# Dynamical Mesoscale Mountain Meteorology

*Richard Rotunno*

*National Center for Atmospheric Research , USA*

- Dynamic

  Of or pertaining to force producing motion

- Meso-(intermediate)scale

  Length ~ 1-100km Time ~1h – 1day

- Mountain Meteorology

  Science of atmospheric phenomena caused by mountains

# Topics

Lecture 1 : Introduction, Concepts, Equations

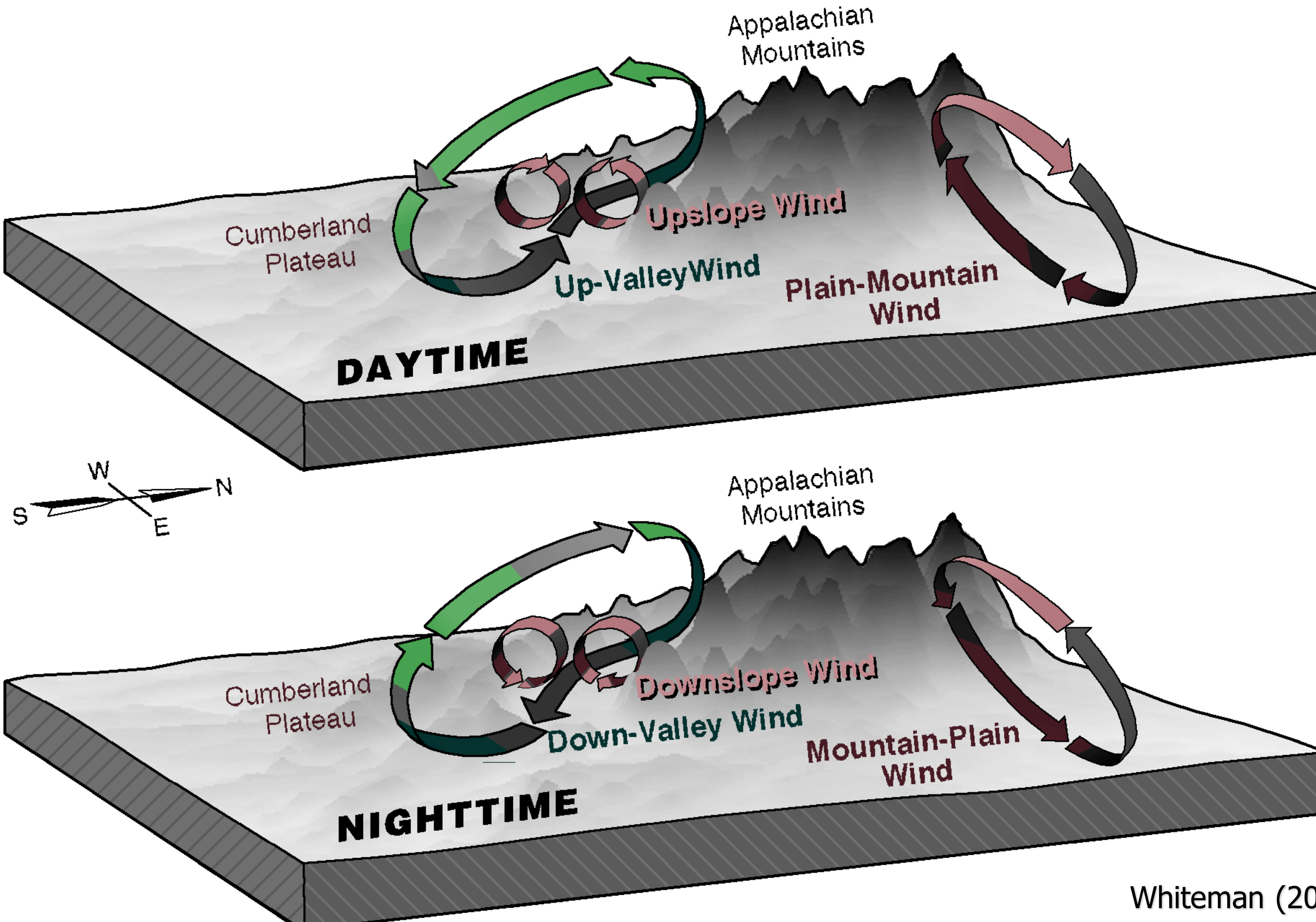
Lecture 2: Thermally Driven Circulations

