Dynamic Mesoscale Mountain Meteorology

Lecture 4: Mountain Lee Vortices

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Topics

Lecture 1 : Introduction, Concepts, Equations

Lecture 2: Thermally Driven Circulations

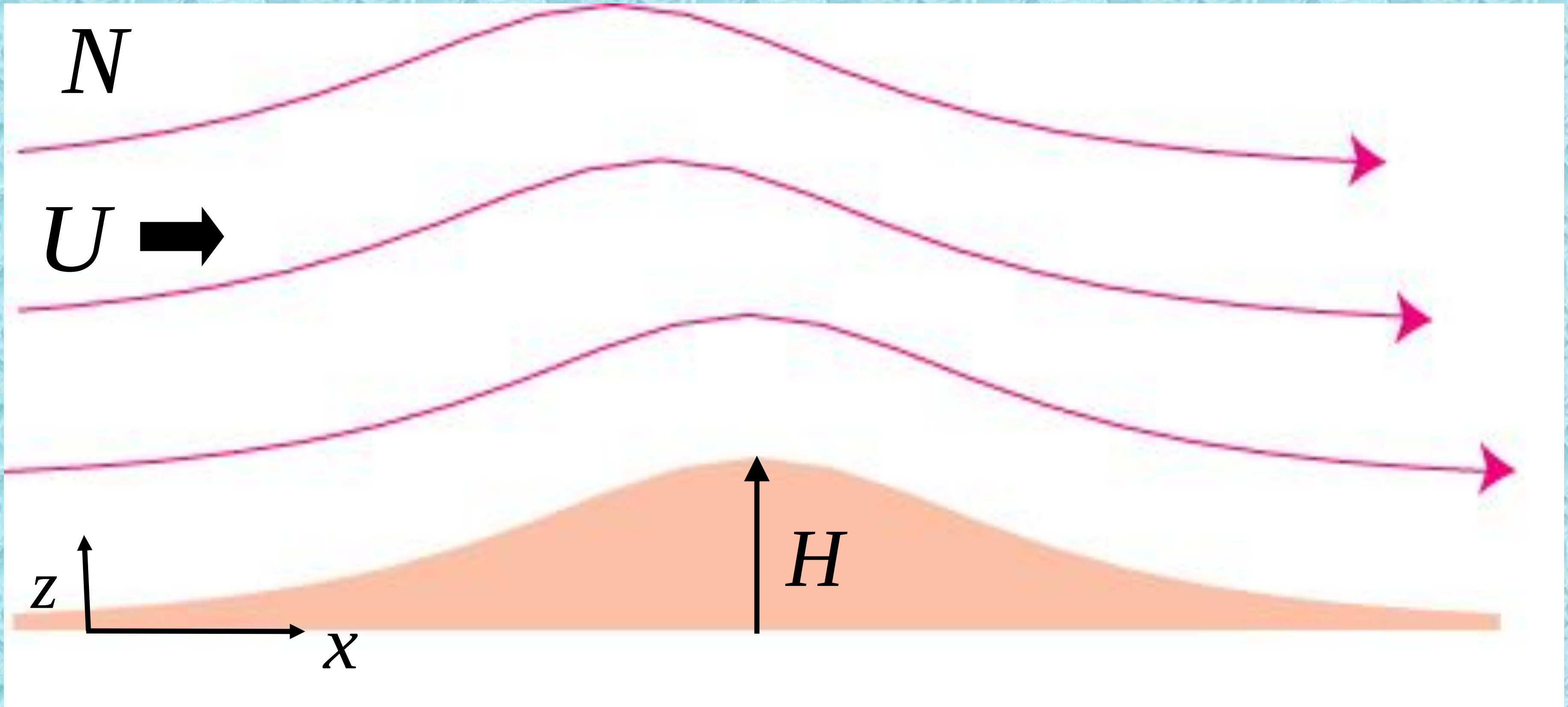
Lecture 3: Mountain Waves

Lecture 4: Mountain Lee Vortices

