Dynamic Mesoscale Mountain Meteorology

Lecture 2: Thermally Driven Circulations

Richard Rotunno

National Center for Atmospheric Research , USA

Topics

Lecture 1 : Introduction, Concepts, Equations

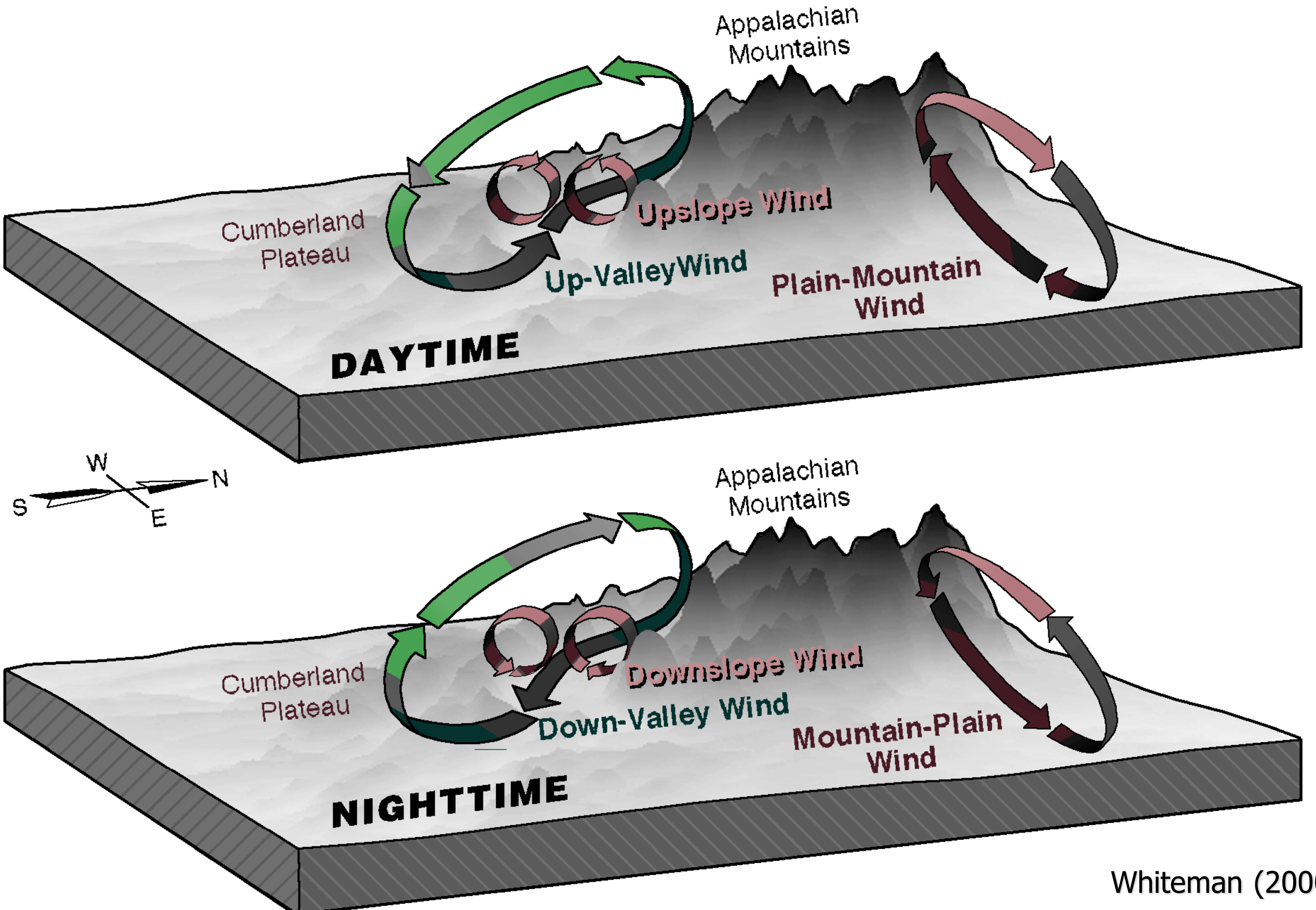
Lecture 2: Thermally Driven Circulations

Lecture 3: Mountain Waves

Lecture 4: Mountain Lee Vortices

Lecture 5: Orographic Precipitation

Thermally Driven Circulations



Whiteman (2000)

Summary of Simplified Governing Equations

Conservation of

momentum

$$\frac{D\vec{u}}{Dt} = -\nabla\varphi + B\hat{k}$$

energy

$$\frac{DB}{Dt} = 0$$

mass

$$\nabla \cdot \vec{u} = 0$$

Vorticity

$$\vec{\omega} = \nabla \times \vec{u}$$

$$\nabla \times \left\{ \frac{D\vec{u}}{Dt} = -\nabla \varphi + B\hat{k} \right\} \Rightarrow$$

$$\frac{D\vec{\omega}}{Dt} = \vec{\omega} \cdot \nabla \vec{u} - \hat{k} \times \nabla B$$

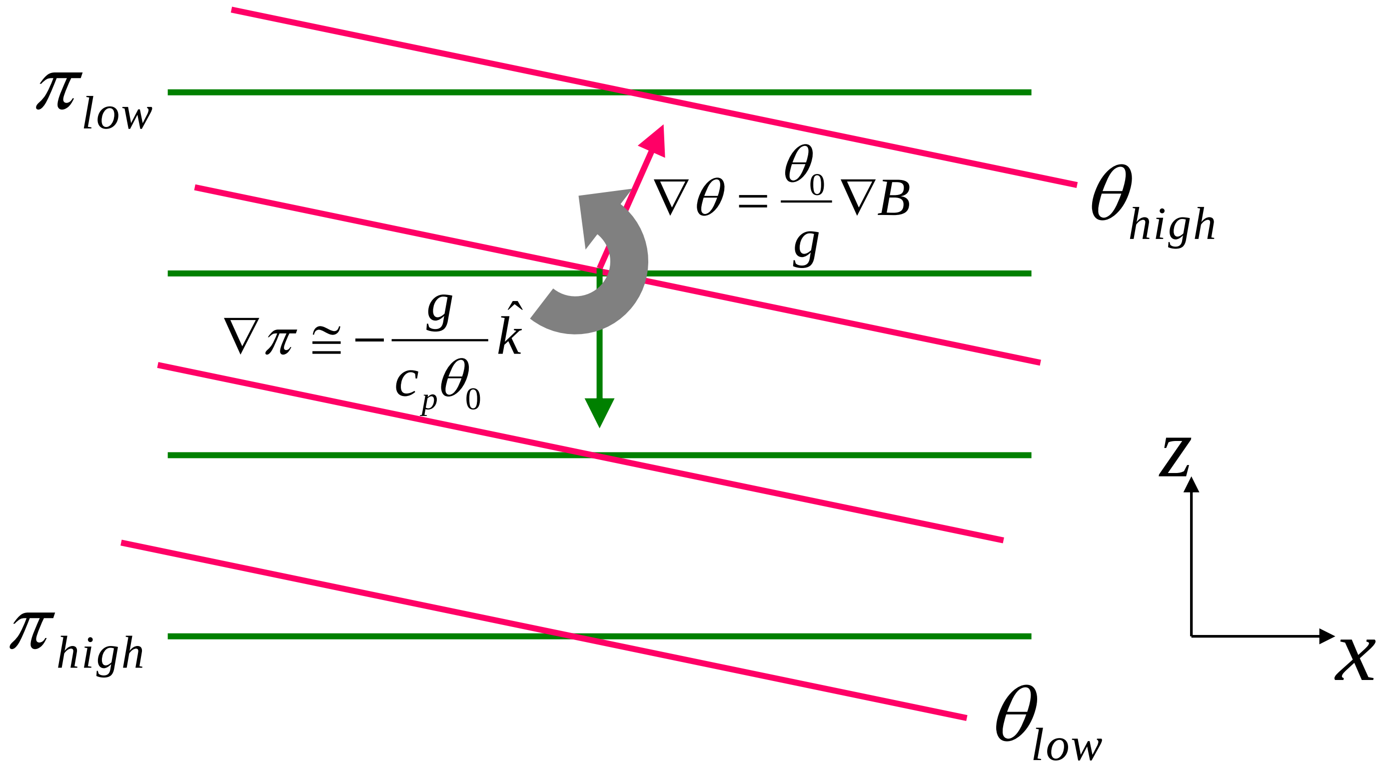
$$\vec{\omega} = (\xi, \eta, \zeta)$$

Batchelor (1967, Chapters 2 and 5)

Baroclinicity

$$-\hat{k}\times\nabla B$$

$$\nabla\times\left(-c_p\theta\nabla\pi\right)=c_p\nabla\pi\times\nabla\theta\cong c_p\nabla\pi_0\times\nabla\tilde{\theta}=-\hat{k}\times\nabla B$$



