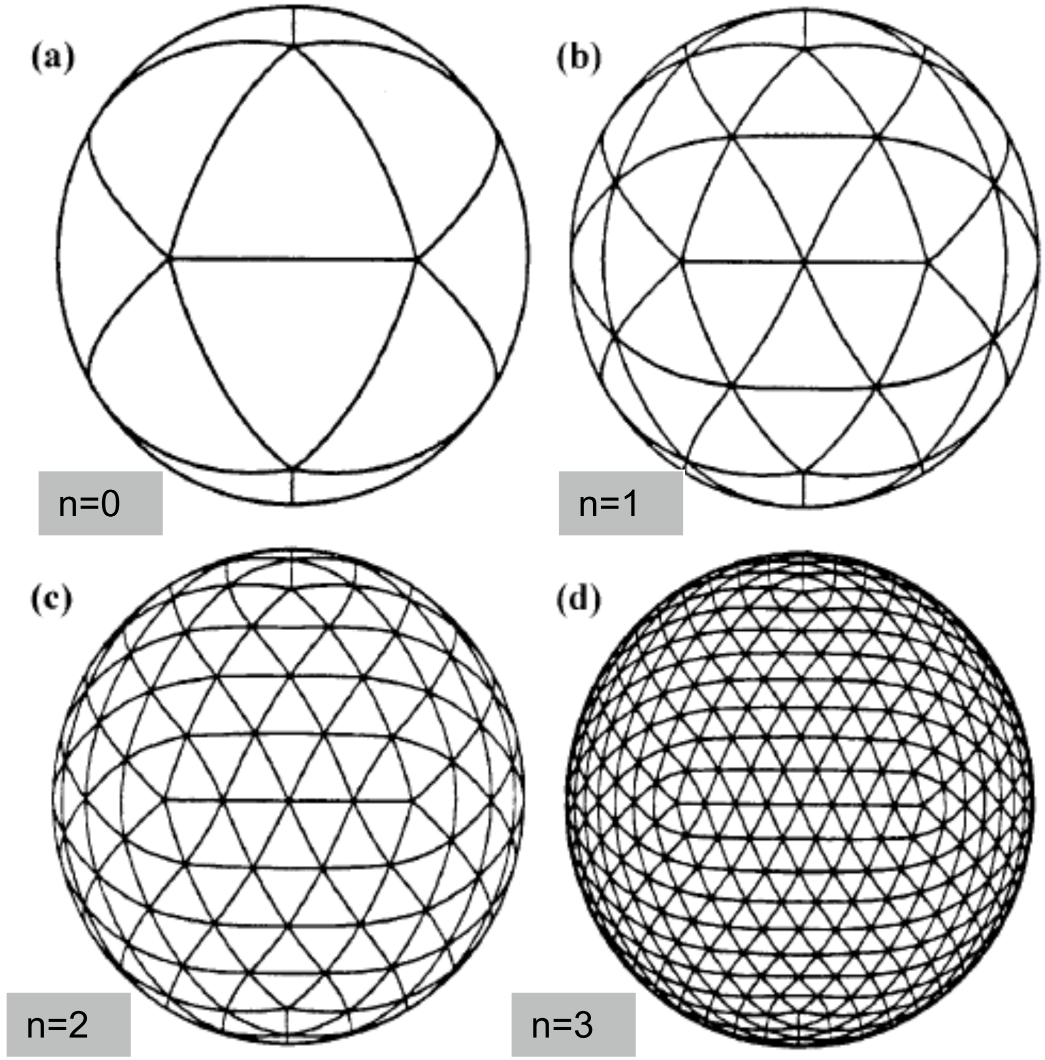
Discretization of Icosahedral Grids on the Sphere

Jin Lee @ GSD/ESRL

Outline of this talk

- Local .vs. Spherical coordinates
- Icosahedral-hexagonal grid inhomogeneity, grid noises, and optimizations: Impacts on numerical accuracy.
- Icosahedral grid descritization and TEs.
- Results on the use of f.-v. Icoshedral model for weather forecasts; FIM and NIM (non-hydrostatic Icosahedral model).
- Preliminary results on NIM
- Numerical ssues related to Icosahedral grid descretizations.

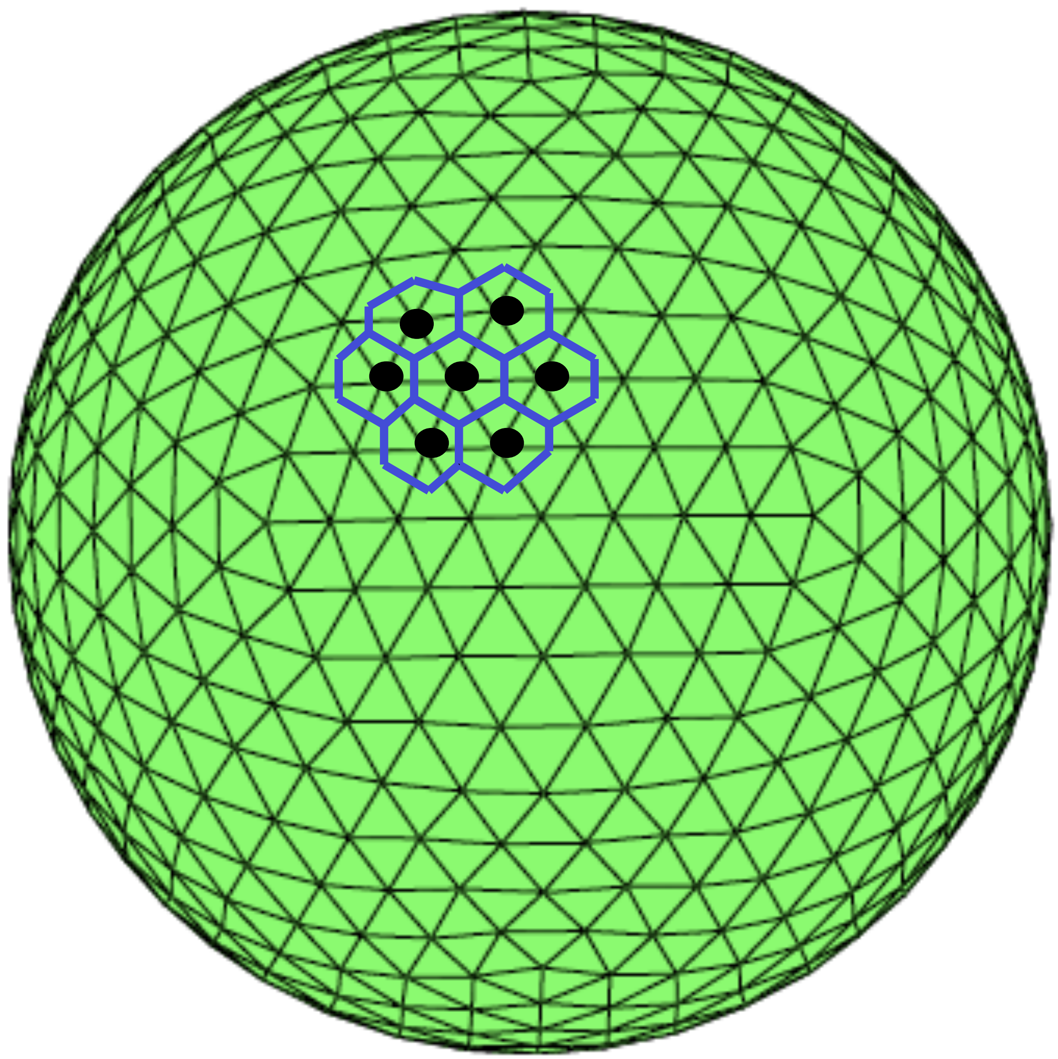
What Is An Icosahedral Model



N=((2**n)**2)*10 + 2 ; 5th level – n=5 → N=10242 ~ 240km; max(d)/min(d)~1.2
6th level – n=6 →N= N=40962 ~ 120km; 7th level – n=7→N=163842   ~60km
8th level – n=8→N=655,362   ~30km;   9th level – n=9→N=2,621,442 ~15km
10th level ~7.km;   11th level ~3.5km , 12th level ~1.7km

Icosahedral-Hexagonal Grid

(Voronoi corner points)