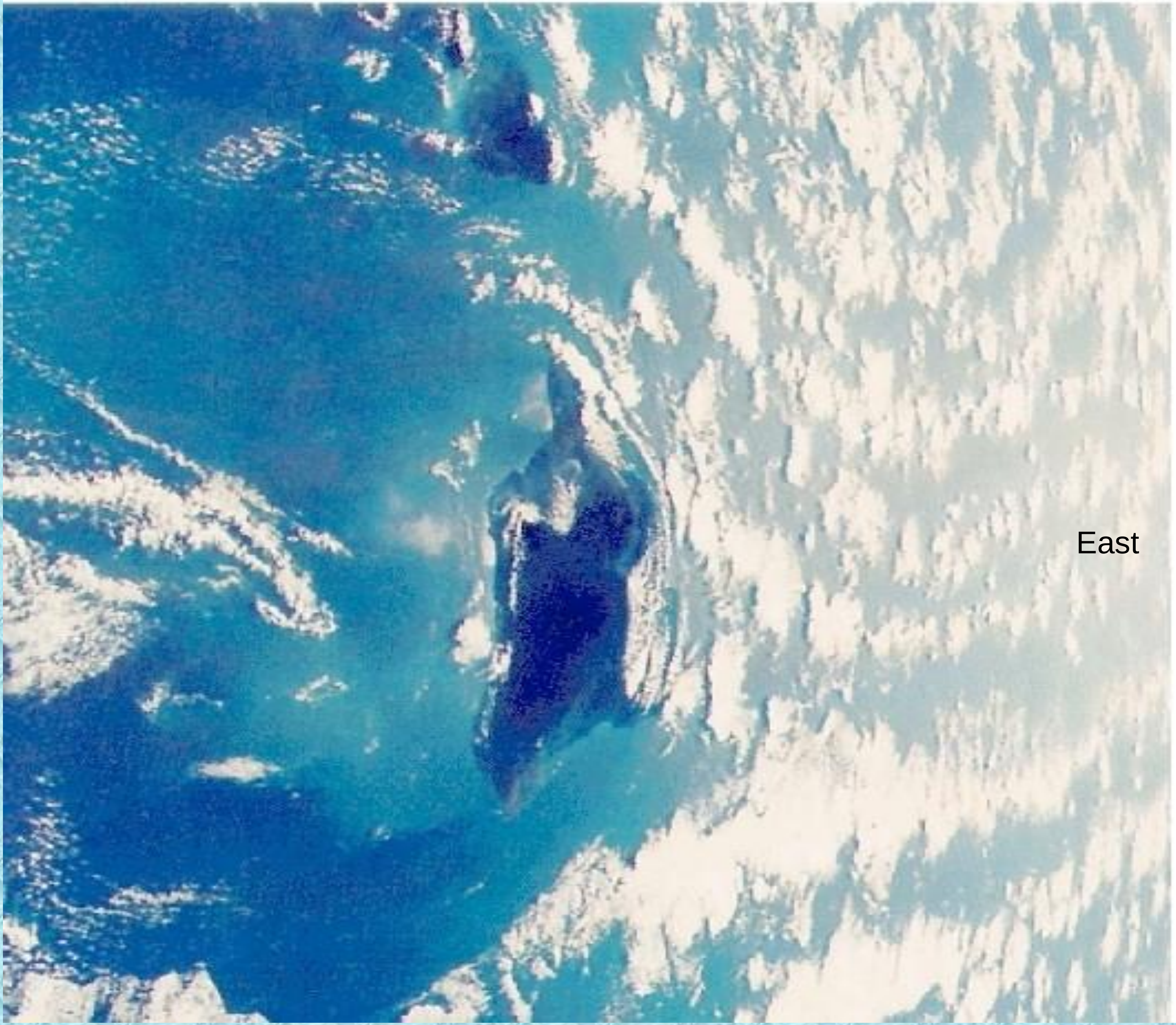
Lecture 5: Orographic Precipitation

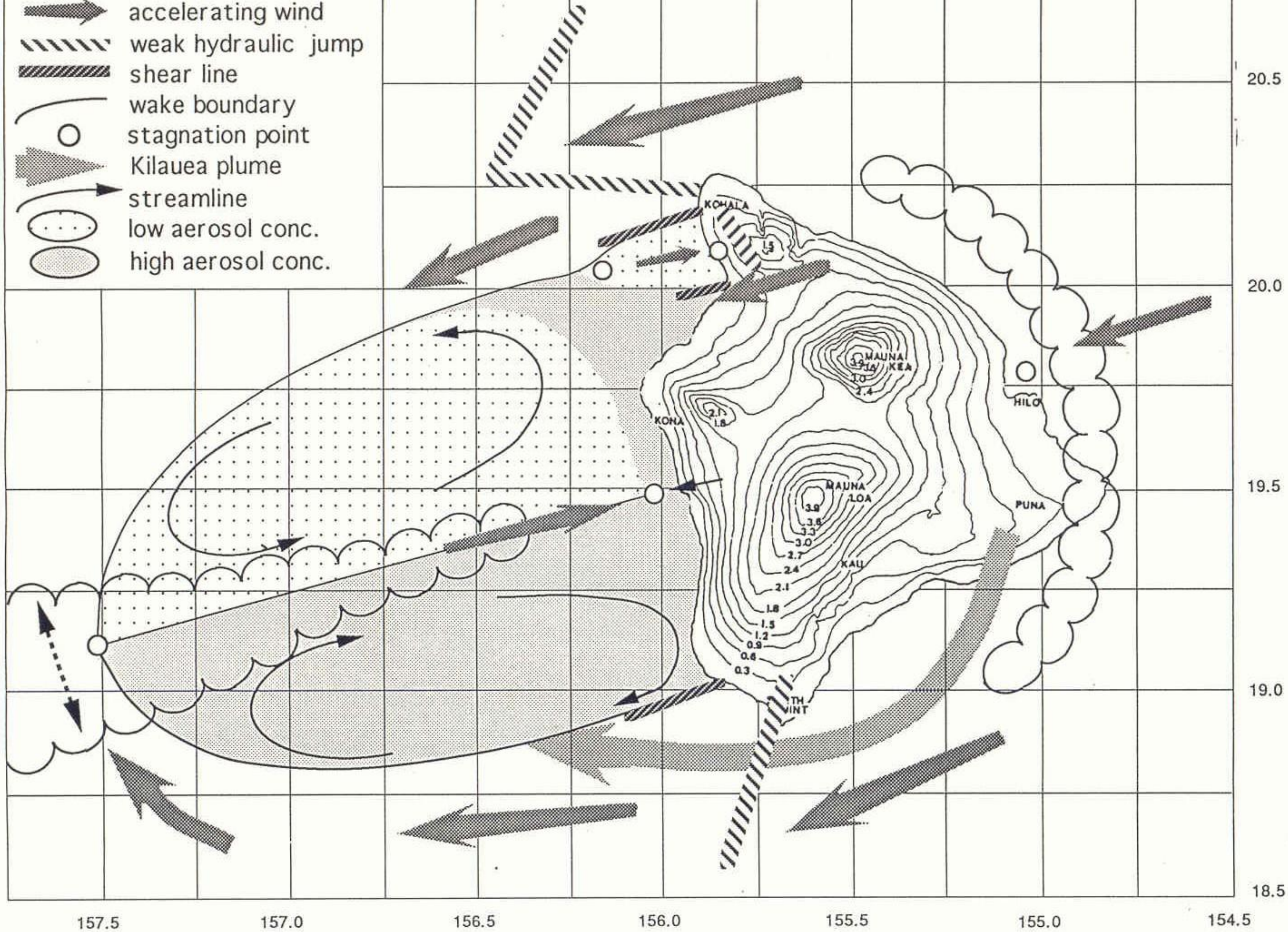
Mountain Lee Vortices

$$H \geq \frac{U}{N} \sim \frac{10m/s}{.01/s} = 1000m$$



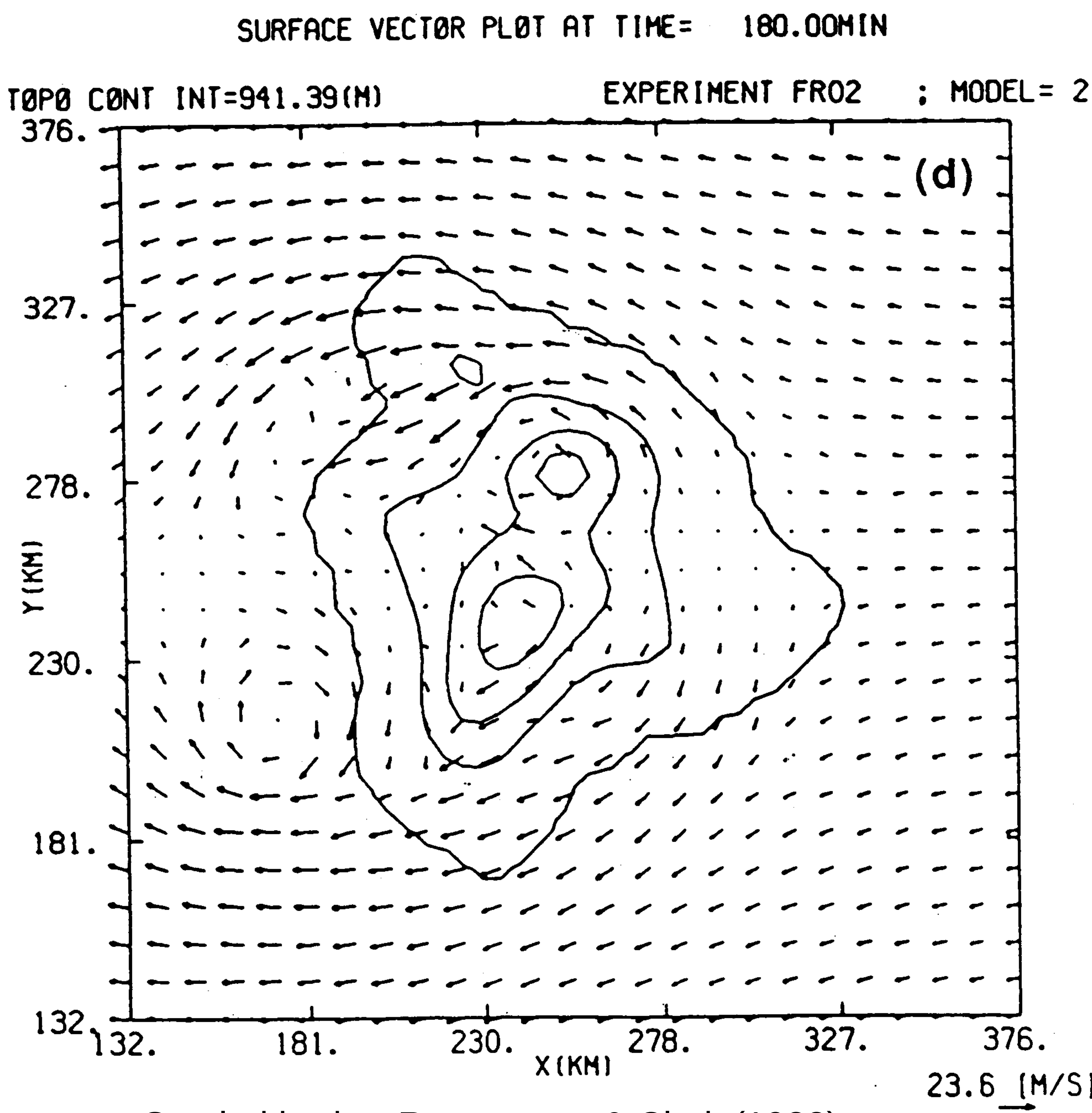
Hawaii
H=3km



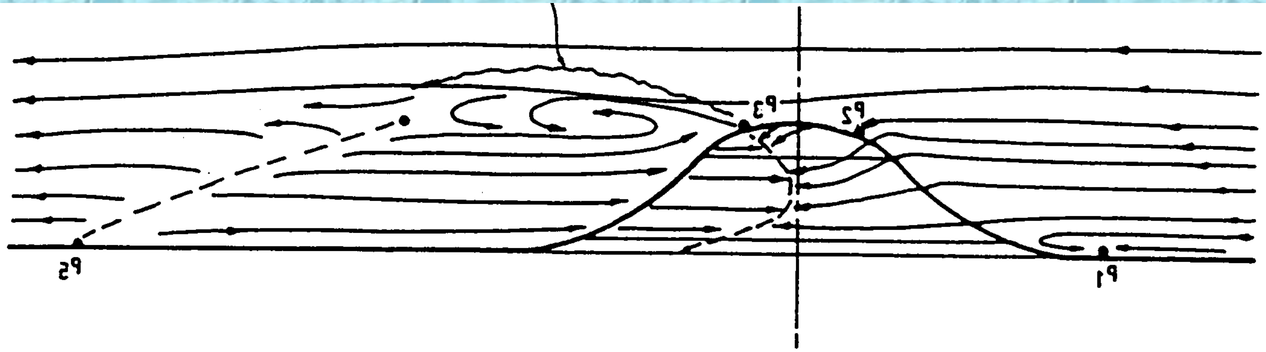


Smith and Grubišić(1993)

