# The Fluid Dynamics of Tornadoes

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# **Keywords**

tornado, supercell, vortex, vortex breakdown, multiple vortices

# Abstract

Because of the difficulty in making measurements under controlled conditions, most of what is known about the fluid dynamics of tornadoes comes from laboratory experiments that produce vortices with features similar to those observed in a tornado. Numerical simulation of laboratory experiments has become a valuable analytical tool owing to the greater ease of extracting data. The success of the numerical simulations has inspired better-defined numerical experiments capable of quantitatively describing the basic features of observed tornado vortices and has motivated simple fluid dynamical explanations. The present article reviews the state of knowledge concerning the fluid dynamics of tornadoes as found in laboratory and numerical analogs.

# **1. TORNADOES**

A tornado is a rapidly rotating column of air that occurs in association with a cumuliform cloud. The most intense variety of tornado arises within a special type of cumuliform cloud known as a supercell thunderstorm, so called because it can persist for several hours in the nearly steady configuration of a rotating updraft. Most of what is understood of the fluid dynamics of tornadoes comes from laboratory and numerical-model idealizations of the supercell thunderstorm as a circular updraft fed by air possessing angular momentum with respect to the updraft center axis. The present article reviews the current understanding of the fluid dynamics of tornadoes based on these idealizations.

#### 1.1. Observations

There is a vast literature on tornado phenomenology and climatology; recent reviews can be found in Bluestein (2007, section 6) and Markowski & Richardson (2010, chapter 10.1.5). Our focus here is on the much smaller body of knowledge of the velocity fields characterizing a tornado, which, because of its extreme violence and capricious nature, renders measurement problematic. Until recently, quantitative knowledge of the tornado velocity field had come only from particle-tracking analysis of motion pictures, a particularly vivid example of which is shown in **Figure 1**. In this example, the most intense part of the wind field is concentrated in the lowest several hundred meters, consistent with other analyses of this type (Davies-Jones 1986, pp. 210–11).

Whereas motion-picture analysis gives a picture of the tornado's exterior flow, it was not until the 1990s that close-range, portable Doppler radar began to reveal the interior flow (Bluestein et al. 2007, Wurman et al. 2007). Figure 2a shows the absolute value of the storm-motionrelative Doppler velocity in the tornado that occurred on May 24, 2011, near El Reno, Oklahoma. As in **Figure 1**, the strongest winds occur near the ground, but the maxima rapidly decay and expand outward with height. Figure 3 shows an analysis of the wind field in cylindrical polar coordinates  $(r, \theta, z)$  from the Doppler on Wheels mobile radar of a tornado near Mulhall, Texas, on May 3, 1999 (Lee & Wurman 2005). By application of the ground-based velocity track display (GBVTD) technique (Lee et al. 1999), one can deduce the axisymmetric component of the velocity field  $(u_0, v_0, w_0)$  from a single radar. Although there are a number of limitations and caveats concerning the application of GBVTD to a tornado (Lee et al. 1999), Figure 3 reliably indicates that the greatest values of  $v_0(r, z)$  are at the lowest observed levels along with intense inflow  $(u_0 < 0)$  and upward motion  $(w_0 > 0)$  near the radius of maximum  $v_0$ , consistent with the motionpicture analysis shown in **Figure 1**. In addition, the analysis suggests the existence of a central downdraft in what appears to be a much larger tornado than those shown in Figures 1 and 2, although smoothing in the GBVTD technique may have contributed to the larger analyzed size. An examination of Figure 2b together with the reflectivity field (not shown) reveals that the debris cloud is relatively empty at its center above a height of  $\simeq 0.5$  km, which might also be indicative of a central downdraft aloft (see also Wurman et al. 1996).

A climatology of Doppler-radar observations (Alexander & Wurman 2008) indicates that the median tornado has a radius of maximum wind of approximately 150 m with an order-of-magnitude variation between the extreme values. The median tornado wind-speed maximum is 55–60 m s<sup>-1</sup>, with speeds of  $\simeq$ 50 m s<sup>-1</sup> and 80–85 m s<sup>-1</sup> at the 25th and 75th percentiles, respectively, and minimum and maximum values of 20 m s<sup>-1</sup> and 135 m s<sup>-1</sup>, respectively. The maximum intensity is found in the lowest few hundred meters. Although tornadoes can last from seconds to more than an hour, a typical tornado lasts approximately 10 min (Bluestein 2007). Occasionally, tornadoes have a multiple-vortex structure (**Figure 4**) with each subvortex associated with a local wind-speed maximum in the Doppler-velocity field (Wurman 2002).



Photogrammetric analysis of the wind field of the Saylor Park, Ohio, tornado of April 3, 1974. The black velocity vectors show the rotation of the column, and the red velocity vectors demonstrate the strong upward motion. Figure taken from Fujita (1992). Abbreviation: AGL, above ground level.

For a more complete picture of the technology brought to bear on measuring the tornado and its parent circulation, the reader is referred to Wurman et al. (2012).

# 1.2. Supercell Tornadoes

The fluid dynamics of supercell thunderstorms (also known as supercells) is reviewed in Klemp (1987) and more recently in Markowski & Richardson (2010, chapter 8). Figure 5 illustrates the three-dimensional storm-motion-relative flow in the nearly steady configuration of a supercell. The illustration is from the point of view of an observer situated at some height above the ground level in the warm, moist air of the environment. The supercell updraft is fed primarily by airstream A originating in the environment, which is also characterized by wind shear. The environmental velocity field of a supercell thus implies a vorticity field such that there is a component of environmental vorticity along the airstream. This vorticity, when tilted vertical by the updraft, produces rotation about a vertical axis at midlevels of the supercell updraft (a mesocyclone) as illustrated in Figure 5. The flow in a supercell is arranged such that precipitation does not fall directly back down the updraft axis, but rather away from it, as illustrated; evaporation of this precipitation cools



Mobile-radar observations of (*a*) the storm-motion-relative Doppler velocity and (*b*) the cross-correlation coefficient  $\rho_{bv}$  between transmitted waves with horizontal and vertical polarizations, identifying locations of debris (values between 0.9 and 1 indicate meteorological scatterers; lower values indicate nonmeteorological debris) of the El Reno, Oklahoma, tornado of May 24, 2011. Observations are displayed at a constant radial distance of 5 km from the radar across a 5-km arc length. The radar beam width is approximately 75 m in the raw data.

the air at low levels and thus sets up a thermal boundary at the surface. The thermal boundary baroclinically produces horizontally oriented vorticity on the forward-flank trajectories (B and C) directed toward the updraft; this vorticity is also tilted upward beneath the main storm updraft and enhances the updraft rotation about a vertical axis at low levels. The typical location of the tornado



#### Figure 3

Axisymmetric component of the wind field of the Mulhall, Texas, tornado of May 3, 1999, derived from mobile-Doppler-radar observations. The radar beam width is approximately 100 m in the raw data. Figure adapted from Lee & Wurman (2005).



An example of a multiple-vortex tornado. Photograph courtesy of H.B. Bluestein.

is indicated by T. Note that the supercell circulation extends for tens of kilometers, whereas the tornado has a scale of at most hundreds of meters.