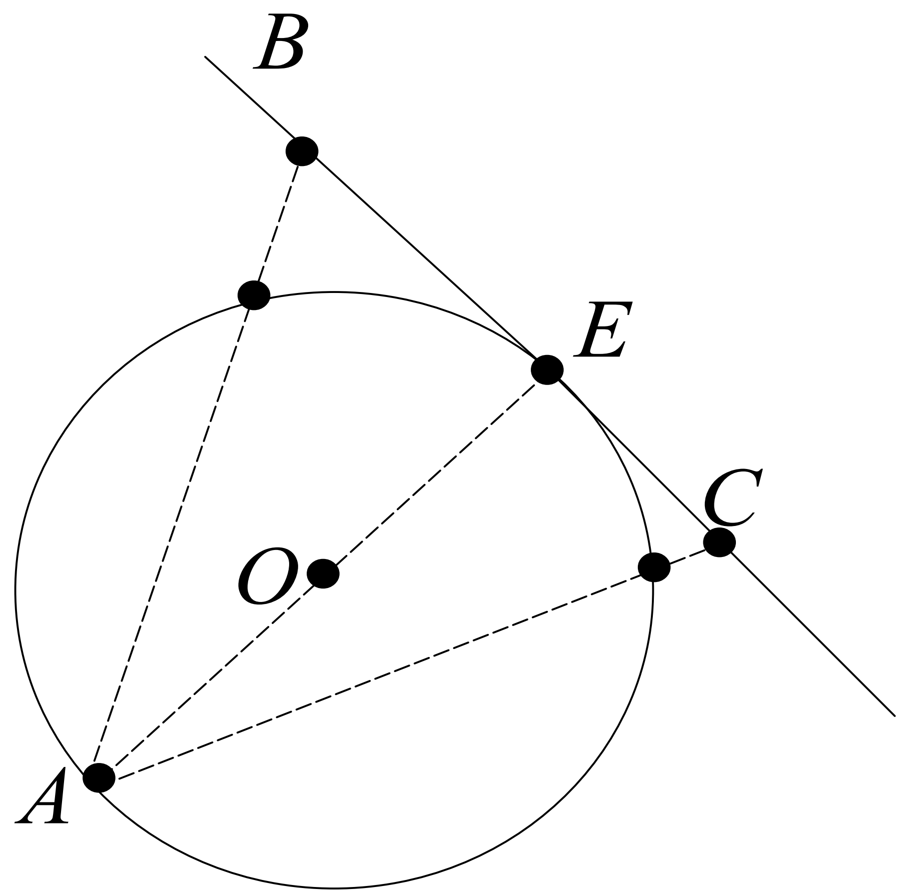


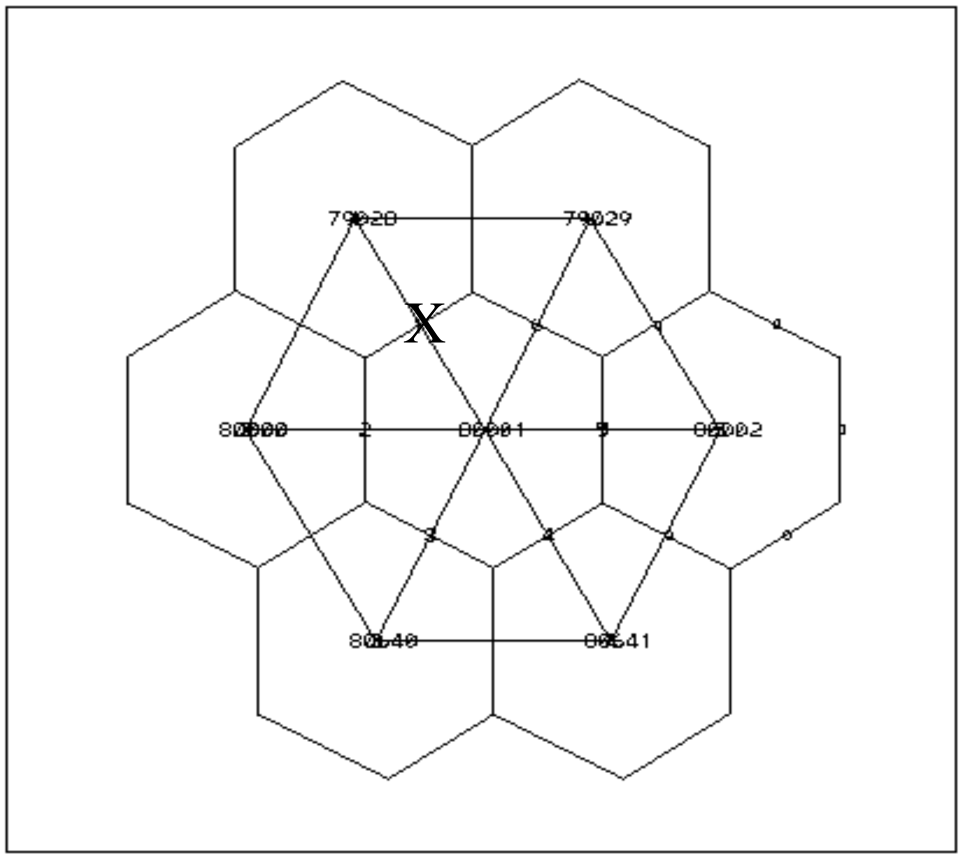
Choice of coordinates for geodesic grids

Spherical .vs. Local coord.

(ping pong .vs. golf balls)

General Stereographic Projection





Finite volume flux computation:

- flux in/out each cell from surrounding donor cells

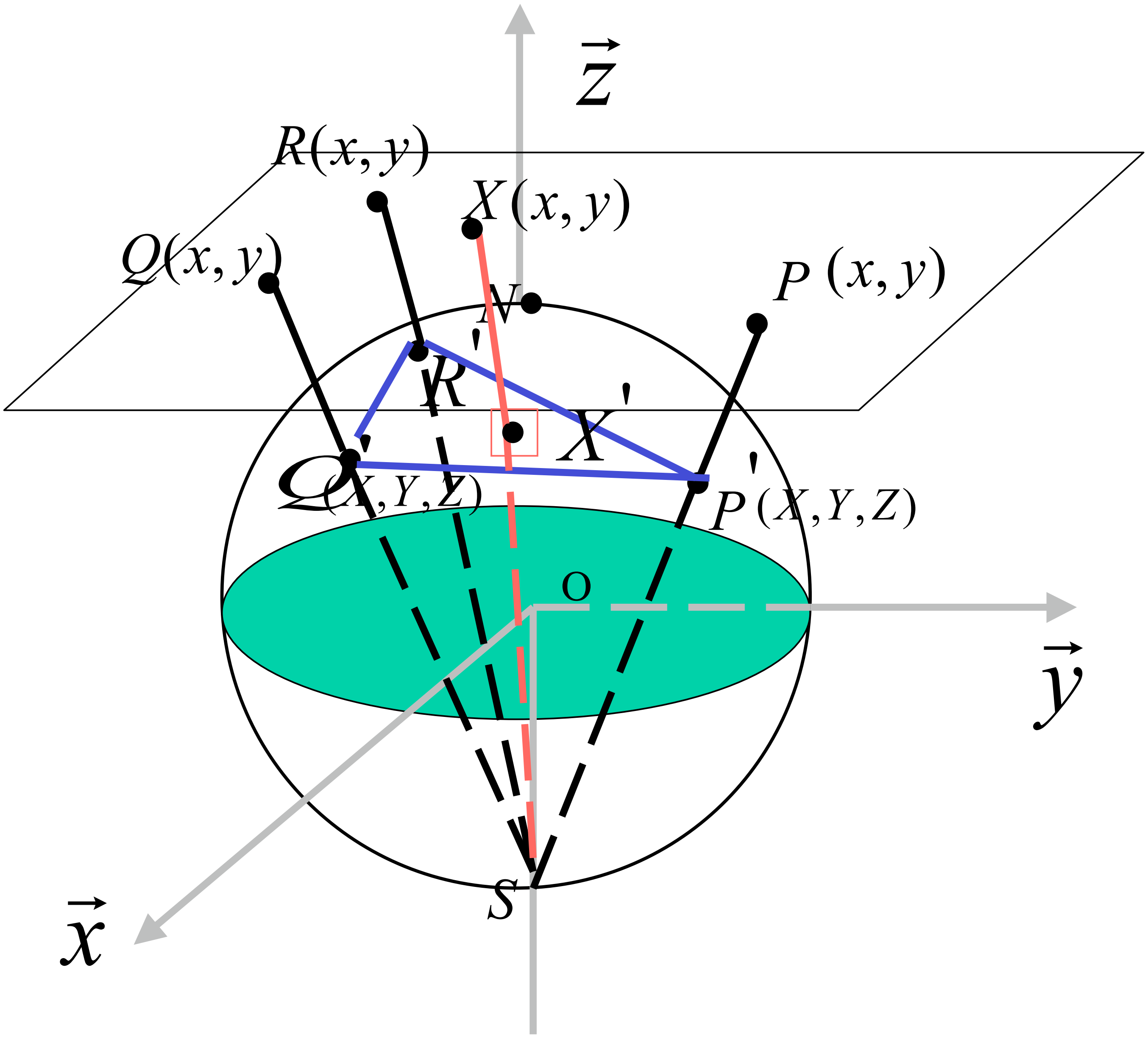
Polynomial interpolation of (n + 1) points means that

$$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{n-1} x^{n-1} + a_n x^n$$

It can be written in a matrix - vector form as follow :

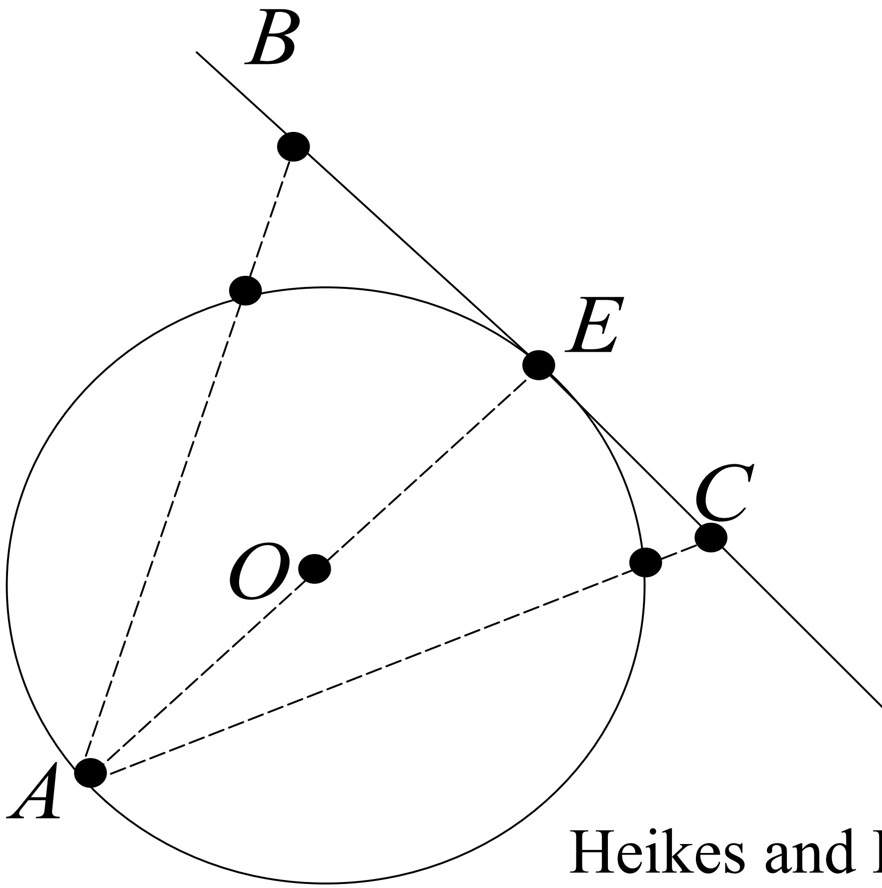
$$\underbrace{\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & \cdots & x_1^{n-1} & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & \cdots & x_2^{n-1} & x_2^n \\ 1 & x_3 & x_3^2 & \cdots & \cdots & x_3^{n-1} & x_3^n \\ \vdots & \vdots & \vdots & \cdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \cdots & \vdots & \vdots \\ 1 & x_{n-1} & x_{n-1}^2 & \cdots & \cdots & x_{n-1}^{n-1} & x_{n-1}^n \\ 1 & x_n & x_n^2 & \cdots & \cdots & x_n^{n-1} & x_n^n \end{bmatrix}}_{\text{Vandermonde Matrix}} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_{n-1} \\ a_n \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ \vdots \\ f_{n-1} \\ f_n \end{bmatrix}$$

A simple 2 - D example : $f(x, y) = a_1 + a_2 x + a_3 y$



$\vec{z}$
$R(x, y)$
$X(x, y)$
$Q(x, y)$
$P\ (x, y)$
N
R'
X'
$Q'(X, Y, Z)$
$P'(X, Y, Z)$
O
$\vec{y}$
S
$\vec{x}$

General Stereographic Projection



Heikes and Randall(1995): used 6-pts stencils on sphere for VDMN with a recursive procedure to establish coplanar stencil points.

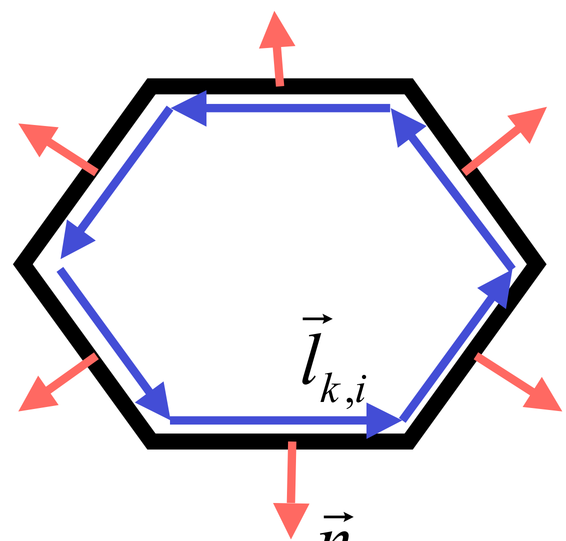
Justification (ii),

The 2-D operator applied to the straight lines, rather than the 3-D operator along the curved lines, e.g.,

Stoke's theorem:

$$\varsigma = \int_A \left(\nabla_h \times \vec{V}_h\right) dA = \oint_s \left(\vec{V}_h \cdot \vec{l}\right) ds$$

$$\varsigma_k = \frac{1}{A_k} \sum_{i=1}^{n} \left(\frac{\vec{V}_{k,i} \cdot \vec{l}_{k,i}}{m_{k,i}} \right) \Delta s_{k,i}$$



