Vertical Coordinate Systems

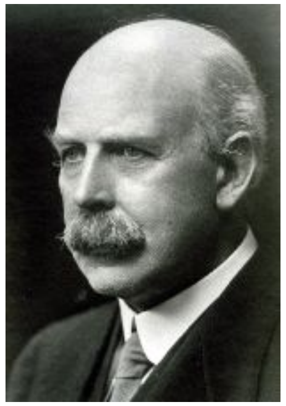
Outline

- General
- Theta and hybrid sigma-theta
- Non-hydrostatic hybrid sigma-theta
- Dealing with the boundary layer

The usual suspects

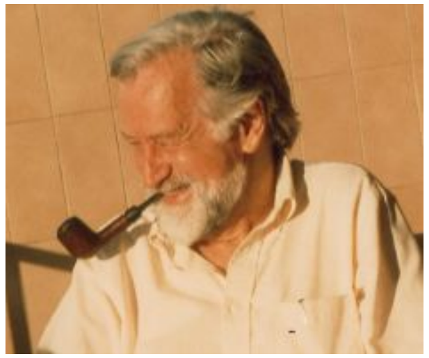
- z
  - Unattractive for hydrostatic atmosphere models because of Richardson's equation
  - Used in many ocean models because of steep bottom topography
  - Simple lower boundary condition
- Sigma, and sigma-p
  - Very widely used in hydrostatic atmosphere models
  - Less attractive with very-high-resolution models
- Theta and sigma-theta hybrids -- discussed below
- ALE
  - Not really a coordinate, more like a set of rules for evolving an adaptive vertical grid
  - Can "try to be" a coordinate, e.g., theta
  - Very flexible
  - Used *only* in numerical models

Advocates of theta coordinates

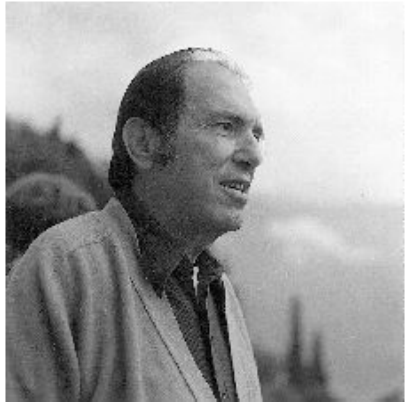


Napier-Shaw

Rossby

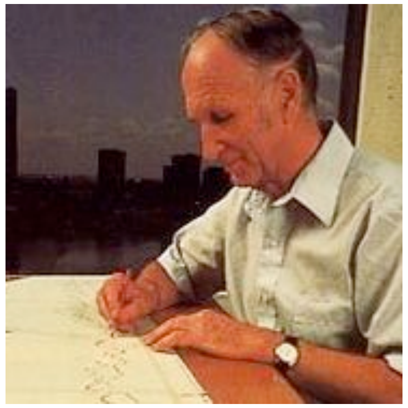


Danielsen



Namias

Starr



Lorenz

Eliassen

Johnson

Bleck

Benjamin

Hoskins

Arakawa

Mass conservation in theta space

$$m \equiv \rho \frac{\partial z}{\partial \theta}$$

$$\left(\frac{\partial m}{\partial t}\right)_\theta + \nabla_\theta \bullet (m\mathbf{v}) + \frac{\partial \left(m\dot{\theta}\right)}{\partial \theta} = 0$$

$$f(\theta)$$

$$\left[\frac{\partial \left(mf\right)}{\partial t}\right]_\theta + \nabla_\theta \bullet (m\mathbf{v}f) + \frac{\partial \left(m\dot{\theta} f\right)}{\partial \theta} = m\frac{df}{d\theta}\dot{\theta}$$

The lure of theta coordinates

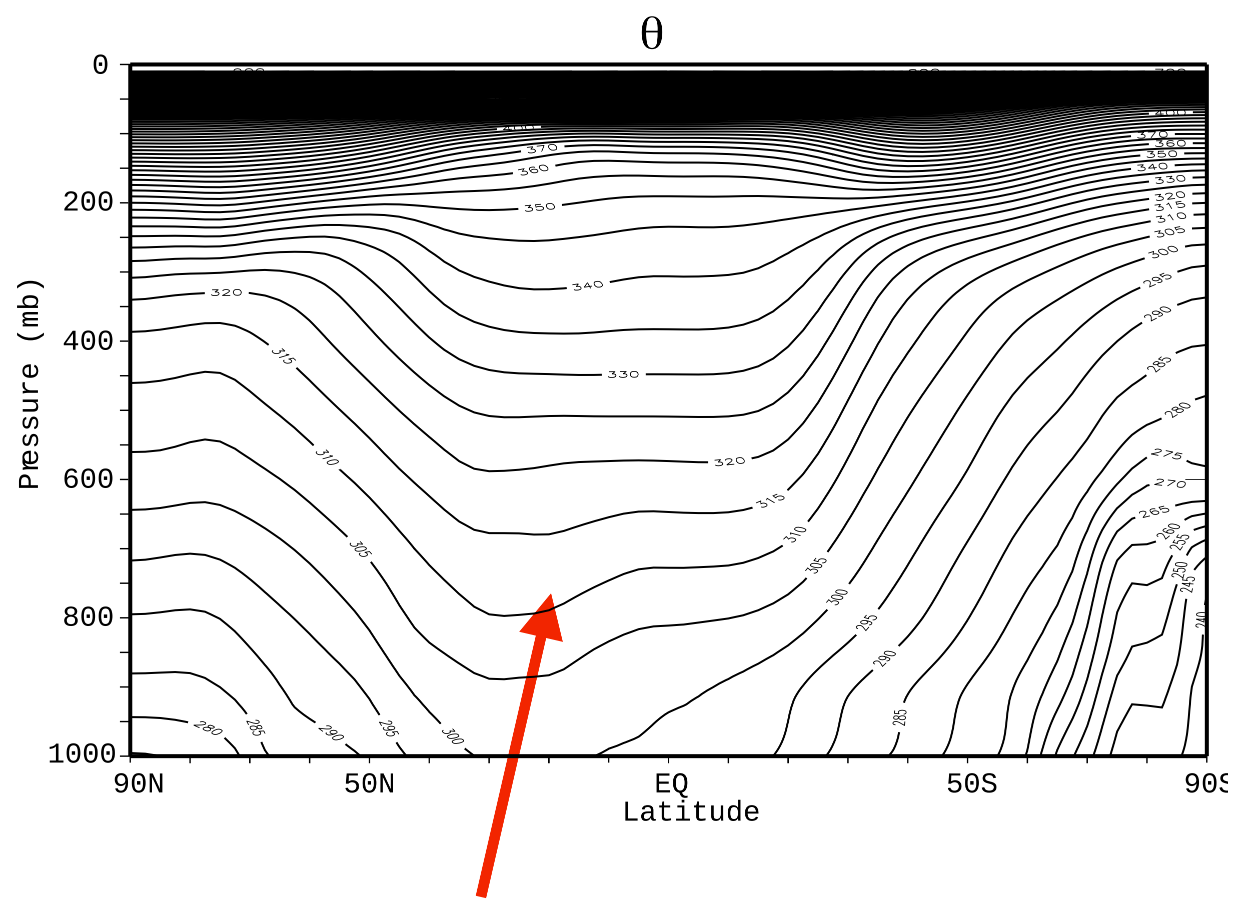
- There is no “vertical motion” in the absence of heating.
  - This minimizes errors associate with vertical advection.
  - Any quasi-Lagrangian system has this property.
- The pressure-gradient force is a gradient.
  - This minimizes pressure-gradient errors near topography, and spurious generation of vorticity.
- The potential vorticity is easily accessible from the wind vector.
- Wave momentum transport occurs via isentropic form drag.
- It is easy to implement diffusion along theta surfaces.
- Both energy and entropy can be conserved (ref Don Johnson).
  - Relevant to the “cold pole” problem?
- Helpful for dealing with the PBL?

But theta surfaces intersect the ground.

Theta coordinates

Where the theta coordinate intersects the Earth's surface, we can define "massless layers," following Lorenz (1955).

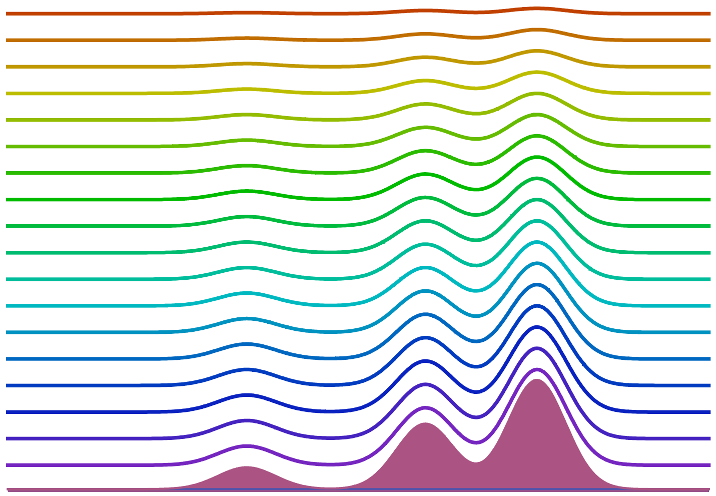
Sign-preserving advection schemes such as FCT can be used to deal with massless layers.