Relation of vorticity and velocity in 2D

by definition

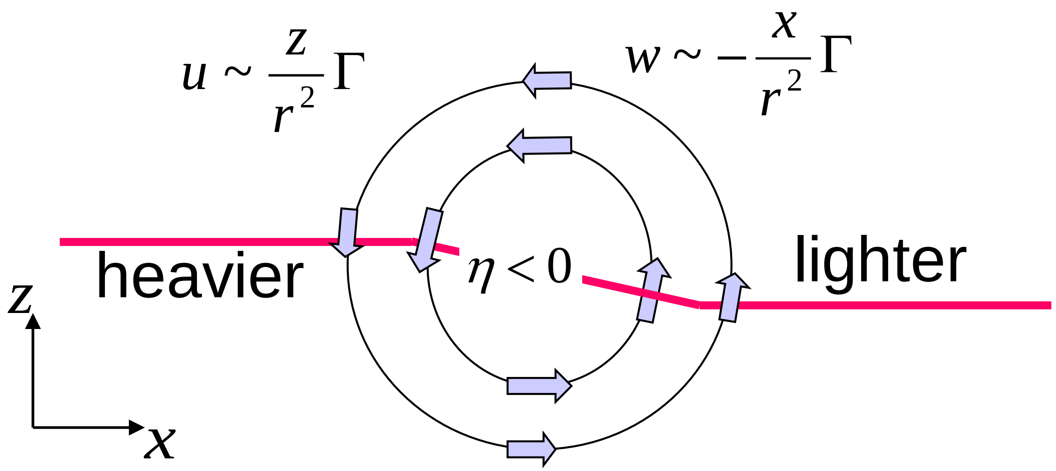
$$\eta = \partial_z u - \partial_x w$$

mass conservation

$$\partial_x u + \partial_z w = 0 \;\Rightarrow u = \partial_z \psi \;,\;\; w = -\partial_x \psi$$

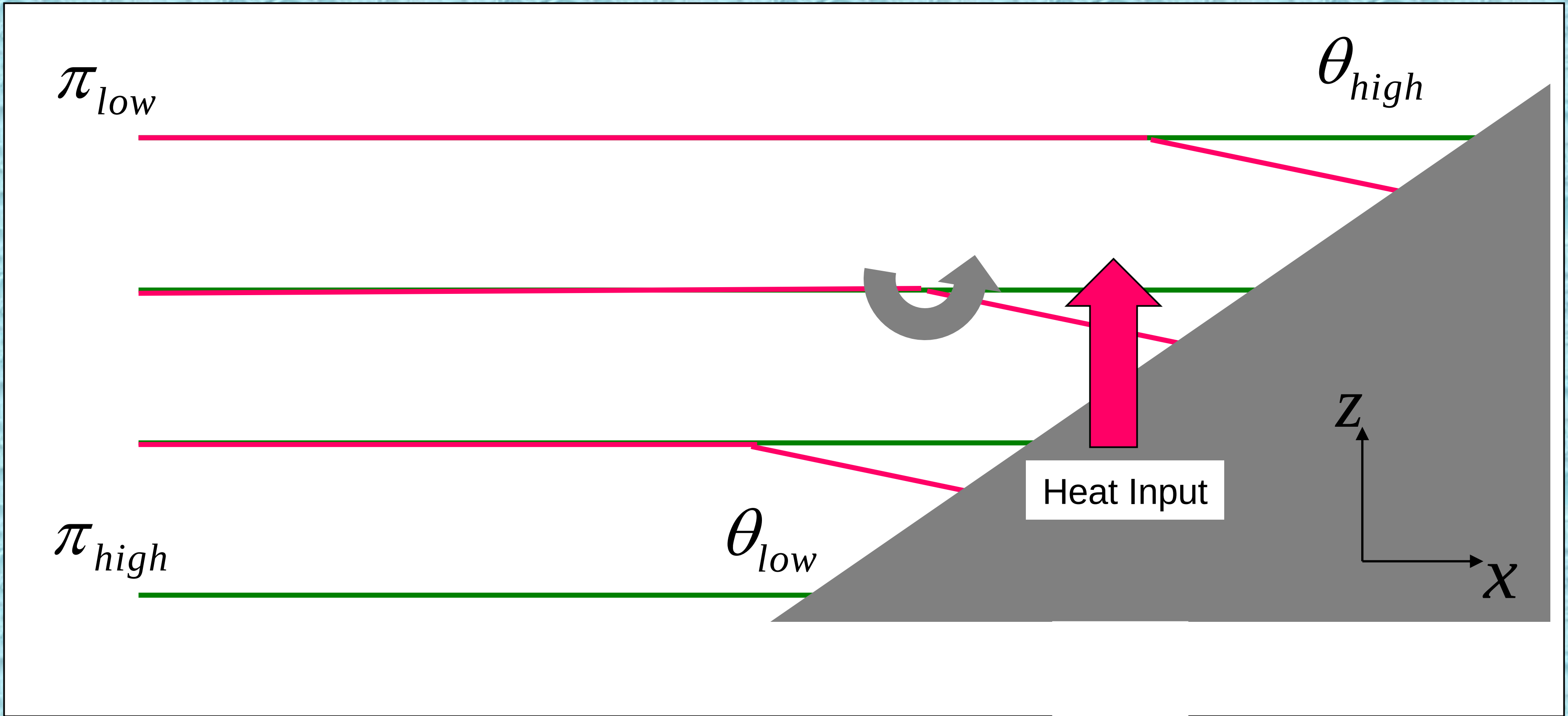
$$\therefore \qquad \boxed{\eta = (\partial_x^2 + \partial_z^2)\psi}$$