Divergence theorem:

$$\int_A \left(\nabla_h \cdot \vec{V}_h \phi\right) dA = \oint_s \left(\vec{V}_h \phi \cdot \vec{n}\right) ds$$

$$\delta_{\phi(k)} = \frac{1}{A_k} \sum_{i=1}^{n} \left(\frac{\phi_{k,i} \vec{V}_{k,i} \cdot \vec{n}_{k,i}}{m_{k,i}} \right) \Delta s_{k,i}$$

Justification (iii): Reduce the number of basis functions in Vandermonde matrix.

$$\underbrace{\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & \cdots & x_1^{n-1} & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & \cdots & x_2^{n-1} & x_2^n \\ 1 & x_3 & x_3^2 & \cdots & \cdots & x_3^{n-1} & x_3^n \\ \vdots & \vdots & \vdots & \cdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \cdots & \vdots & \vdots \\ 1 & x_{n-1} & x_{n-1}^2 & \cdots & \cdots & x_{n-1}^{n-1} & x_{n-1}^n \\ 1 & x_n & x_n^2 & \cdots & \cdots & x_n^{n-1} & x_n^n \end{bmatrix}}_{\text{Vandermonde Matrix}} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_{n-1} \\ a_n \end{bmatrix} = \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ \vdots \\ f_{n-1} \\ f_n \end{bmatrix}$$

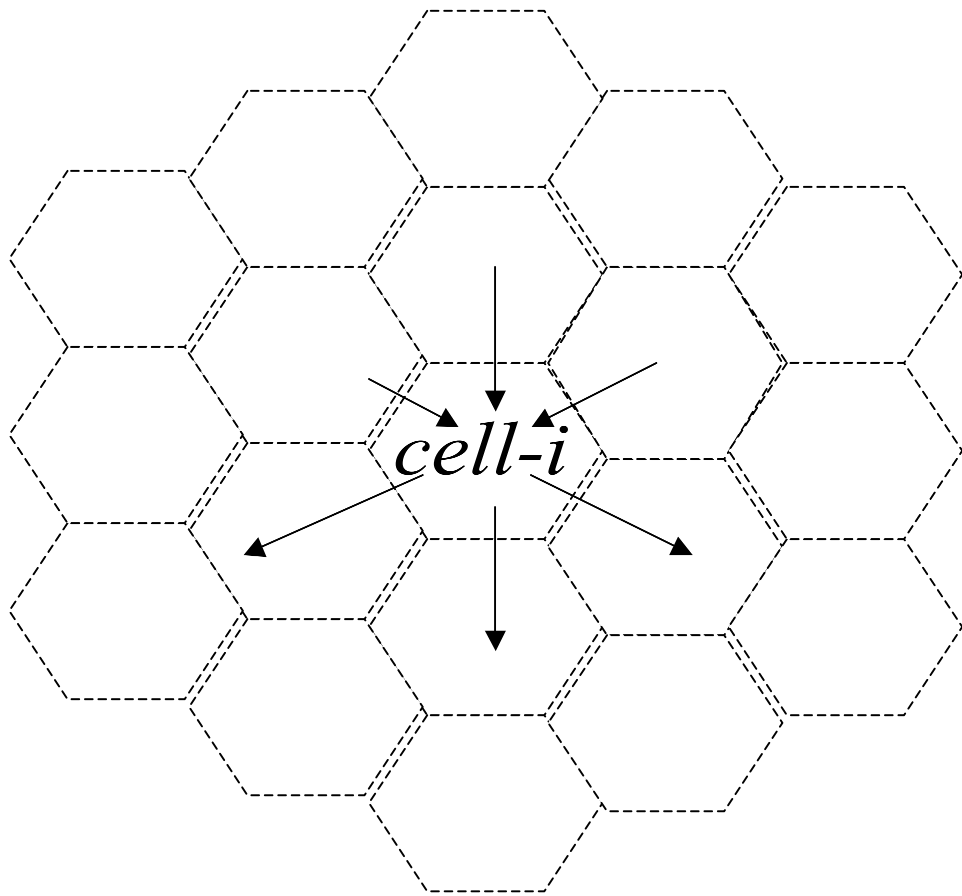
Justification (iv): It requires only a few extra fast product operators

(~DWD MWR 2000: rotate with two great circles)

Monotonicity and Positive Definitness on Icos-grids with AB 3rd scheme

Flux-Correct-Transport (Zalesak, 1979)

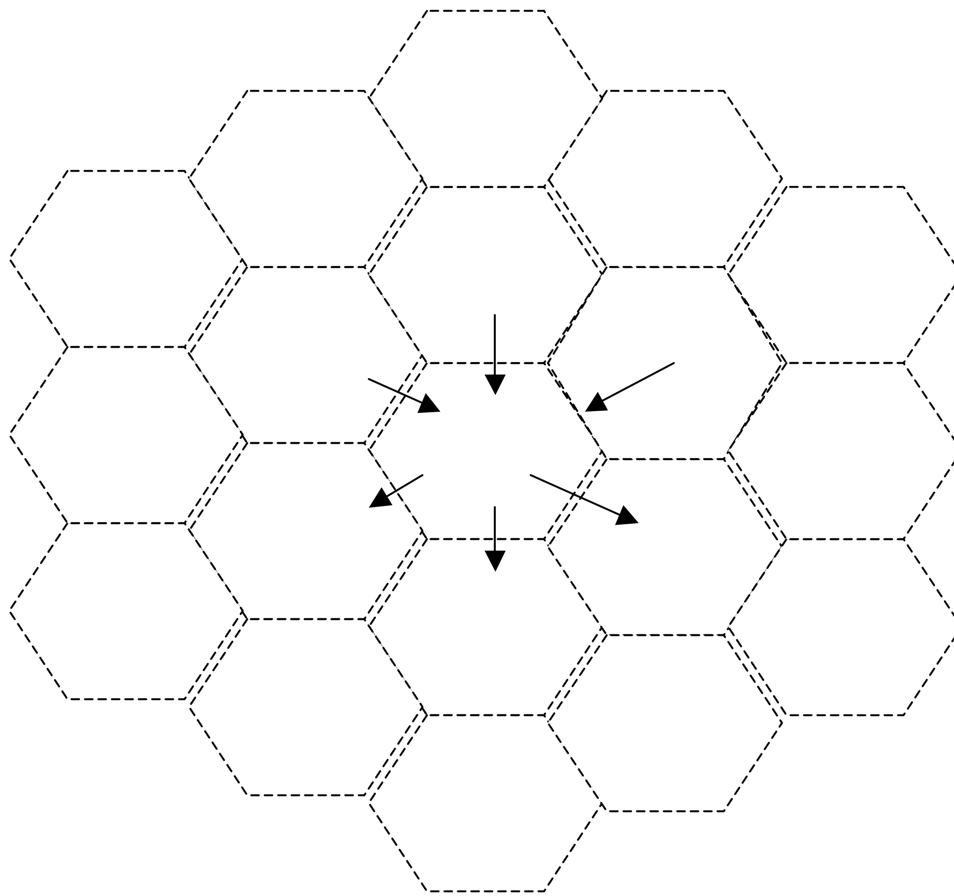
Compute high order fluxes
flxh over all edges
in the domain.



Compute low order fluxes

flxl over all edges

in the domain.

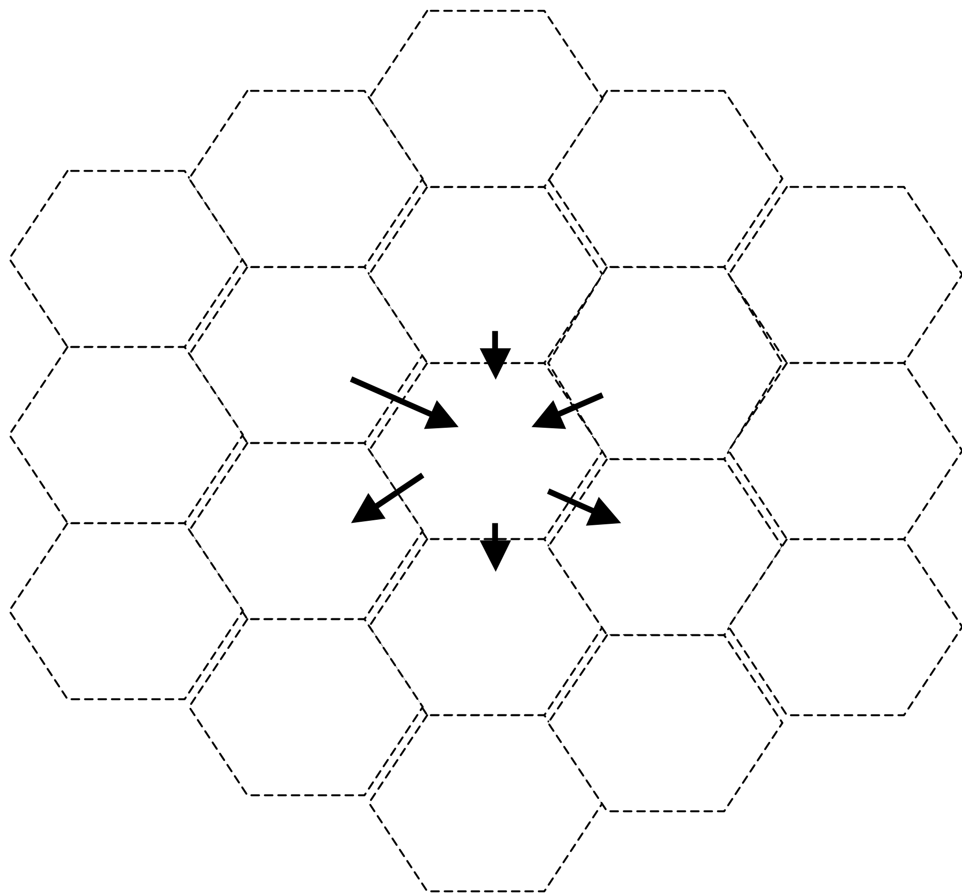


Define antidiffusive fluxes

$fad = flxh - flxl$

over all edges

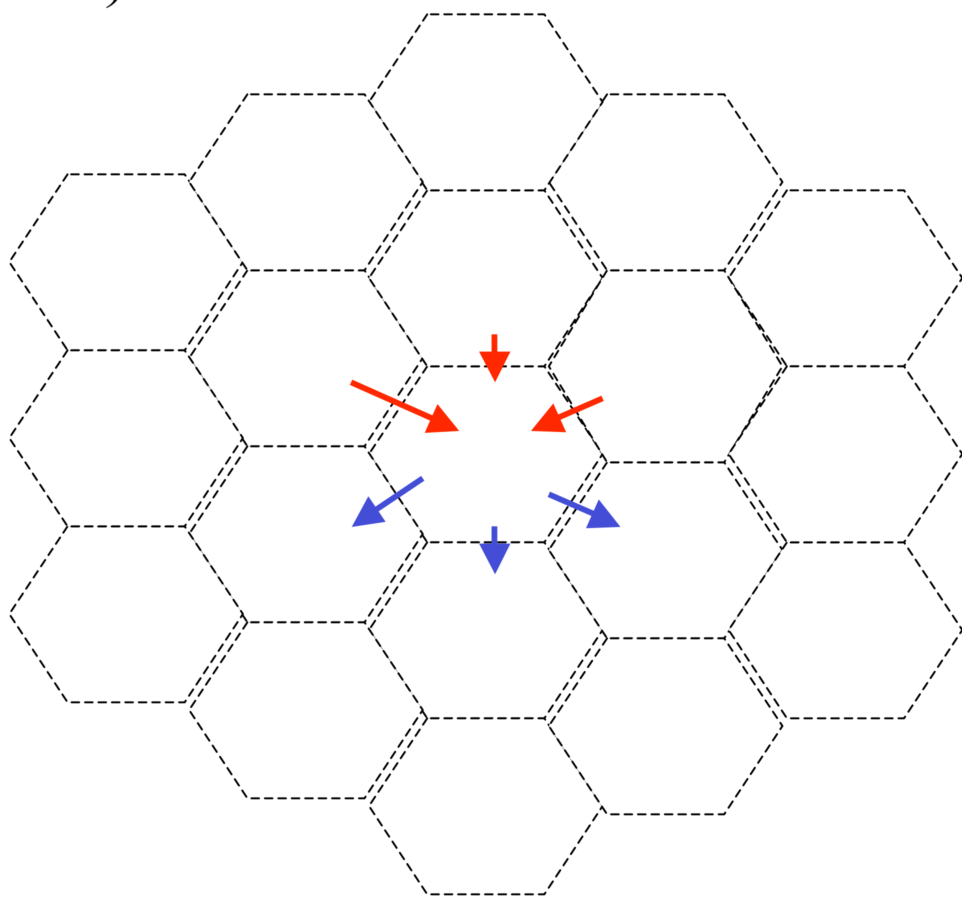
in the domain.



