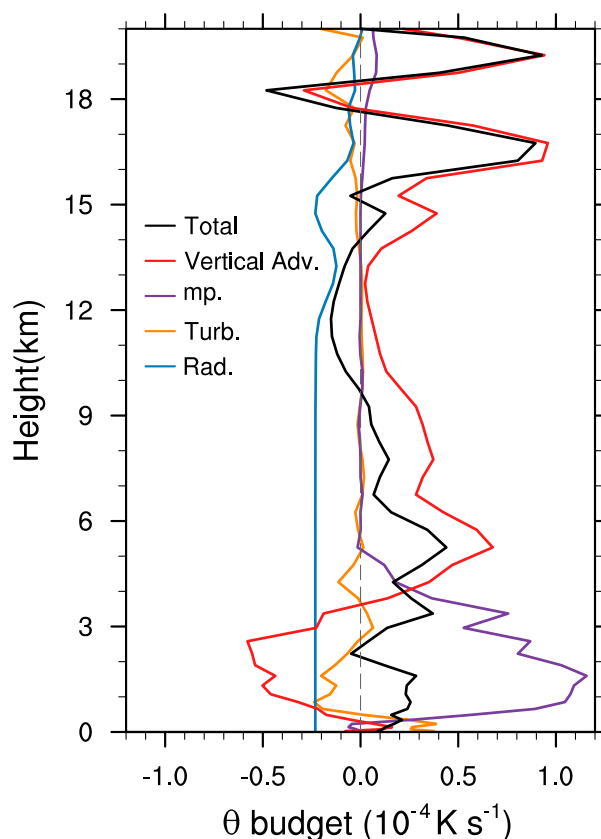


FIG. 18. Terms in time-averaged (from 6 to 30 h) θ budget at the center of TC ($r = 0$): θ tendency (the sum of all the calculated budget terms; black); vertical advection (red); tendency from the microphysical scheme (purple); turbulence tendency (orange); and the radiation term (blue).