Example: Localized Vorticity

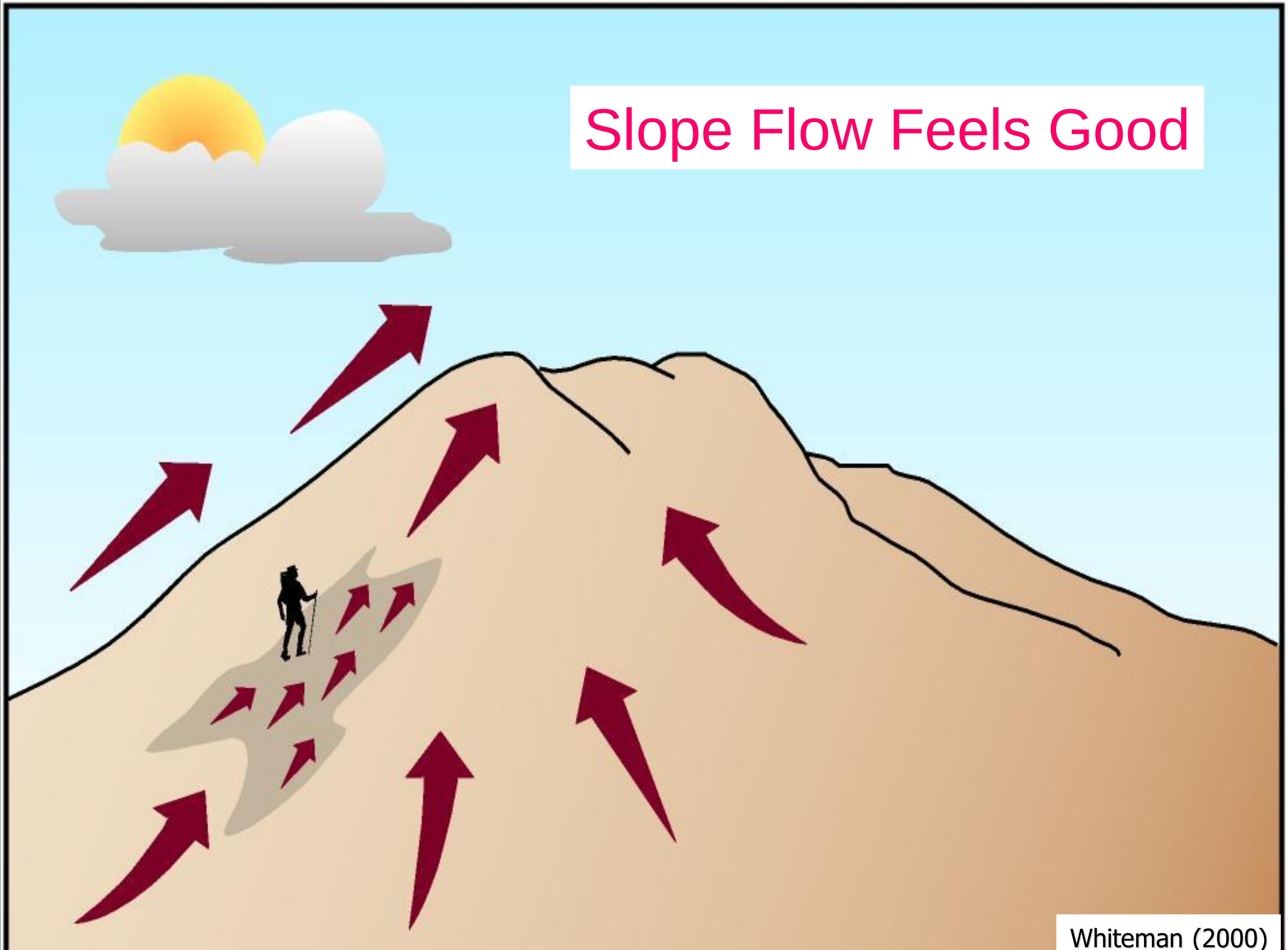


Slope Flow

$$-\hat{k}\times\nabla B = -\partial_x B\,\hat{j}$$



Slope Flow Feels Good

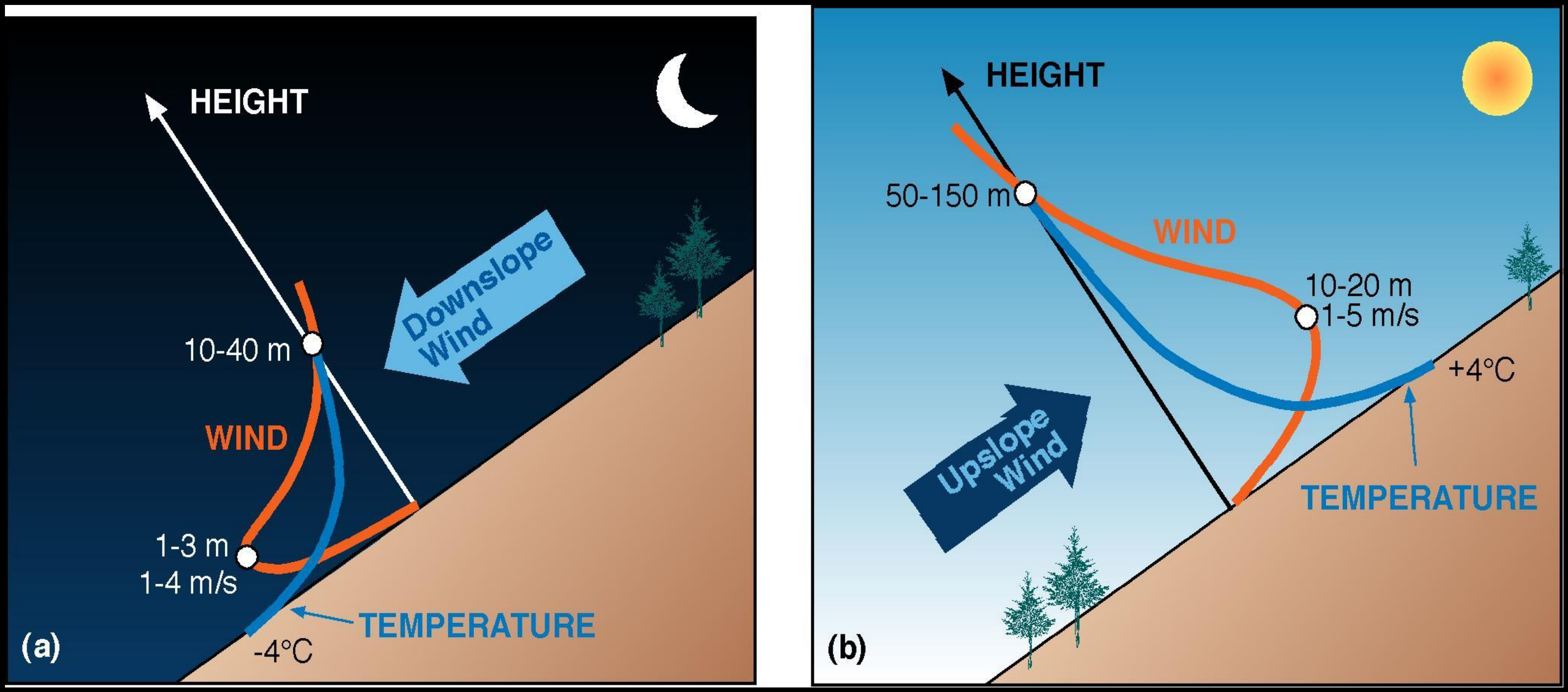


Whiteman (2000)

Clouds Make Slope Flow Visible

Bernhard Muehr photos, Karlsruher Wolkenatlas © 2000

Slope Flow Theory



Whiteman (2000)

Modify Simplified Governing Equations

Conservation of

momentum

$$\frac{D\vec{u}}{Dt} = -\nabla\varphi + B\hat{k} + \nu\nabla^2\vec{u}$$

energy

$$\frac{DB}{Dt} = \kappa\nabla^2 B$$

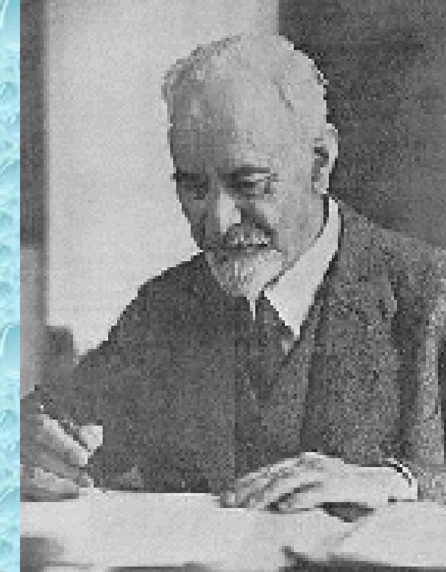
mass

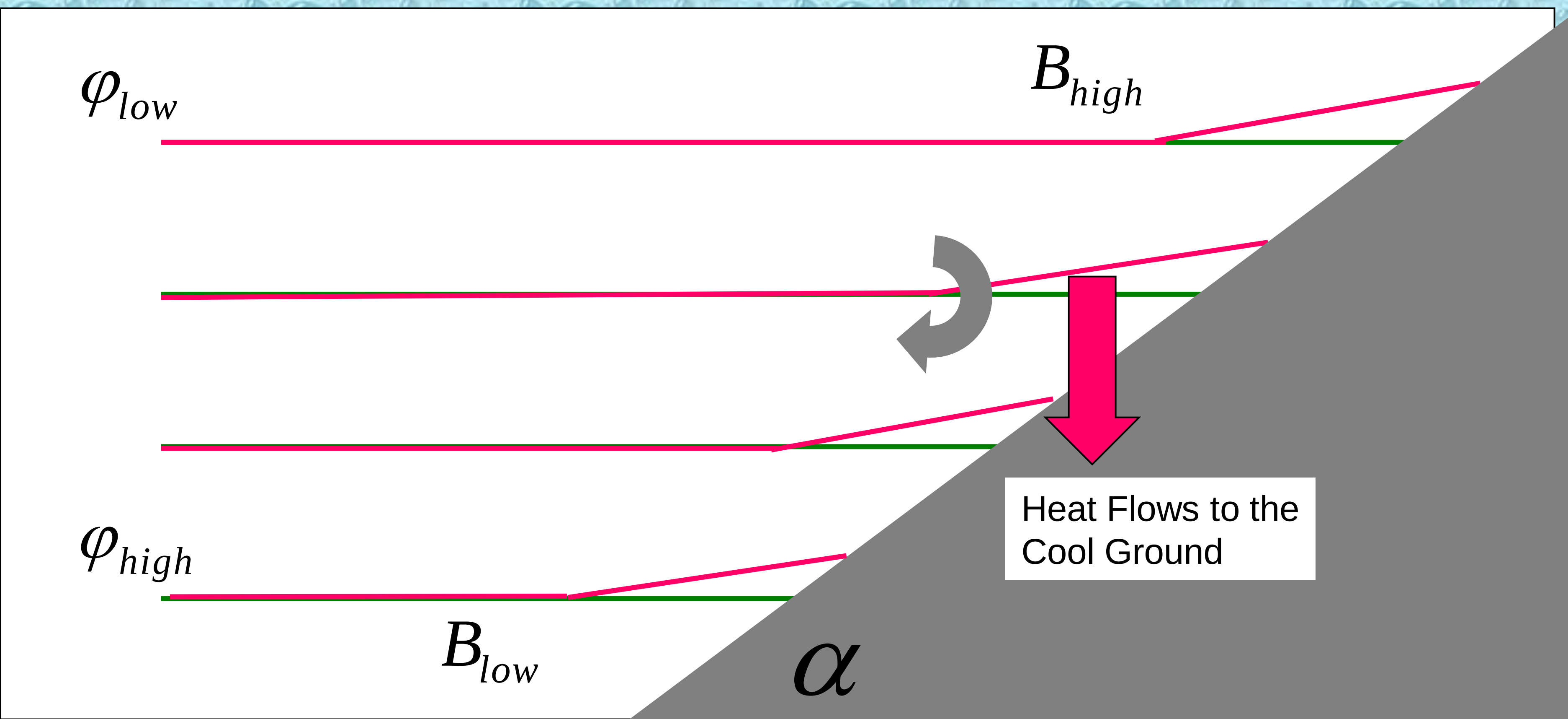
$$\nabla\cdot\vec{u} = 0$$

Prandtl's Model

Inifinitely long slope/Constant Stratification

$$B = N^2 z + b$$
$$\varphi = N^2 z^2 / 2 + \psi$$





Governing Equations

momentum

$$\frac{D\vec{u}}{Dt} = -\nabla\psi + b\hat{k} + \nu\nabla^2\vec{u}$$

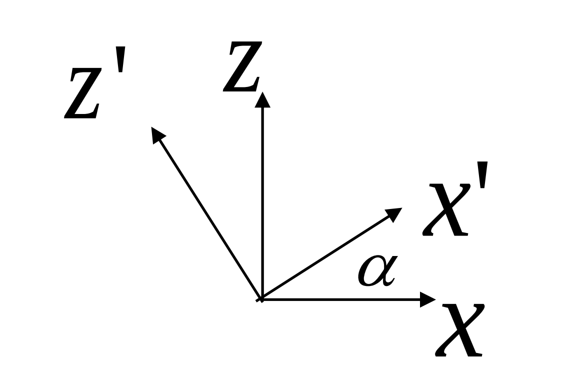
energy

$$\frac{Db}{Dt} + wN^2 = \kappa\nabla^2 b$$

mass

$$\nabla\cdot\vec{u} = 0$$

Governing equations in slope coordinates



momentum

$$(\partial_t + u'\partial_{x'} + w'\partial_{z'})u' = -\partial_{x'}\psi + b\sin\alpha + \nu(\partial_{x'}^2 + \partial_{z'}^2)u'$$

$$(\partial_t + u'\partial_{x'} + w'\partial_{z'})w' = -\partial_{z'}\psi + b\cos\alpha + \nu(\partial_{x'}^2 + \partial_{z'}^2)w'$$