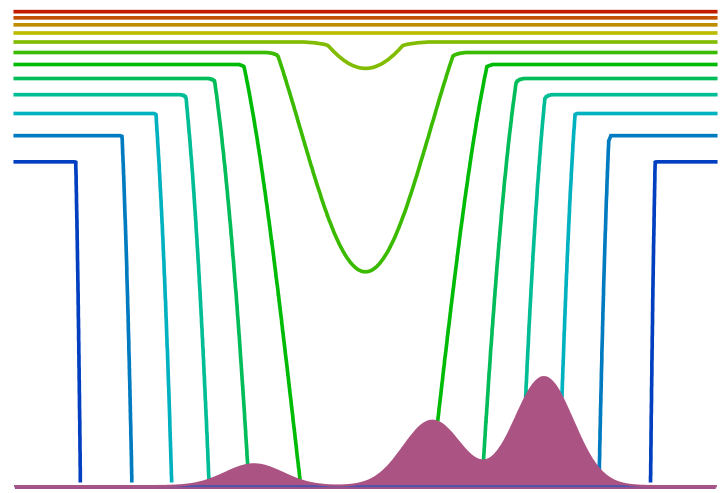
Note spreading of isentropes in the tropical troposphere. *Many theta layers are "wasted" in the tropics.*

In order to have adequate vertical resolution over the entire globe, it is necessary to allocate many theta layers. A hybrid sigma-theta coordinate helps with this problem.

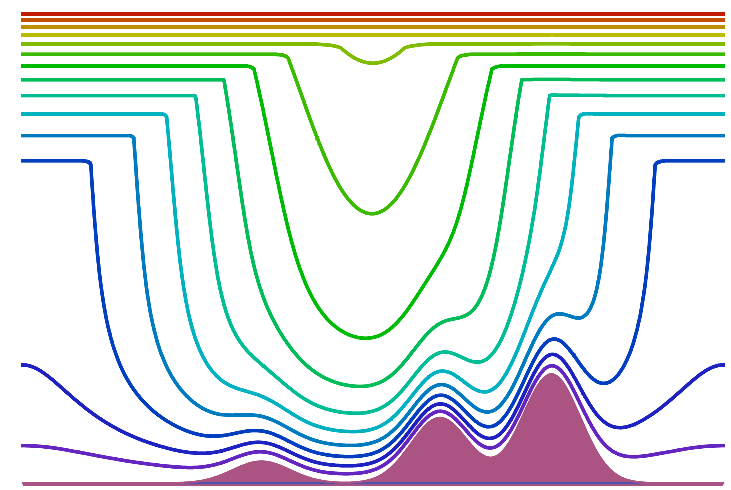
The three amigos



sigma coordinate



isentropic coordinate



hybrid coordinate

Lower boundary is a coordinate surface.

Mass flows freely across layer edges.

Pressure-gradient force is not a gradient.

Lower boundary is not a coordinate surface.

Mass flows stays within layers except for heating.

Pressure-gradient force is a gradient.

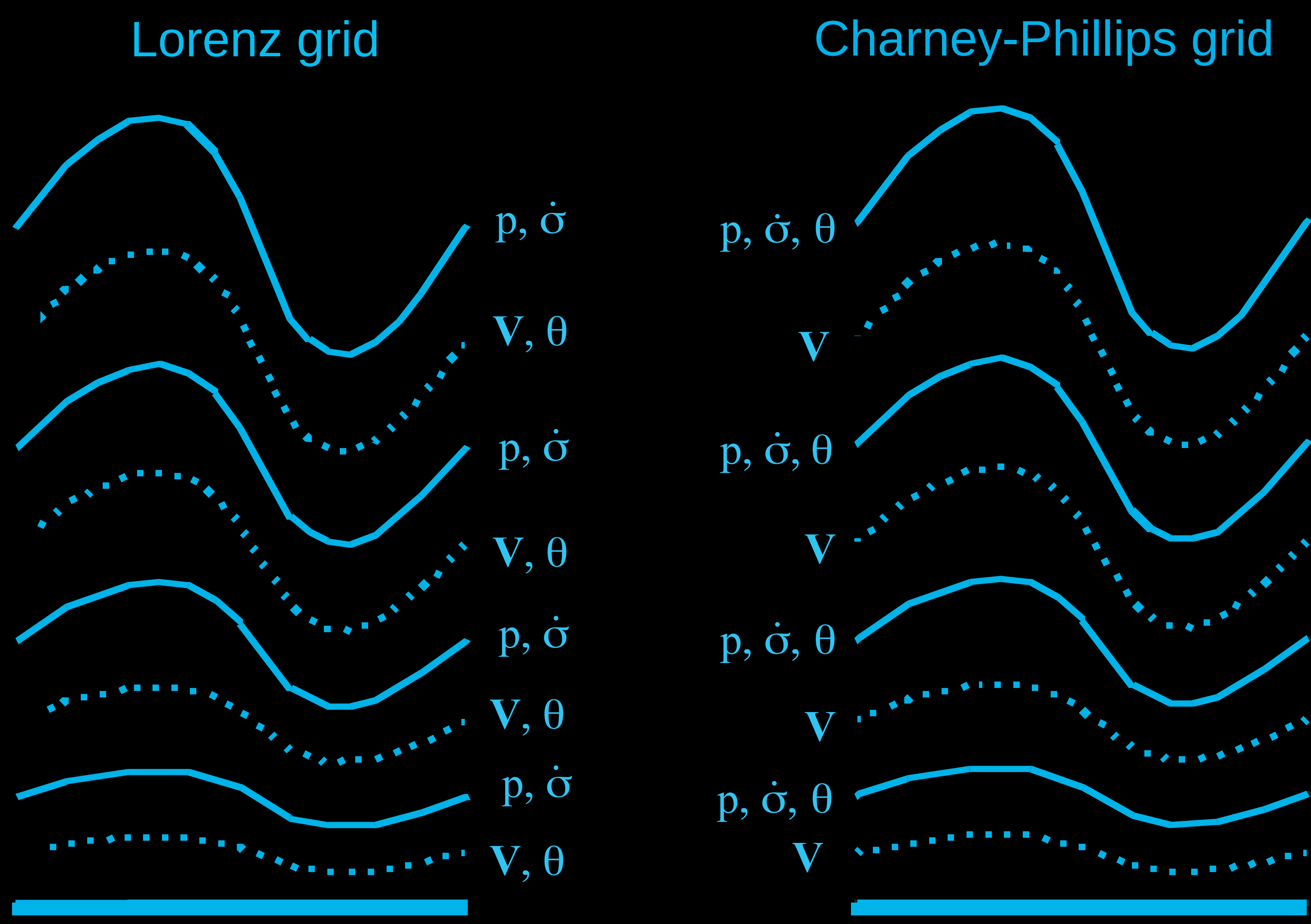
Massless layers are "lost" in the tropics.

Behaves like sigma near the lower boundary.

Becomes theta smoothly and naturally throughout most of the troposphere and all of the stratosphere.

High-res lower troposphere.

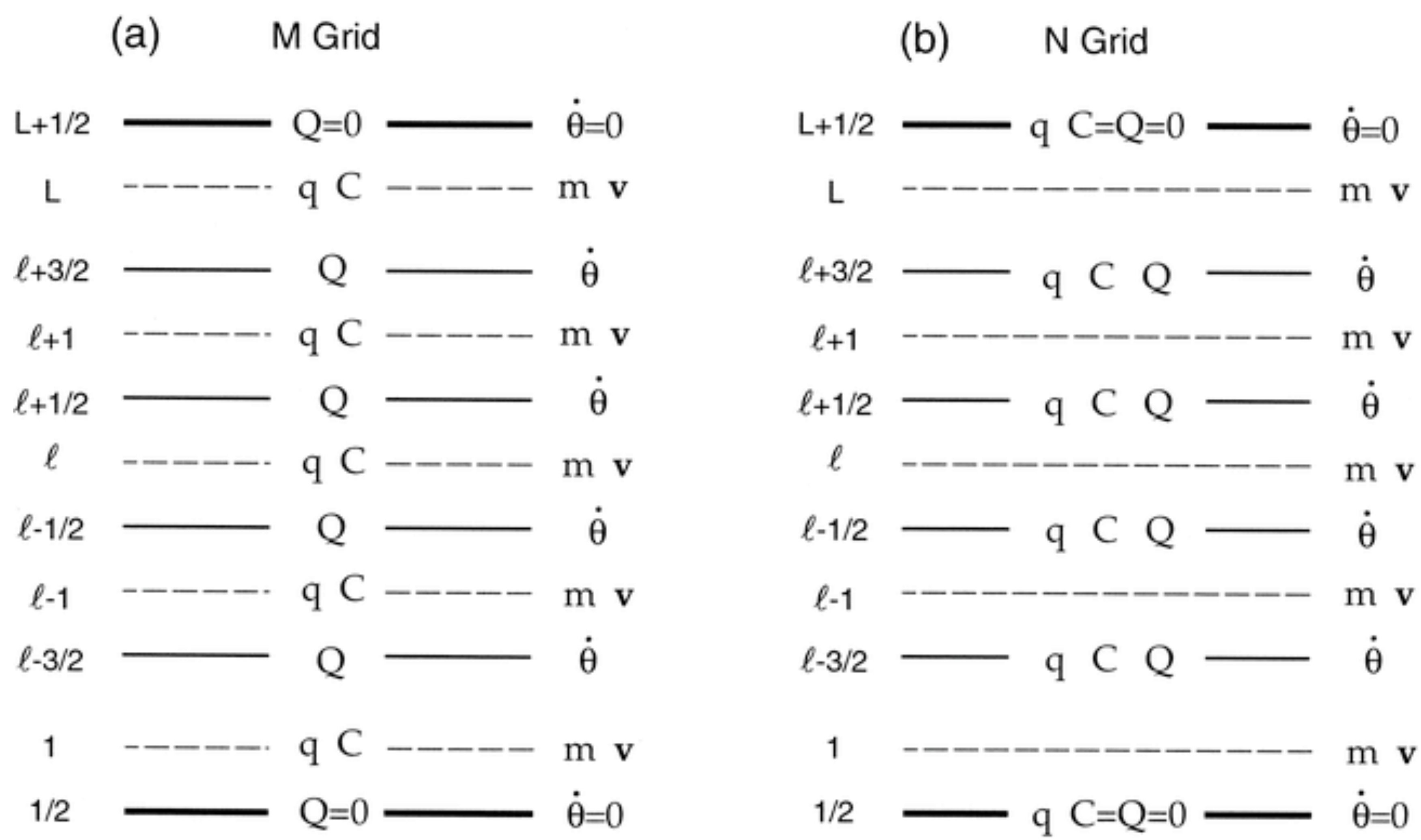
Vertical staggering



The L-grid has a stationary computational mode in the temperature, while the CP grid does not.