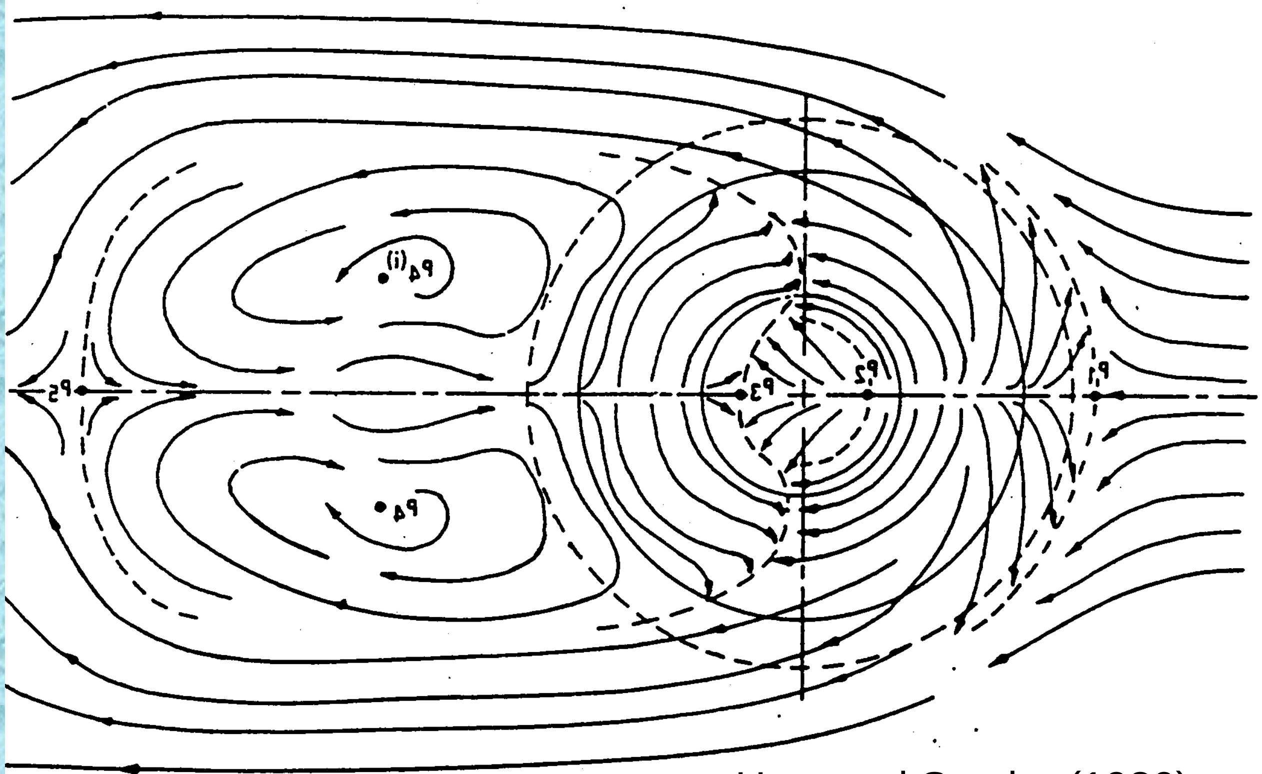
Numerical Simulations



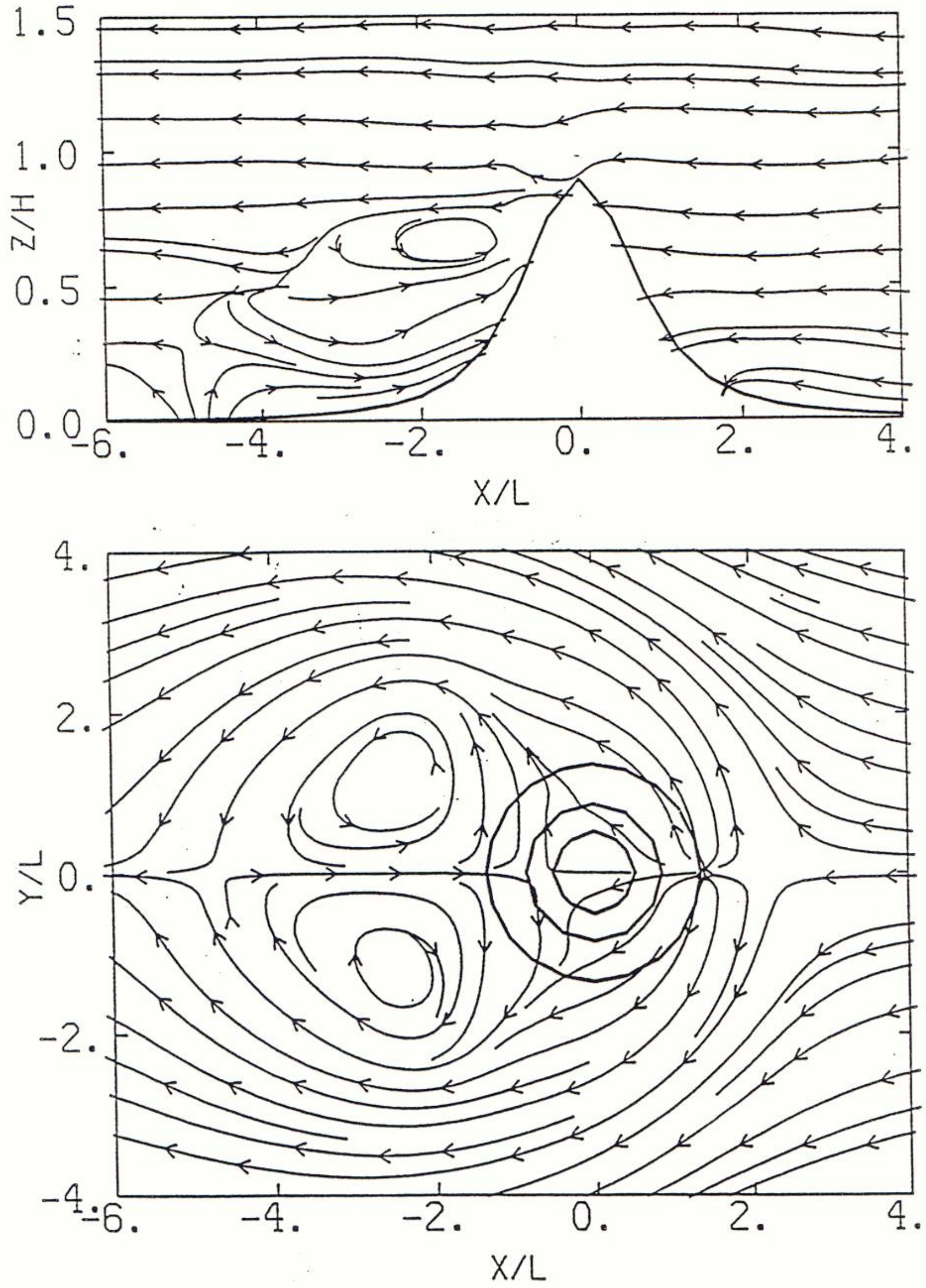
Smolarkiewicz, Rasmussen & Clark (1988)



Laboratory Simulations



Hunt and Snyder (1980)



Idealized Numerical Simulations

Smolarkiewicz and Rotunno (1989a)

Source of Vorticity?

momentum

$$\frac{D\vec{u}}{Dt} = -\nabla\varphi + B\hat{k} + \vec{F}$$

energy

$$\frac{DB}{Dt} = Q$$

continuity

$$\nabla\cdot\vec{u} = 0$$

vorticity

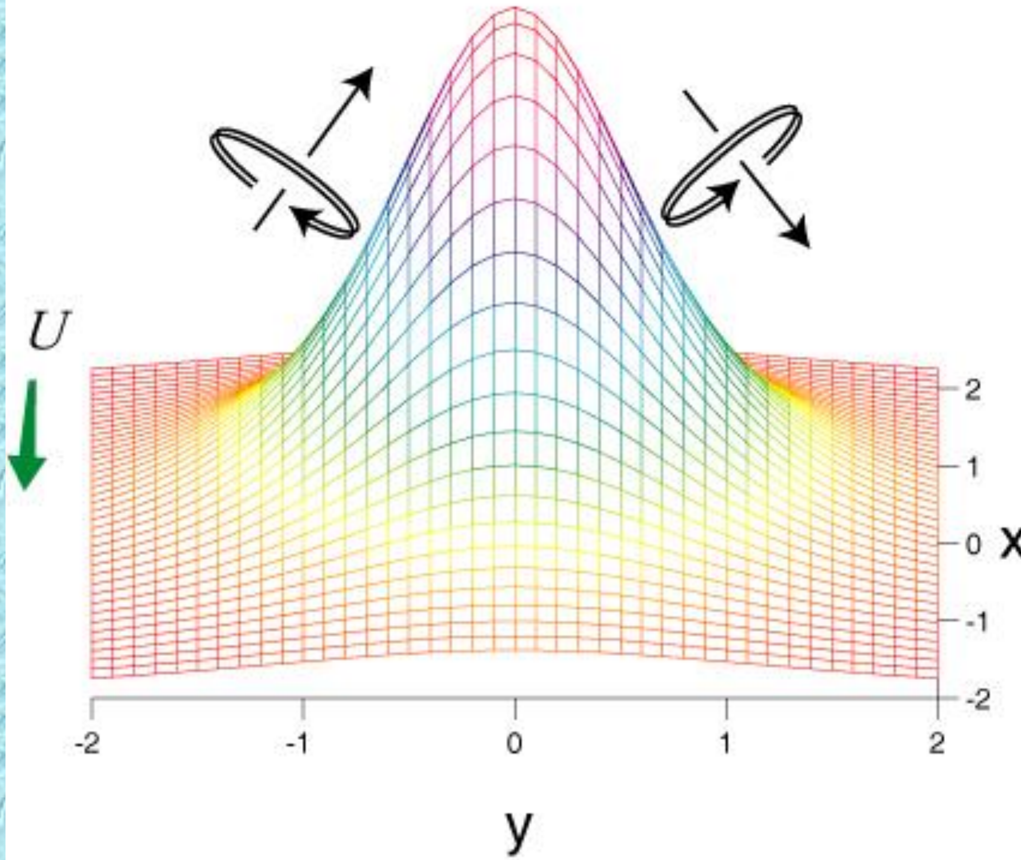
$$\frac{D\vec{\omega}}{Dt} = \vec{\omega}\cdot\nabla\vec{u} - \hat{k}\times\nabla B + \nabla\times\vec{F}$$

Assume Newtonian fluid with constant viscosity $\Rightarrow \quad \nabla\times\vec{F} = \nu\nabla^2\vec{\omega}$

$$\vec{u} = (u, v, w) \qquad \vec{\omega} = (\xi, \eta, \zeta)$$

Frictional Contact with Mountain

$$\frac{D\vec{\omega}}{Dt} = \nu \nabla^2 \vec{\omega}$$

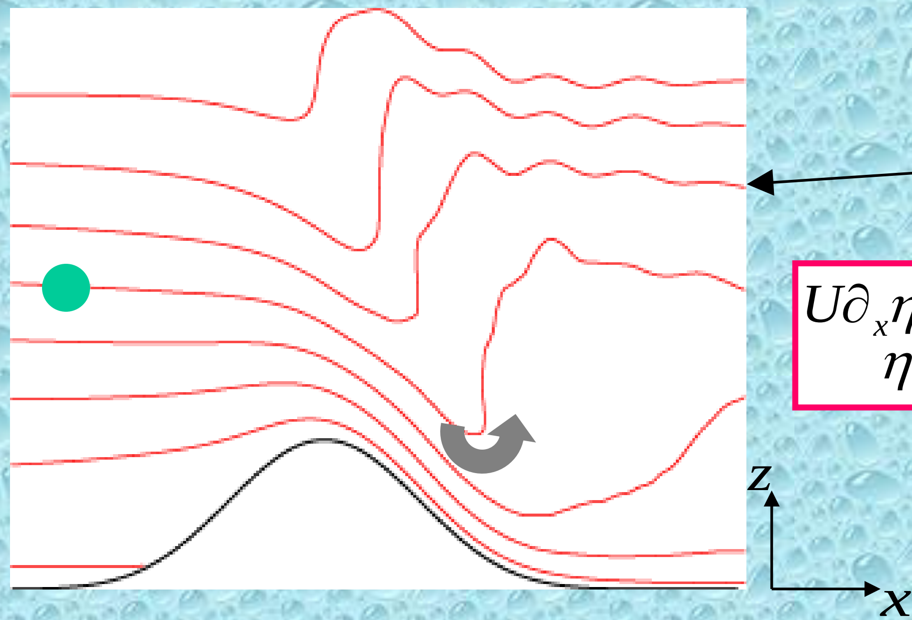


Baroclinicity produces horizontal vorticity

$$\frac{D\vec{\omega}_h}{Dt} = (\vec{\omega}\cdot\nabla)\vec{u}_h - \hat{k}\times\nabla B$$