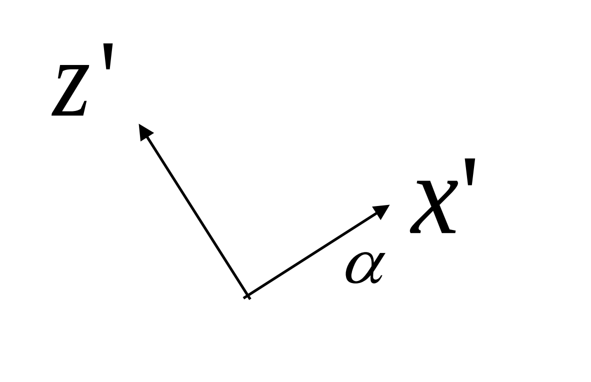
energy

$$(\partial_t + u'\partial_{x'} + w'\partial_{z'})b + (u'\sin\alpha + w'\cos\alpha)N^2 = \kappa(\partial_{x'}^2 + \partial_{z'}^2)b$$

mass

$$\partial_{x'}u' + \partial_{z'}w' = 0$$

Look for solutions with $w' = 0, \partial_{x'} = 0, \partial_t = 0$



momentum

$$x': \quad 0 = b \sin\alpha + \nu \partial_{z'}^2 u'$$

$$z': \quad 0 = -\partial_{z'} \psi + b \cos\alpha$$

energy

$$u' \sin\alpha \, N^2 = \kappa \partial_{z'}^2 b$$

combine 1st and 3rd equations →

$$\partial_{z'}^4 b + \frac{N^2 \sin^2\alpha}{\nu\kappa} b = 0$$

/ boundary conditions

$$b\big|_{z'=0} = b_s, \partial_{z'}^2 b\big|_{z'=0} = 0 \ (u'\big|_{z'=0} = 0), \text{solution bounded as } z' \to \infty$$

Solution

$$b(z') = b_s \exp\left(-\frac{z'}{\lambda}\right)\cos\left(\frac{z'}{\lambda}\right) \; ; \; \lambda \equiv \left(\frac{4\nu\kappa}{N^2 \sin^2 \alpha}\right)^{\frac{1}{4}}$$

$$u'(z') = \frac{b_s}{N}\sqrt{\frac{\kappa}{\nu}}\exp\left(-\frac{z'}{\lambda}\right)\sin\left(\frac{z'}{\lambda}\right)$$