Lecture 3:  Mountain Waves

Lecture 4:  Mountain Lee Vortices

Lecture 5:  Orographic Precipitation

# Thermally Driven Circulations




Whiteman (2000)

# Mountain Waves

Mt. Shasta

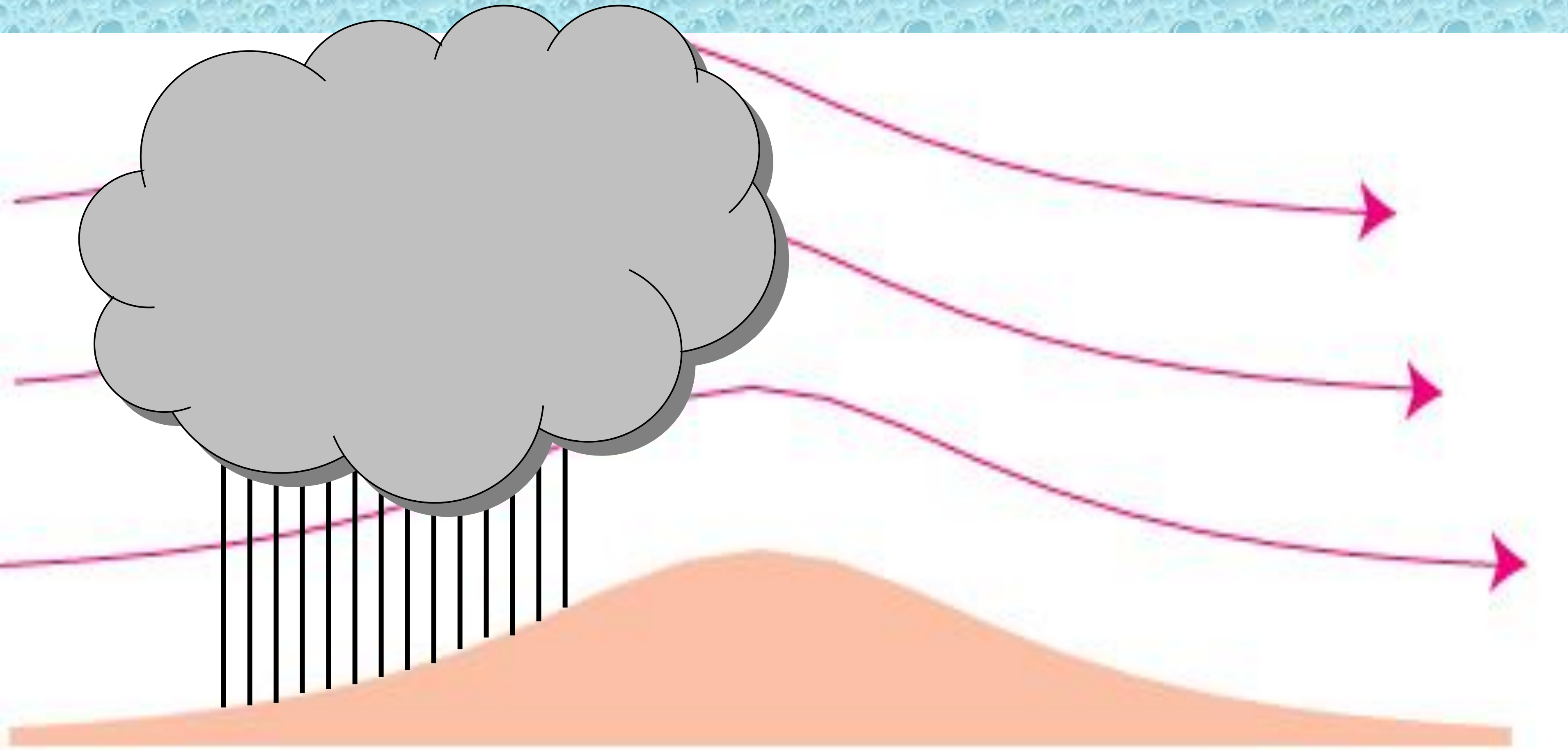
Jane English

# Mountain Lee Vortices


Hawaii

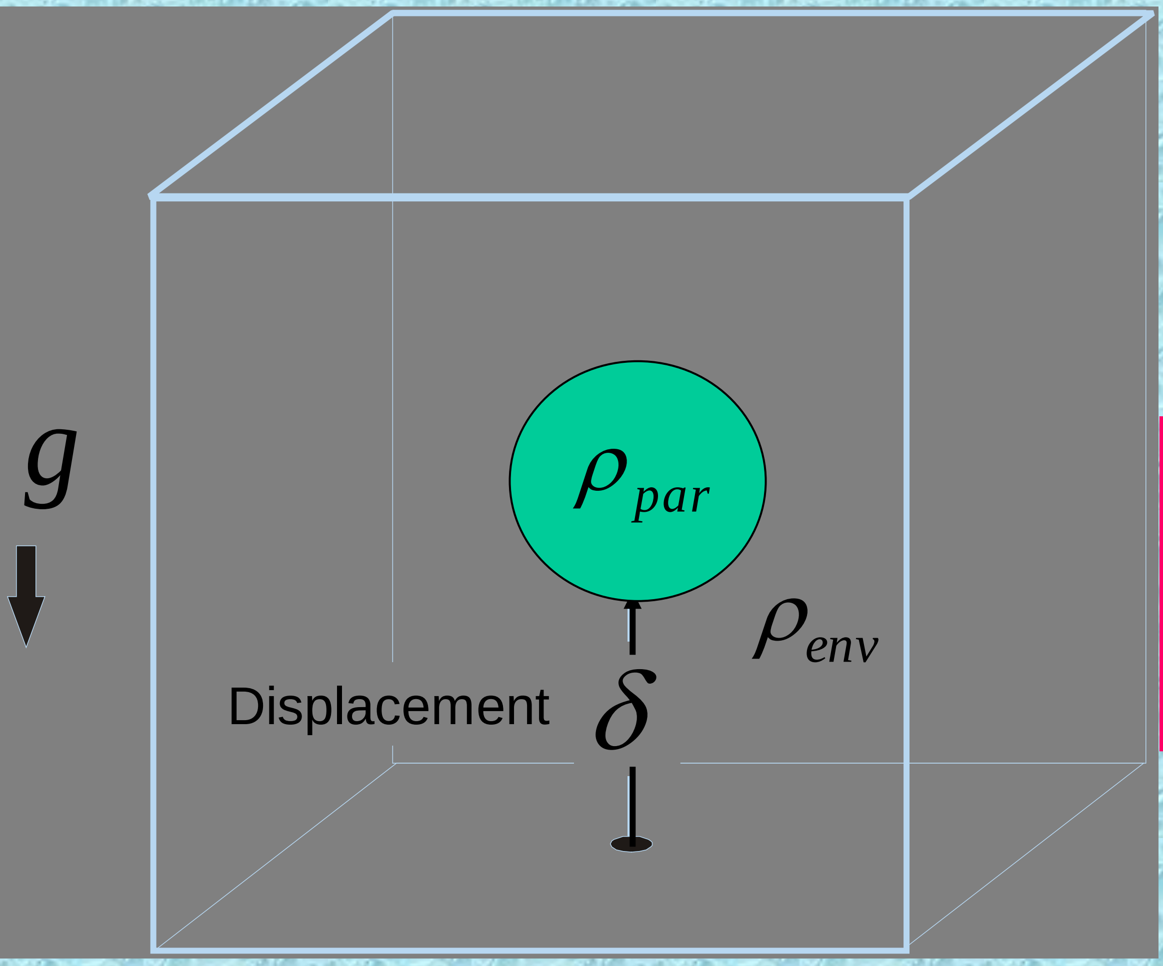
Space Shuttle

# Orographic Precipitation



# What do all these phenomena have in common?

## Buoyancy





$$B = g\,\frac{\rho_{env} - \rho_{par}}{\rho_{par}}$$

$\rho$ = density
"env" = environment
"par" = parcel

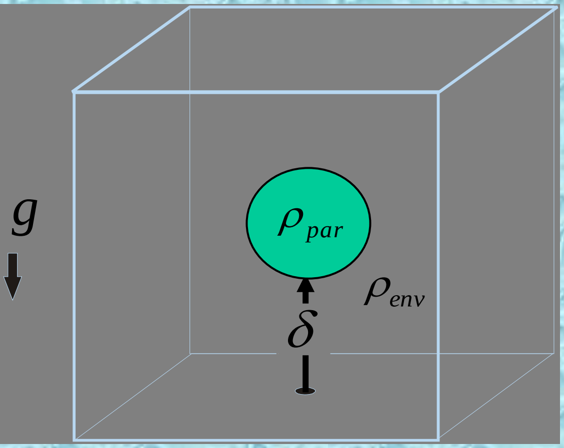