The focus of much current research on tornadoes is on the precise nature of how the thermalboundary-associated rotation gives rise to tornado formation (e.g., Markowski et al. 2008). **Figure 6** illustrates several of the complex interactions considered. In very brief summary, as noted above, the thermal boundary is recognized as a source of low-level rotation for the tornado; however, the rain-cooled air, being negatively buoyant, resists uplift and most often surrounds the tornado, leading to its eventual demise (**Figure 6***a*). A leading hypothesis is that there is an optimal balance at which there is enough cool air to produce low-level rotation but not so much as to weaken the updraft and thus prevent tornado formation (**Figure 6***b*).

Although the nature of the low-level rotation giving rise to a tornado is critical to the overall understanding of the tornado and the ability to make short-term (minutes) predictions, it is an area in which research has yet to reveal definitive answers. Moreover, because of the disparity in scale between the tornado and its parent circulation, research on the fluid dynamics of tornadoes has progressed through the study of the mechanisms by which an idealized supercell, in the form of a circularly symmetric updraft with a given source of low-level rotation, produces smaller-scale intense vortices with features common to tornadoes.

**Figure 7** uses a superposition of photographs and mobile-Doppler-radar data to illustrate the time sequence of a tornado developing within a pre-existing rotating updraft. In the earliest stage, there is an updraft (indicated by the cumuliform cloud) and evidence of updraft-scale rotation (indicated by the Doppler-radar data); as the tornado forms, the subcloud layer becomes increasingly obscured by the so-called rain curtain that wraps around the tornado, as illustrated in **Figure 6***a*.

#### 2. VORTEX CHAMBERS

Vortex chambers have been used to isolate and simplify aspects of the supercell thunderstorm thought to be important for tornado formation. We distinguish here two types of chambers: those in laboratory experiments and those in numerical-simulation experiments.



Schematic diagram of the flow within a supercell thunderstorm. The tornado (indicated by the T) forms between the warm inflow and rain-cooled surface outflow. The curled black arrows containing the straight black arrows represent the horizontal component of vorticity. The horizontal dimensions of the plane are roughly 30 km  $\times$  30 km, and the vertical dimension (exaggerated by a factor of  $\simeq$ 3) is roughly 10 km. Figure adapted from Klemp (1987).

#### 2.1. The Ward Chamber

The most influential laboratory experiment in the field of tornado dynamics is the one conceived by Ward (1972) and further explored in an updated version reported by Church et al. (1977, 1979). The essence of the experiment is illustrated in **Figure 8***a*, which shows an upward-directed fan (representing the supercell updraft) producing a volume flow rate  $2\pi Q$  through an orifice of radius  $r_o$ ; inflow occurs over a layer of depth *b* (representing the subcloud layer), and angular momentum  $2\pi \Gamma_s$  (representing subcloud circulation around the updraft center) is imparted to it by means of a rotating screen at  $r = r_s$ . The working fluid is taken to have constant density  $\rho$ and dynamic viscosity  $\mu$  and thus can be characterized by a single value of its kinematic viscosity  $v = \mu/\rho$ . Hence there are six external dimensional parameters,  $r_s$ ,  $r_o$ , *b*, *v*, *Q*, and  $\Gamma_s$ , from which four nondimensional parameters can be formed: the swirl ratio,  $S = r_o \Gamma_s / (2Q)$ ; the Reynolds number, Re = Q/(vb); the aspect ratio,  $a = b/r_o$ ; and the ratio of inflow to the updraft radius,  $r_o/r_s$ . The Ward-type chamber is distinguished by the generally small values of *a* and values of  $r_o/r_s \simeq 1/2$ , which are so chosen to satisfy the requirements of geometrical similarity to the flow in a supercell.

Analysis of the flow in the Ward-type chamber (Davies-Jones 1973) indicates that for fixed geometrical parameters, certain aspects of the flow depend mainly on *S*, with a weaker dependence on *Re*. An example of the dependence of the flow on *S* is given in Church et al. (1977, figure 4)



Illustration of a hypothesis on the effects of the cold-air boundary on tornado formation. Figure adapted from Marquis et al. (2012).

and is illustrated in Figure 9. For low values of S, the flow takes the form of a single vortex in an updraft extending from the surface through the depth of the chamber (Figure 9a). At somewhat larger values of S, the single vortex in an updraft undergoes a transition at some level above the surface to a single vortex with a central downdraft surrounded by updraft (Figure 9b). For larger values yet of S, the central downdraft penetrates to the lower surface (Figure 9c). Finally, at the largest values of S, the single two-celled vortex undergoes a transition to multiple vortices revolving about a common center (Figure 9d). The explanation of these experimental outcomes is the main objective of the remainder of this review.

Numerical simulations of flow in the Ward chamber were carried out by Harlow & Stein (1974) and Rotunno (1977, 1979, 1984), varying S over the experiment range, but at much smaller Re. These simulations qualitatively reproduced the experimental results with respect to the changing vortex type as a function of swirl ratio, S, as illustrated in Figure 9.

# 2.2. The Fiedler Chamber

Although it is relatively straightforward to understand the relation of the Ward-type chamber to the rotating flow beneath a thunderstorm updraft, the presence of open boundaries at which the flow is not known before the experiment is run presents a special challenge for the analyst. For this reason, Fiedler (1995) suggested the idealization of the supercell updraft in a closed domain with ambient rotation as one that could provide the basis for a more definitive analysis. In this idealization, shown in Figure 8b, the height of the chamber encloses the entire thunderstorm updraft; therefore, the height of the cylinder represents the distance between the ground and the tropopause, and the rotation of the domain is analogous to the entire column of rotating updraft in a supercell thunderstorm.



**Azimuth angle** 



Two vortex chambers used in the study of tornadoes: (a) Ward (1972) and (b) Fiedler (1994). Panel a adapted from Davies-Jones (1986, figure 10.26).

As in the Ward-type chamber, the height *b* and radius  $r_s$  of the cylinder enter as external dimensional parameters; however, in the Fiedler chamber, the role of the volume flow rate is played by a prescribed upward (buoyancy) force per unit mass b(r, z), which has the radial and vertical scales,  $l_r$  and  $l_z$ , respectively, and the location of the maximum  $z_m$  as external parameters; the imposed buoyancy force also gives the velocity scale  $W = \sqrt{2 \int_0^b b(0, z) dz}$ . The ambient angular momentum distribution is  $\omega r^2$ . From the eight external dimensional variables  $(r_s, l_r, l_z, z_m, b, v, W,$  and  $\omega$ ), dimensional analysis indicates that the solution can depend on up to six nondimensional groupings; with the first four parameters fixed by the requirements of geometrical similarity to a supercell, the solutions depend solely on two nondimensional numbers:  $\Omega = \omega b/W$ , a swirl ratio, and  $Re_F = W b/v$ , a Reynolds number. The Fiedler experiment forms the basis for the theoretical explanations given in Section 3.