$$z' = z\cos\alpha - x\sin\alpha$$

$$B(x,z) = N^2 z + b(x,z)$$

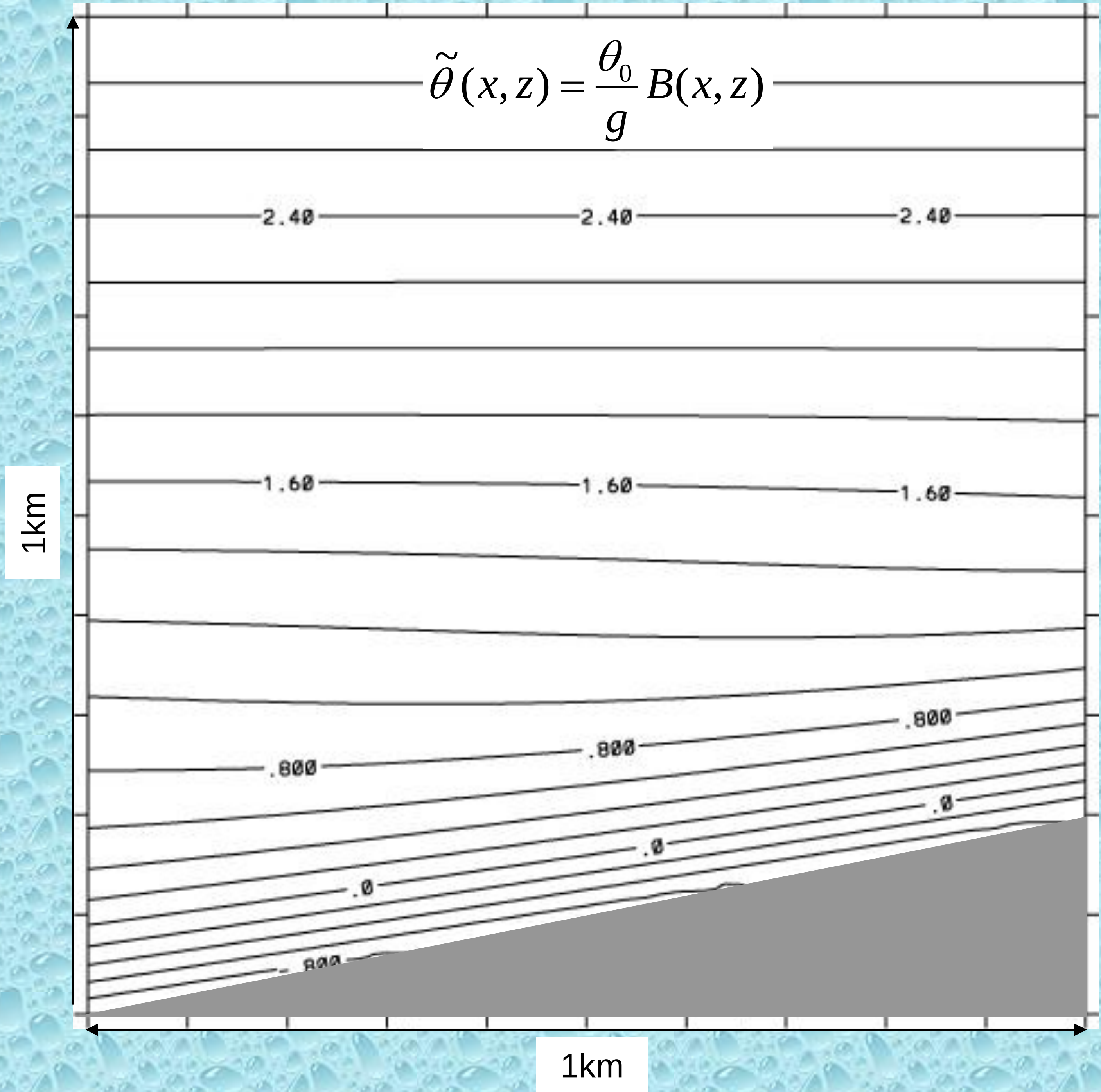
Example

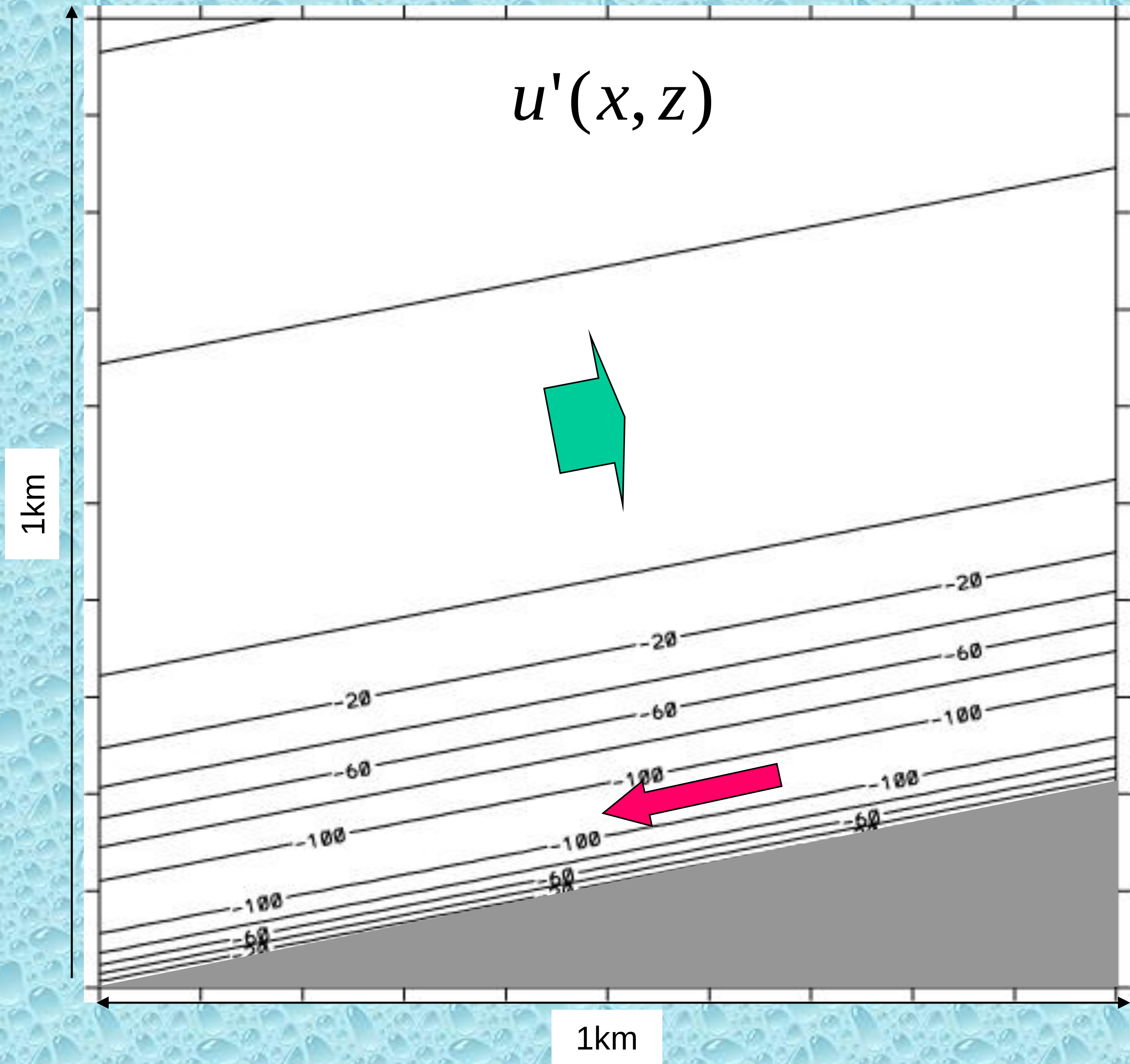
$$\nu = \kappa = 5 m^2 / s$$

$$N = .01 s^{-1} \Rightarrow \partial_z \bar{\theta} \approx 3^\circ C / km$$

$$\alpha = 0.2$$

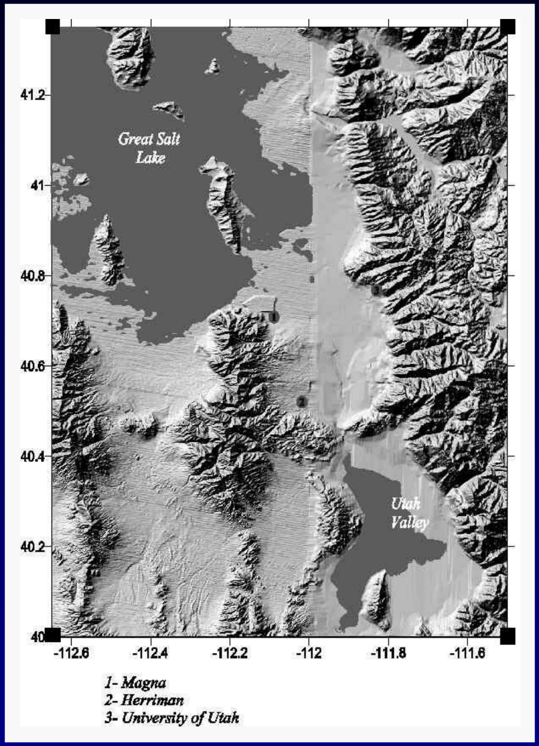
$$\theta_s = -1^\circ C$$



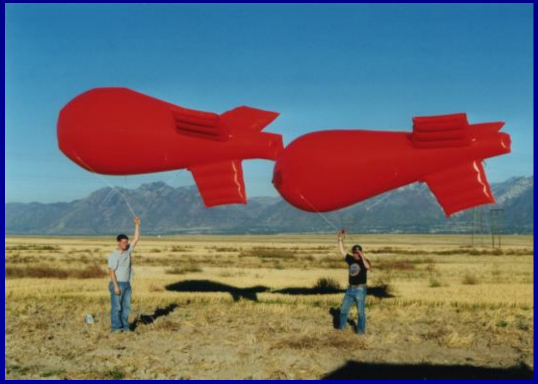


$u'(x,z)$
-20
-20
-20
-60
-60
-60
-100
-100
-100
-100
-100
-100
-60
-60
-60
1km
1km