Group antidiffusive fluxes as

$In - fluxex$(in red) and

$out - fluxex$(in blue).

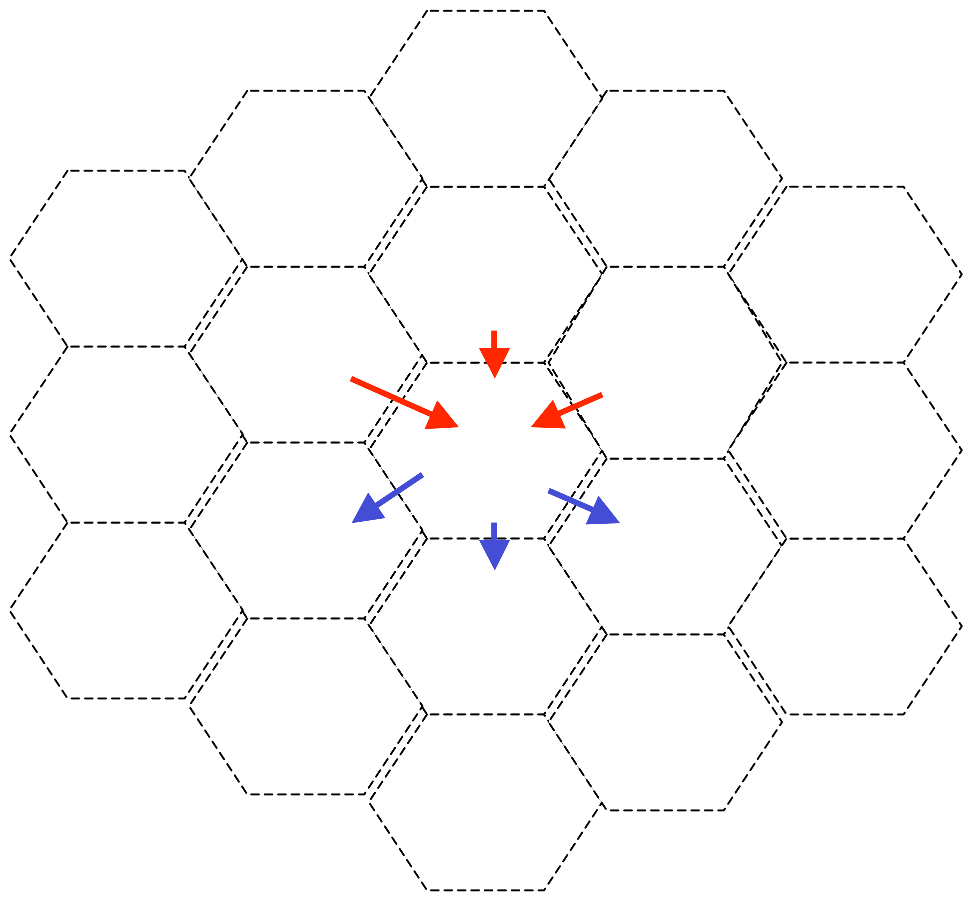


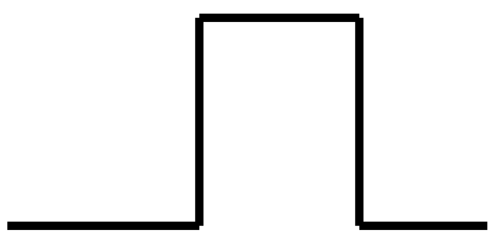
Group antidiffusive fluxes as

$In - fluxex$(in red) and

$out - fluxex$(in blue).

(2 - D formulation, no directional splitting)





Icosahedral SWE :

$$\begin{cases} \dfrac{\partial u}{\partial t} - (f + \zeta)v + m\dfrac{\partial(E_k + \phi)}{\partial x} = 0 \\ \dfrac{\partial v}{\partial t} + (f + \zeta)u + m\dfrac{\partial(E_k + \phi)}{\partial y} = 0 \\ \dfrac{\partial \phi}{\partial t} + m^2\left[\dfrac{\partial}{\partial x}\left(\dfrac{\phi u}{m}\right) + \dfrac{\partial}{\partial y}\left(\dfrac{\phi v}{m}\right)\right] = 0 \end{cases}$$

where, ζ : vorticity, $E_k = \dfrac{u^2 + v^2}{2}$

Numerics of the Icosahedral SWM

- Each Icosahedral cell is solved on a local coordinate.
- Model variables are defined on a non-staggered A-grid.
- Vandermonde Interpolation to estimate edge variables.
- All differentials evaluated as line integrals around the cells using finite-volume conservative discretization.
- Grid points in a linear horizontal loop that allow any horizontal point sequence
- Explicit 3rd-order Adams-Bashforth time differencing.
- Flux Corrected Transport formulated based on the high-order (3rd Order) Adams-Bashforth scheme to maintain conservative positive definite transport.

Issues of Grid In-homogeneity

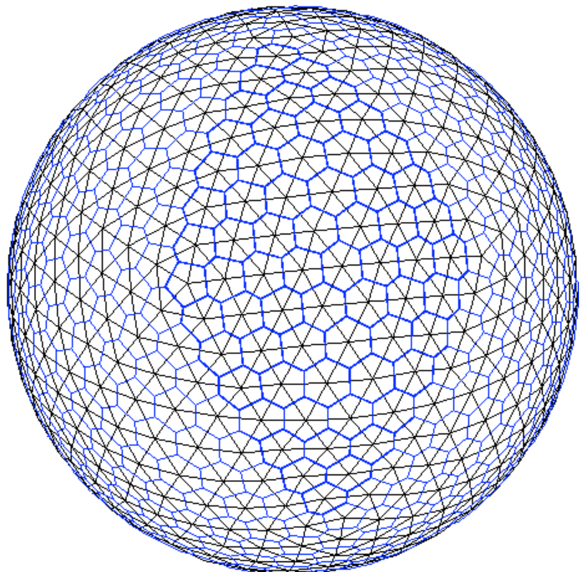
Grid optimization/
impact on numerical accuracy

Geometric properties of Icosahedral-Hexagonal grid on sphere (in preparation)

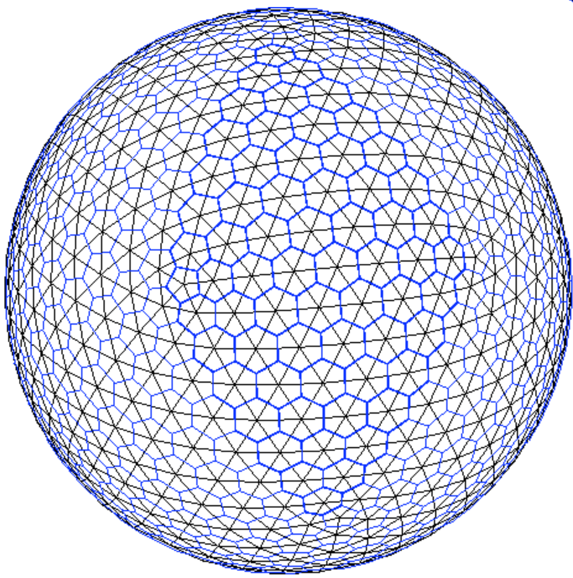
Ning Wang and Jin Lee

Comparisons of three Icosa-grids

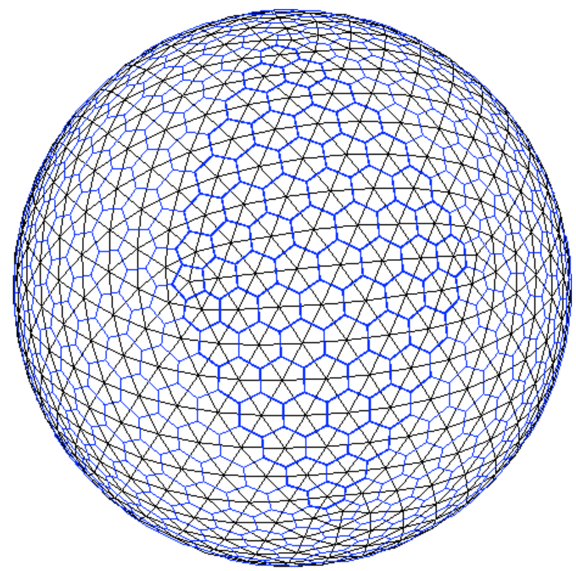
STD: Edge_max/min=1.84



SPN: Edge_max/min=1.30



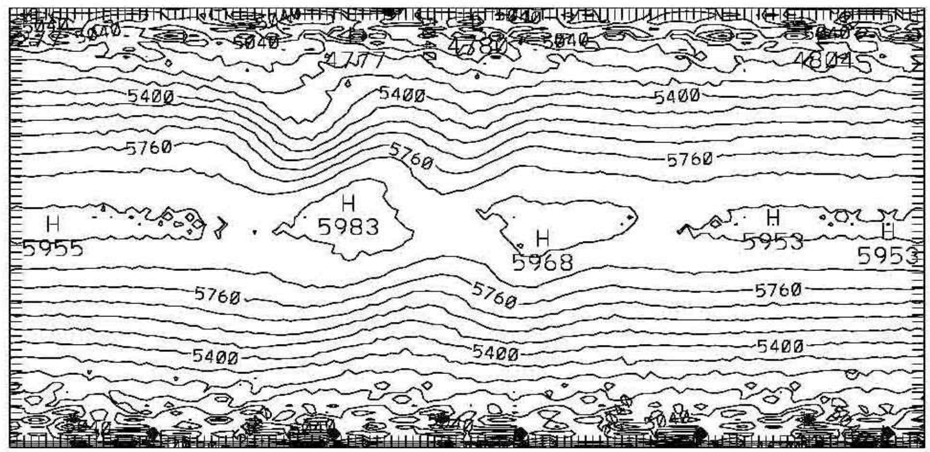
MGC: Edge_max/min=1.32



Williamson etal.(1992) Case V: *Zonal flow over Mountain*

PHI (6 DAY)