# Buoyancy is acceleration

mass $\times$ acceleration
$=$ sum of forces

$$\rho_{par}\ddot{\delta} = -\frac{\partial p_{par}}{\partial z} - \rho_{par} g$$

To a good approximation...

$$\frac{\partial p_{par}}{\partial z} \approx \frac{\partial p_{env}}{\partial z} = -\rho_{env} g$$




$$\ddot{\delta} = g\,\frac{\rho_{env} - \rho_{par}}{\rho_{par}} \equiv B$$

$p$ = pressure
$z$ = vertical coordinate

# Stability

$$\delta \times B < 0 \text{ (stable) }, \; \delta \times B > 0 \text{ (unstable)}$$

$$B = g \frac{\rho_{env} - \rho_{par}}{\rho_{par}} \equiv -N^2 \delta$$

$$N^2 > 0 \text{ (stable) }, \; N^2 < 0 \text{ (unstable)}$$

$$\rho_{env} \cong \rho_{env}(0) + \left.\frac{\partial \rho}{\partial z}\right|_{env} \delta \; ; \; \rho_{par} \cong \rho_{env}(0) + \left.\frac{\partial \rho}{\partial z}\right|_{par} \delta$$

$$N^2 = -\frac{g}{\rho}\left( \left.\frac{\partial \rho}{\partial z}\right|_{env} - \left.\frac{\partial \rho}{\partial z}\right|_{par} \right)$$

= 0 for incompressible density-stratifed fluid

$N^2$ = "static stability"

$N$ = "Brunt-Väisälä Frequency"