Numerical simulations following the Fiedler model have been carried out for large ranges of  $\Omega$  and  $Re_F$  (Fiedler 1994, 1998, 2009; Nolan 2005; Nolan & Farrell 1999). For a fixed value of  $Re_F$ , these experiments produce a range of vortex types as a function of  $\Omega$  analogous to that produced in the Ward-type experiments. Three-dimensional experiments in a box version of **Figure 8b** with  $Re_F = 40,000$  by Fiedler (2009) indicate turbulent flow with multiple vortices for larger values of  $\Omega$ .

Note that the Fiedler chamber is a virtual chamber for use in numerical experimentation, and there is no actual laboratory experiment that is quite the same as that pictured in **Figure 8**b, although there are some closely related ones. For example, the Turner & Lilly (1963) experiment consists of carbonated water with a free surface in a cylindrical container on a rotating turntable; an upward force is created on the center axis by the instigated release of CO<sub>2</sub> bubbles. The Phillips (1985) chamber uses water as the working fluid and has a free surface, rotating side walls reaching partway up to the free surface and a stationary bottom boundary; water is extracted at the top center and is reintroduced into a larger tank containing the rotating side walls. The Maxworthy (1972) chamber is closed as in **Figure 8**b, and the upward force is produced by a propeller; however,

#### Figure 7

Doppler velocity superimposed on photographs of a tornado forming in a supercell thunderstorm. Radial velocities less than  $-50 \text{ m s}^{-1}$  are shaded light red. Figure adapted from Wakimoto et al. (2011).



For increasing swirl ratio, S, Ward (1972) showed that the form of the vortex changes from (a) single-celled to (b) single-celled below and doubled-celled above to (c) doubled-celled to (d) multiple vortices. Figure adapted from Davies-Jones (1986).

the bottom boundary is held fixed while the top and sides rotate. In the latter two experiments, the parameters were set in the range to produce a vortex like that in **Figure** *9b*.

# 2.3. The Lewellen Chamber

Lewellen et al. (1997) performed a large-eddy simulation (LES) using an open (as in **Figure 8***a*) box-shaped computational domain with external-parameter settings meant to emulate those that might be found in a supercell rotating updraft. These experiments are distinguished by the use of LES (Wyngaard 2010, chapter 6) to represent atmospheric turbulence. Lewellen et al. (2000) showed that the LES experiments can also produce a range of vortex types as a function of the equivalent swirl ratio that is analogous to that produced in the Ward-type experiments. However, they also identified parameters other than the swirl ratio and Reynolds number (e.g., surface roughness and inflow velocity profiles) that affect the experimental outcomes in the atmospheric analog to **Figure 9**. We return to these experiments in the following section.

#### **3. THEORETICAL ANALYSIS**

#### 3.1. Numerical Simulations in the Fiedler Chamber

The dynamics of the flows illustrated in **Figure** *9a–c* can be described by the axisymmetric, incompressible, constant-density Navier-Stokes equations in a rotating frame of reference in cylindrical coordinates. The nondimensional equations for the radial, azimuthal, and vertical accelerations and continuity are, respectively,

$$\frac{du}{dt} = -\frac{\partial\phi}{\partial r} + \frac{v^2}{r} + 2\Omega v + \frac{1}{Re_F} \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (ru) \right) + \frac{\partial^2 u}{\partial z^2} \right],\tag{1}$$

$$\frac{dv}{dt} = -\frac{uv}{r} - 2\Omega u + \frac{1}{Re_F} \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (rv) \right) + \frac{\partial^2 v}{\partial z^2} \right],\tag{2}$$

$$\frac{dw}{dt} = -\frac{\partial\phi}{\partial z} + b + \frac{1}{Re_F} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial w}{\partial r} \right) + \frac{\partial^2 w}{\partial z^2} \right],\tag{3}$$

$$\frac{1}{r}\frac{\partial ru}{\partial r} + \frac{\partial w}{\partial z} = 0,\tag{4}$$

where  $\phi$  is the nondimensional pressure variable, and the other variables have already been introduced in dimensional form; lengths are nondimensionalized by *h*, velocities by *W*, and time by *h/W*. For the simulations discussed presently,

$$b(r, z) = 0.5[1 + \cos(2\pi r_b)] \tag{5}$$

for  $r_b = [r^2 + (z - 0.5)^2]^{1/2} \le 0.5$ ; otherwise b(r, z) = 0. This definition gives  $\int_0^1 b(0, z)dz = 0.5$ and thus a nondimensional velocity scale of 1. The chamber boundaries at  $r = r_s/b \equiv R = 2$ and z = 0, 1 are impermeable and, unless otherwise mentioned, no-slip. The Reynolds number  $Re_F$  equals 10,000 but decreases in the upper half of the chamber with the viscous terms in Equations 1–3 appropriately modified, as discussed by Fiedler (1994, pp. 339–42).

Figure 10 shows the velocity and pressure fields for a sequence of simulations with increasing  $\Omega$ . The top row of **Figure 10** shows the steady-state solution for  $\Omega = 0.01$ . For this relatively small value, the upward central force from Equation 5 produces an in-up-and-out flow and a stagnation-point pressure maximum located at (r, z) = (0, 1), whereas the azimuthal flow, v(r, z), takes the form of a tall, slender vortex, with the maximum azimuthal velocity  $v_m$  located at upper levels. The middle row of **Figure 10** indicates that with  $\Omega = 0.025$ , the flow in the *r*-*z* plane changes such that w(r, z) is now characterized by a low-pressure, centerline jet at low levels and a central downdraft at higher levels, whereas the radial velocity u(r, z) now has strong inflow (u < 0) concentrated in the boundary layer; the height of  $v_m$  descends and is associated with the upward jet. Near  $z \simeq 0.45$ , there is an abrupt transition from a centerline upward jet to a central downdraft and a widening of the swirling flow. For  $\Omega = 0.08$ , the last row of Figure 10 indicates that the central downdraft extends to the surface, whereas  $v_m$  is located at the top of the boundary layer, as indicated by the vertical extent of the intense inflow jet. For this value of  $\Omega$  and larger, the flow is less steady as axisymmetric instabilities arise on the interface of the inner downdraft and outer updraft (e.g., Nolan & Farrell 1999). The correspondence of these solutions with the schematic in Figure 9*a*-*c* is apparent.

#### 3.2. The Two-Celled Vortex

**Figure 10** shows a progression from a one-celled vortex (centerline updraft) to a two-celled vortex (central downdraft surrounded by updraft) with increasing swirl ratio, and the coexistence of both at an intermediate swirl ratio. The low-pressure, swirling, centerline jet seen for the case  $\Omega = 0.025$  in **Figure 10** is obviously rooted in the boundary layer. However, the two-celled flow at upper levels exists with or without the boundary layer, as we now demonstrate. To remove the lower boundary layer and all attendant effects, it is sufficient to perform the simulations as described above, except now with free-slip (instead of no-slip) conditions at z = 0. **Figure 11** shows that with free-slip lower boundary conditions, two-celled vortices occur in the range of  $\Omega$  investigated in **Figure 10**, the vertical velocity maxima remain at higher levels, and the  $v_m$  occur at z = 0. The relative simplicity of these solutions without the effects of the frictional lower boundary makes them a convenient starting point for analysis.