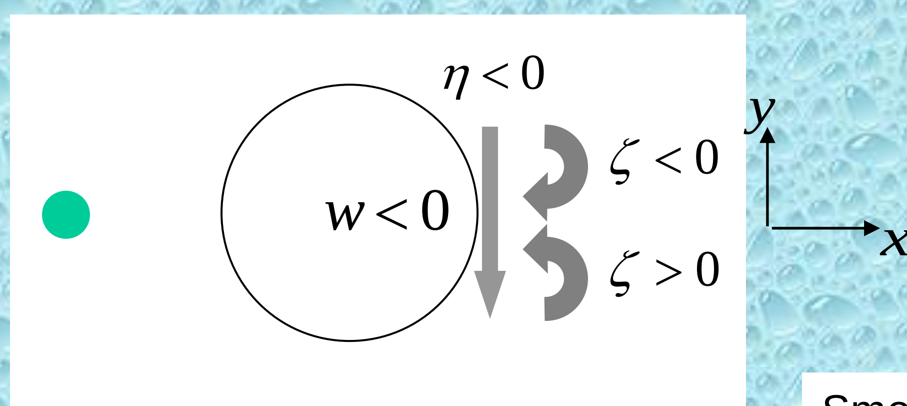
$$\frac{NH}{U} > 1$$

$B = const.$



$$U\partial_x \eta = -\partial_x B + \cdots \Rightarrow$$
$$\eta = -B/U < 0 \quad (B > 0)$$

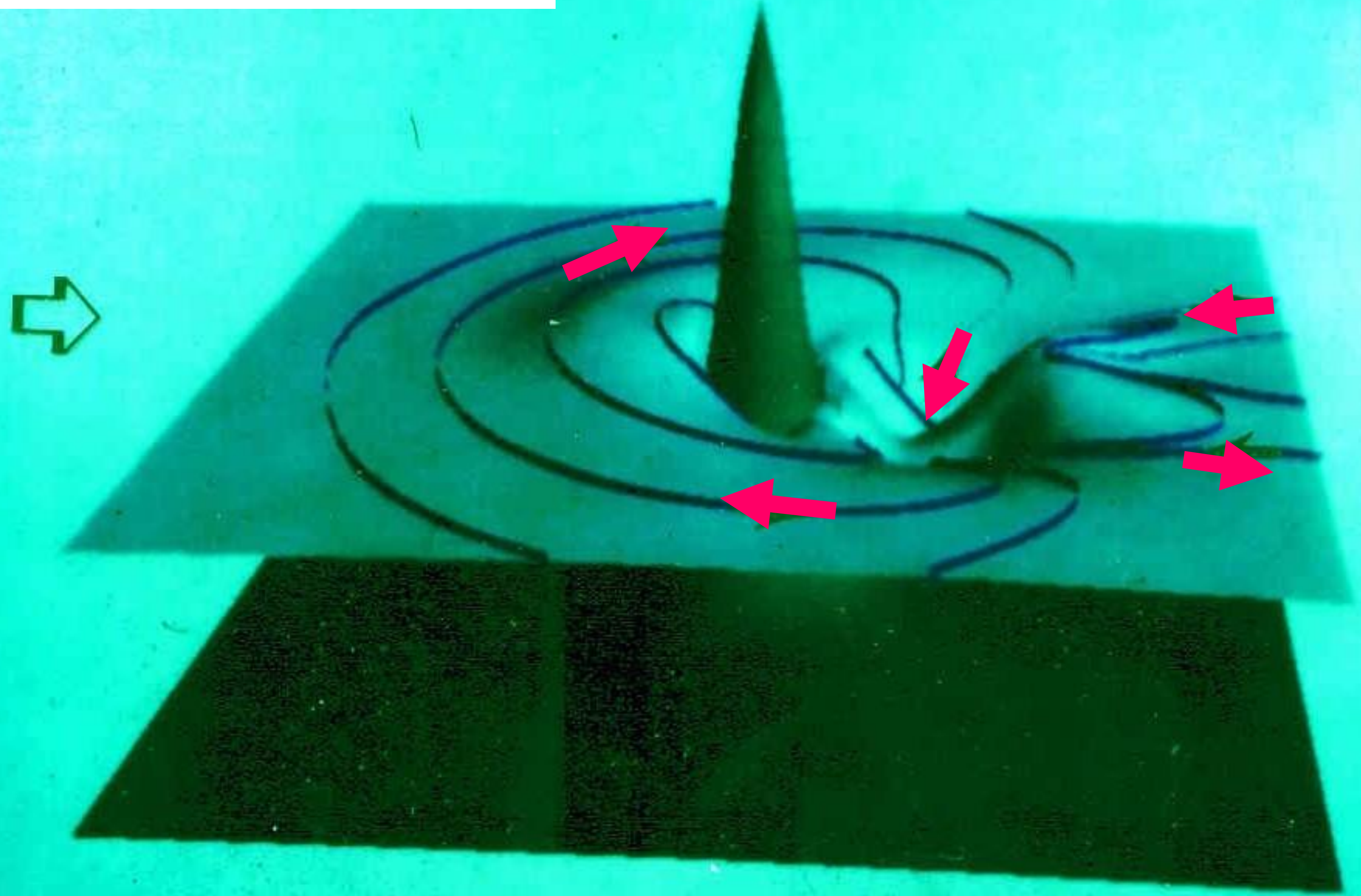
tilting produces vortex pair



$$\frac{D\zeta}{Dt} = \eta\frac{\partial w}{\partial y} + \cdots$$

Smolarkiewicz and Rotunno (1989a)

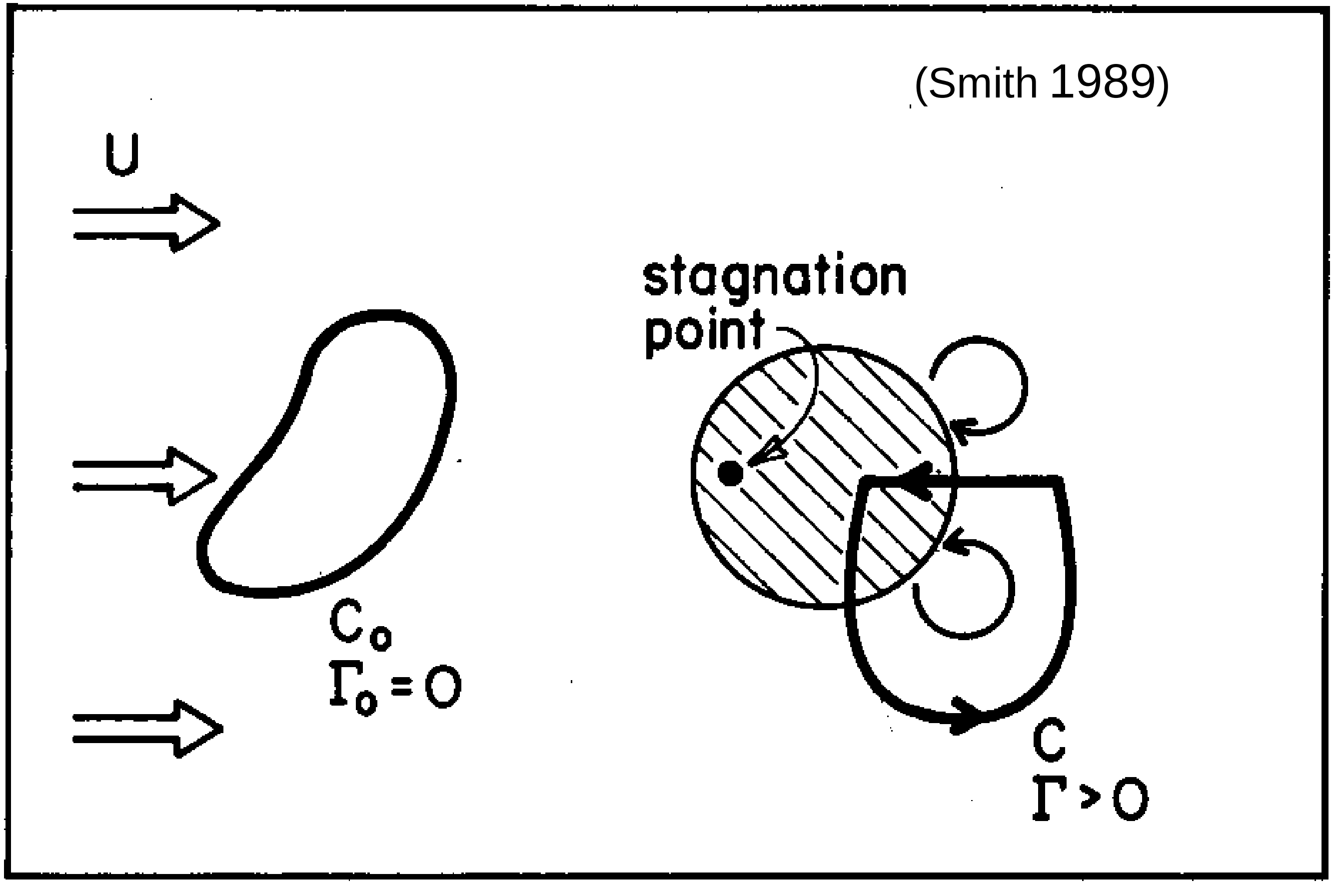
Vortex Lines on B surface



$$q = \vec{\omega} \cdot \nabla B \cong 0$$

Smolarkiewicz and Rotunno (1989a)

A different interpretation: Creation of potential vorticity *q* defines lee vortex