The CP-grid knows about potential vorticity.

Moisture staggering



Konor & Arakawa 2000

Hybrid vertical coordinate

Motto: Theta where possible, sigma where necessary.

Konor and Arakawa (1997) invented a simple and elegant way to transition from theta to sigma, near the surface and wherever theta decreases upward.

The method works without any if-statements.

Generalized vertical coordinate

(Konor and Arakawa)

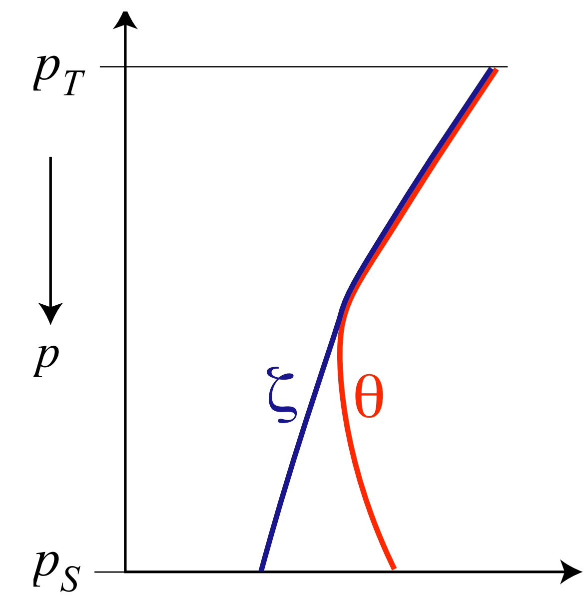
Suppose that ζ is of the form

$$\zeta = f(\sigma) + g(\sigma)\theta$$

where σ increases upward, so that

$$g(\sigma) \to 0 \quad \text{as} \quad \sigma \to \sigma_S$$

$$f(\sigma) \to 0 \quad \text{and} \quad g(\sigma) \to 1 \quad \text{as} \quad \sigma \to \sigma_T$$



We require monotonicity:

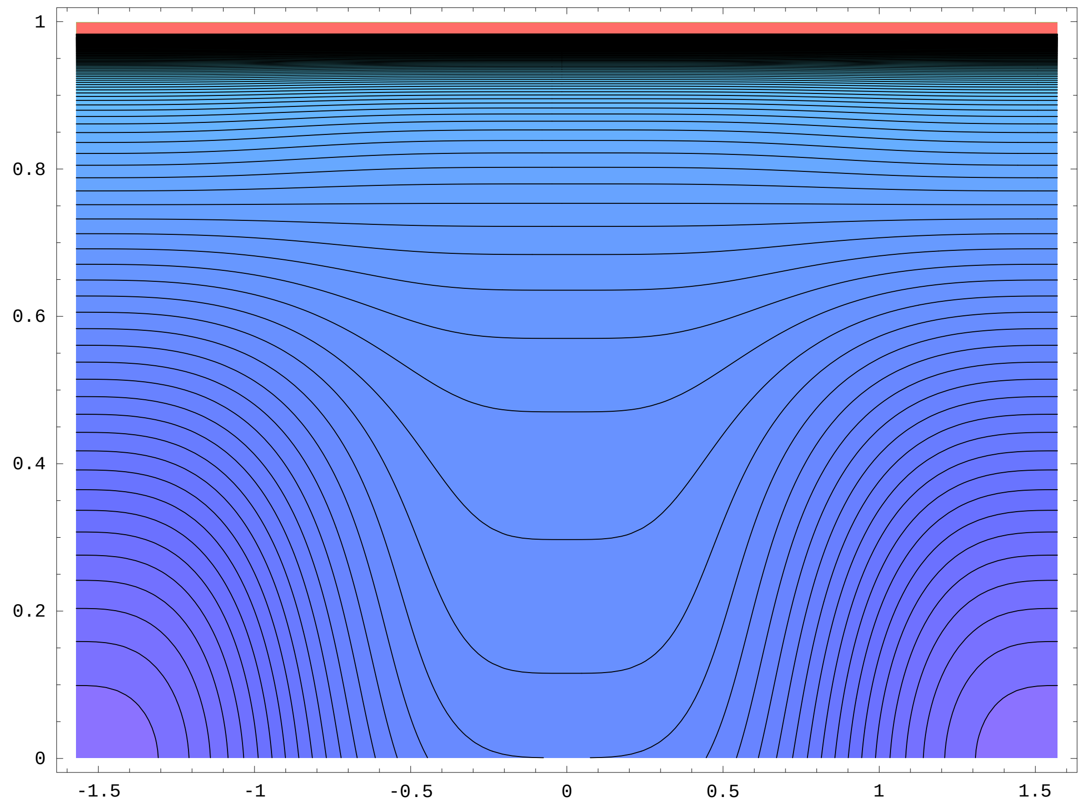
$$\frac{\partial \zeta}{\partial \sigma} = \frac{\partial f}{\partial \sigma} + \frac{\partial g}{\partial \sigma}\theta + g\frac{\partial \theta}{\partial \sigma} > 0$$

which can be ensured by choosing

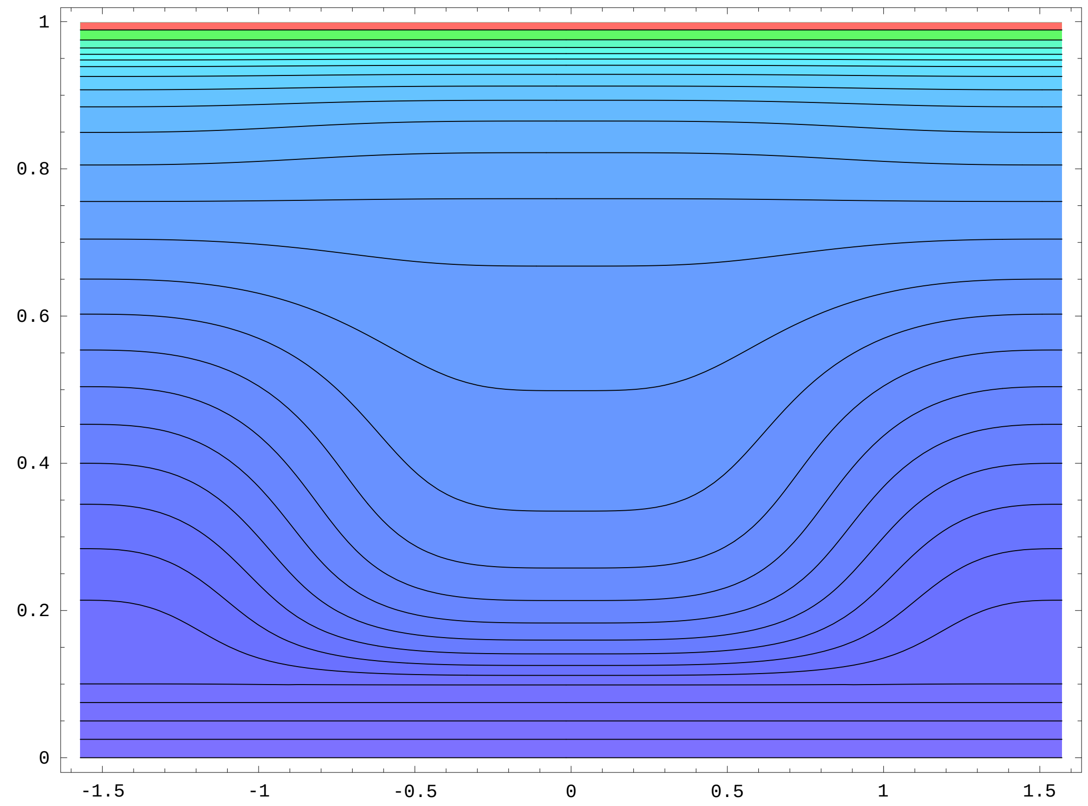
$$\frac{\partial f}{\partial \sigma} + \frac{\partial g}{\partial \sigma}\theta_{\min} + g\left(\frac{\partial \theta}{\partial \sigma}\right)_{\min} = 0, \quad g > 0, \quad \frac{\partial g}{\partial \sigma} > 0$$

We specify $g(\sigma)$ and solve for $f(\sigma)$.