Let us consider the equation for the circulation  $C = \oint \mathbf{u} \cdot d\mathbf{l}$  on the rectangular circuit ABCDA, where the corner points A, B, C, and D are located at (r, z) = (0, 0), (0, 1), (R, 1), and (R, 0), respectively. With Equations 1 and 3, one obtains

$$\frac{\partial C}{\partial t} = \int_0^1 b(0, z) dz - \int_0^R \left[ r^{-1} v^2(r, 0) + 2\Omega v(r, 0) \right] dr + \oint \mathbf{D} \cdot d\mathbf{l},\tag{6}$$

where **D** represents the boundary friction terms. The first term on the right-hand side (the buoyancy force per unit mass) creates positive (in-up-and-out) circulation, whereas the last term represents the opposing tendency of boundary friction. With a free-slip condition at z = 0, the middle



Numerical solutions displaying u, v, w, and  $\phi$  for increasing  $\Omega$  and  $Re_F = 10^4$  for the Fiedler chamber. The contours are given in increments of 0.10 starting at  $\pm 0.05$  (zero contour not plotted); red indicates positive values, whereas blue indicates negative ones. The minimum and maximum for each field are displayed on the upper left and right, respectively, of each contour plot. The display window is  $[(r, z)|0 \le r \le 0.5, 0 \le z \le 1]$ . Figure courtesy of B.H. Fiedler, using the model described in Fiedler (1994).

term (the centrifugal force per unit mass) is not zero and also produces an opposing tendency to the positive circulation created by the buoyancy force.

A simplification of Equation 6 allows the following estimate of an upper limit for  $v_m$ . Because  $v \sim O(1)$  and we are operating in a regime in which  $\Omega \ll 1$ , the second term in the radial integral in Equation 6 can be neglected. Setting the friction term in Equation 6 to zero to derive the upper



Numerical solutions displaying u, v, w, and  $\phi$  for increasing  $\Omega$  and  $Re_F = 10^4$  for the Fiedler chamber, as in **Figure 10**, except with free-slip conditions at the lower boundary.

limit gives

$$\int_{0}^{1} b(0, z) dz = \int_{0}^{R} r^{-1} v^{2}(r, 0) dr,$$
(7)

which represents a balance between the hydrostatic pressure drop and the cyclostrophic pressure drop. Using standard vortex profiles, one can approximate the right-hand side of Equation 7 by  $\beta^{-1}v_m^2$ , where, for example,  $\beta = 2.0$  or 1.0 for a hollow-core or Rankine vortex, respectively

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(Fiedler 1994, pp. 338–39). With these approximations and Equation 5, the steady-state upper limit derives from Equation 6 as

$$v_c \simeq (\beta/2)^{1/2},\tag{8}$$

which is sometimes referred to as the thermodynamic speed limit.

To estimate the radius,  $r_c$ , at which this velocity would be attained, we consider the inviscid, steady form of Equation 2, which implies the material conservation of angular momentum,  $\Gamma = rv + \Omega r^2$ . Applying the latter between a radius  $r = r_c (< R)$ , where the velocity becomes much less than  $\Omega r_c$ , and  $r = r_c$ , at z = 0, yields

$$r_c v_c \simeq \Omega r_e^2, \tag{9}$$

which when combined with Equation 8 gives the relation

$$r_c \simeq (2/\beta)^{1/2} \Omega r_e^2. \tag{10}$$

Analysis of  $\sqrt{r_e v_e/\Omega}$  from these free-slip numerical experiments (with  $0 < \Omega \le 0.2$ ) indicates a relatively constant value of  $r_e \simeq 1.5 \pm 0.1$ . Thus Equation 10 indicates an expanding vortex core with increasing swirl ratio, consistent with **Figure 11**. We note in passing that further experimentation indicates that  $r_e$  increases with the horizontal scale of the forcing function,  $l_r$ , consistent with Nolan (2005).

The significance of  $r_c$  as the defining feature of the two-celled vortex can be appreciated with the following considerations. The free-slip solutions shown in **Figure 11** suggest that a cylindrical vortex sheet divides the slowly rotating, downward-flowing inner cell from the rapidly rotating updraft of the outer cell. With a free-slip condition at z = 0, Equation 2 can be written as

$$u\zeta = \frac{1}{Re_F} \frac{\partial \zeta}{\partial r},\tag{11}$$

where  $\zeta = r^{-1}(\partial\Gamma/\partial r)$  is the vertical component of vorticity, and the vertical diffusion terms are negligible. From Equation 11, it is immediately clear that the radius of the vortex sheet (where  $\zeta$ takes its maximum) coincides with that of u = 0, the defining feature of a two-celled vortex. With convergent inflow  $u \propto -r$  at the surface, a simple scale analysis of Equation 11 indicates that the thickness of the vortex sheet  $\delta_r$  is proportional to  $Re_F^{-1/2}$ . Hence for large  $Re_F$ ,  $\delta_r$  is small, and the radii of  $v = v_c$  and u = 0 are close to each other.

In summary, the two-celled vortex is a result of the buoyancy-produced, in-up-and-out flow transporting angular-momentum-conserving parcels inward at low levels: The finite buoyancy force (Equation 5) puts an upper limit on the swirling velocity (Equation 8), and the conservation of angular momentum (Equation 9) puts a nonzero lower limit (Equation 10) on the radius to which these parcels can penetrate. Thus a two-celled vortex is produced.

#### 3.3. The End-Wall Vortex

The central feature of rotating-flow boundary layers is their interaction with the rotating flow away from the boundary (Rott & Lewellen 1966). Referring to Equation 1, the basic explanation of this interaction is that away from the frictional boundary, there is an outer rotating flow v(r) in balance with the pressure field, i.e.,  $\partial \phi / \partial r = v^2 / r$ . According to boundary-layer theory (Batchelor 1967, chapter 5.7), the vertical variation of  $\partial \phi / \partial r$  is small in the boundary layer; hence as v diminishes near the frictional boundary, the pressure gradient must be balanced by  $Re_F^{-1}\partial^2 u/\partial z^2$ , which in the present case implies inflow (u < 0) in the boundary layer. If the inflow is convergent, then flow emerging from the boundary layer occurs, carrying with it the properties of the boundary layer to the outer flow.



Radial profiles through the end-wall vortex from (*a*) the case  $\Omega = 0.025$  of **Figure 10a** at z = 0.17 and (*b*) the solution of Equation 19 neglecting the vertical derivative term. The dashed line is the test for supercriticality described in the text.