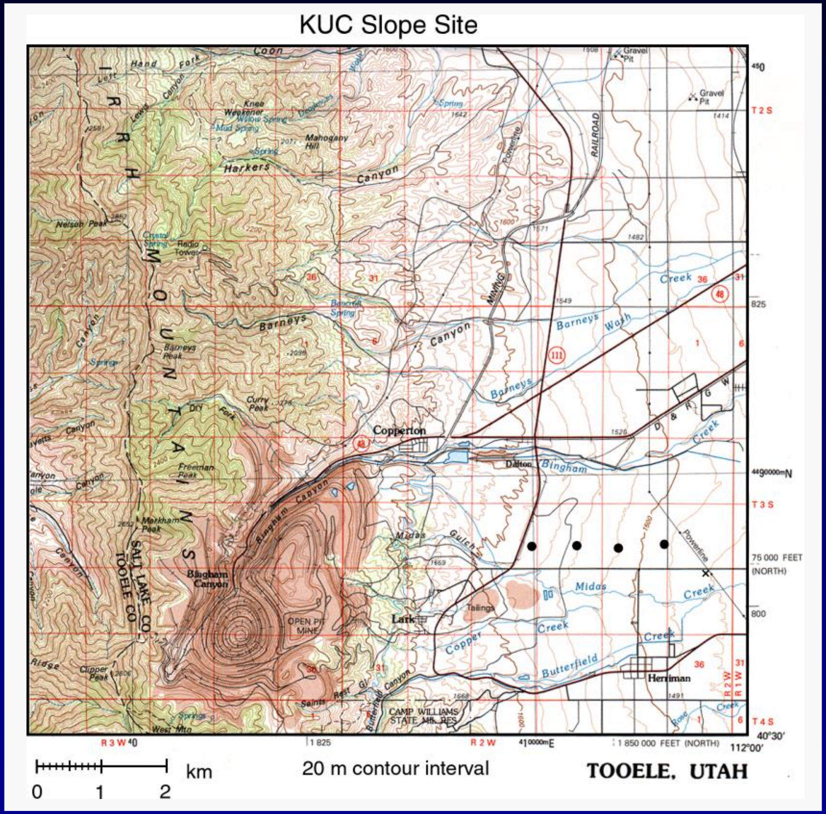
VTMX, Salt Lake Valley, UT

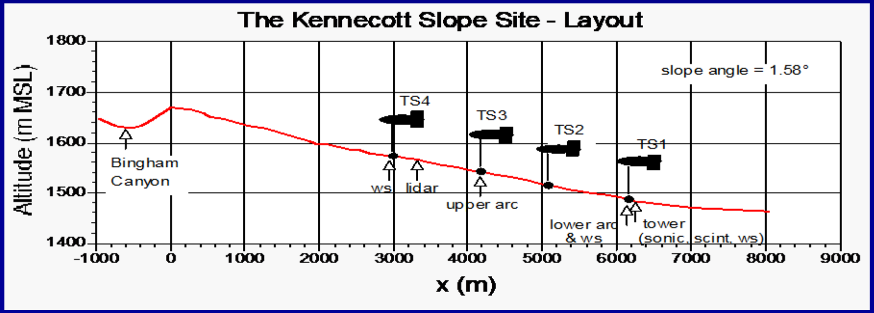


Whiteman photos



Topographic map



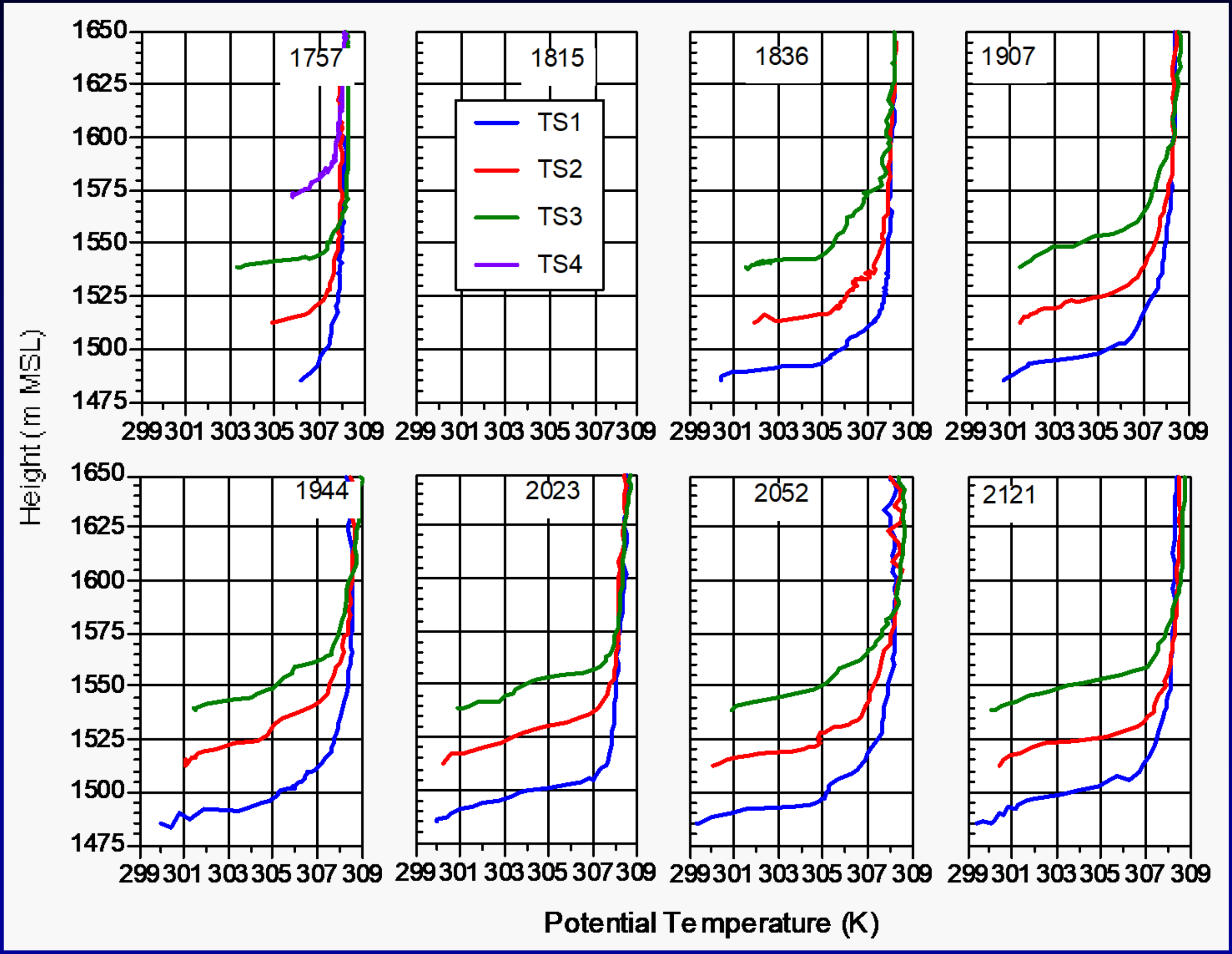


Sharon Zhong

Thomas Haiden

Eric Skyllingstad

Potential temperature soundings, 2 Oct 2000



7K temp deficit in 25-50 m

temp deficit strengthens with downslope distance

coldest air at lowest elevations

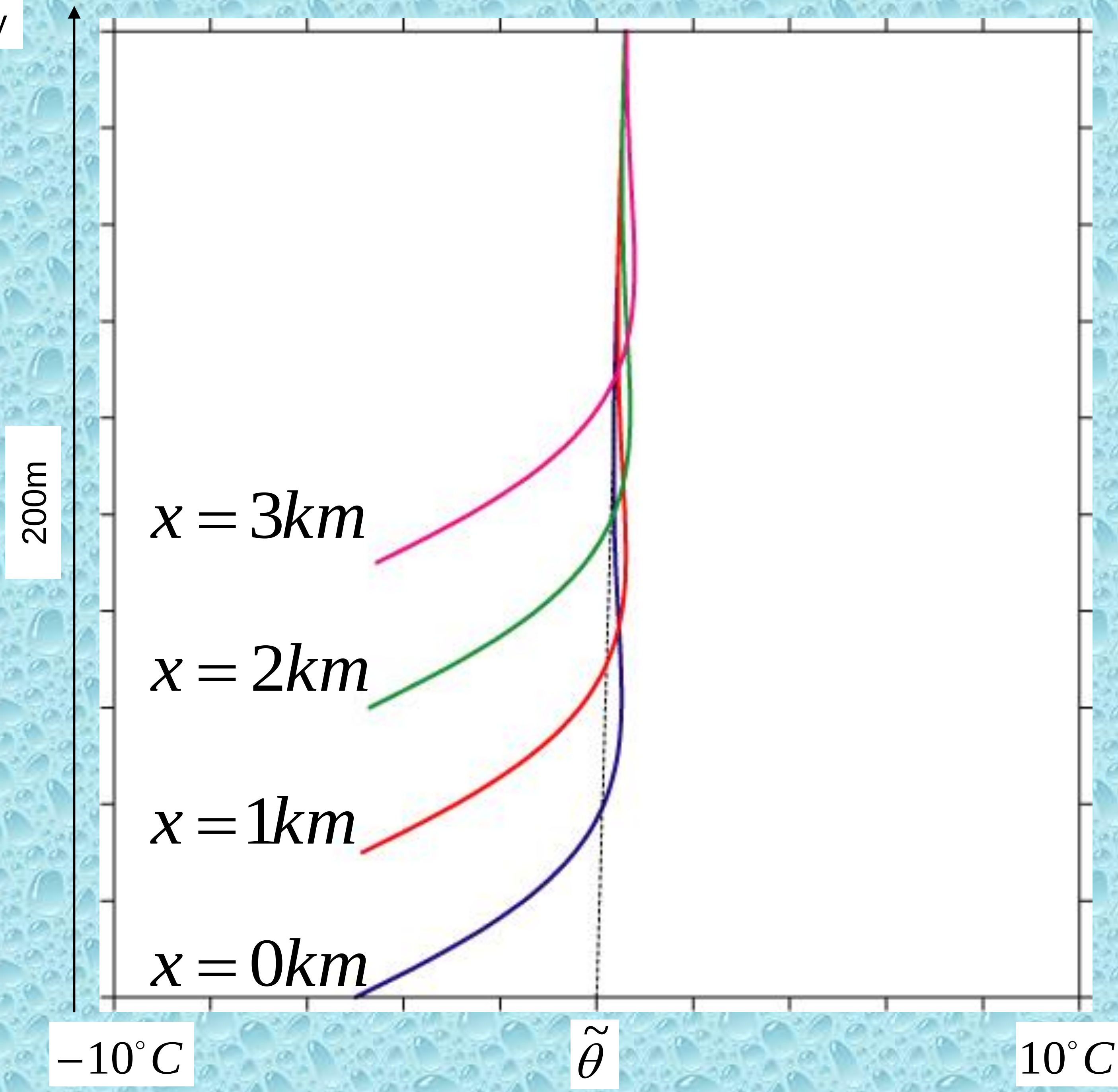
Compare / Prandtl Theory

$$\nu = \kappa = 10 m^2 / s$$

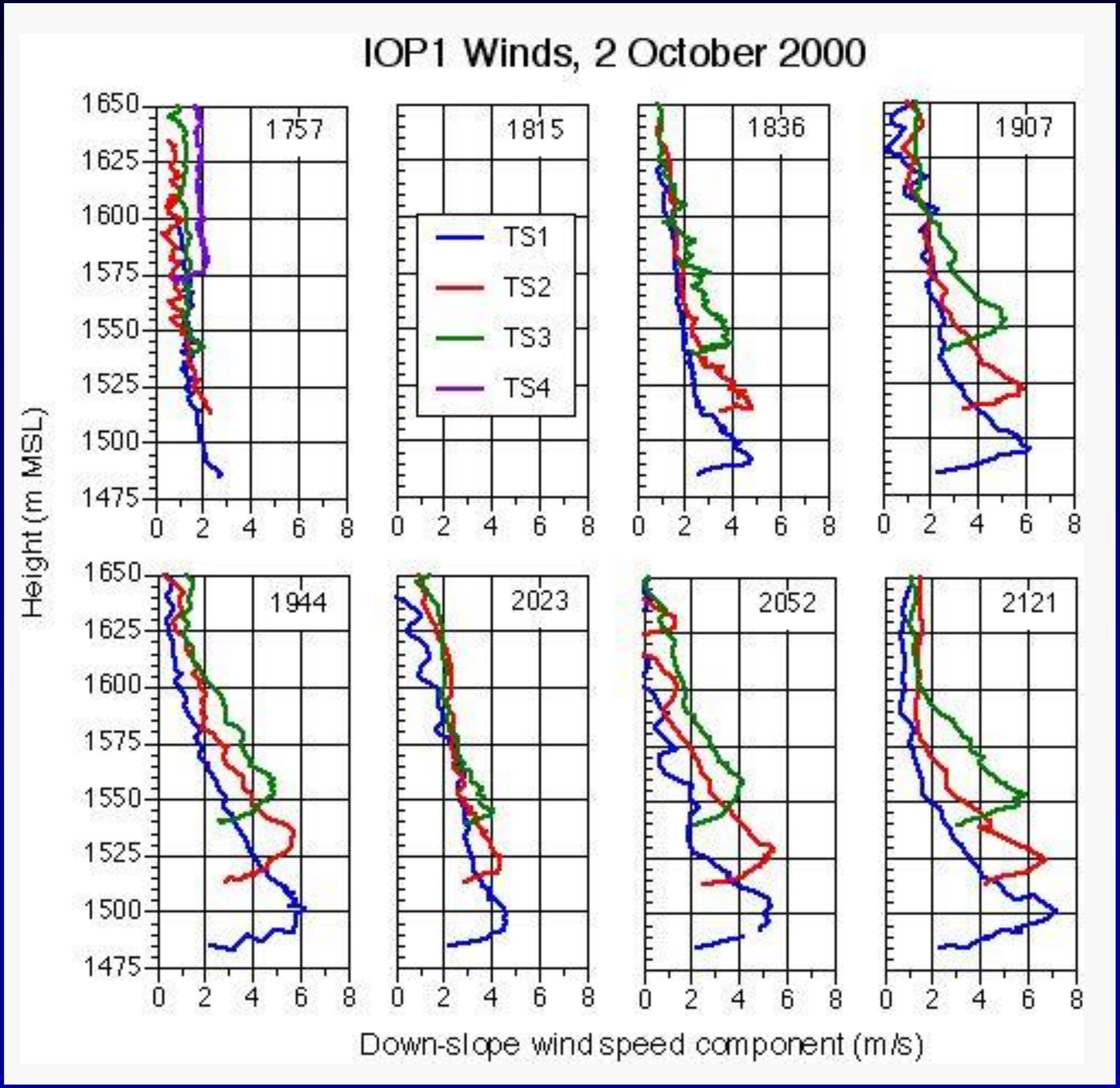
$$N = .01 s^{-1} \Rightarrow \partial_z \bar{\theta} \approx 3^\circ C / km$$