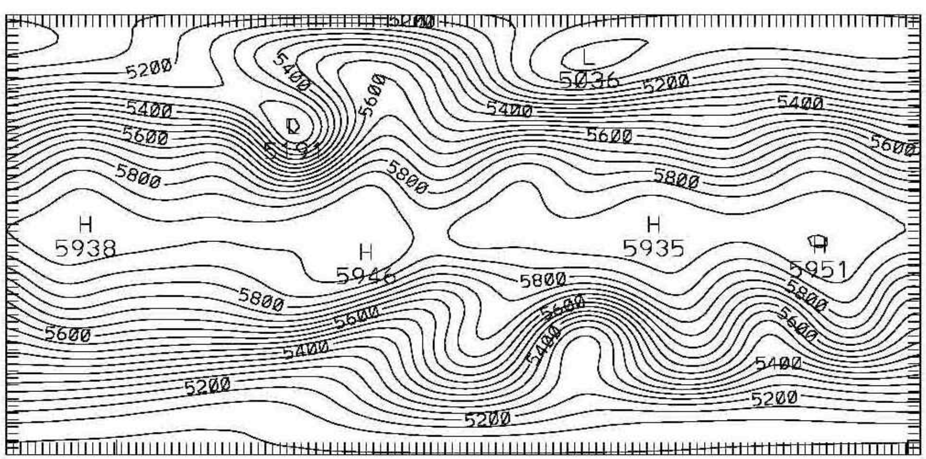
STD



CONTOUR FROM 4410.0 TO 5940.0 CONTOUR INTERVAL OF 90.000 PT(3,3)= 4958.3

PHI (15 DAY)

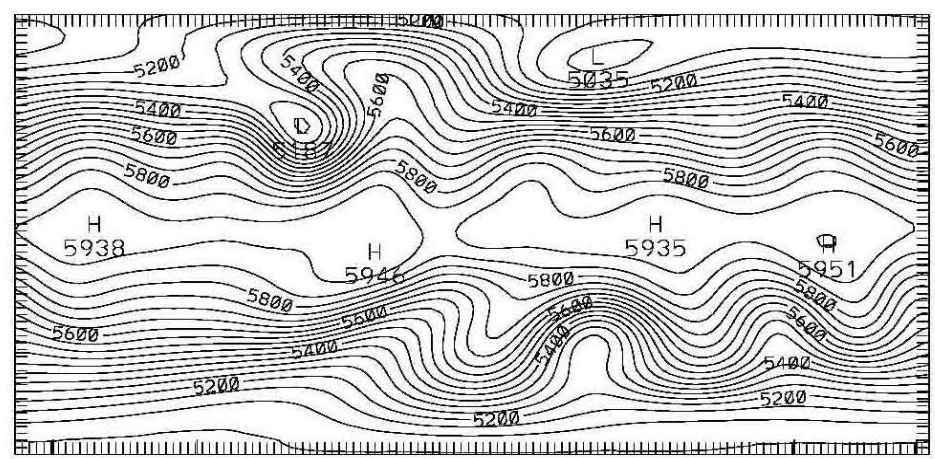
SPN



CONTOUR FROM 5000.0 TO 5950.0 CONTOUR INTERVAL OF 50.000 PT(3,3)= 5051.1

PHI (15 DAY)