**3.3.1. Boundary layer of a potential vortex.** The vortical jet discussed in Section 3.1 is called an end-wall vortex as it originates from the end-wall boundary layer of an envisioned outer vortex  $v_{\infty}(r)$ . Figure 12*a* shows radial profiles of  $v, w, \phi$ , and  $\Gamma$  and total head,  $H = \phi + (u^2 + v^2 + w^2)/2$ , through the end-wall vortex (at z = 0.17, the level of  $v_m$ ) for the case of  $\Omega = 0.025$  shown in Figure 10. The profile of  $\Gamma(r)$  tends to move toward a near constant with increasing radius, indicating a small vertical component of vorticity  $\zeta$ . Observations such as these motivate the identification of the outer flow with the potential vortex,  $v_{\infty} = \Gamma_{\infty}/r$ , where, based on Equation 9,  $\Gamma_{\infty} \propto \Omega r_e^2$ .

The boundary layer of a potential vortex forming on a disc of finite radius was studied theoretically by Burggraf et al. (1971). According to these authors, the height of the boundary layer grows from the edge of the disc inward and reaches the terminal height  $r_d \sqrt{\nu/\Gamma_d}$ , where  $r_d$  is the dimensional disk radius and  $\Gamma_d$  represents the dimensional outer value. With  $\Gamma_d = W h \Gamma_\infty \propto W h \Omega r_e^2$ and  $r_d/h = r_e$ , Burggraf et al.'s (1971) estimate for the nondimensional height of the boundary layer of a potential vortex in the present context is

$$\delta \propto (Re_F \Omega)^{-1/2}.$$
 (12)

To assess the applicability of Burggraf et al.'s (1971) theory, **Figure 13***a* shows the profiles for  $u, v, \phi$ , and H at r = 0.125 taken from **Figure 10** ( $\Omega = 0.025$ ). **Figure 13***b* shows Burggraf et al.'s (1971) solution, which is characterized by v(z) monotonically increasing with height from v(0) = 0 to  $v(\infty) = \Gamma_{\infty}/r$  and u(z) monotonically increasing from  $u(0) = -\Gamma_{\infty}/r$  (the no-slip condition on u is satisfied in a vanishingly small surface layer) to  $u(\infty) = 0$  in a way that keeps H constant. Comparison of Burggraf et al.'s (1971) terminal similarity solution with the Fiedler numerical solution in **Figure 13** indicates that the theory gives a good description of the profile v(z) with  $\delta \simeq 0.06$ . The u(z) profile is less well described as the viscous sublayer is finite in the numerical solutions, and there is at higher levels inward flow that is associated with the upper-level forcing. The head H, however, is relatively constant except for the smaller values in the viscous sublayer. At this radius,  $\phi$  is nearly independent of z, consistent with the traditional boundary-layer approximation.

Using laser-Doppler anemometers, investigators have shown that Burggraf et al.'s (1971) solution gives an excellent description of the boundary layer of a laboratory potential vortex



Vertical profiles through the end-wall boundary layer from (*a*) the case  $\Omega = 0.025$  of **Figure 10***a* at r = 0.125 and (*b*) the Burggraf et al. (1971, table 7) terminal similarity solution (*thin lines*) and an analytical approximation thereof (*thick lines*).

(Phillips & Khoo 1987) and a good description of the end-wall boundary layer in a Ward-typechamber flow (Wilson & Rotunno 1986).

**3.3.2. The corner flow.** Taking the inviscid Burggraf et al. (1971) terminal boundary layer as a description of the flow entering the corner region (near the origin), Fiedler & Rotunno (1986) computed the end-wall vortex from the equation (Batchelor 1967, chapter 7.5)

$$r\eta = r^2 \frac{dH}{d\psi} - \frac{d}{d\psi} \frac{\Gamma^2}{2},\tag{13}$$

where the streamfunction  $\psi$ , by virtue of Equation 4, is given by

$$u = \frac{1}{r} \frac{\partial \psi}{\partial z}; \quad w = -\frac{1}{r} \frac{\partial \psi}{\partial r}.$$
 (14)

For the purposes of the present exposition, we repeat Fiedler & Rotunno's (1986) calculation using a simple analytical approximation to Burggraf et al.'s (1971) terminal boundary-layer solution.

At the radius  $r_t$  of the inflowing terminal boundary, the boundary-layer approximations ( $\phi$  constant with height and  $w^2 \ll u^2 + v^2$ ) allow Equation 13 to be expressed simply as

$$r\eta = r_t \frac{\partial u_t}{\partial z},\tag{15}$$

where, here and in the following, inflow variables are denoted by the subscript *t*. We consider here the expression for  $u_t(z)$  given by

$$u_t(z) = -\frac{\Gamma_{\infty}}{r_t} \frac{1}{1 + (z/\delta)^2},$$
(16)

and, to have H constant with height, we let

$$v_t(z) = \frac{\Gamma_\infty}{r_t} \sqrt{1 - \left(\frac{r_t u_t}{\Gamma_\infty}\right)^2}.$$
(17)

These profiles are shown in **Figure 13***b* and are evidently a fairly good representation of Burggraf et al.'s (1971) terminal solution. With Equations 14 and 16, and letting  $\psi(0) = 0$ , we find

$$\frac{z}{\delta} = \tan\left(-\frac{\psi}{\delta\Gamma_{\infty}}\right). \tag{18}$$

Substituting Equation 16 into Equation 15, and then eliminating  $z/\delta$  using Equation 18, we have

$$\frac{\partial^2 \tilde{\psi}}{\partial \tilde{r}^2} - \frac{1}{\tilde{r}} \frac{\partial \tilde{\psi}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{\psi}}{\partial \tilde{z}^2} = -\frac{2 \tan \tilde{\psi}}{(1 + \tan^2 \tilde{\psi})^2},\tag{19}$$

where the definition of  $\eta$ , Equation 14, and the substitutions  $\tilde{\psi} = -\psi/(\delta \Gamma_{\infty})$ ,  $\tilde{r} = r/\delta$ , and  $\tilde{z} = z/\delta$  have been used.

Equation 19 represents the flow in the corner region with the assumed inflow conditions in Equations 16 and 17. Here we suppose that there is a height at which the flow is approximately independent of  $\tilde{z}$  and look for solutions to Equation 19 for the outflow profiles  $[0, \tilde{v}(\tilde{r}), \tilde{w}(\tilde{r})] = \delta \Gamma_{\infty}^{-1}[0, v(r), w(r)]$ . Neglecting  $\partial^2 \tilde{\psi} / \partial \tilde{z}^2$ , Equation 19 becomes an ordinary differential equation for  $\tilde{\psi}(\tilde{r})$  with boundary conditions  $\tilde{\psi}(0) = 0$  and  $\tilde{\psi}(\tilde{r}_o) = \pi/2$ , where  $\tilde{r}_o$  is the radius of a hypothetical container. **Figure 12b** shows the numerical solution for the velocity and pressure profiles on a finite domain of radius  $\tilde{r}_o = 6$  and is a very good match to the solutions found in Fiedler & Rotunno (1986, figure 7*a*) using the actual Burggraf et al. (1971) terminal boundary layer to define the right-hand side of Equation 15. The qualitative comparison between the theoretical (constant-head) profiles shown in **Figure 12***b* and those of the full numerical model in **Figure 12***a* is good. To make a quantitative comparison, we estimate from **Figure 12***a* that  $\Gamma_{\infty} \simeq 0.05$ . The *v* profile in **Figure 13***a* indicates  $\delta \simeq 0.06$ ; however, excluding the frictional sublayer (seen in the *u* profile), we estimate that  $\delta \simeq 0.04$ . Hence we arrive at  $(v_m, w_m) \simeq \delta^{-1} \Gamma_{\infty}(\tilde{v}_m, \tilde{w}_m) \simeq 1.25 \times (0.5, 1.0) = (0.625, 1.25)$  and  $r_m = \tilde{r}_m \delta \simeq 1.1\delta \simeq 0.04$ , which are close to the simulated values shown in **Figure 12***a*.