# $B, N^2$ in terms of Temperature

Air is a compressible fluid…

$$\left.\frac{\partial \rho}{\partial z}\right|_{par} \neq 0$$

Gas Law →

$$\rho = \frac{p}{RT}$$

$$B = g\,\frac{T_{par} - T_{env}}{T_{env}}$$

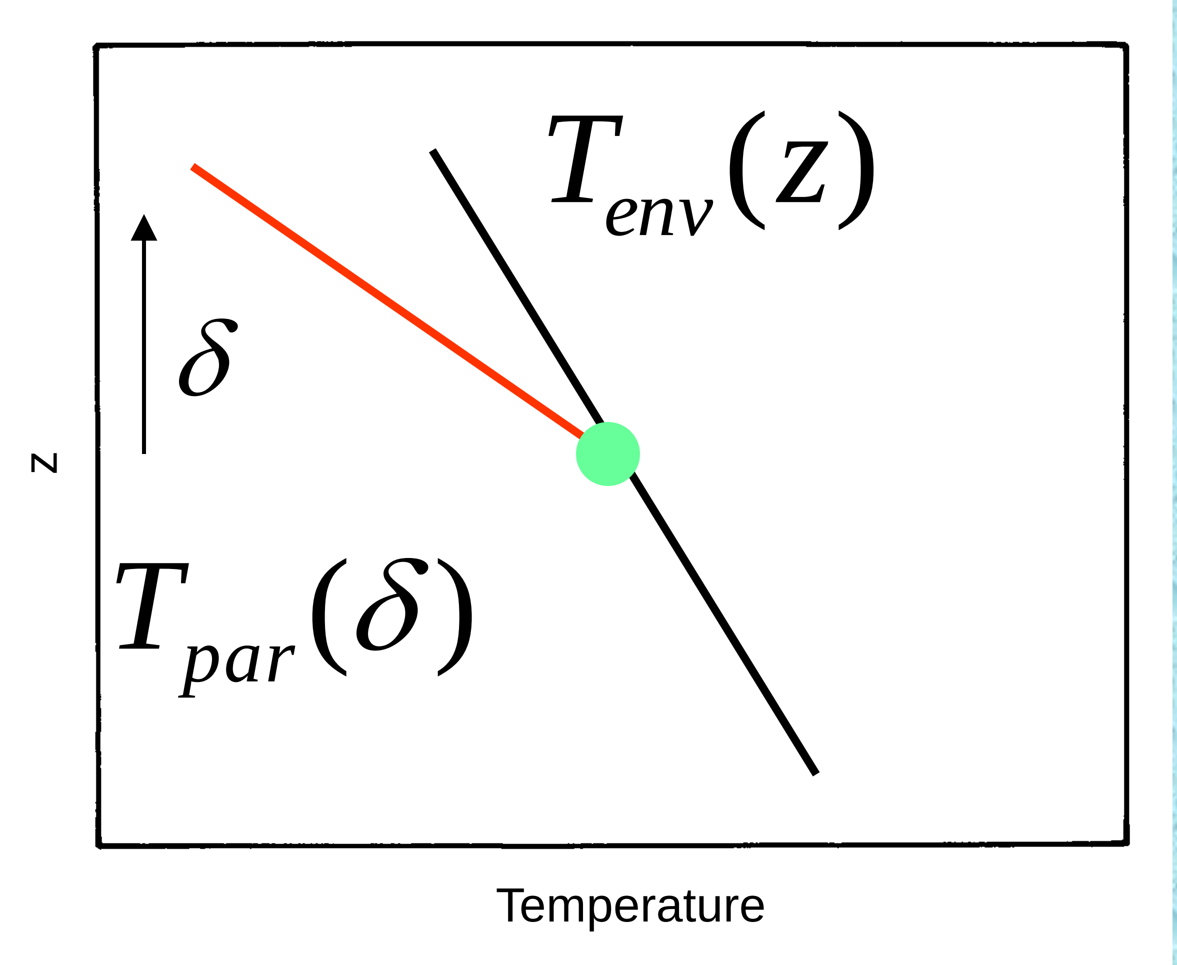
$$N^2 = \frac{g}{T}\left(\left.\frac{\partial T}{\partial z}\right|_{env} - \left.\frac{\partial T}{\partial z}\right|_{par}\right)$$

1st Law of Thermo (adiabatic)→

$$c_p \left.\frac{\partial T}{\partial z}\right|_{par} = \frac{1}{\rho_{par}}\frac{\partial p_{par}}{\partial z} \approx -g \Rightarrow \left.\frac{\partial T}{\partial z}\right|_{par} \cong -9.8^{\circ} C\, km^{-1}$$