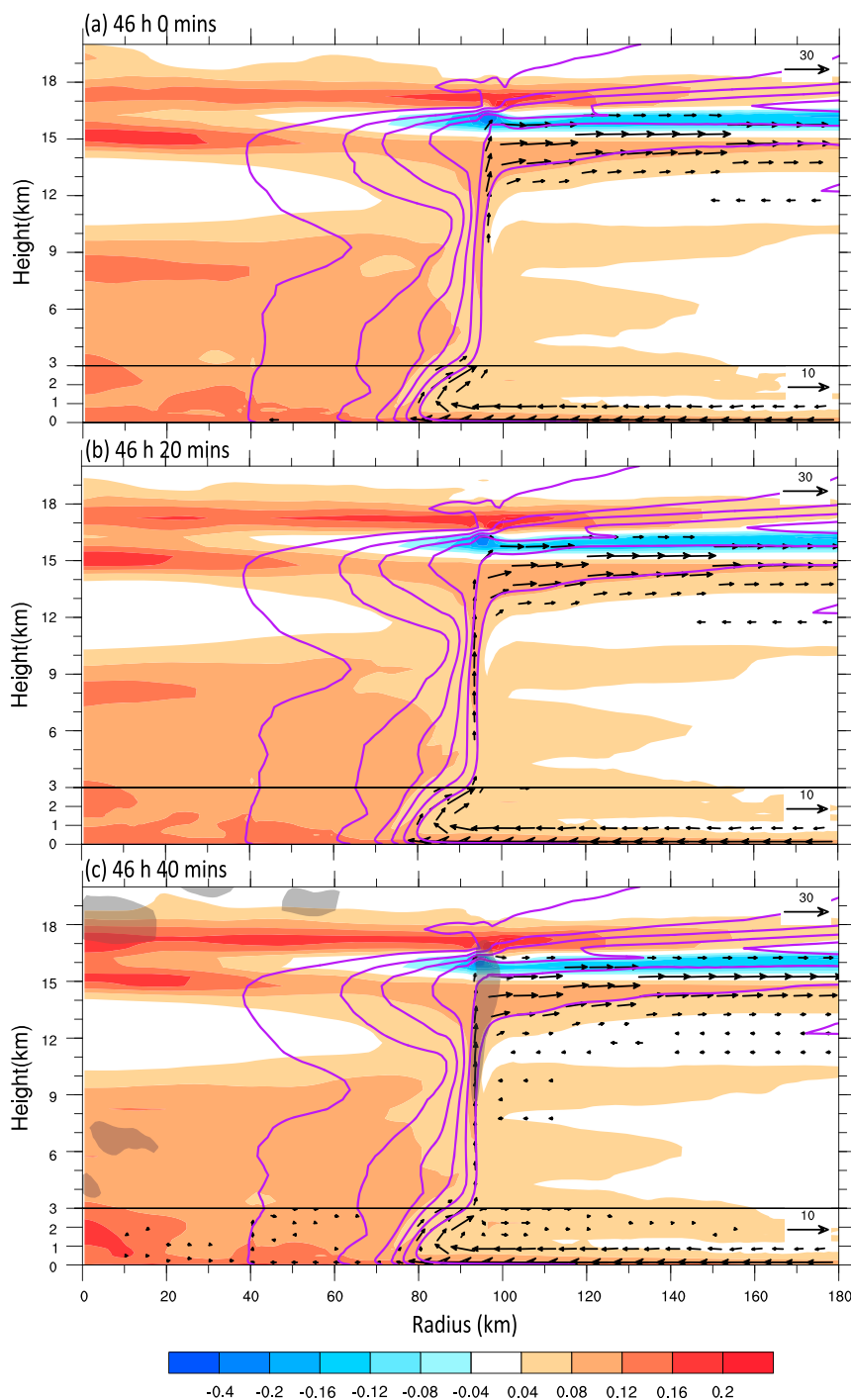


FIG. 19. As in Fig. 15, but at (a) 46 h 0 min, (b) 46 h 20 min, and (c) 46 h 40 min. Vectors are not shown when the wind speed is less than 5 m s^{-1} above 3 km and less than 2 m s^{-1} below 3 km.

be considered as occurring in several phases (e.g., Miyamoto and Takemi 2013, 2015; Kilroy et al. 2017; Xu and Wang 2018). Xu and Wang (2018) classified the intensification process of their axisymmetric simulated TC into two periods: the initial spinup period and the

primary intensification period. The primary intensification period is further divided into three subphases: 1) a slowly intensifying phase, 2) an RI phase, and 3) an adjustment phase toward the quasi-steady state (Miyamoto and Takemi 2013, 2015; Xu and Wang 2018). The initial