MGC

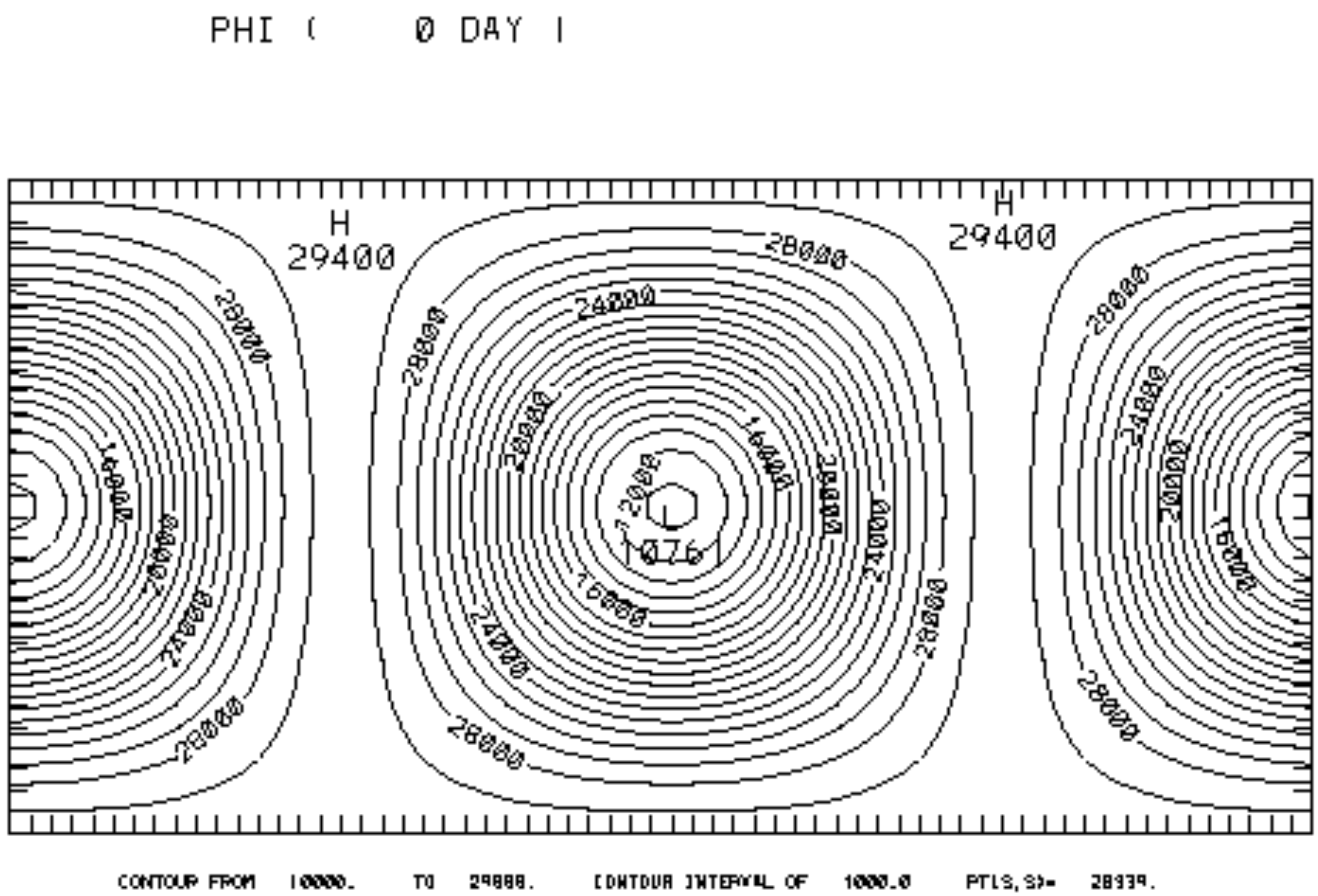


CONTOUR FROM 5000.0 TO 5950.0 CONTOUR INTERVAL OF 50.000 PT(3,3)= 5052.1

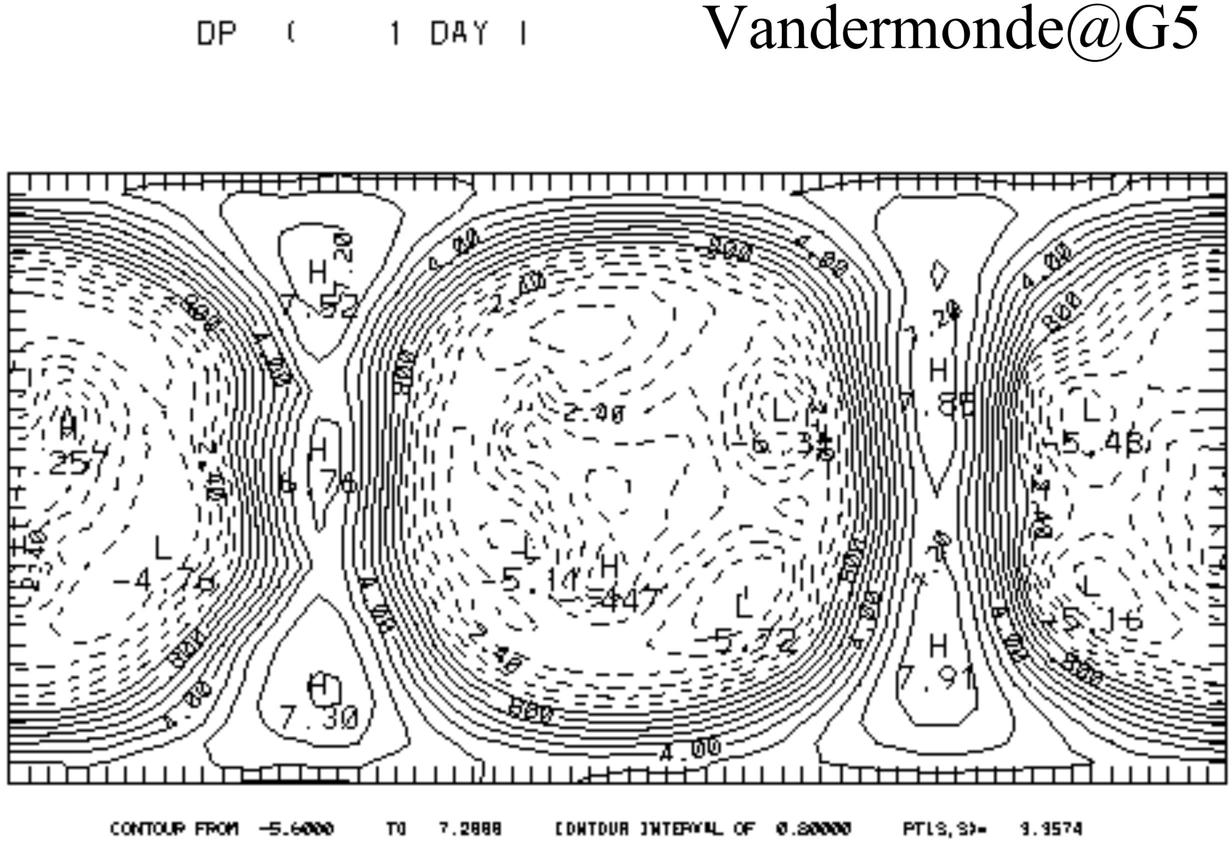
Grid Noises / TEs Convergence

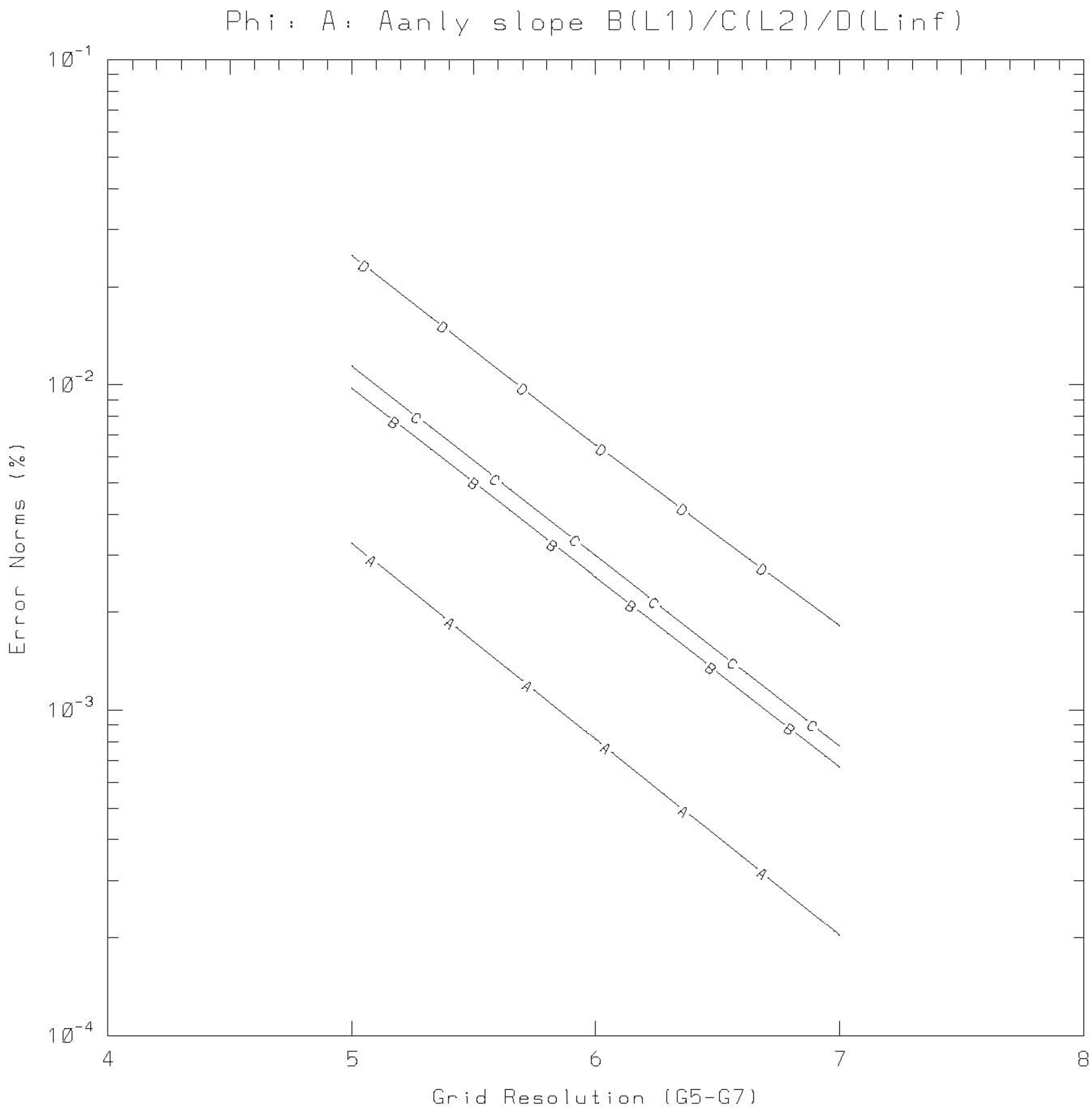
Grid optimization/
impact on numerical accuracy

CASE II (steady state nonlinear geostrophic flow)



Vandermonde@G5




Phi: A: Aanly slope B(L1)/C(L2)/D(Linf)
10^-1
10^-2
10^-3
10^-4
Error Norms (%)
4
5
6
7
8
Grid Resolution (G5-G7)
A
B
C
D

FIM and NIM model components

ESMF

1. Collab. with NCEP/EMC
(Icos- for NCEP model ensemble)

- **FIM** (flow-following finite-volume Icosahedral model):

  model components:
  - finite-volume,
  - Icosahedral grid,
  - theta-sig hybrid coord,
  - hydrostatic,
  - physics (GFS, WRF…)

2. Collab. among OAR Labs.
(ESRL/GSD, GFDL, AOML)

- **NIM** (Non-hydrostatic Icosahedral model) is a global cloud resolving model (GCRM) for Wx/Clmt fcsts.

  model components:
  - finite-volume,
  - Icosah-grid, cubed sphere,..
  - largrangian vertical coord,
  - non-hydrostatic,deep-atm,
  - full coriolis terms,
  - physics(GFDL, GFS, …)

FIM model/ system

-Contributors

FIM DESIGN

Jin Lee

Sandy MacDonald

Rainer Bleck

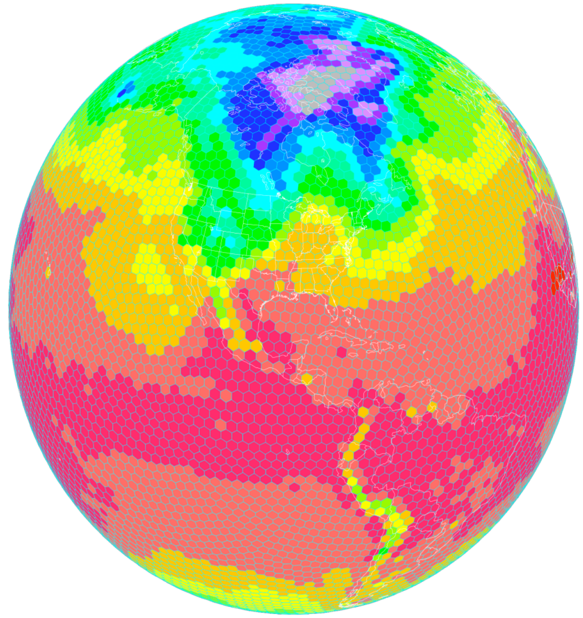
Jian-Wen Bao

John M. Brown

Jacques Middlecoff

Ning Wang

Stan Benjamin

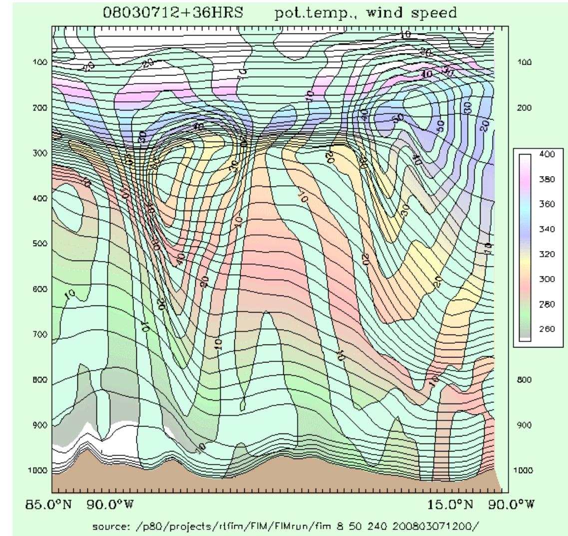


Tom Henderson — ESMF, Subversion

Chris Harrop — WorkFlow Manager, xml real-time scripts

Bill Moninger
Brian Jamison
Susan Sahm
Ed Szoke — Verification, Web page, Evaluation

Georg Grell — FIM- chemistry, aerosols



FIM display capabilities for Mary Glackin visit

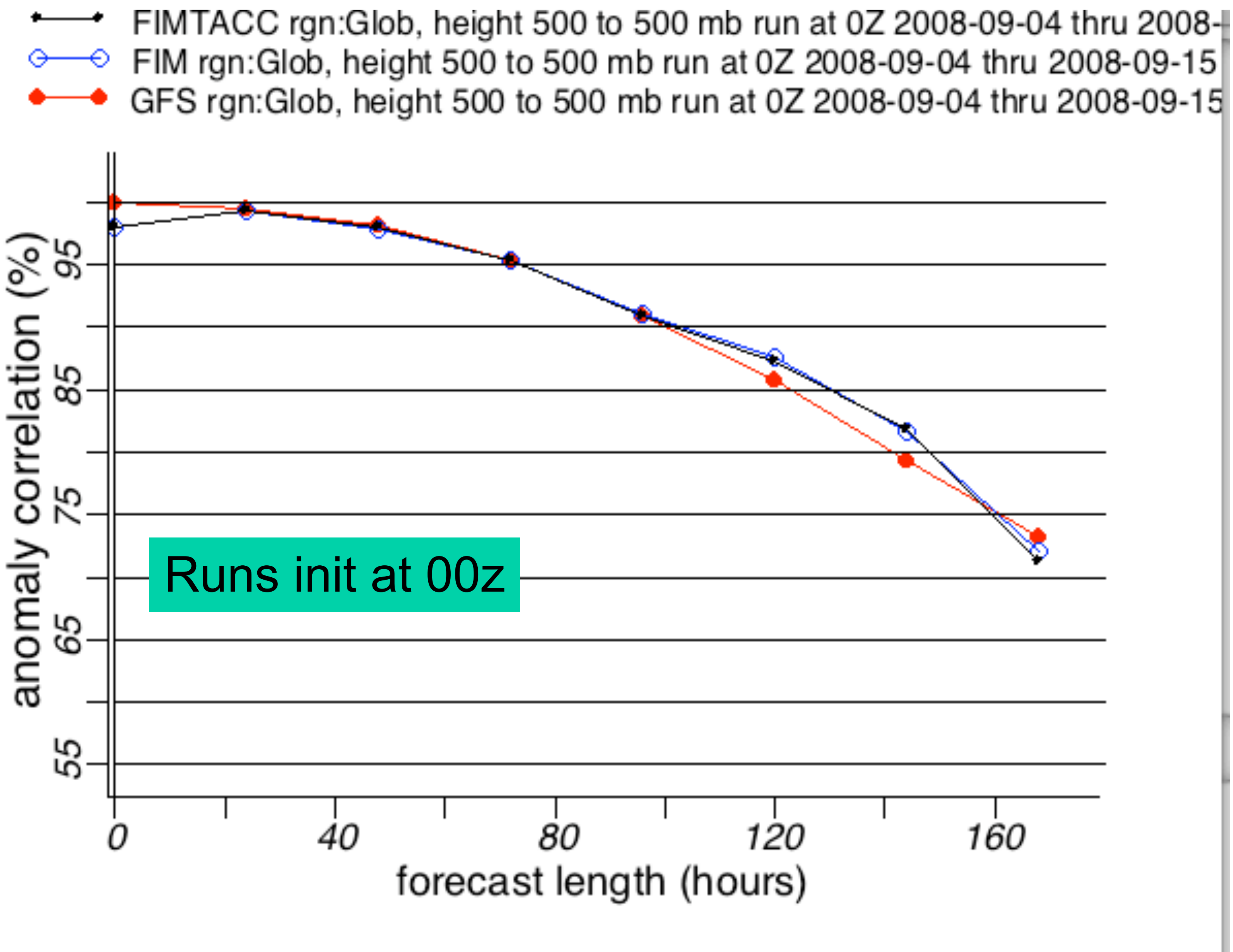
Bob Lipschutz
David Himes
Beth Russell
Steve Albers
Tom Kent
+ dozens more…