Example

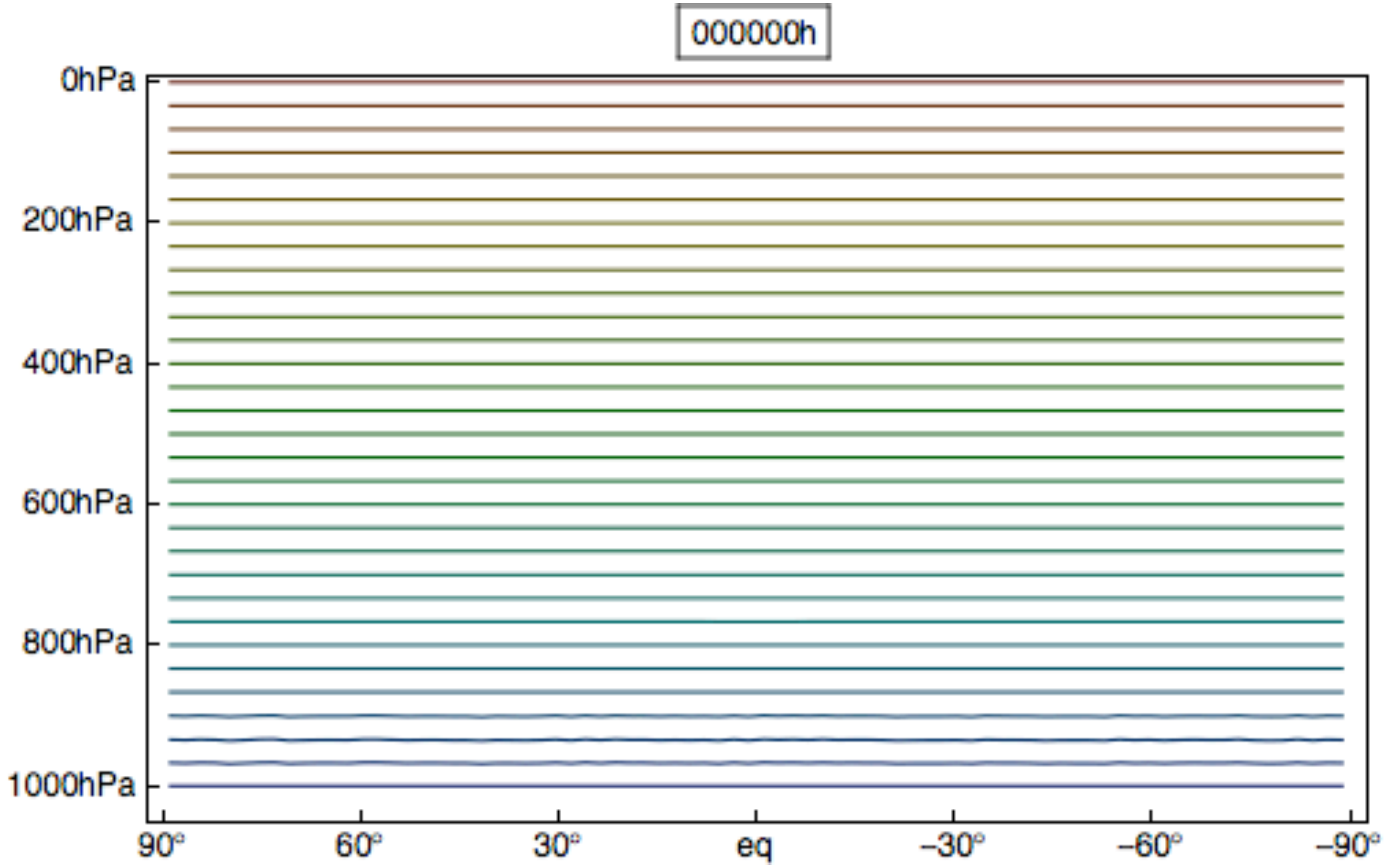


Theta



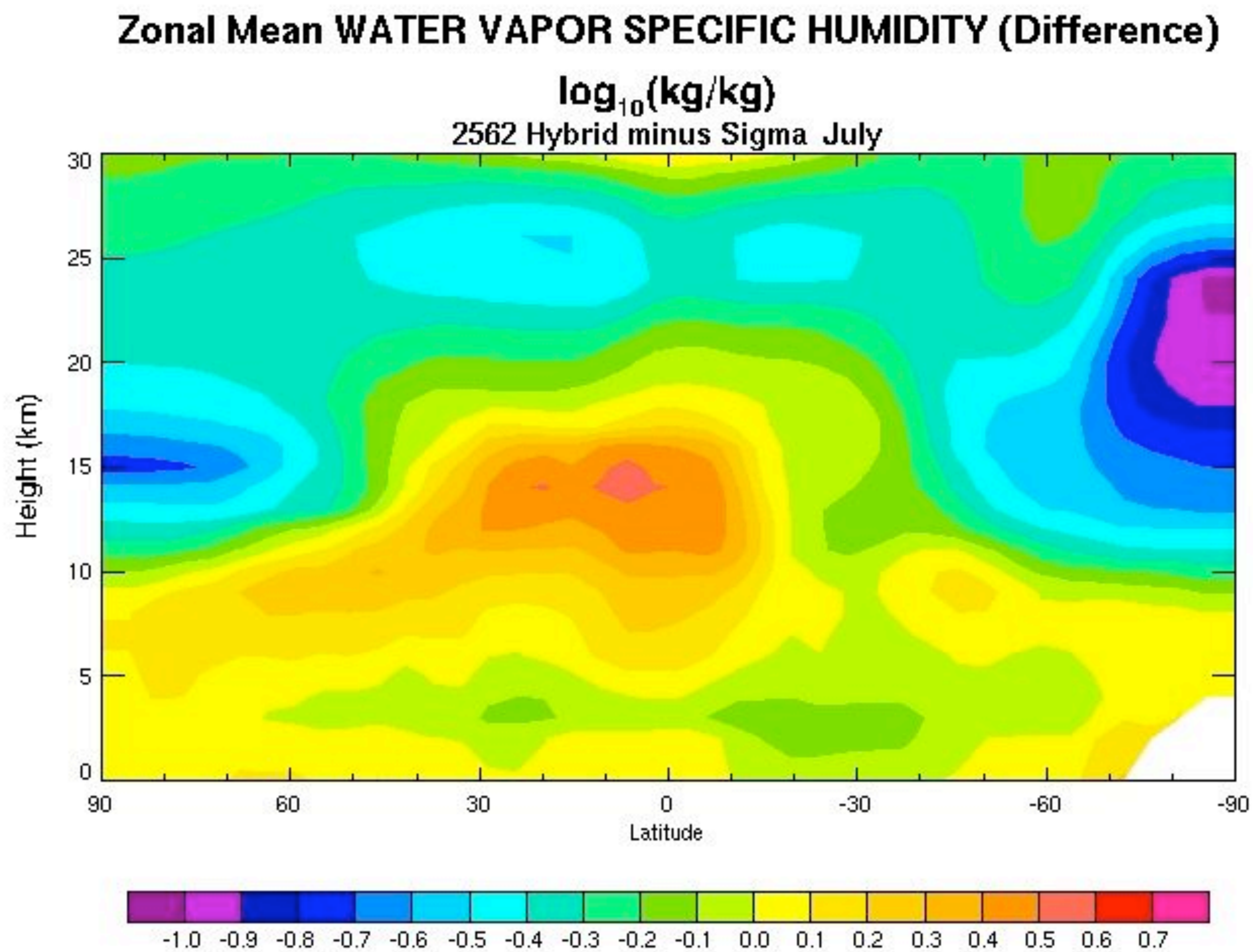
Zeta

Starting HS

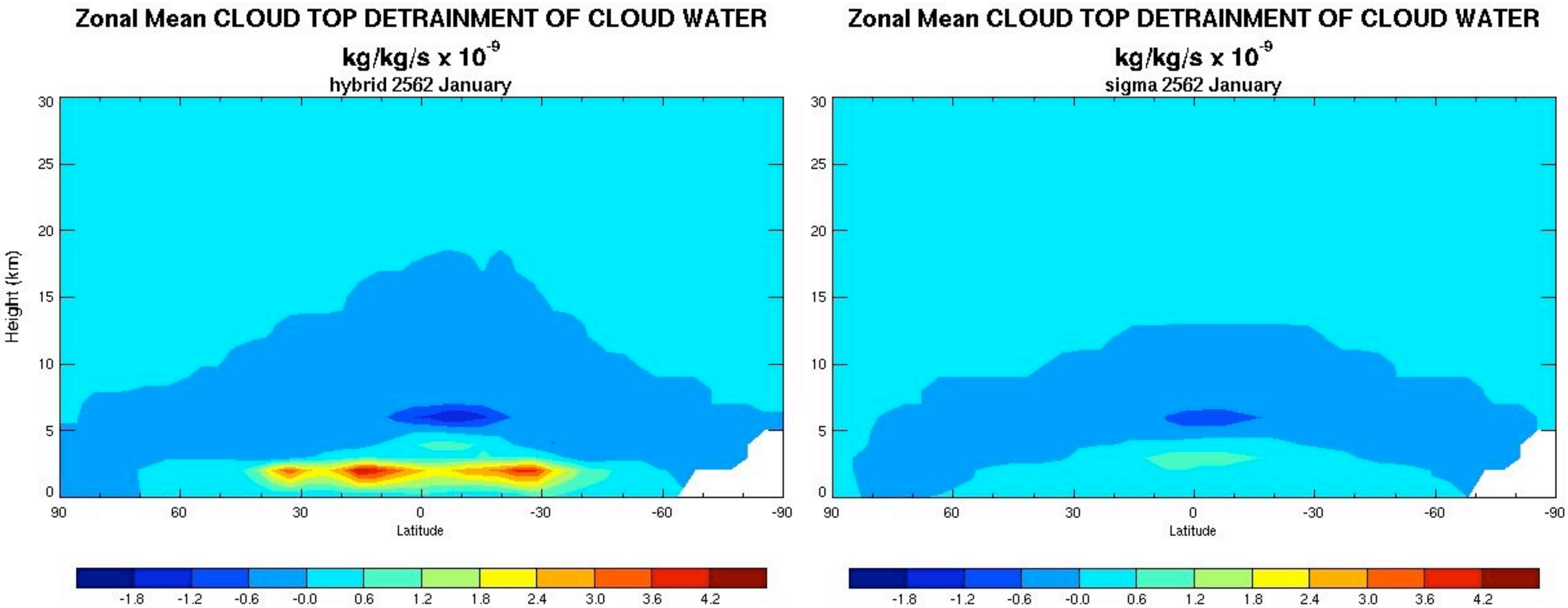


000000h
0hPa
200hPa
400hPa
600hPa
800hPa
1000hPa
90°
60°
30°
eq
−30°
−60°
−90°

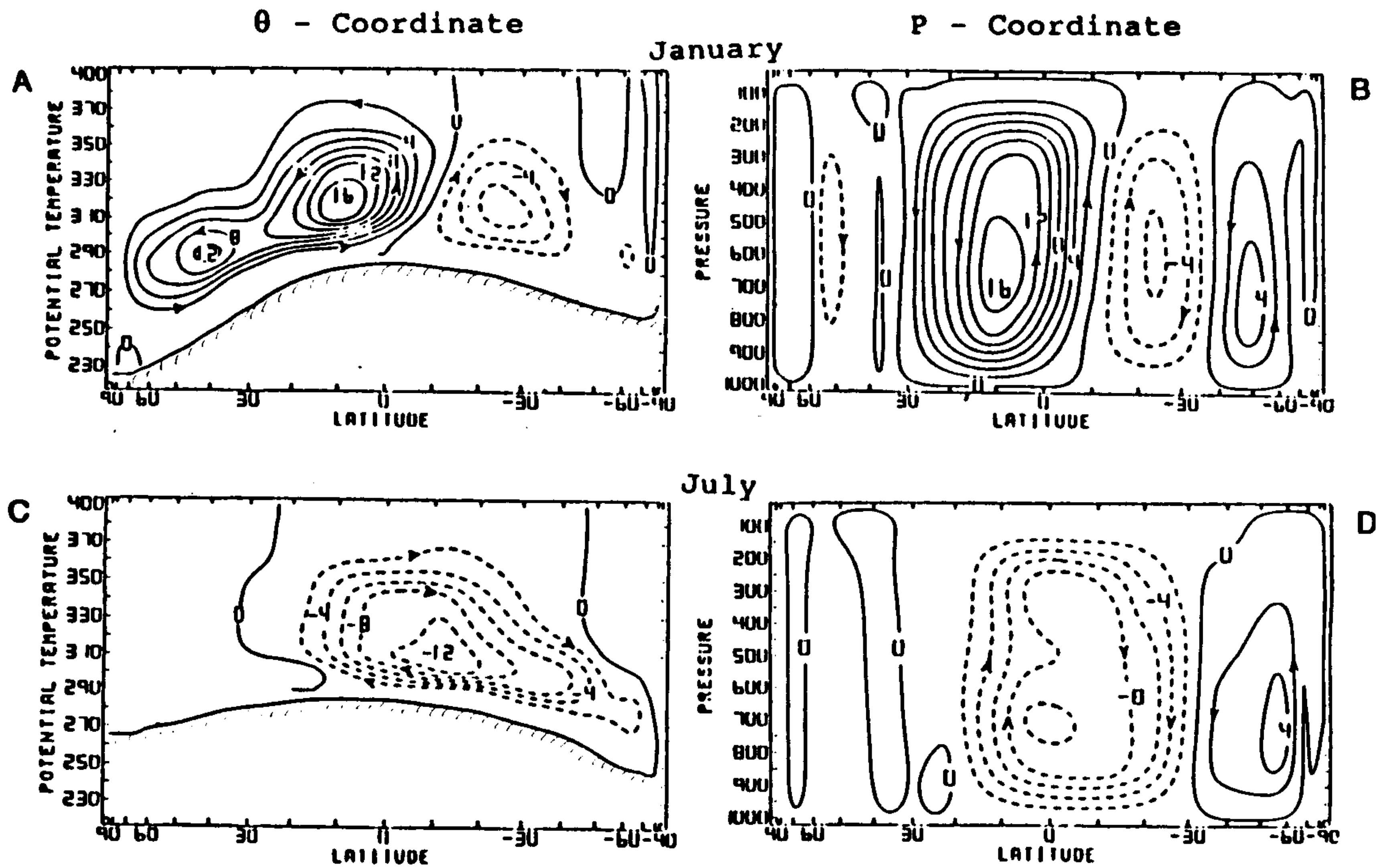
H2O stays in the troposphere.



Lots of shallow cumulus clouds



Meridional mass circulation