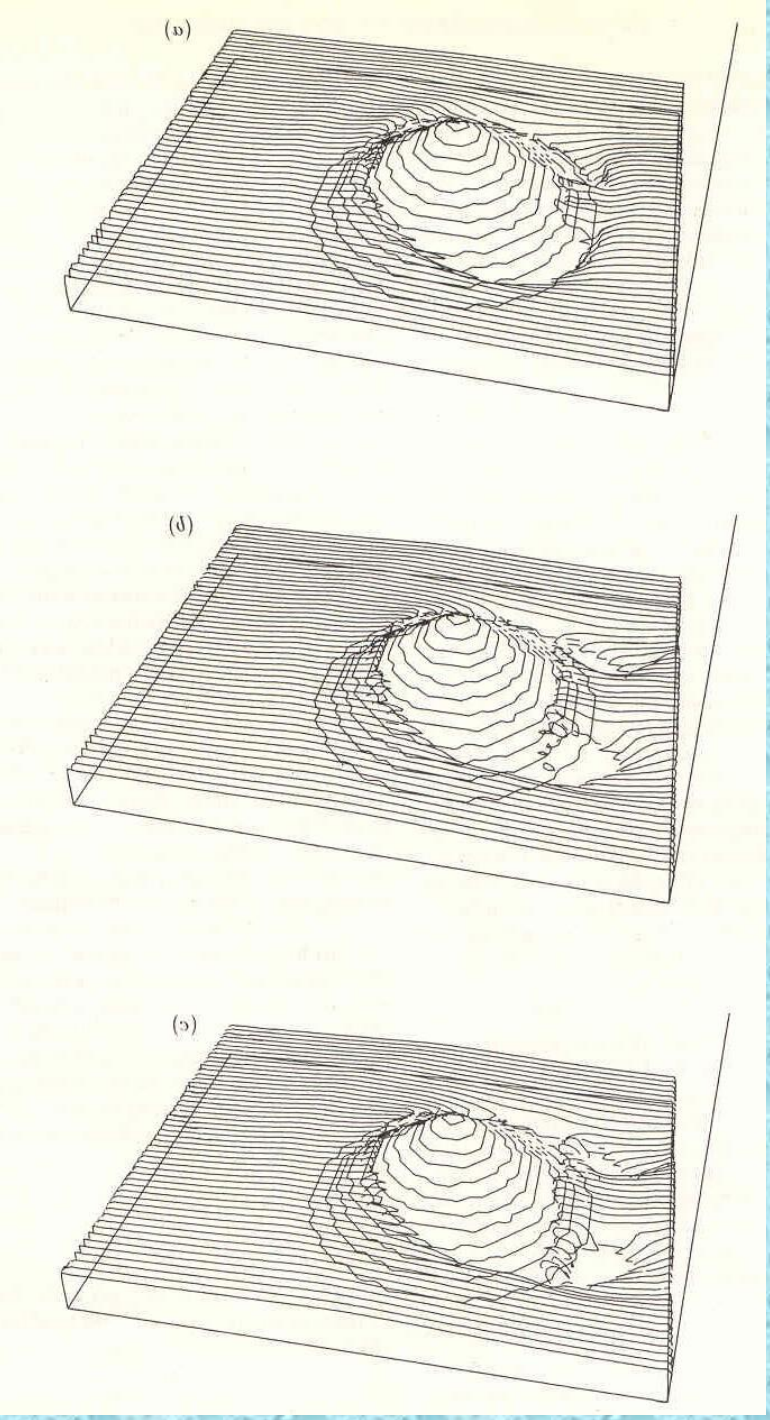
upstream $q = \vec{\omega} \cdot \nabla B = 0$

downstream $q = \vec{\omega} \cdot \nabla B \neq 0$

$$\frac{Dq}{Dt} + \nabla \cdot \vec{J} = 0 \quad , \quad \vec{J} = -\vec{F} \times \nabla B - \vec{\omega} Q$$

Initial Value Problem:
Vortex formation precedes mixing

time



Smolarkiewicz and Rotunno (1989b)

Recent Work

Epifanio and Rotunno (2005,JAS)

-New high–resolution simulations

-Zero stress on surface rigorously enforced

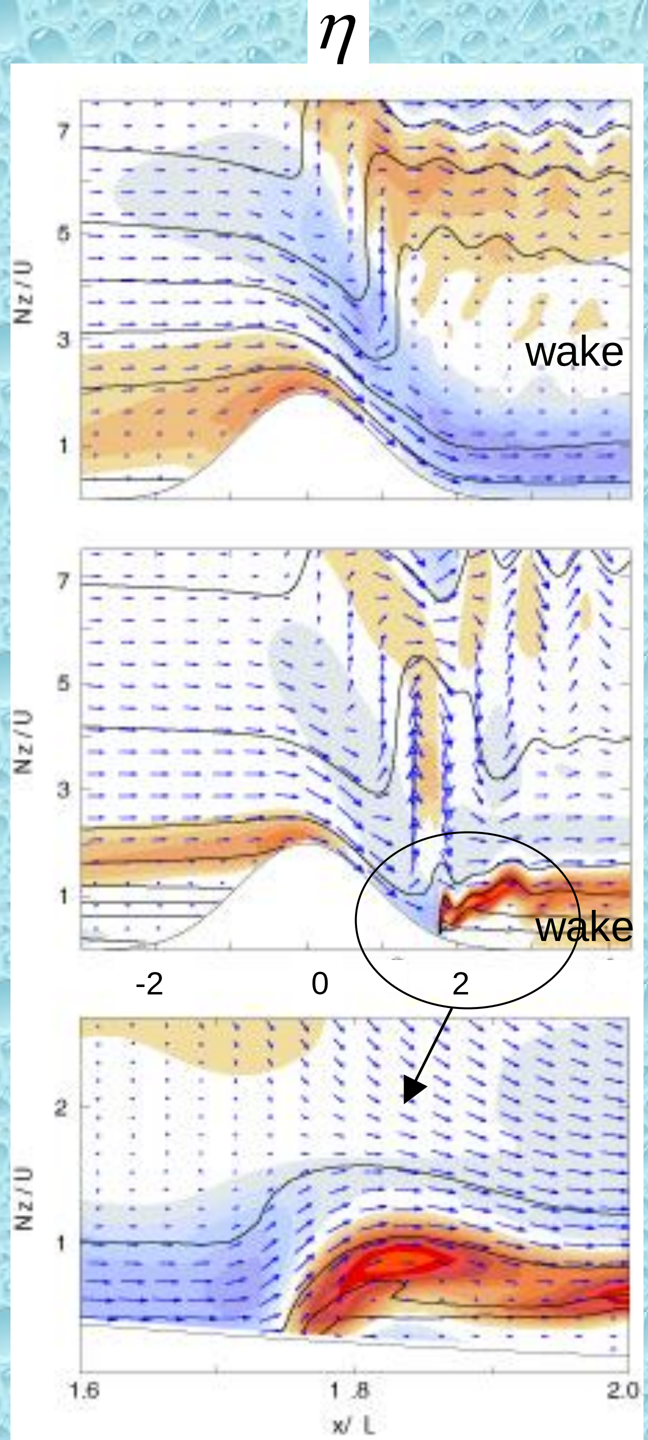
-Contribution to total flow from 3D vorticity deduced