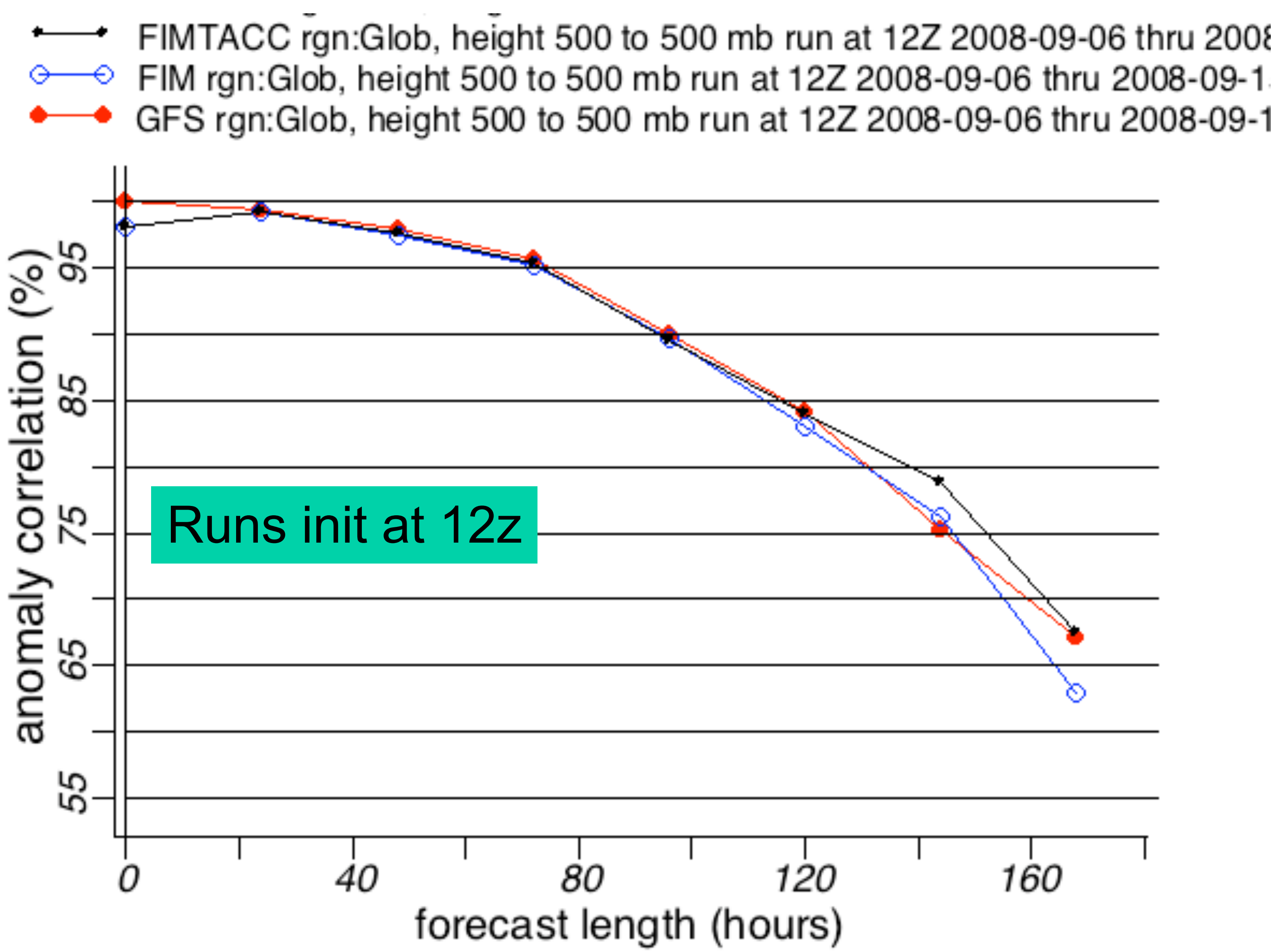
Results on the use of f.-v. Icosahedral model for weather forecasts

FIM real time forecasts initialized with GFS initial condition at 00 and 12 UTC

(comparisons of FIM and GFS 500 mb ACC)

- Interpolate GFS initial data to Icosahedral grid.
- Perform hydrostatic initialization.
- Perform 7-day fcst with dx~30km without explicit dissipation.
- Experimental FIM runs with dx~15km resolution.
- Same initial condition, terrain & sfc parameters, physics package runing at similar model resolutions (dx~30 km)


FIMTACC rgn:Glob, height 500 to 500 mb run at 0Z 2008-09-04 thru 2008-
FIM rgn:Glob, height 500 to 500 mb run at 0Z 2008-09-04 thru 2008-09-15
GFS rgn:Glob, height 500 to 500 mb run at 0Z 2008-09-04 thru 2008-09-15
anomaly correlation (%)
95
85
75
65
55
Runs init at 00z
0
40
80
120
160
forecast length (hours)

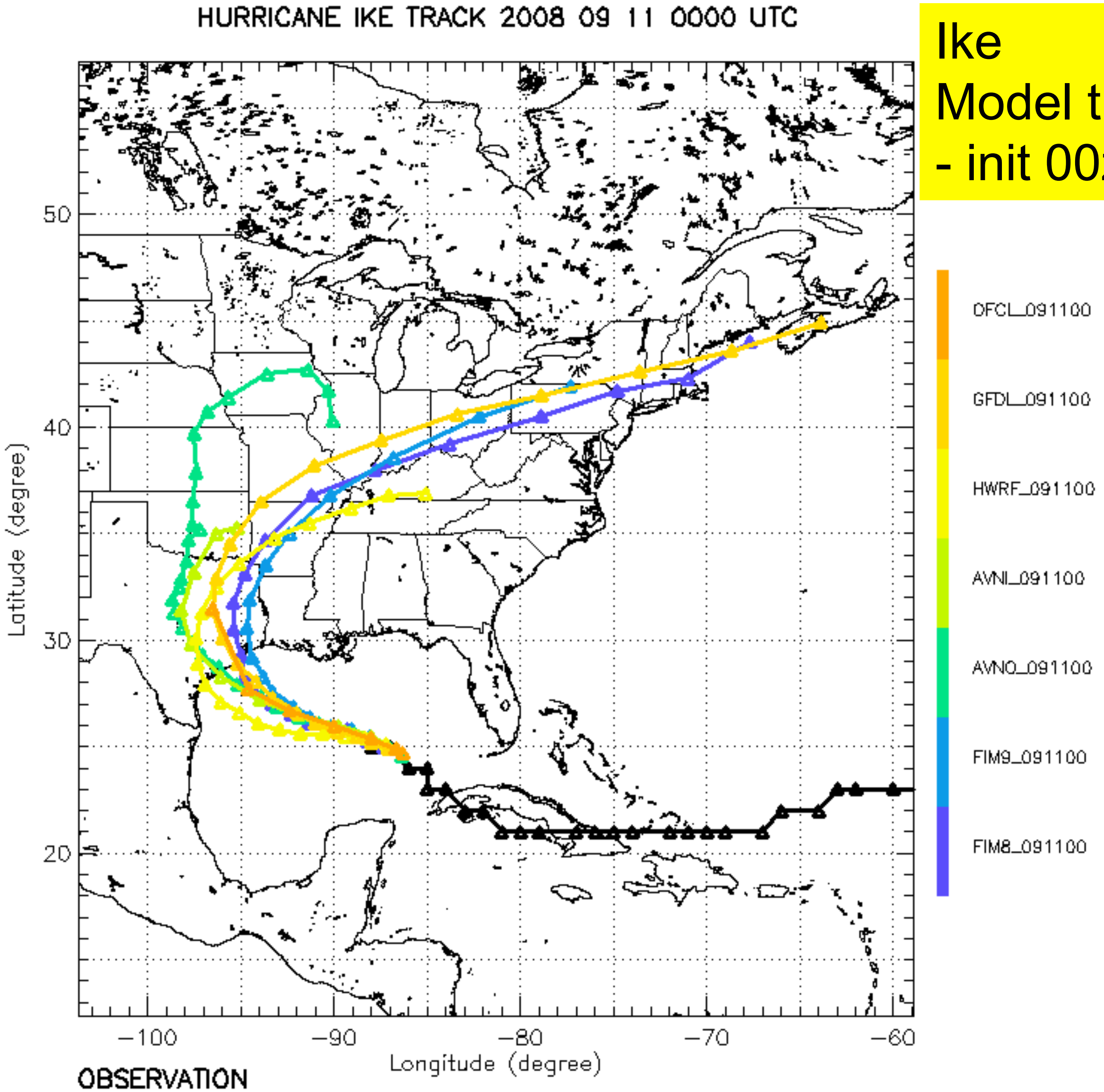


FIMTACC rgn:Glob, height 500 to 500 mb run at 12Z 2008-09-06 thru 2008
FIM rgn:Glob, height 500 to 500 mb run at 12Z 2008-09-06 thru 2008-09-1
GFS rgn:Glob, height 500 to 500 mb run at 12Z 2008-09-06 thru 2008-09-1
anomaly correlation (%)
95
85
75
65
55
Runs init at 12z
0
40
80
120
160
forecast length (hours)

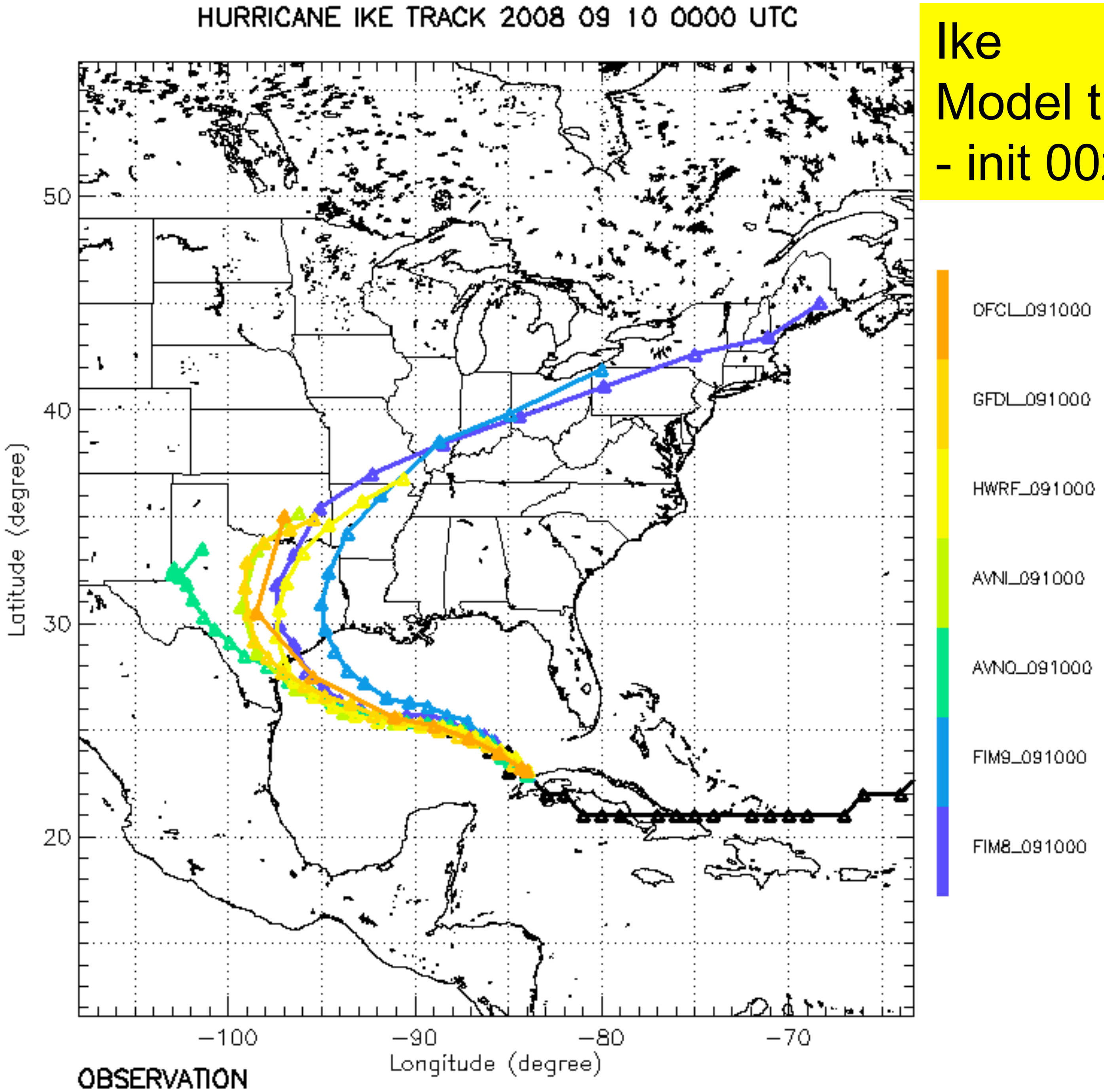
Mass Conservation and Positive Definiteness