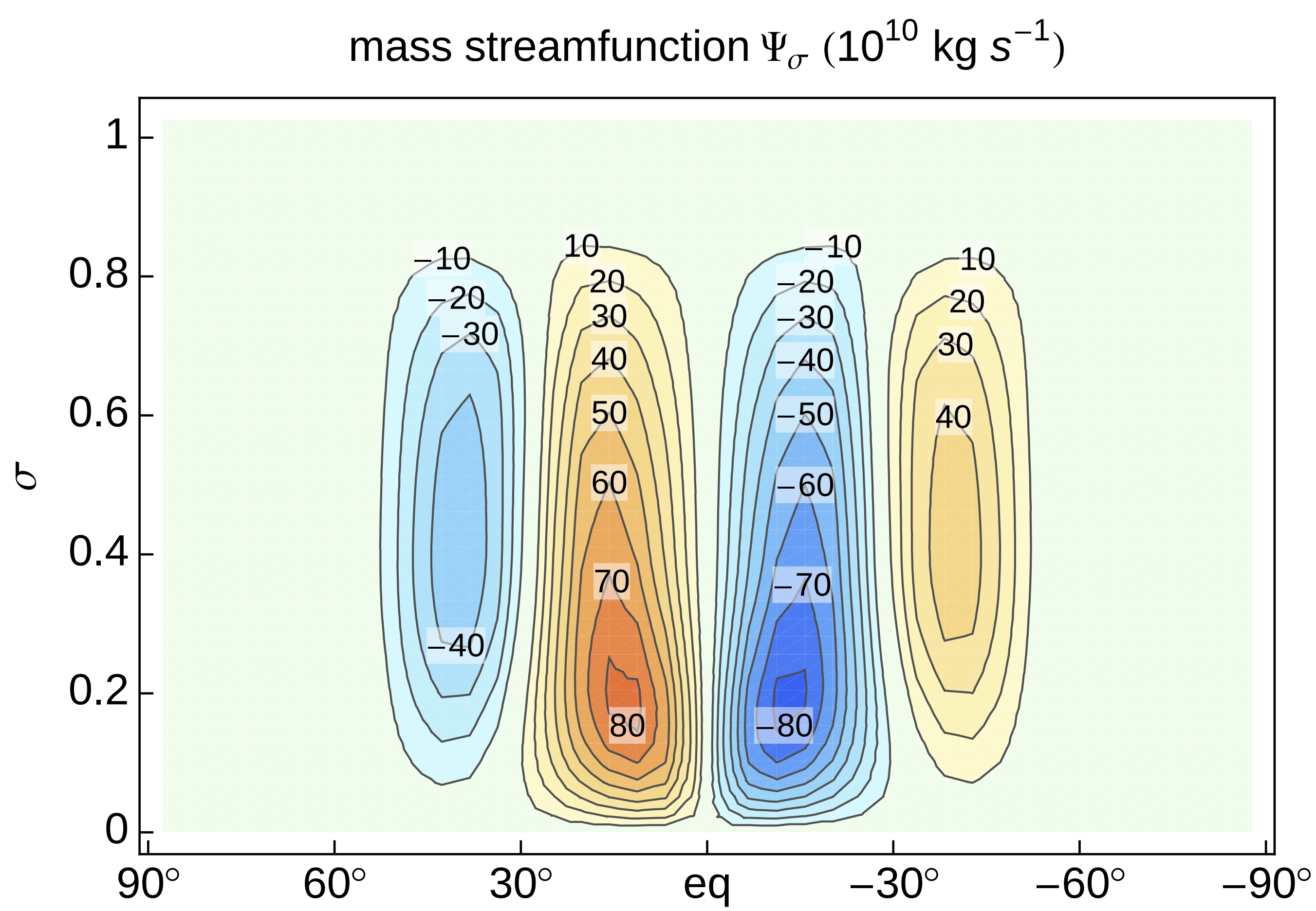


Townsend & Johnson 1985

The mass streamfunction in sigma coordinates for the zonally averaged mass transport is defined by

$$\frac{\partial \Psi_\sigma}{\partial \varphi} = 2\pi a^2 \cos\varphi \overline{\left[m_\sigma \dot{\sigma} \right]} \quad \text{and} \quad \frac{\partial \Psi_\sigma}{\partial \sigma} = -2\pi a \cos\varphi \overline{\left[m_\sigma v \right]}$$