To summarize, one can think of the end-wall vortex as the result of the inviscid Burggraf et al. (1971) terminal boundary layer turning the corner (in accordance with Equation 13) near the origin and becoming an upward-flowing swirling jet of radius  $r_m \simeq \delta$  and maximum upward velocity  $w_m \simeq \Gamma_{\infty}/\delta$  and maximum azimuthal velocity  $v_m \simeq 0.5\Gamma_{\infty}/\delta$ . For numerical solutions of Equation 13 for  $\psi(r, z)$  in the corner region with comparison to laboratory data, the reader is referred to Wilson & Rotunno (1986). Further details on the vorticity dynamics of the solutions to Equation 13 for the corner flow may be found in Rotunno (1980).

#### 3.4. End-Wall-Vortex Breakdown

As shown in **Figure 10** ( $\Omega = 0.025$ ), the end-wall vortex terminates abruptly near  $z \simeq 0.45$ ; the latter transition is generally identified with the phenomenon of vortex breakdown (Benjamin 1962, Leibovich 1978). Benjamin's (1962) theory is that a vortex with axial velocity can be "supercritical" in the sense that axially propagating disturbances cannot propagate upstream (downward in the present case) against the axial flow; therefore, a transition to conditions downstream (upward in the present context) must be abrupt. Exploiting an analogy to hydraulic jumps in open-channel flow, Benjamin (1962) developed the idea that the supercritical vortex undergoes a transition to a subcritical "conjugate" state with same volume flow rate  $= 2\pi \int_0^{\tilde{r}_0} \tilde{w}\tilde{r}d\tilde{r}$  and total axial momentum flux  $2\pi \int_0^{\tilde{r}_0} (\tilde{w}^2 + \tilde{\phi})\tilde{r}d\tilde{r}$ .

The Benjamin supercriticality test is based on the steady, inviscid versions of Equations 1–4, linearized about a cylindrical vortex  $[0, \tilde{v}(\tilde{r}), \tilde{w}(\tilde{r})]$  with vertical derivatives set to zero,

$$\tilde{r}\frac{d}{d\tilde{r}}\left(\frac{1}{\tilde{r}}\frac{d\tilde{\psi}^{t}}{d\tilde{r}}\right) + \left(\frac{1}{\tilde{w}^{2}\tilde{r}^{3}}\frac{d\tilde{\Gamma}^{2}}{d\tilde{r}} + \frac{1}{\tilde{r}\tilde{w}}\frac{d\tilde{w}}{d\tilde{r}} - \frac{1}{\tilde{w}}\frac{d^{2}\tilde{w}}{d\tilde{r}^{2}}\right)\tilde{\psi}^{t} = 0,$$
(20)

where  $\tilde{\psi}^t$  is a test function that satisfies one of the boundary conditions on  $\tilde{\psi}$ . In the present case, we set  $\tilde{\psi}^t(0) = 0$  and integrate Equation 20 outward; if there is at least one zero in the interval  $[0, \tilde{r}_0]$ , a standing wave solution is possible, and therefore the flow is subcritical; otherwise, it is



Radial profiles through the end-wall vortex from the solution of Equation 19 neglecting the vertical derivative term, as in **Figure 12**b except for the subcritical conjugate state.

supercritical. Applying this procedure to the end-wall-vortex profile shown in **Figure 12***b* shows that it is supercritical. Hence the transition to downstream conditions is expected to be abrupt, as found in the numerical simulation [**Figure 10** ( $\Omega = 0.25$ )].

To calculate the downstream subcritical conjugate state, Fiedler & Rotunno (1986) outlined several methods, the simplest one being to assume a constant vertical velocity  $\tilde{w}^*$  between  $\tilde{r} = 0$  and  $\tilde{r} = \tilde{r}^*$ , solve Equation 19 between  $\tilde{r}^*$  and  $\tilde{r}_o$ , and then adjust  $(\tilde{w}^*, \tilde{r}^*)$  until  $\tilde{\psi}(\tilde{r}_o) = \pi/2$  and the total axial momentum flux is the same as that in the upstream supercritical vortex. The result of this calculation is shown in **Figure 14** from which it may be inferred that  $\tilde{r}_m \simeq 3$  and  $\tilde{v}_m \simeq 0.3$  in the subcritical conjugate state, and Benjamin's (1962) test confirms the subcritical nature of this vortex.

In summary, the end-wall vortex, produced by Burggraf et al.'s (1971) terminal boundary layer of a potential vortex, is supercritical and adjusts to downstream conditions through a vortex breakdown. Calculations using the nonlinear inviscid Equation 13 indicate

$$r_m^{super} \simeq \delta, \quad v_m^{super} \simeq 0.5\Gamma_\infty/\delta,$$
(21)

for the supercritical end-wall vortex, whereas calculations using Equation 19, but allowing head loss, give

$$r_m^{sub} \simeq 3\delta, \quad v_m^{sub} \simeq 0.3\Gamma_\infty/\delta,$$
 (22)

for the downstream subcritical conjugate state.

#### 3.5. An Optimal Vortex

The numerical experiments shown in **Figures 10** and **11** suggest that the vortex far downstream of the end-wall vortex is the two-celled vortex that occurs in the absence of boundarylayer effects. Under this hypothesis, the condition for the coexistence of the supercritical endwall vortex, its subcritical conjugate, and the two-celled vortex occurs when the latter two are compatible. A comparison of the estimates of the maximum winds from the two-celled vortex (Equation 8) and subcritical vortex (Equation 22) (i.e.,  $v_c \simeq v_m^{sub}$ ) gives  $(\beta/2)^{1/2} \simeq 0.3\Gamma_{\infty}/\delta$ , which, with Equation 12 and  $\Gamma_{\infty} \propto \Omega r_e^2$ , gives the optimal condition

$$\Omega^{\dagger} = \alpha (Re_F r_e^4)^{-1/3}, \qquad (23)$$

where  $\alpha$  is an order-unity proportionality constant. Taking  $\alpha = 1$  with  $r_e = 1.5$  and  $Re_F = 10,000$ , one obtains  $\Omega^{\dagger} = 0.027$ , which is in the range of the transition among solution types shown in **Figure 10**. The variation of the critical swirl ratio with the inverse Reynolds number was emphasized by Nolan & Farrell (1999).