Integrated mass	=	5.022915528916858E+018	at time step=	1
Integrated mass	=	5.022915528916743E+018	at time step=	60
Integrated mass	=	5.022915528916771E+018	at time step=	120
Integrated mass	=	5.022915528916695E+018	at time step=	180
Integrated mass	=	5.022915528917074E+018	at time step=	240
Integrated mass	=	5.022915528917066E+018	at time step=	300
Integrated mass	=	5.022915528916768E+018	at time step=	360
Integrated mass	=	5.022915528916948E+018	at time step=	420
Integrated mass	=	5.022915528916820E+018	at time step=	480
Integrated mass	=	5.022915528916910E+018	at time step=	540
Integrated mass	=	5.022915528916967E+018	at time step=	600
Integrated mass	=	5.022915528916953E+018	at time step=	660
Integrated mass	=	5.022915528916784E+018	at time step=	720
Integrated mass	=	5.022915528916850E+018	at time step=	780
Integrated mass	=	5.022915528916886E+018	at time step=	840
Integrated mass	=	5.022915528916961E+018	at time step=	900
Integrated mass	=	5.022915528916819E+018	at time step=	960
Integrated mass	=	5.022915528916939E+018	at time step=	1020
Integrated mass	=	5.022915528916824E+018	at time step=	1080
Integrated mass	=	5.022915528916941E+018	at time step=	1140
Integrated mass	=	5.022915528916814E+018	at time step=	1200

Ike
Model tracks
- init 00z 11 Sep



Ike
Model tracks
- init 00z 10 Sep



Development of NIM on Z-coord.
(NIM: non-hydrostatic Icosahedral Model)

Preliminary Resutls

Compressible 2D Boussinesq eqs cast in flux form on Z-coord:

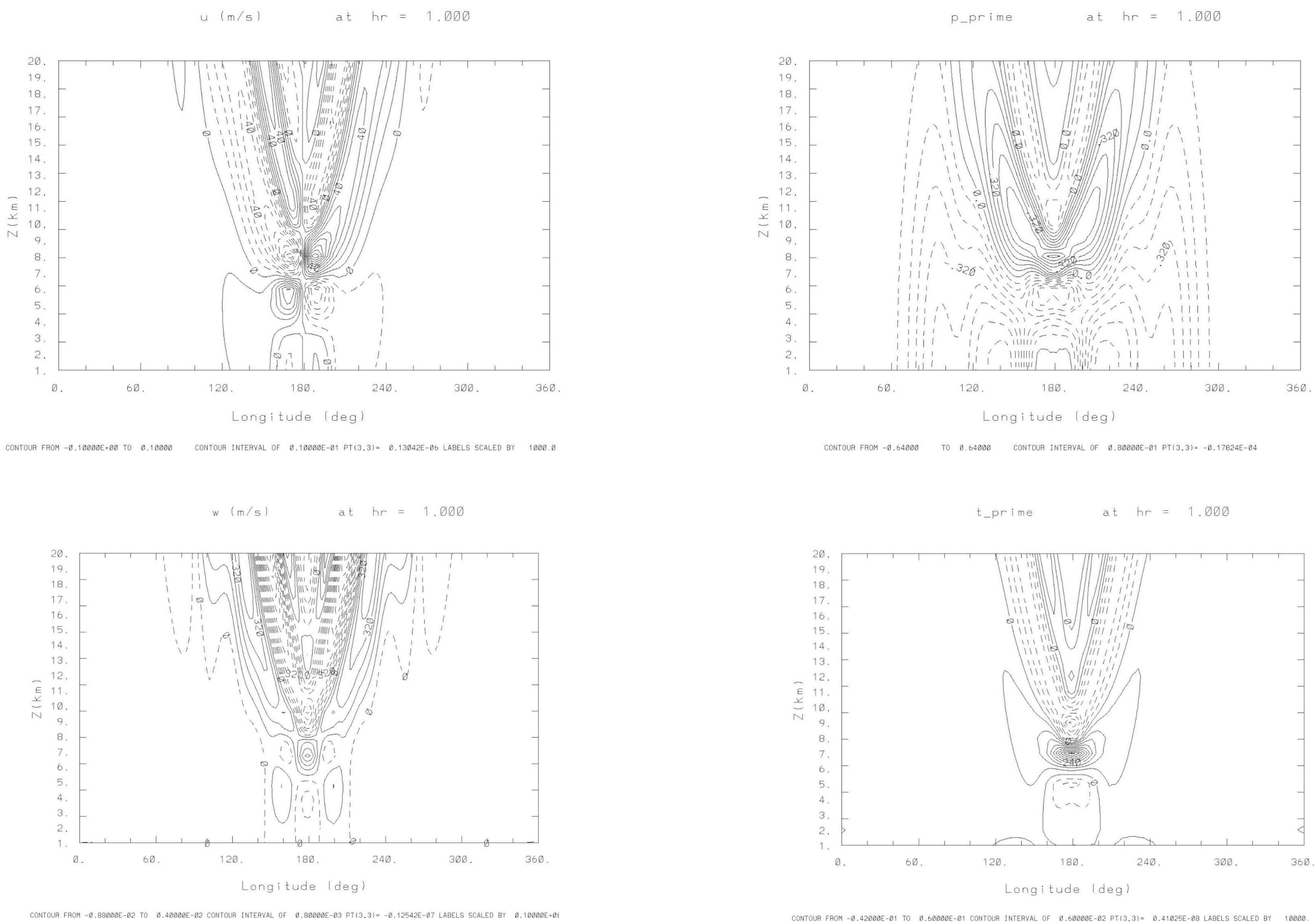
(A ***simple*** system which permits both gravity and acoustic waves)

$$\begin{cases} \dfrac{\partial U}{\partial t}+\dfrac{\partial(Uu)}{\partial x}+\dfrac{\partial(Wu)}{\partial z}+\dfrac{\partial p'}{\partial x}= 0 \\ \dfrac{\partial W}{\partial t}+\dfrac{\partial(Uw)}{\partial x}+\dfrac{\partial(Ww)}{\partial z}+\left(\dfrac{\partial p'}{\partial z}-g\dfrac{\Theta'}{\overline{\Theta}(z)}\right)= 0 \\ \dfrac{\partial \Theta'}{\partial t}+\dfrac{\partial(U\theta')}{\partial x}+\dfrac{\partial(W\theta')}{\partial z}+\dfrac{N^2}{g}W\overline{\theta}(z)=\dfrac{\Theta}{C_p}\dfrac{\dot{H}}{T} \\ \dfrac{\partial P'}{\partial t}+\dfrac{\partial(Up')}{\partial x}+\dfrac{\partial(Wp')}{\partial z}+\gamma P\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial w}{\partial z}\right)= 0 \end{cases}$$

$$(U,W,\Theta,P)=(\rho u,\rho w,\rho\theta,\rho p);\quad \theta(x,z,t)=\overline{\theta}(z)+\theta'(x,z,t)$$

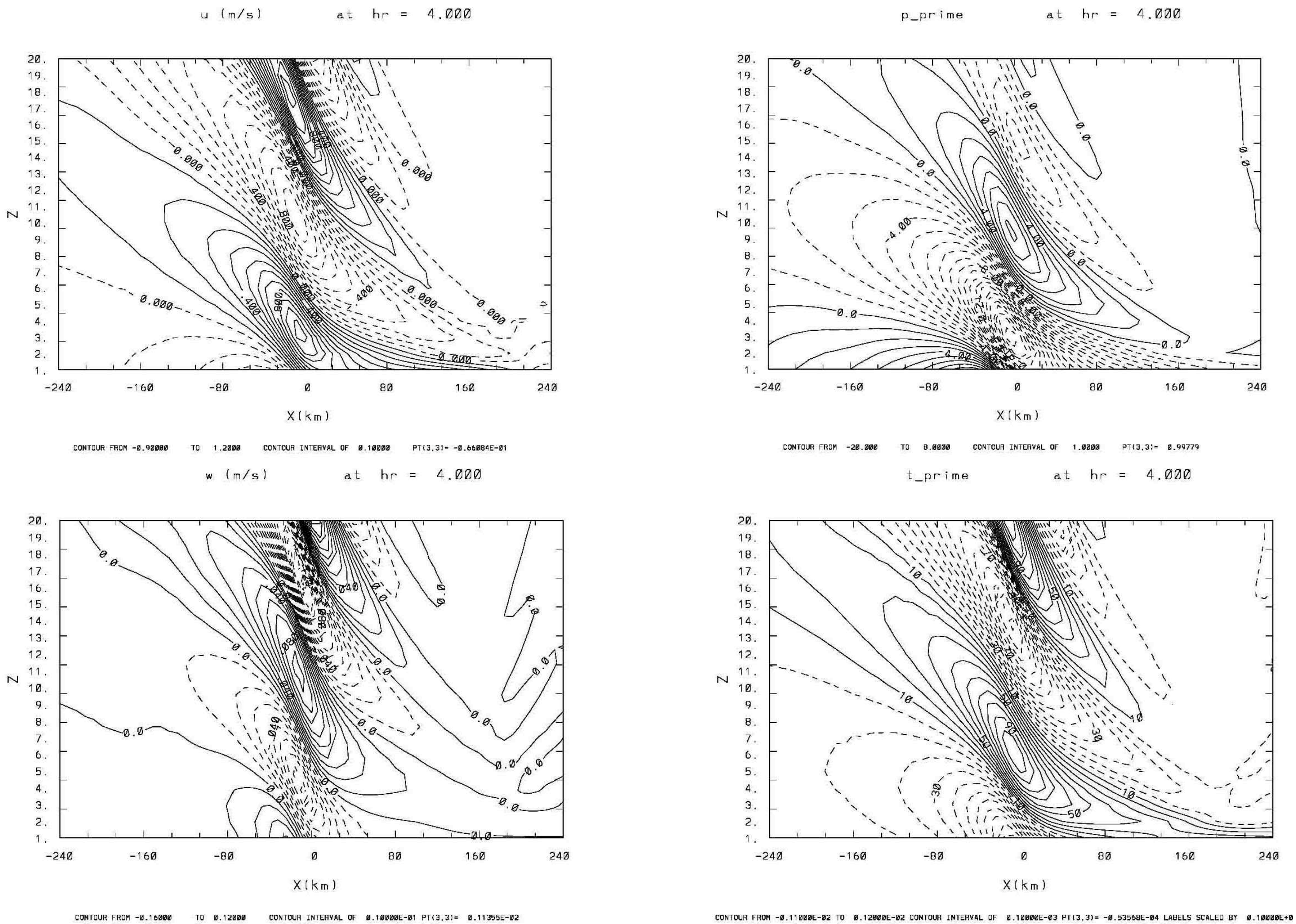
$$p(x,z,t)=\overline{p}(z)+p'(x,z,t);\quad \rho(x,z,t)=\overline{\rho}(z)+\rho'(x,z,t)$$

Heating excited vertical fast waves (explicit treatment)



dx=dz=1-km, dt=1sec, I.C: state of rest.

Numerical experiment on Mountain waves



<u>dx=8km, dz=1-km, dt=1sec, U=20m/s, Zs=100m, L=30km</u>

Issues related to Icosahedral-Hexagonal grids

- Icos-grid noises (grid imprinting):

  1. grid optimization algorithms available to reduce grid noises,

  2. numerical accuracy dependent,

  3. relatively small compared to other geodesic grids.
- Conservation .vs. Accuracy

  1. Mass conservation.

  2. Typically, the 2nd-order numerical accuracy.
  The 3rd-order scheme is desirable.
- Grid staggering issues :

  A-grid with derivatives estimated using Vandermonde stencil points including center points (linear analysis?), C-grid ?
- Grid nesting issues:

  Nesting ? Stretching grid ?