$$\alpha = 0.03$$

$$\theta_s = -5^\circ C$$



Downslope wind comp soundings, 2 Oct 2000



jet profile at all sites

max velocity approx 15m above slope

max velocity increases with downslope distance

max velocity reaches 7 m/s

downslope flow layer extends to ~150 m above slope

volume(mass) flux increase with downslope distance

Mass flux =
300 kg/s/m * 20 km =
6000 t/s (22 km^3/h)

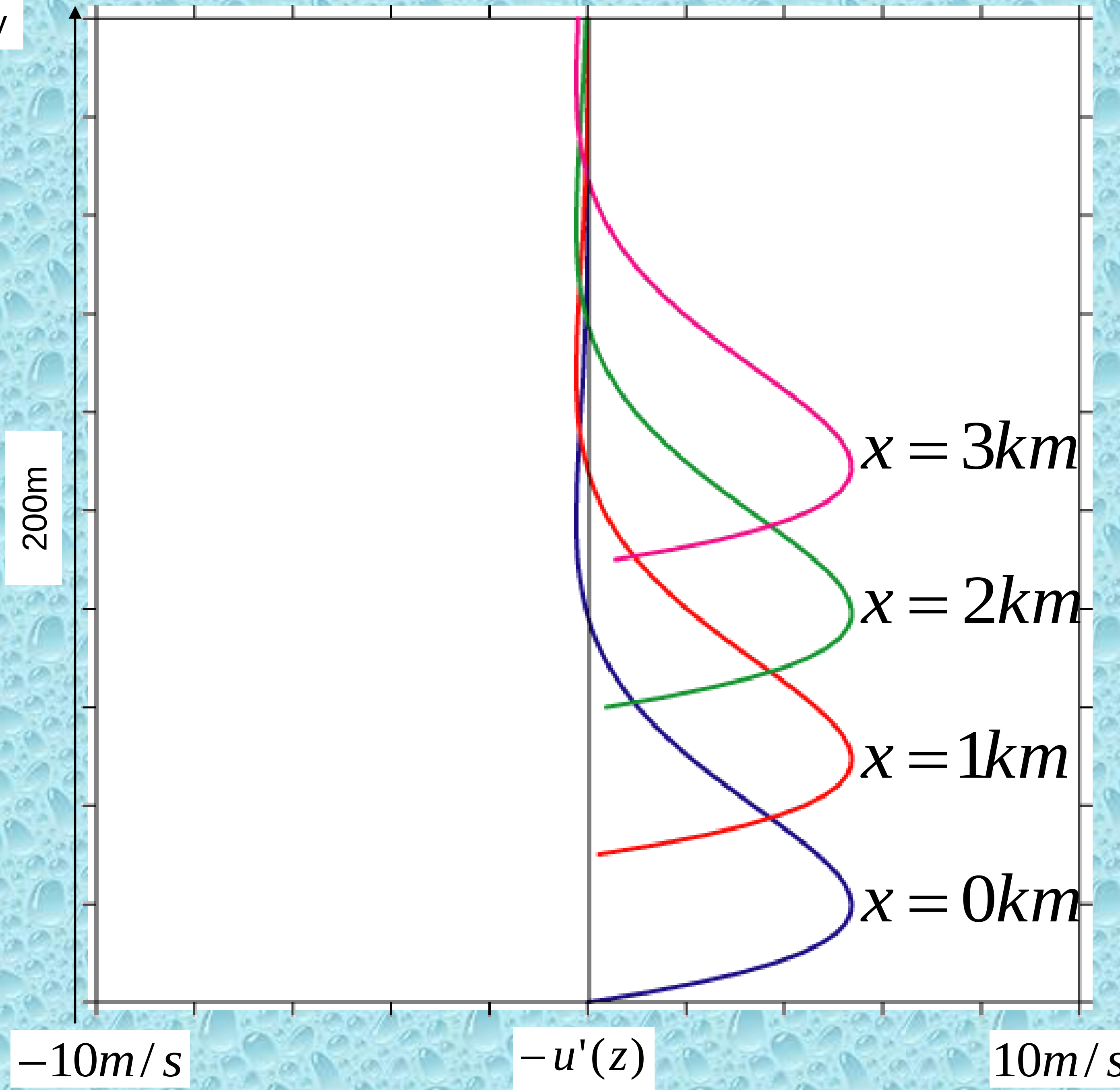
Compare / Prandtl Theory

$\nu = \kappa = 10 m^2 / s$

$N = .01 s^{-1} \Rightarrow \partial_z \bar{\theta} \approx 3^\circ C / km$

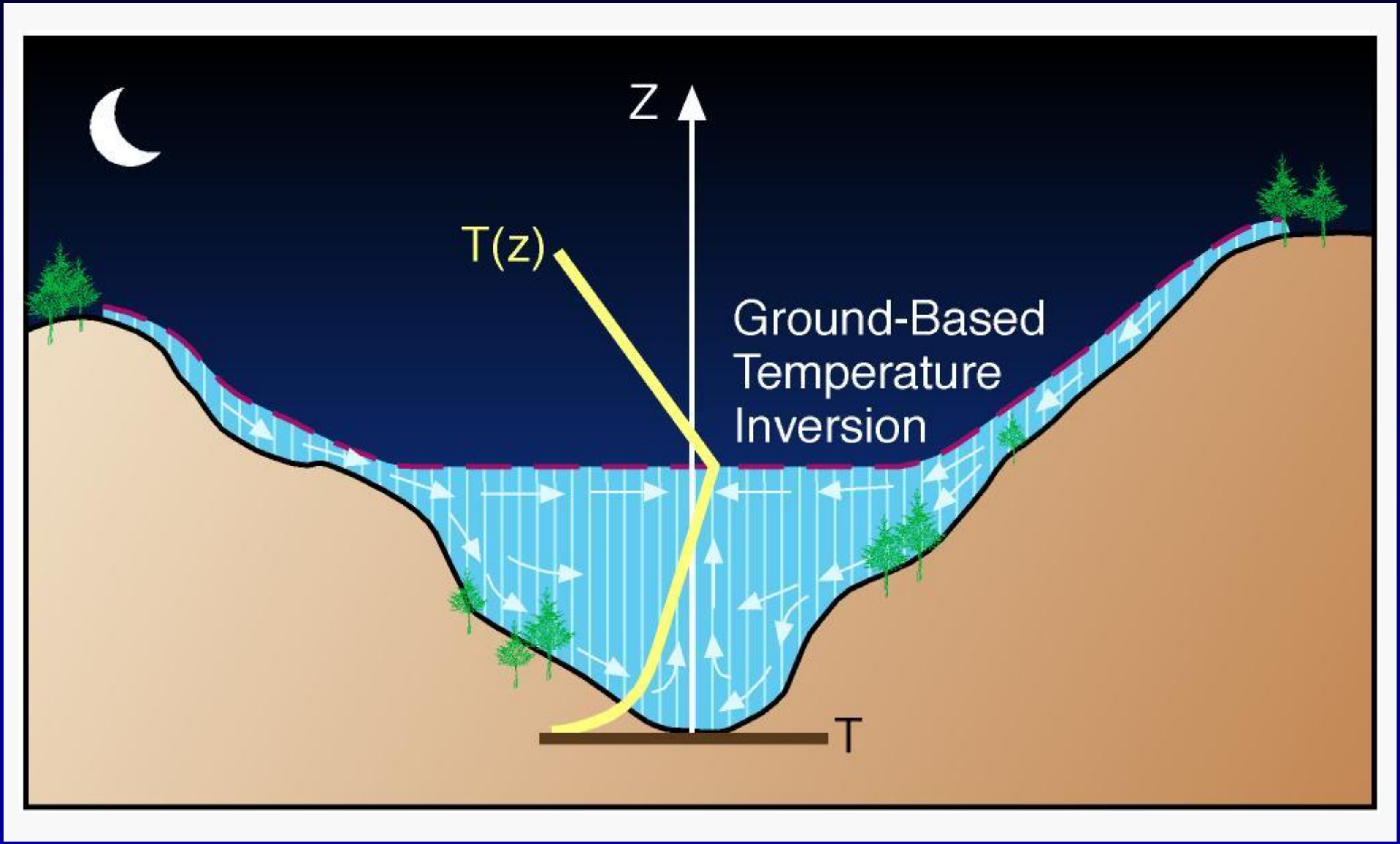
$\alpha = 0.03$

$\theta_s = -5^\circ C$



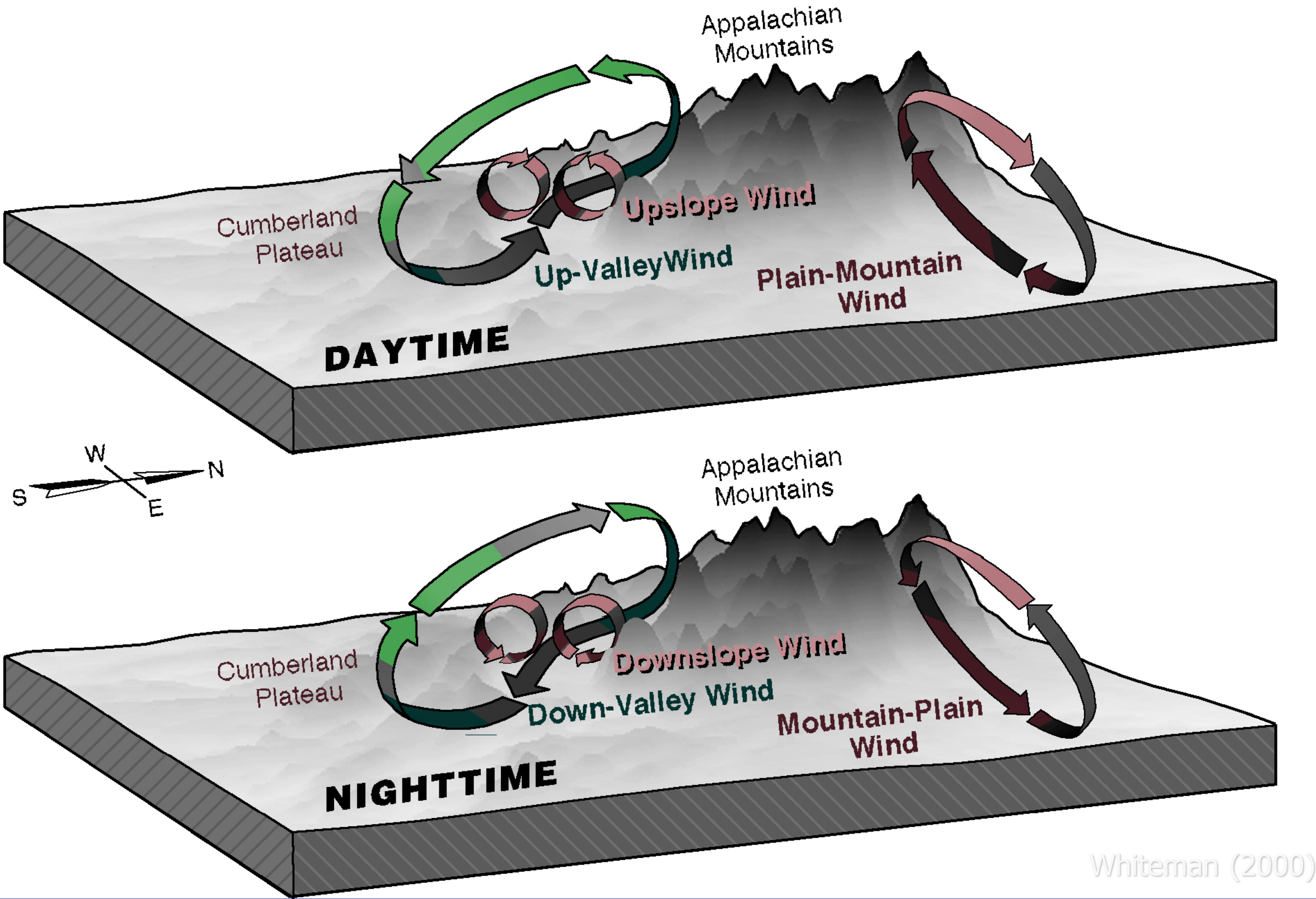
Further Complications

Downslope flows leave the slope…