It is convenient at this juncture to bring in other views on the nondimensional parameters that characterize the numerical solutions based on the Fiedler chamber. As noted in the discussion following Equation 10, Nolan (2005) argued that the most relevant length scale is the horizontal length scale associated with the forcing function b(r, z) as it determines the distance over which angular-momentum-conserving parcels travel to the vortex core. Pursuing that idea, we let  $r_e \propto l_r/b$  in Equation 23 to obtain

$$Re_V S_r^2 = \alpha', \tag{24}$$

where, following Nolan (2005), the vortex Reynolds number  $Re_V = \omega l_r^2 / v$ , and the swirl ratio  $S_r = \omega l_r / W$  ( $\alpha'$  is a different coefficient of proportionality); note that the dimensional length scale *h* drops out of the equation. For a limited range of variation of  $S_r$ , the optimal condition would depend mainly on  $Re_V$  (Nolan 2005). Another possibility is to define a Reynolds number based on  $l_r$  ( $Re_r = W l_r / v$ ), in which case Equation 24 can be expressed as  $S_r^3 Re_r = \alpha'$ , which is analogous in form to Equation 23. Finally, one could define a single parameter  $V = \omega^3 l_r^4 / v W^2$  and express Equation 24 as  $V = \alpha'$ .

In the optimal condition (Equation 23), the maximum velocity in the boundary layer  $v_m^{super} \simeq (5/3)v_m^{sub} \simeq (5/3)v_c$  and therefore significantly exceeds the velocity  $v_c$  that results from the conservation of angular momentum in the simple in-up-and-out circulation driven by a fixed forcing.

For  $\Omega > \Omega^{\dagger}$ , the downstream two-celled vortex is too large to be matched by the subcritical vortex that emerges from the breakdown of the supercritical end-wall vortex. [Maxworthy (1972) refers to this as a "drowned" vortex jump, again in analogy with hydraulic flow.] A comparison of the pressure distributions between the no-slip and free-slip simulations at  $\Omega = 0.08$  (Figures 10 and 11) indicates the dominance of the free-slip two-celled vortex. However, the velocity field in the no-slip simulation continues to exhibit strong flow in all components in the boundary layer. In fact, the boundary-layer effect can produce  $v_m^{no-slip} > v_m^{free-slip}$  (see Figures 10 and 11 for  $\Omega = 0.08$ ) as has been pointed out (Howells et al. 1988, Lewellen & Teske 1977, Rotunno 1979). For yet larger  $\Omega$ , the two-celled vortex becomes unstable to nonaxisymmetric perturbations, which we discuss below.

For  $\Omega < \Omega^{\dagger}$ , the single-celled vortex evident in **Figure 10** ( $\Omega = 0.01$ ) can still be considered an effect of the lower boundary layer because the free-slip solutions in **Figure 11** indicate a two-celled vortex for the same  $\Omega$ . However, for decreasing  $\Omega$ , Equation 12 indicates an increasing boundary-layer thickness and, together with Equation 9, a smaller end-wall-jet velocity,  $\Gamma_{\infty}/\delta \propto r_e^2 Re_F^{1/2}\Omega^{3/2}$ .

**Figure 15***a* summarizes the theory reviewed up to this point. Matching the two-celled vortex to the subcritical vortex downstream of the breakdown of the end-wall vortex gives a condition for the latter to exist. This end-wall vortex is optimal in the sense that it gives a swirling-flow maximum in the boundary layer that is about twice (~5/3) that of the swirling-flow maximum in the flow aloft, which is determined by the buoyancy of the updraft. **Figure 15***b* shows an example of this optimal vortex from the Purdue Tornado Vortex Simulator (Pauley & Snow 1988). The optimal vortex should not be confused with the most-intense vortex in an absolute sense as a vortex with  $\Omega > \Omega^{\dagger}$  may have a greater  $v_m$  in the boundary layer yet a smaller amplification with respect to the  $v_m$  aloft.

#### 3.6. Generalizations

The objective of both practical and theoretical interest is to predict the occurrence of intense vortices such as those in **Figure 10** in more realistic environments. The question of how to translate the more-or-less definitive ideas from the steady laminar axisymmetric vortex-chamber simulations to the unsteady and turbulent real atmosphere remains a topic of active research. A typical supercell



(*a*) Schematic diagram of an optimal vortex in which the subcritical, two-celled vortex can remain suspended above the supercritical end-wall vortex produced by the end-wall boundary layer in the corner region and (*b*) a laboratory example thereof. Panel *b* taken from Pauley & Snow (1988).

updraft may have  $W = 50 \text{ m s}^{-1}$ ; with a tropopause depth of  $b = 10^4 \text{ m}$  and the kinematic viscosity of air  $\simeq 2.0 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ , the atmospheric Reynolds number  $Re_F$  equals  $2.5 \times 10^{10}$ . If the atmosphere remained laminar with steady input conditions in the parent circulation, then the only change to note is the smaller value of the critical value of  $\Omega^{\dagger}$  (Equation 23). However, some tornadoes (e.g., **Figure 1**) appear to be turbulent, and the turbulence in such tornadoes presumably impacts the mean dynamical structure. Conversely, some tornadoes appear to have a very laminar structure (Fiedler 2009, figure 1).

One approach toward taking turbulence into account is to represent the effects of turbulence in an axisymmetric model (e.g., Lewellen & Sheng 1981). Recently Fiedler & Garfield (2010) used a commercially available code with a sophisticated suite of parameterizations to perform simulations like those in **Figure 10** except with  $Re_F$  as large as  $10^8$ , which, although less than atmospheric values, is outside the reach of common laboratory experiments, which operate in the range  $Re_F \simeq 10^3-10^5$  (Church et al. 1979). With one of the models, Fiedler & Garfield (2010, figure 7) found that the simulated vortex intensity became independent of  $Re_F$  for large values; however, the intensity varied strongly with the chosen parameterization.

Another approach is to use LES (Wyngaard 2010), which, as the name suggests, solves a lowpass-filtered version of the Navier-Stokes equations to simulate the largest structures and uses a parameterization for the effects of subfilter turbulence based on the theory of small-scale turbulence. Using an open domain similar to a box-shaped version of that shown in **Figure 8a**, Lewellen et al. (1997) performed an LES study of a tornado-like flow with external parameters set to yield a vortex like that in **Figure 9c**,*d*. They found that with a grid resolution of several meters, the resulting solutions were not very sensitive to changes in grid resolution or to the parameterization of subgrid-scale turbulence. Multiple vortices (see below) were found to be the primary turbulent structures acting on the axisymmetric mean flow. Subsequent work by Lewellen et al. (2000) using a range of external settings (giving different swirl ratios) showed that the range of structures in **Figure 9** could be reproduced by the LES model; an example is shown in Annu. Rev. Fluid Mech. 2013.45:59-84. Downloaded from www.annualreviews.org Access provided by National Center for Atmospheric Research -NCAR on 06/01/15. For personal use only.



#### Figure 16

A large-eddy simulation of a tornado-like vortex. The lengths are scaled by a core-size estimate  $r_c (\approx 200 \text{ m})$  and an upper-vortex maximum-velocity estimate  $V_c (\approx 50 \text{ m s}^{-1})$ . The maximum length for the velocity vectors in the r - z plane is 3.8 $V_c$ . Figure adapted from Lewellen et al. (2000).