Two types of wake formation

Wave Breaking
N,U constant

Upstream blocking
2-layer *N* with *U* constant

2D Obstacle / 3D *y*-periodic domain



Case with Upstream Blocking

-Following graphs shows evolution of a surface of *B*=constant

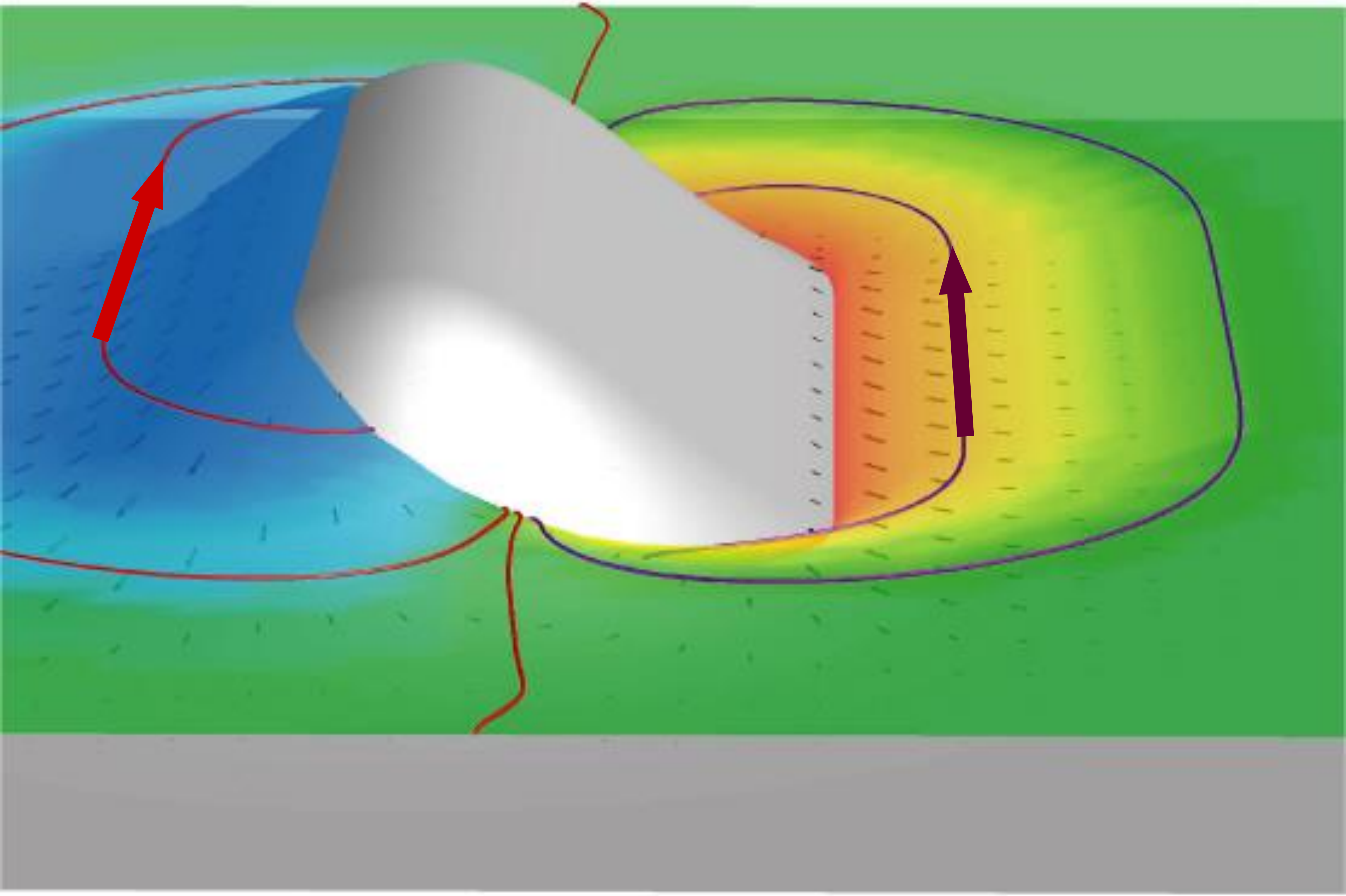
- Flow induced by 3D vorticity field

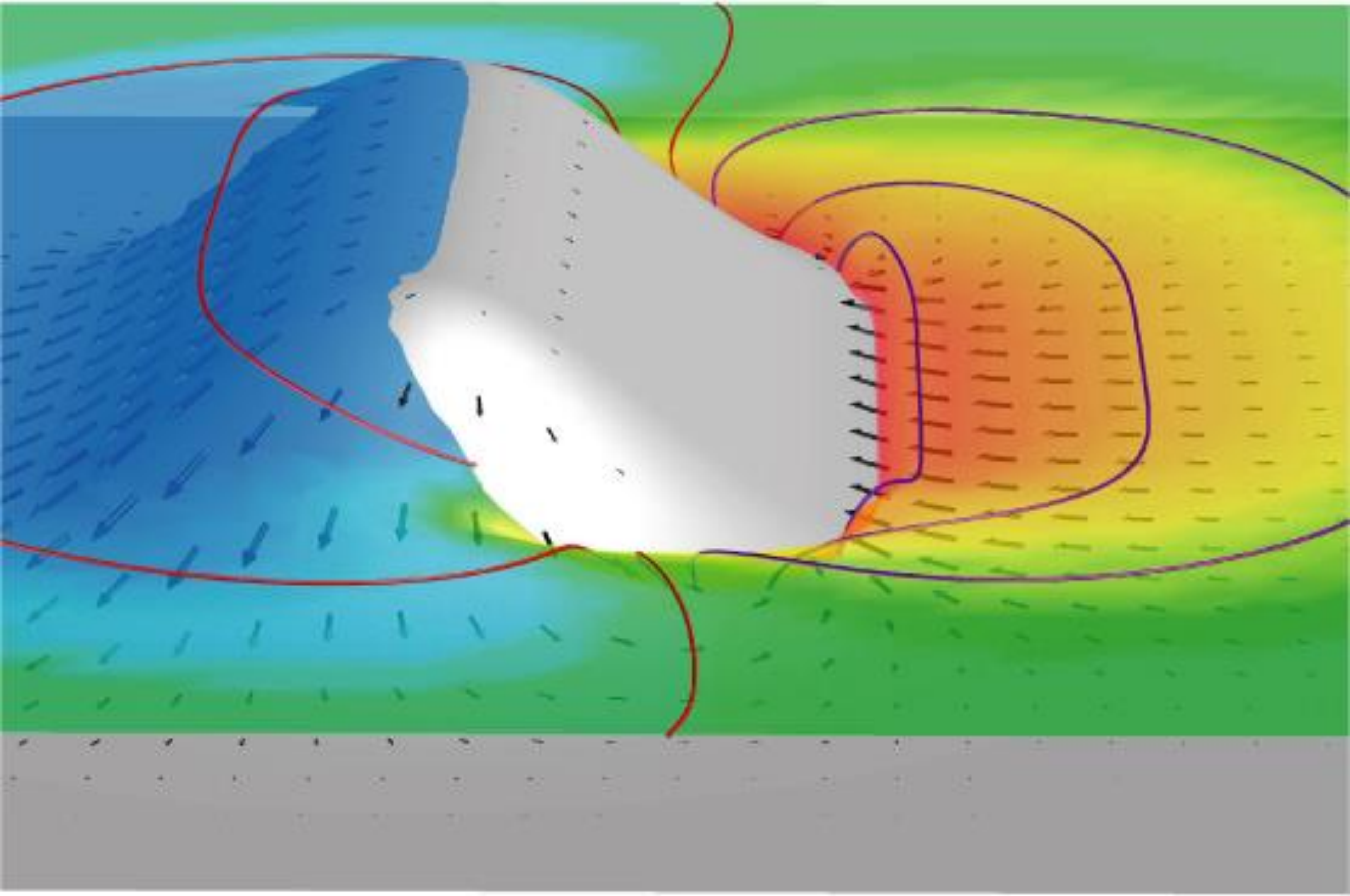
$$\vec{u} = \nabla \chi + \nabla \times \vec{\psi}$$

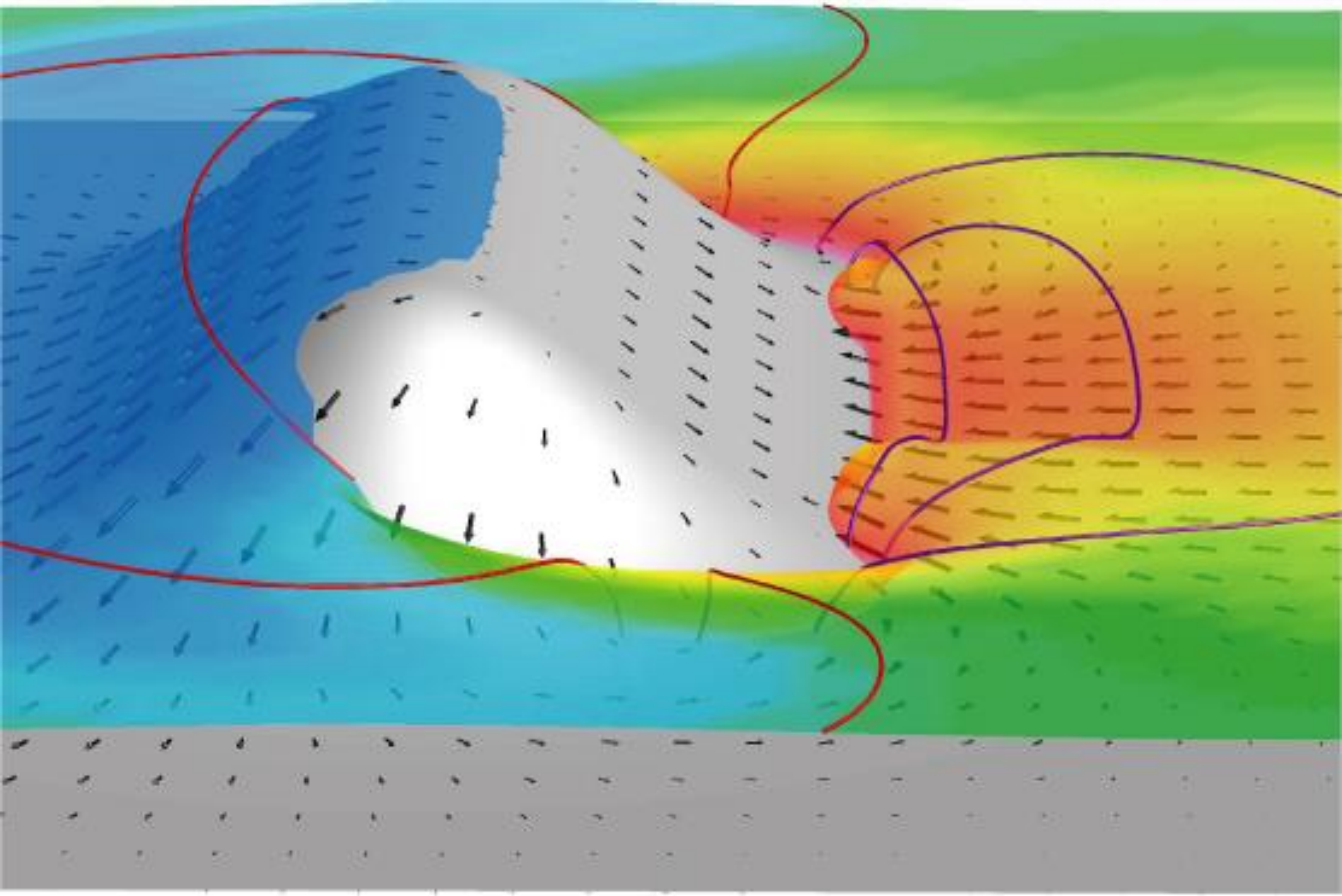
$$\nabla \cdot \vec{u} = 0 \Rightarrow \qquad \nabla^2 \chi = 0$$

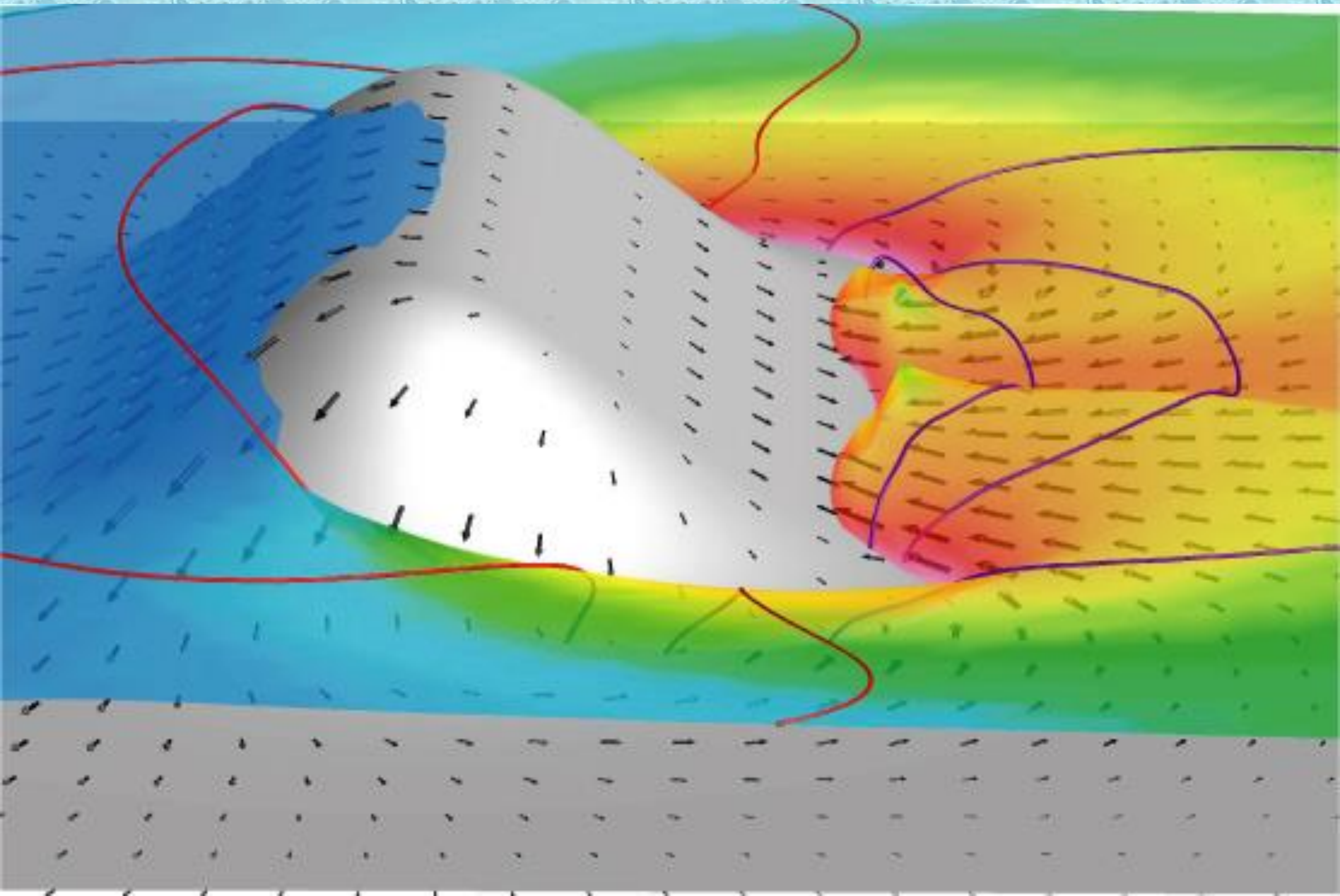
$$\nabla \times \vec{u} = \vec{\omega} \Rightarrow \nabla(\nabla \cdot \vec{\psi}) - \nabla^2 \vec{\psi} = \vec{\omega}$$