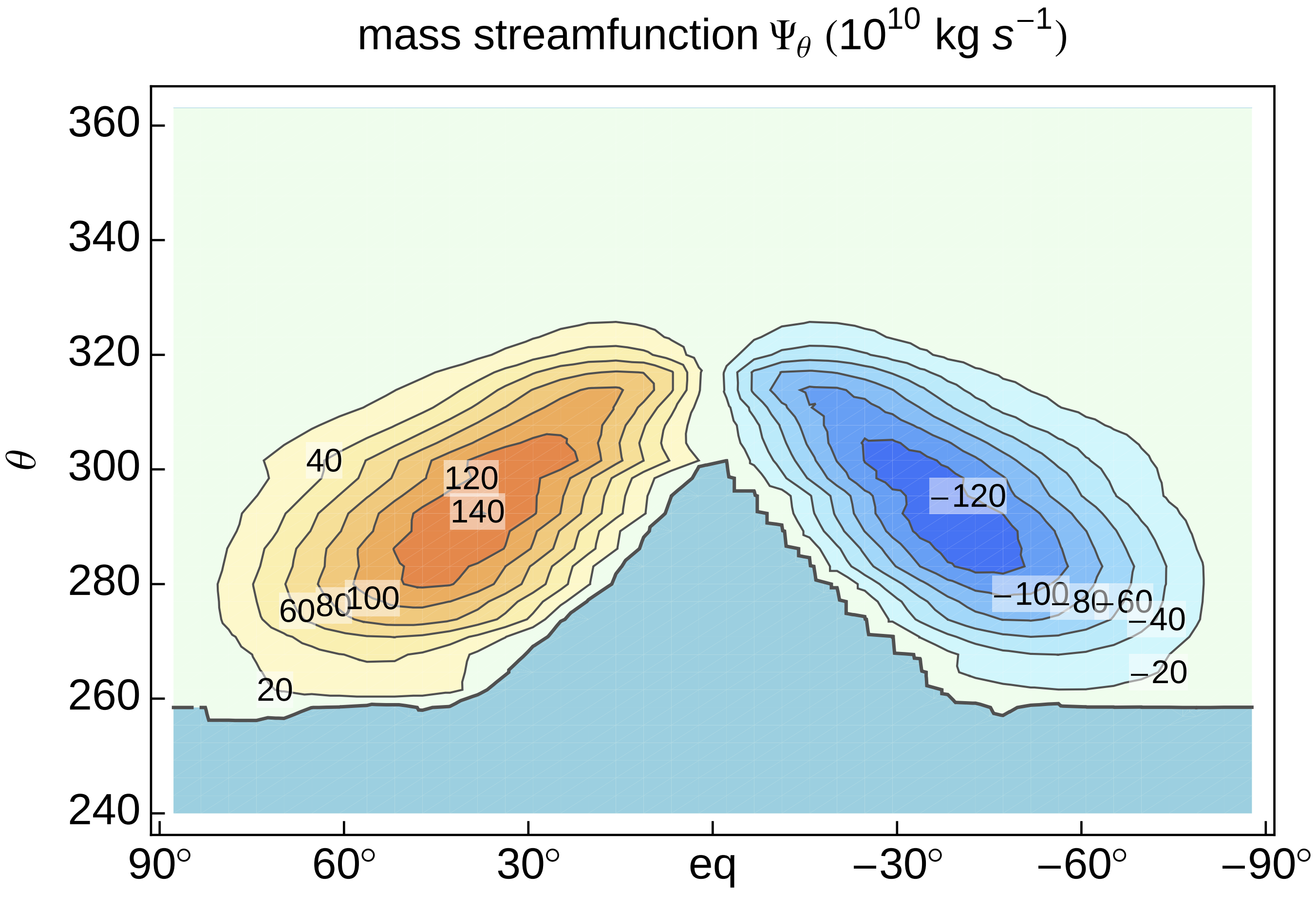
The Hadley circulations extend from equator to mid-latitudes. The Ferrell circulations are poleward of the Hadley circulations.



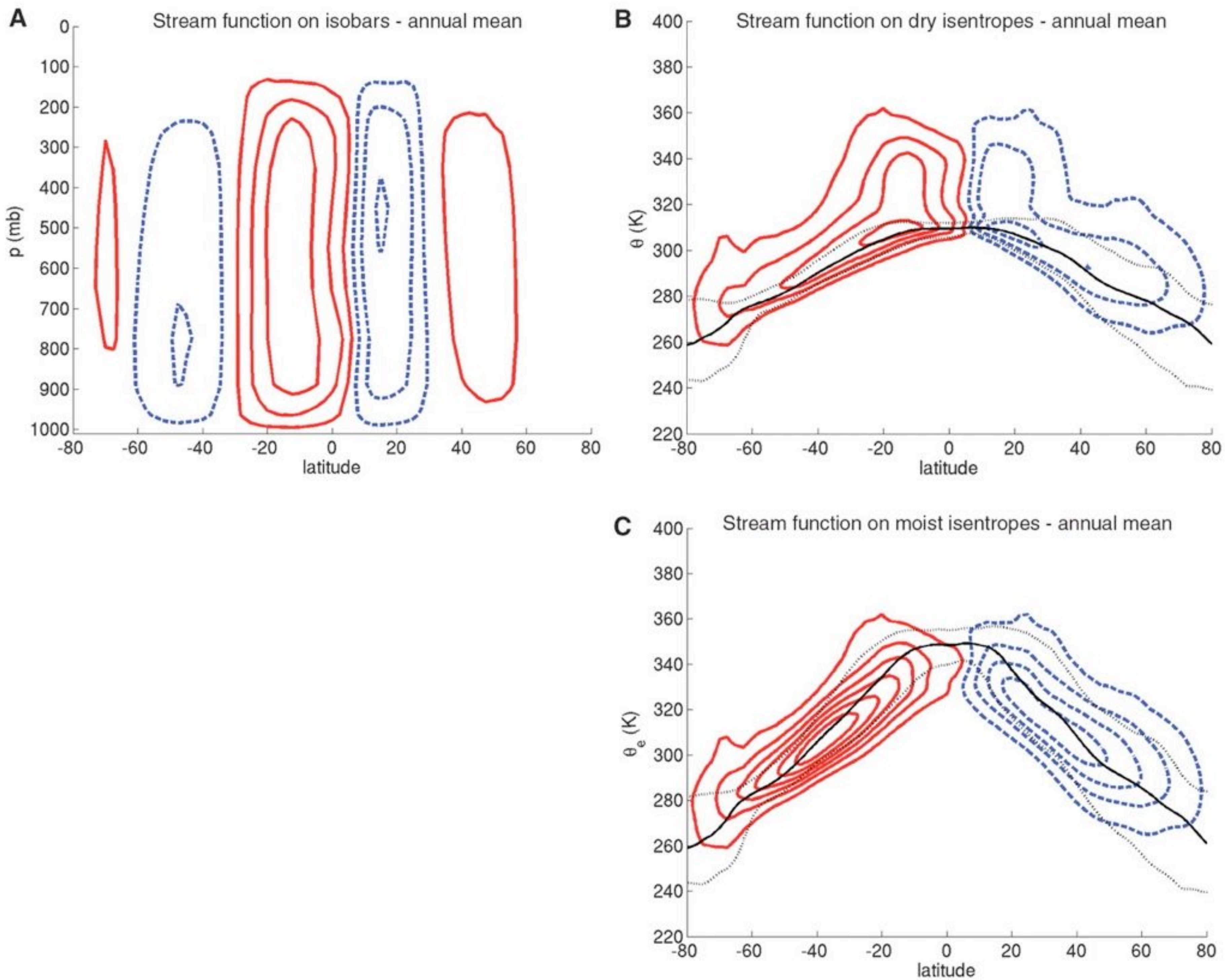
The isentropic mass streamfunction for the zonally averaged mass transport is defined by

$$\frac{\partial \Psi_\theta}{\partial \varphi} = 2\pi a^2 \cos\varphi \overline{\left[m_\theta \dot{\theta} \right]} \quad \text{and} \quad \frac{\partial \Psi_\theta}{\partial \theta} = -2\pi a \cos\varphi \overline{\left[m_\theta v \right]}$$