$c_p$ = specific heat at constant pressure , $R$ = gas constant for dry air

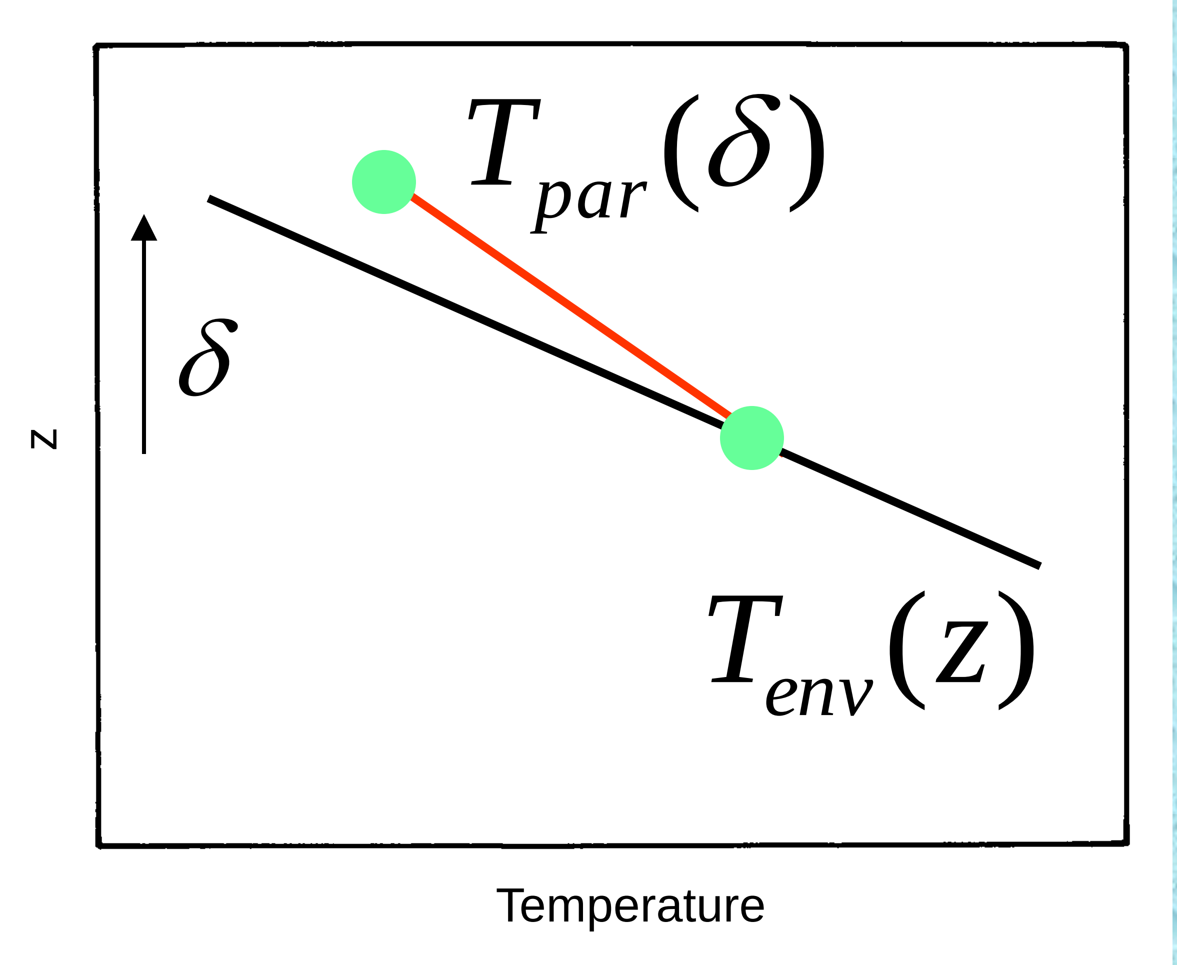
# Air Parcel Behavior in a Stable Atmosphere