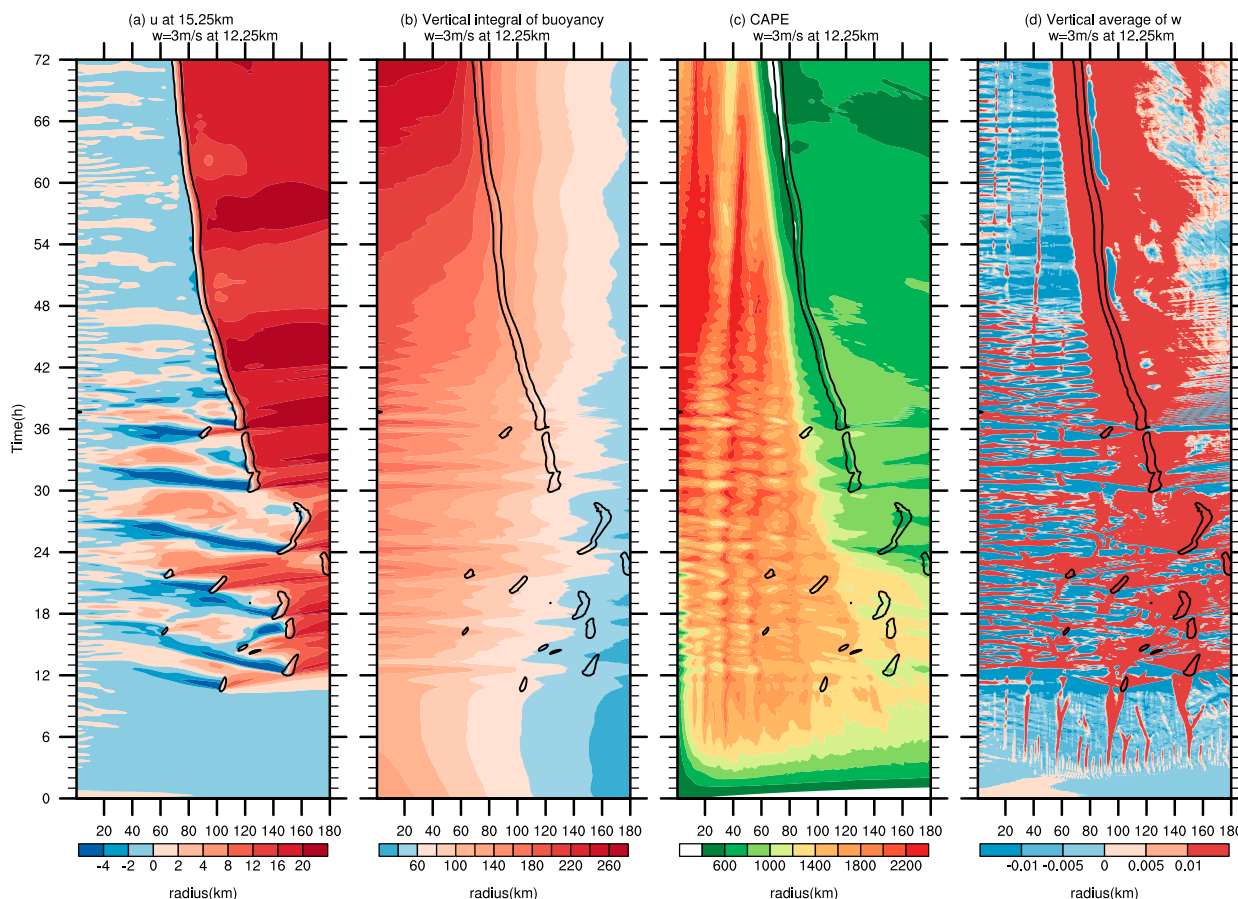


FIG. 20. Time-radius plots of (a) radial velocity at the height of 15.25 km, (b) vertical integral of buoyancy (from $z = 0$ to 25 km), (c) CAPE, and (d) vertical average of vertical velocity (from $z = 0$ to 16.25 km). The black lines denote a vertical velocity larger than 3 m s^{-1} at the height of 12.25 km.

spinup period and the slowly intensifying phase can roughly be related to the phase I herein, and the RI phase can be regarded as the phase II herein. Their simulations undergo a similar sequence of initial convection associated with Ekman pumping that subsequently contracts to the familiar narrow eyewall updraft as in our study. Motivated by [Xu and Wang \(2018\)](#), we have done several sensitivity tests varying the initial vortex depth. Our experiments (not shown) indicate that the duration of phase I is generally shorter for a deeper the initial vortex. We interpret this sensitivity as evidence that a deeper vortex needs fewer episodes of convection to loft the angular momentum surfaces to the tropopause where the [E12](#) solutions apply.

[Kilroy et al. \(2017\)](#) conducted a three-dimensional numerical simulation to study tropical cyclogenesis and intensification using CM1. In their Fig. 6a, one observes that the three-dimensional convective cells tend to exist in rings beyond the radius of 40 km, consistent with the present study; this structure is also reflected in the

azimuthally averaged flow (their Fig. 7), which is also consistent with our axisymmetric flow in that there is a general area of cumulus convection between 40 and 100 km (their Fig. 7d) that eventually contracts into a structure that looks like the Emanuel-type vortex (their Fig. 7f). We believe the present study provides a benchmark against which these more complicated three-dimensional TC flows can be compared.

In summary, the present study provides a detailed analysis of the evolution of an axisymmetric TC before it reaches a state of moist slantwise neutrality. The high time resolution of our analysis provides new insight into the effect of convection on the enhancement of the warm core. A similar analysis of a three-dimensional TC vortex is a topic we plan to investigate in the future. Preliminary results indicate the preference for the 3D cells to form in annular rings (as in [Kilroy et al. 2017](#)) and that the 3D cells systematically contribute to outflow at their tops in the manner explained herein.

Acknowledgments. The authors are grateful for constructive comments by three anonymous reviewers of the manuscript. This research was conducted during the first author's visit to NCAR, which was supported by the MMM visitor program of NCAR. NCAR is sponsored by the National Science Foundation. This research was supported in part by the National Key Research and Development Program of China under Grant 2017YFC1501601 and National Natural Science Foundation of China Grant 41875067.

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