Figure 16, wherein an instantaneous view of the velocity field indicates a strong qualitative similarity to the optimal  $\Omega = 0.025$  solution of Figure 10.

Lewellen et al. (1997) noted that achieving the optimal vortex can depend on factors beyond the swirl ratio, citing surface roughness and boundary-layer profiles at the inflow boundaries as prime examples of the kinds of variations that could have a strong effect on the intensity of the simulated tornado. Using LES, Lewellen et al. (1997) and Lewellen & Lewellen (2007) performed a great number of numerical experiments, varying inflow and outflow conditions, in some cases prescribing their variation with time. In an attempt to identify the physical variables in the near-corner-flow environment that determine the corner flow, Lewellen et al. (2000) defined the "corner-flow swirl ratio"

$$S_c = r_c \, \Gamma_\infty^2 / \, \Upsilon, \tag{25}$$

where  $\Upsilon = -2\pi \int_0^{z_1} \langle u(r_1, z)[\Gamma_{\infty}(r_1, z_1) - \Gamma(r_1, z)] \rangle r_1 dz$  is the volume inflow rate of boundary-layer air with an angular momentum deficit; the station  $r = r_1$  is just upstream of the corner region, and  $z_1$  is the boundary-layer height. In a generalization of the analysis of vortex breakdown given in Section 3.4, Lewellen & Lewellen (2007) demonstrated that the optimal (in the sense defined above) vortices should occur near their theoretically derived critical value  $S_c^*$ , which they found is in the range 0.70–1.78.

The parameter  $S_c$  focuses attention on the relation between the inflow boundary just upstream of the corner region (the denominator) and the subcritical vortex above the vortex breakdown (the numerator). One can think of Equation 25 as the ratio of the subcritical vortex core size  $r_c$  to a measure of the inflow boundary-layer thickness  $\Upsilon/\Gamma_{\infty}^2$  (Fiedler 2009). As illustrated by Equations 21 and 22, the inflow boundary-layer thickness determines the radius of the subcritical vortex that can remain suspended above the supercritical end-wall vortex. For the profiles in Equations 16 and 17, one can compute  $\Upsilon/\Gamma_{\infty}^2 = \pi [-\sqrt{2} + \pi - \cosh^{-1}(\sqrt{2})]\delta \approx 2.6578\delta$ , which is close to that calculated numerically by Fiedler (2009) ( $\Upsilon/\Gamma_{\infty}^2 \approx 2.63\delta$ ) for Burggraf et al.'s (1971) terminal boundary-layer solutions shown in **Figure 13b**. With  $r_c \simeq 3\delta$  (Equation 22), one obtains  $S_c \approx 1.13$ .

#### 3.7. Multiple Vortices

Returning to the schematic diagram in **Figure 9**, **Figure 9***d* illustrates that for a larger swirl ratio, the two-celled vortex is no longer axisymmetric and breaks into satellite vortices revolving around a common center. To isolate the physical features of the instability, Rotunno (1984) carried out axisymmetric and three-dimensional numerical simulations of the Ward-chamber flow for swirl ratio S = 1 and low Reynolds number (Re = 150), under free-slip lower boundary conditions. The axisymmetric solutions were similar to the two-celled vortices discussed in relation to **Figure 11**, and the three-dimensional counterparts exhibited a nonaxisymmetric instability leading to two satellite vortices (Rotunno 1984) (**Figure 17***a*). The vortices were found to retrograde with respect to the maximum axisymmetric mean of the azimuthal velocity and, moreover, spiral clockwise with height, whereas the streamlines of the azimuthal-mean vortex spiral counterclockwise with height. Multiple vortices were later simulated at much higher Re and frictional lower boundary conditions (Fiedler 1998, 2009; **Figure 17***b*) and in the LES context (Lewellen et al. 1997) (**Figure 17***c*). Although exhibiting much greater time variability and complex spatial structure in the later more realistic experiments, the basic features are qualitatively similar to those found in the low–Reynolds number free-slip experiments.

Although  $\zeta > 0$  in the two-celled vortex, the azimuthal component of vorticity  $\eta$  equals  $\partial u/\partial z - \partial w/\partial r < 0$  because |u| is small and w increases outward; hence the vortex lines in the vortex sheet that separates the updraft from downdraft in the two-celled vortex also spiral clockwise with height. In a stability analysis of the two-celled vortex, Rotunno (1978) found that radial shear of the vertical velocity was needed to destabilize disturbances with azimuthal wave numbers m = 1, 2 and suggested that the unstable motions for these wave numbers that spiral clockwise with height were the preferred orientation for the finite-amplitude multiple vortices observed in the Ward chamber. This basic picture was reinforced by more detailed stability calculations for the Ward-chamber flow by Walko & Gall (1984). However, recent calculations reported by Nolan (2012) on the stability of the time- and azimuthal-mean flow produced in the three-dimensional simulations from Fiedler (2009) indicate that the rapid change in vertical velocity with height through the vortex breakdown (**Figure 15**) is a major source of perturbation energy for the most unstable mode. This mode is illustrated in **Figure 17d**.

Rotunno



Three-dimensional perspectives of pressure surfaces showing multiple vortices from (*a*) direct numerical simulation of a low–Reynolds number case with a free-slip lower boundary condition (Rotunno 1984), (*b*) a direct numerical simulation of a high–Reynolds number case with a no-slip lower boundary condition (Fiedler 2009), (*c*) a large-eddy simulation (Lewellen et al. 1997), and (*d*) the most unstable mode calculated by Nolan (2012) of the time and azimuthal-mean flow from the three-dimensional simulations of Fiedler (2009).

Three-dimensional simulations with frictional boundary conditions show that the satellite multiple vortices can be microcosms of the intense end-wall jet axisymmetric vortex and have very high velocities locally in time and space (Fiedler 2009).

#### 4. SUMMARY AND OUTLOOK

Most of what is known of the fluid dynamics of tornadoes derives from laboratory and numerical experiments that produce vortices that share similarities with what can be observed in a tornado. Tornado vortices have been observed with a range of structures (Figures 1-4) that compare favorably with that found in laboratory and numerical experiments (Figure 9). In particular, observations (Section 1) and simulations (Section 3) indicate that all three components of the velocity field are most intense at the lowest levels, suggesting an important role for the boundary layer of the thunderstorm-associated rotating flow above (the mesocyclone). As detailed in this article, the fluid dynamical interaction between a laboratory mesocyclone and the lower frictional boundary that produces intense tornado-like vortices is well understood for laminar and, to some extent, turbulent flows. However, the general inability to distinguish which supercell storms will produce a tornado indicates that important pieces of information are still missing. It is the present author's opinion that further progress awaits a deeper understanding of the connection between laboratory flows, for which boundary-layer effects are critically important and effects of cool boundaries are absent, and studies of natural mesocyclones, for which boundary-layer effects are typically given secondary consideration (and are very difficult to observe) to the effects of cool-air boundaries.

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# **DISCLOSURE STATEMENT**

The author is not aware of any biases that might be perceived as affecting the objectivity of this review.

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# Errata

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