$$N^2 > 0$$

$$\delta > 0 \Rightarrow \ddot{\delta} = B = -N^2\delta < 0$$

# Air Parcel Behavior in an Unstable Atmosphere




$$N^2 < 0$$

$$\delta > 0 \Rightarrow \ddot{\delta} = B = -N^2\delta > 0$$

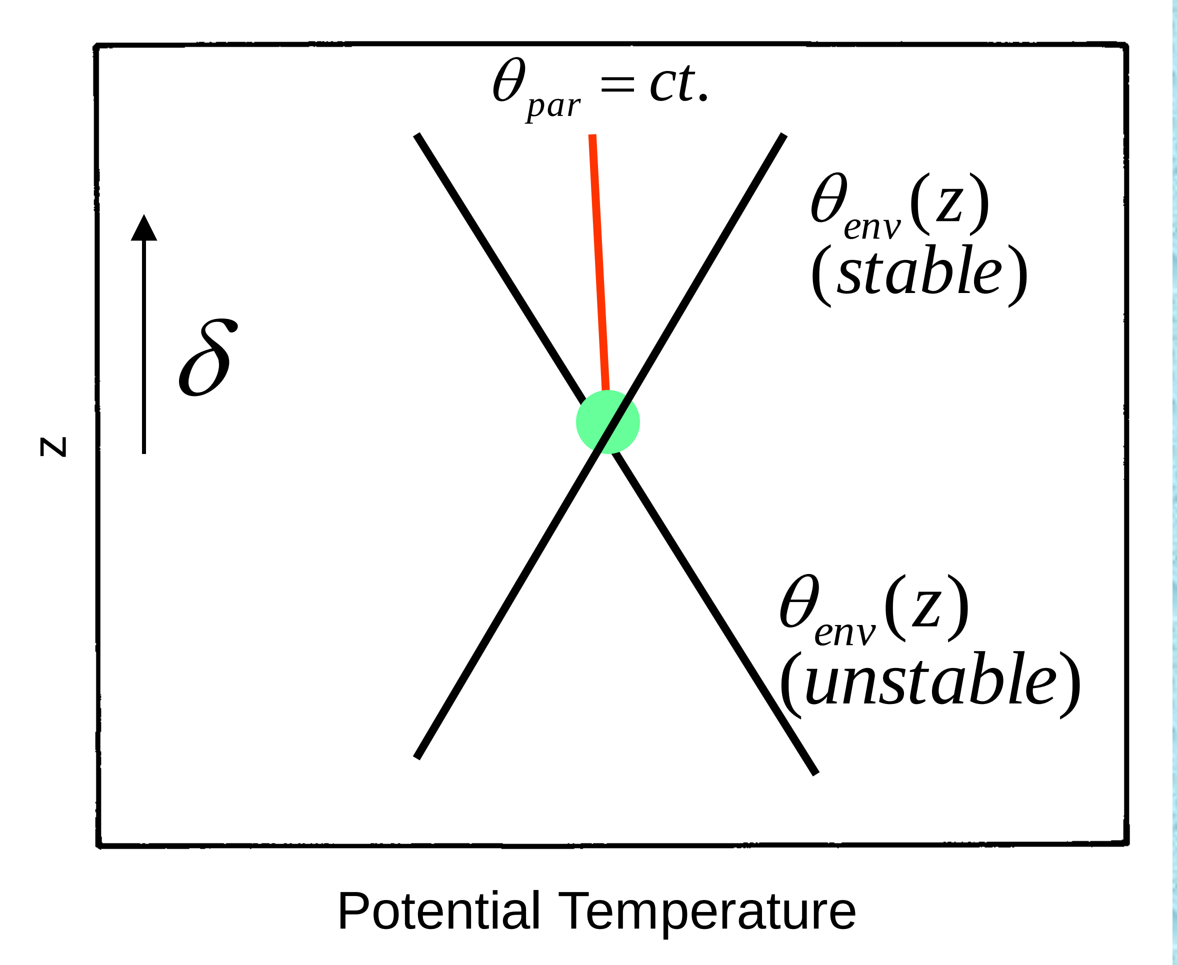
# $N^2, B$ in terms of potential temperature $\theta$

$$\theta \equiv \frac{T}{\pi} \; ; \; \pi \equiv \left( \frac{p}{1000mb} \right)^{R/c_p}$$

$$B = g \frac{\theta_{par} - \theta_{env}}{\theta_{env}}$$

$$N^2 = \frac{g}{\theta} \left( \left. \frac{\partial \theta}{\partial z} \right|_{env} - \left. \frac{\partial \theta}{\partial z} \right|_{par} \right)$$

# Air Parcel Behavior in Stable or Unstable Atmosphere




$$N^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z}\bigg|_{env}$$

# Dynamic Mesoscale Mountain Meteorology

## Governing Equations

# 1st Law of Thermodynamics

$$c_p \frac{DT}{Dt} = \frac{DQ}{Dt} + \frac{1}{\rho}\frac{Dp}{Dt}$$


With previous definitions →

Common form…

$$T \frac{D(c_p \ln \theta)}{Dt} = \frac{DQ}{Dt}$$

In terms of $\pi$ and $\theta$ …..