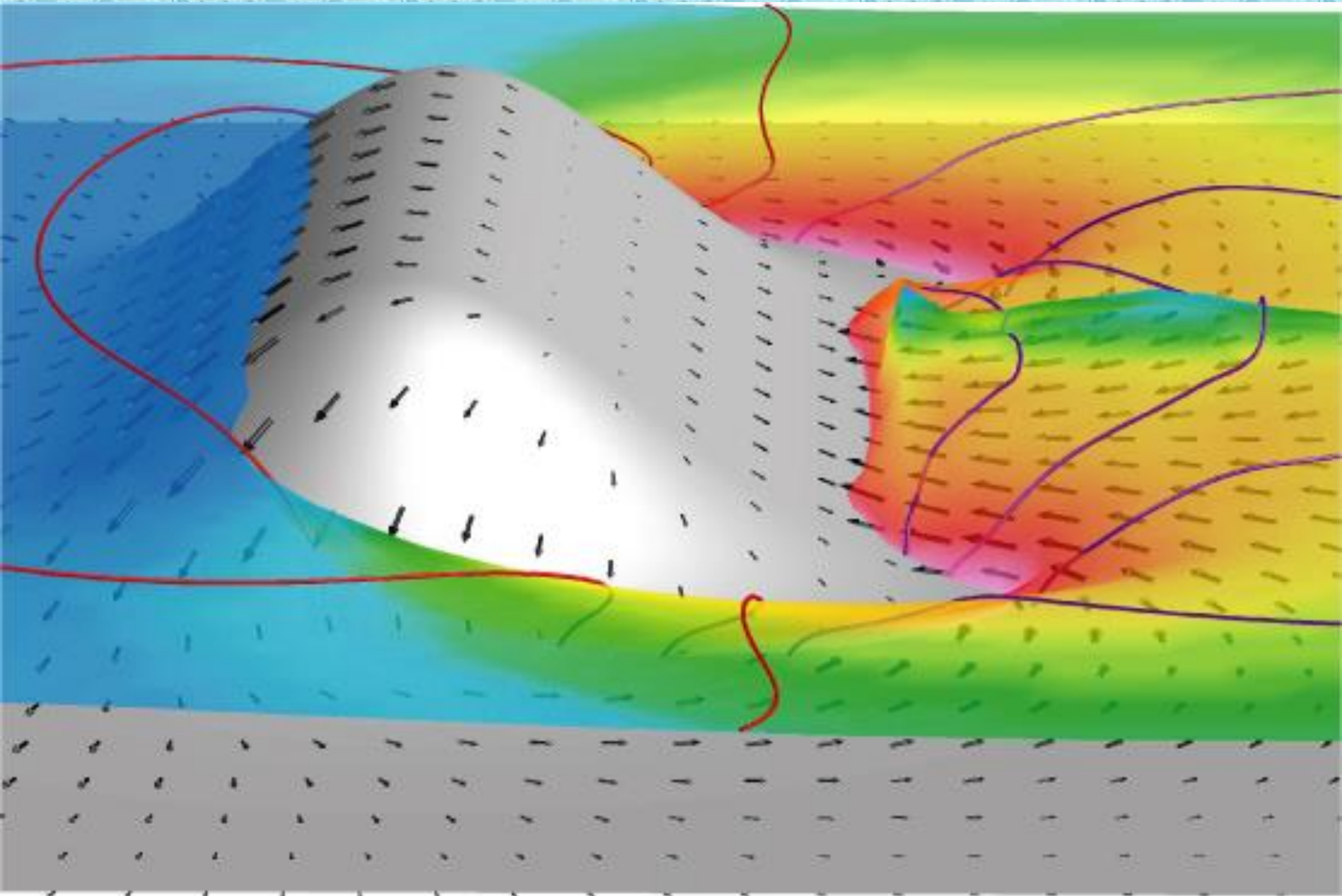
$$\frac{D\vec{\omega}}{Dt} = \vec{\omega} \cdot \nabla \vec{u} - \hat{k} \times \nabla B + \nabla \times \vec{F}$$