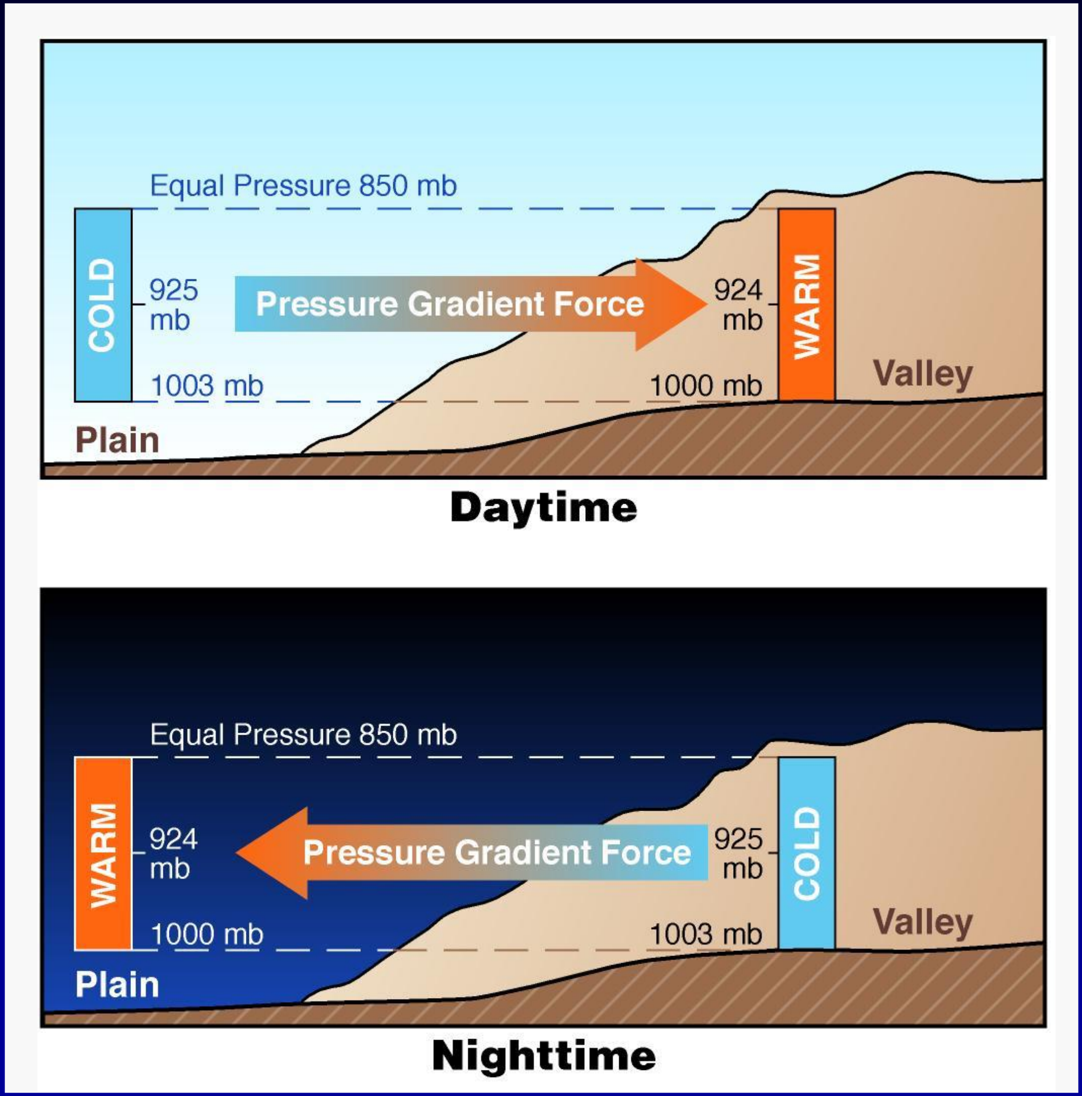


Whiteman (2000)

Multi-Scale Topography



Along-valley flows



Whiteman (2000)

Valley winds are closed circulations that attempt to equalize horizontal pressure gradients that are built up hydrostatically between the valley and plain caused by the greater temperature range of a column of air within the valley compared to a similar column of air over the plain at the same elevation.

Summary

- Baroclinity (Buoyancy Gradient) Produces Motion

- Ambient Stratification and Turbulent Mixing Modulates Motion

Warning Label: Prediction for slope/valley winds for any real valley requires knowledge of insolation, heat absorption, surface characteristics (e.g. vegetation), large-scale weather conditions. (Whiteman 2000)