$$\frac{D\theta}{Dt} = \frac{1}{c_p \pi}\frac{DQ}{Dt}$$

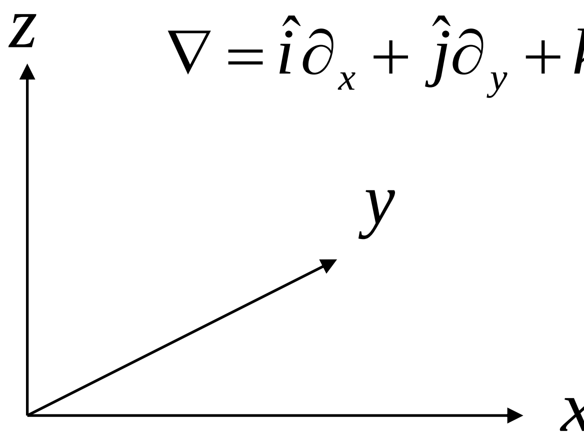
# Newtons 2nd Law

$$\frac{D\vec{u}}{Dt} = -c_p \theta \nabla \pi - g\hat{k} + \vec{F}$$


With previous definitions →

$$c_p \theta \nabla \pi = \frac{1}{\rho} \nabla p$$

$$\nabla = \hat{i}\partial_x + \hat{j}\partial_y + \hat{k}\partial_z$$




$$\vec{u} = (u, v, w)$$

$$\frac{D}{Dt} = \partial_t + u\partial_x + v\partial_y + w\partial_z$$

$\vec{F}$ = frictional force/unit mass

# Mass Conservation

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot \vec{u} = 0$$

With previous definitions →

$$\rho = \rho(\pi, \theta)$$

$$\frac{D}{Dt}\left( \ln\theta + \left(1 - \frac{1}{\kappa}\right)\ln\pi \right) = \nabla \cdot \vec{u}$$

$$\kappa \equiv R/c_p$$

# Summary of Governing Equations

Conservation of

momentum

$$\frac{D\vec{u}}{Dt} = -c_p \theta \nabla \pi - g\hat{k} + \vec{F}$$

energy

$$\frac{D\theta}{Dt} = \frac{1}{c_p \pi} \frac{DQ}{Dt}$$

mass

$$\frac{D}{Dt}\left( \ln\theta + \left(1 - \frac{1}{\kappa}\right) \ln\pi \right) = \nabla \cdot \vec{u}$$

# Simplify Governing Equations I

Neglect molecular diffusion → $Q = 0,\ \vec{F} = 0$

Conservation of

momentum

$$\frac{D\vec{u}}{Dt} = -c_p \theta \nabla \pi - g\hat{k}$$

energy

$$\frac{D\theta}{Dt} = 0$$

mass

$$\left(1 - \frac{1}{\kappa}\right) \frac{D \ln \pi}{Dt} = \nabla \cdot \vec{u}$$

# Simplify Governing Equations II

Conservation of momentum

Boussinesq Approximation

$$\theta = \theta_0 + \tilde{\theta} \quad , \quad \pi = \pi_0 + \tilde{\pi} \quad \left|\tilde{\theta}\right| \big/ \theta_0 << 1 \, , \quad \left|\tilde{\pi}\right| \big/ \pi_0 << 1$$

$$c_p \theta_0 \, \partial_z \pi_0 = -g \;\Rightarrow\; \pi_0(z) = 1 - gz \big/ c_p \theta_0$$

$$\frac{D\vec{u}}{Dt} \cong -\nabla (c_p \theta_0 \, \tilde{\pi}) + \frac{g\tilde{\theta}}{\theta_0} \hat{k} = -\nabla \varphi + B \hat{k}$$

# Simplify Governing Equations III

Conservation of energy

With $$\theta = \theta_0 + \tilde{\theta}$$

$$\frac{DB}{Dt} = 0$$

# Simplify Governing Equations IV

Conservation of mass

$$\left(1-\frac{1}{\kappa}\right)\frac{D\ln\pi}{Dt}=\nabla\cdot\vec{u}$$

By definition →

$$\ln\pi=\kappa\ln(p/1000mb)$$

$$-\frac{1}{\rho c^2}\frac{Dp}{Dt}=\nabla\cdot\vec{u}$$

$c$ = speed of sound

3 conditions for effective incompressibility (Batchelor 1967 pp. 167-169)

$$\frac{U^2}{c^2}<<1\ ,\ \frac{\omega^2L^2}{c^2}<<1,\ \frac{gL}{c^2}<<1$$

$$\nabla\cdot\vec{u}\cong 0$$

$U, L, \omega$ = velocity, length, frequency scales

# Summary of Simplified Governing Equations

Conservation of

momentum

$$\frac{D\vec{u}}{Dt} = -\nabla\varphi + B\hat{k}$$

energy

$$\frac{DB}{Dt} = 0$$

mass

$$\nabla \cdot \vec{u} = 0$$

Still nonlinear (advection)

Filtering → equations for mean / turbulent fluxes of *B* and *u* (see Sullivan lectures)

# Summary

- Buoyancy is a fundamental concept for dynamical mountain meteorology
- Boussinesq approximation simplifies momentum equation
- For most mountain meteorological applications, velocity field approximately solenoidal ($\nabla \cdot \vec{u} = 0$)