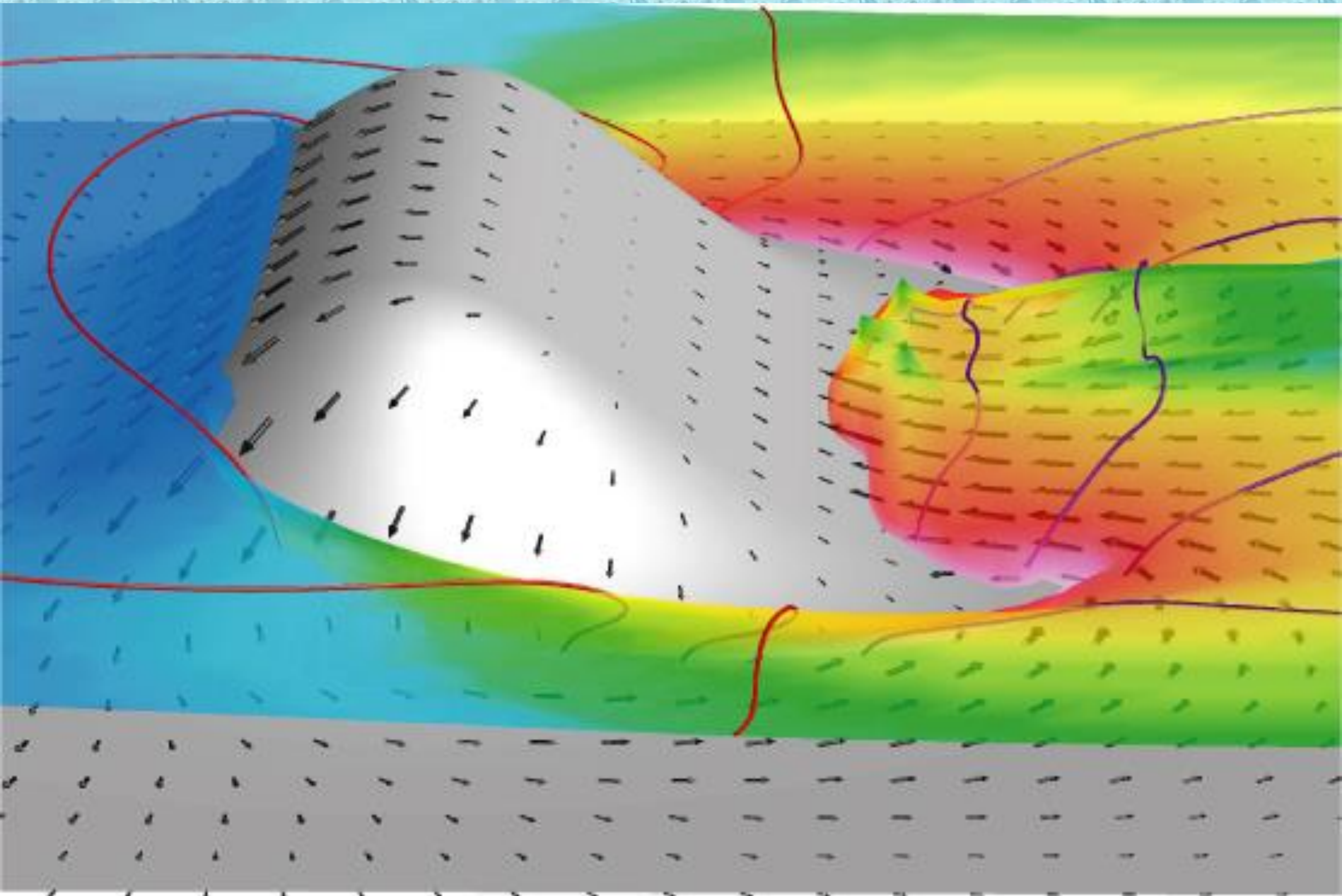


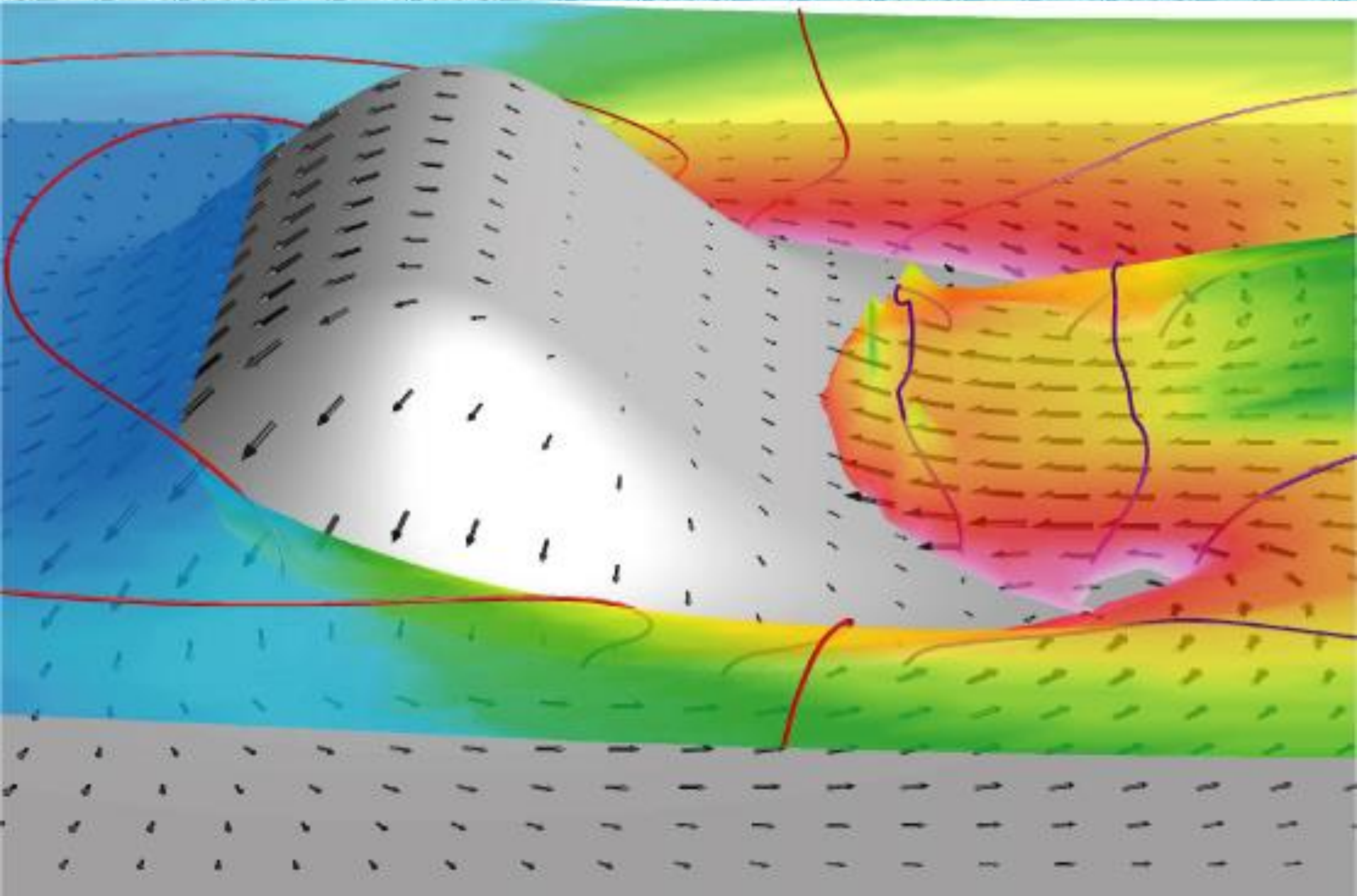


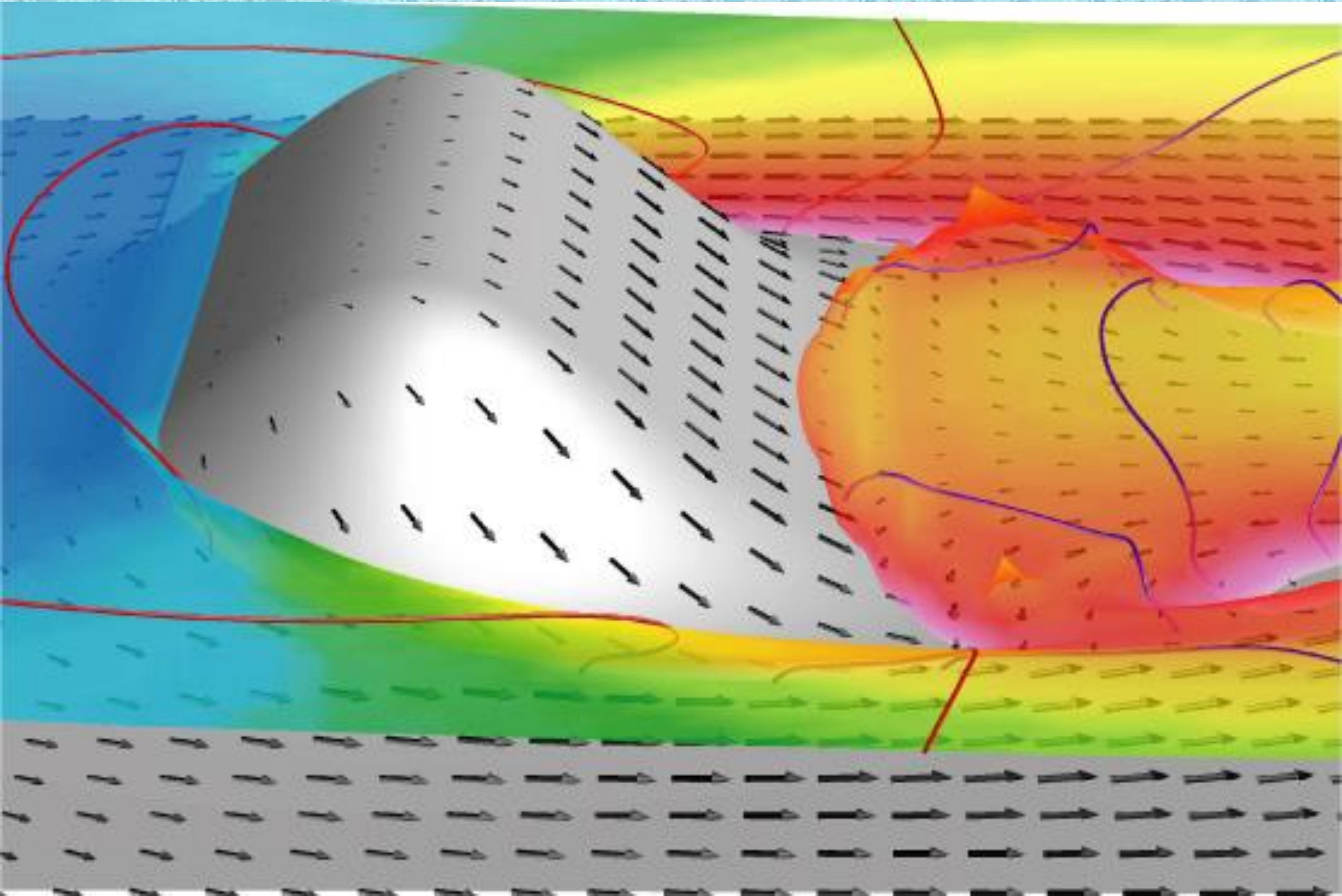










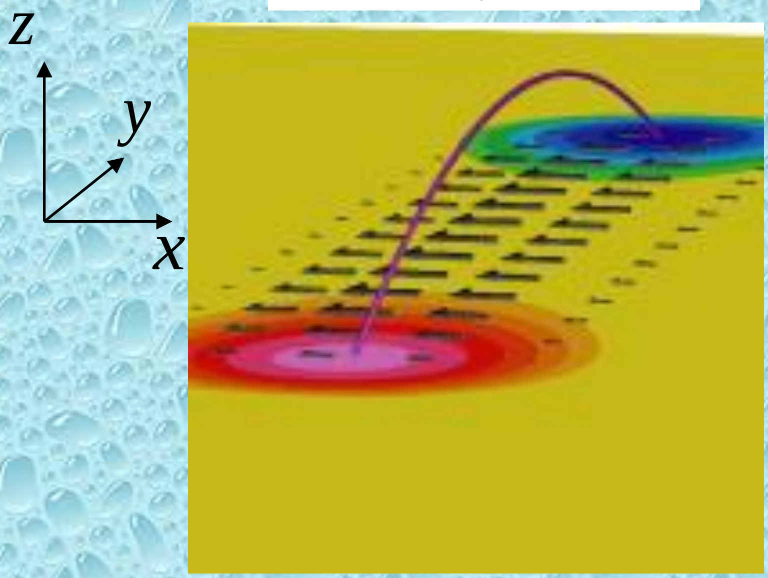


$$\frac{D\vec{\omega}}{Dt} = (\vec{\omega} \cdot \nabla \vec{u}) - \hat{k} \times \nabla B + \nu \nabla^2 \vec{\omega}$$

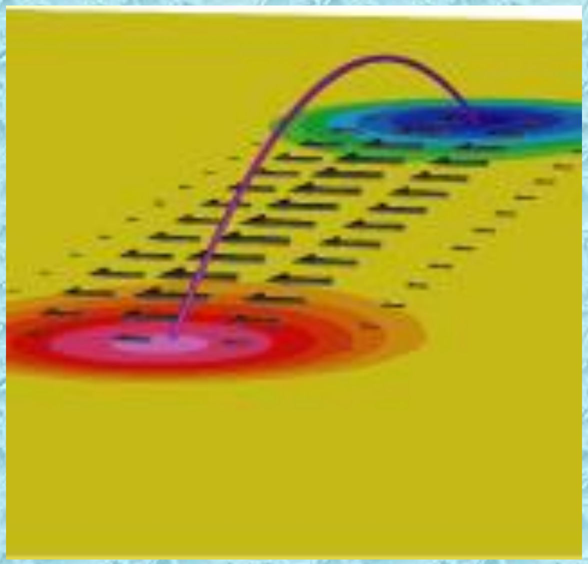
-All the vortical flow originates with baroclinicty

-For low aspect ratio obstacles *H / L << 1*
almost all of the rotational flow induced by the horizontal vorticity components

full vorticity inversion



small *H/L* inversion



Summary

- Baroclinicity is a fundamental source of lee-vortex vorticity in stratified flow
- Numerical simulations of an initial-value problem show vortex formation as an adjustment under gravity of constant density surfaces displaced by the motion of the obstacle relative to the fluid.