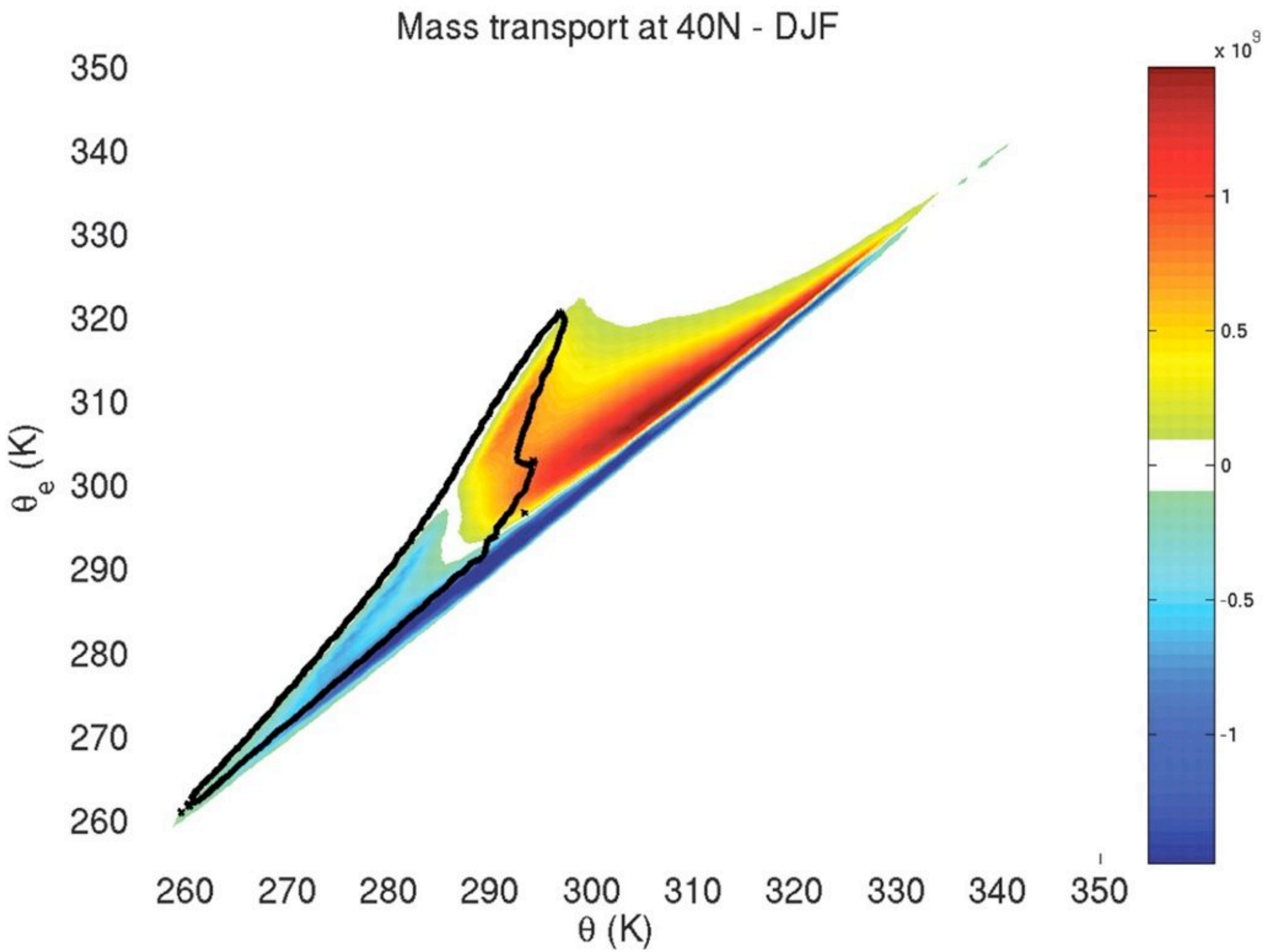
The Hadley circulations extend from equator to pole. No Ferrell circulations.



The effects of moisture

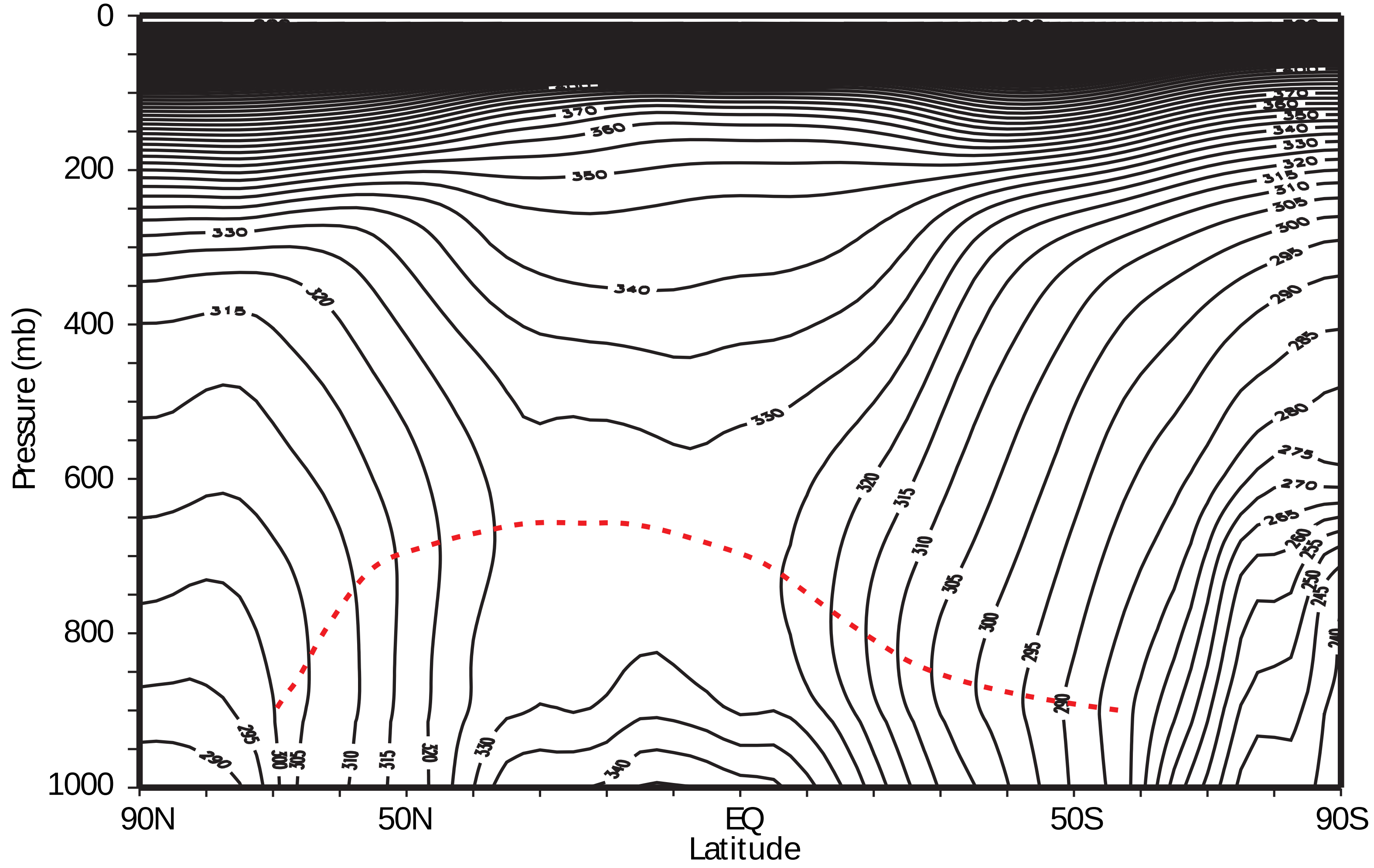


Pauluis et al. 2008



Pauluis et al. 2008

Equivalent potential temperature



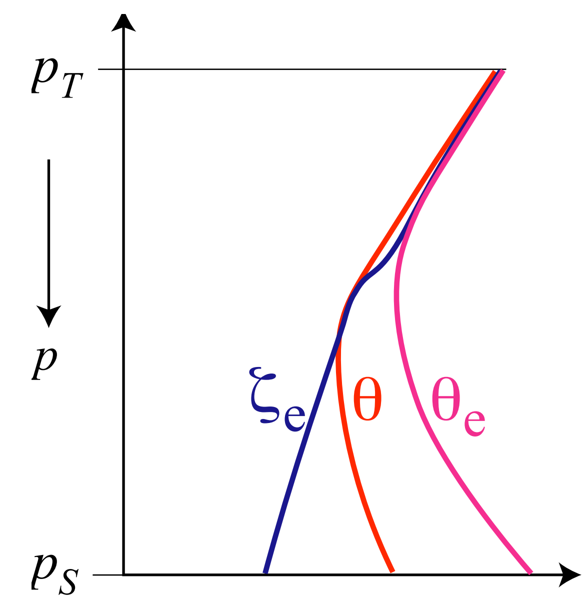
Recursive definition of ζ_e

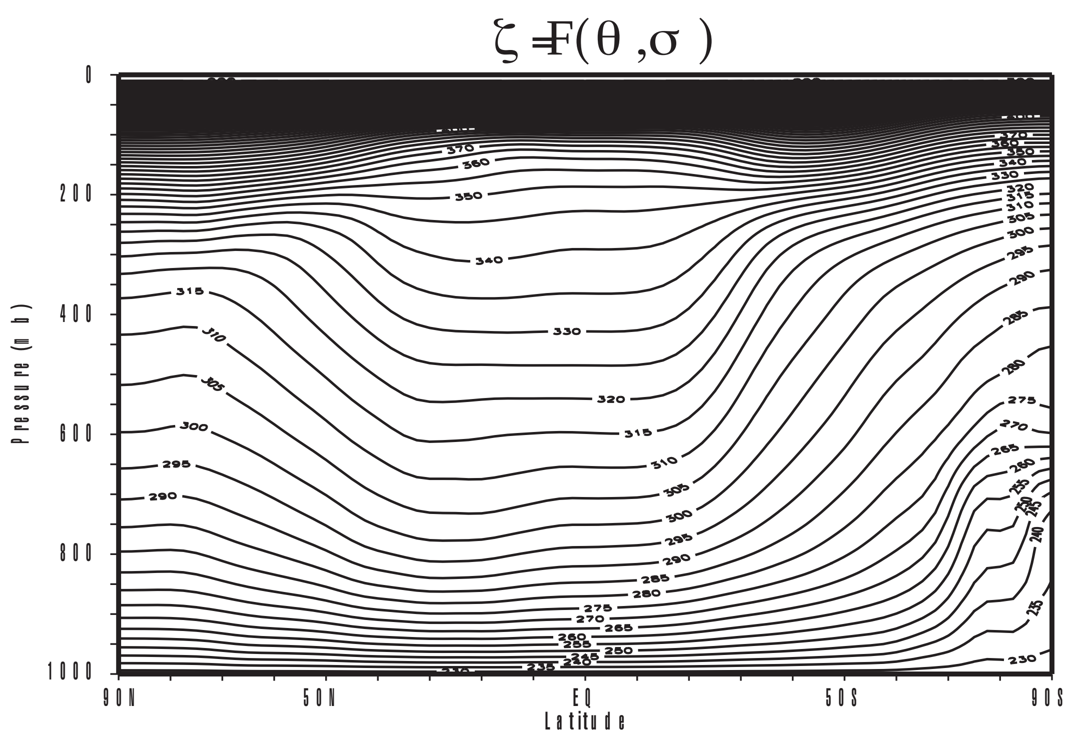
Having defined

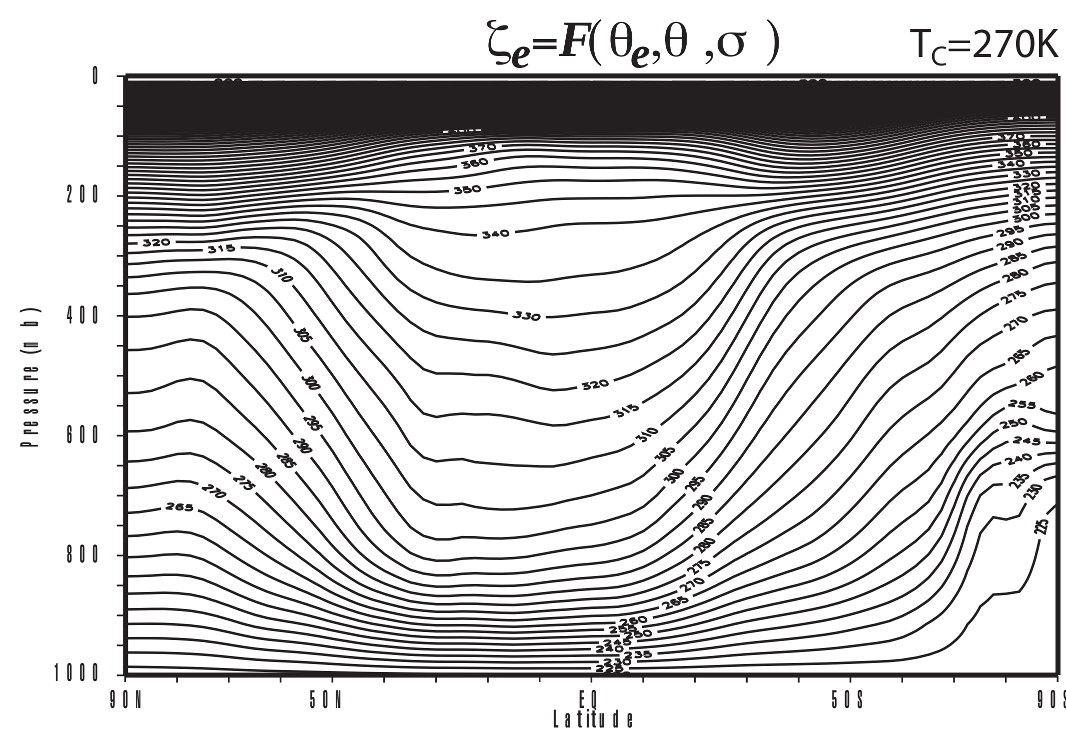
$$\zeta = f(\sigma) + g(\sigma)\theta$$

we now write

$$\zeta_e = f(\zeta) + g(\zeta)\theta_e$$







Non-hydrostatic hybrid-coordinate regional model

- Mike Toy's disertation
- Tested on Boulder windstorm case -- breaking mountain waves
- ALE, but tries to be Konor-Arakawa coordinate

Slide from Mike Toy

Vertical momentum transport

- In z coordinates:

$\overline{(\ )}$ Zonal mean

$(\ )'$ Perturbation

$$\frac{\partial}{\partial t}\bar{u} = -\frac{1}{\bar{\bar{\rho}}}\frac{\partial}{\partial z}\left[\overline{(\rho w)' u'}\right]$$

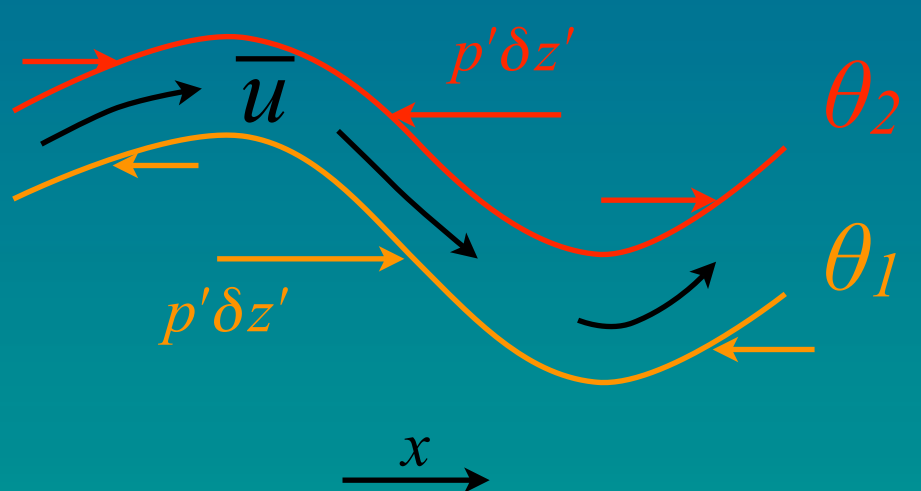
An eddy flux

- In H coordinates:

$$\frac{\partial}{\partial t}\bar{u} = \frac{1}{\bar{\bar{m}}}\frac{\partial}{\partial \theta}\left[\overline{p' \frac{\partial z'}{\partial x}}\right]$$

Pressure drag on isentropic (material) surfaces

A "quasi-Lagrangian view"



Slide from Mike Toy

Vertical momentum transport

- In a generalized vertical coordinate (η):

$$\frac{\partial}{\partial t}\bar{u} = \frac{1}{\overline{m}}\frac{\partial}{\partial \eta}\left[\overline{p'\frac{\partial z'}{\partial x}} - \overline{(m\dot{\eta})'u'}\right]$$

Vertical momentum flux

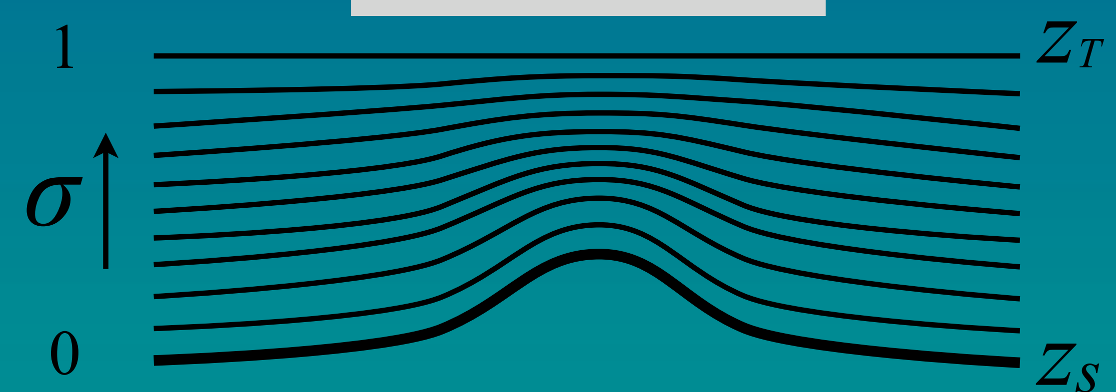
$$\dot{\eta} \equiv \frac{D\eta}{Dt}$$

Generalized vertical velocity

Another coordinate is the terrain-following σ coordinate, in which both terms of the momentum flux are retained:

$$\left[\overline{p'\frac{\partial z'}{\partial x}} - \overline{(m\dot{\sigma})'u'}\right]$$

$$\sigma \equiv \frac{z - z_S}{z_T - z_S}$$



Slide from Mike Toy

Diagnosed momentum fluxes

Nonhydrostatic gravity waves in an isothermal, uniform flow over a small mountain

- Mountain height = 10 m
- Mountain half-width = 2 km
- Mean zonal wind = 20 m s^{-1}
- Steady state reached in ~ 1.11 hours

$$\frac{\partial}{\partial t}\bar{u} = \frac{1}{\bar{m}}\frac{\partial}{\partial \eta}\left[\overline{p'\frac{\partial z'}{\partial x}} - \overline{(m\dot{\eta})'u'}\right] = 0$$

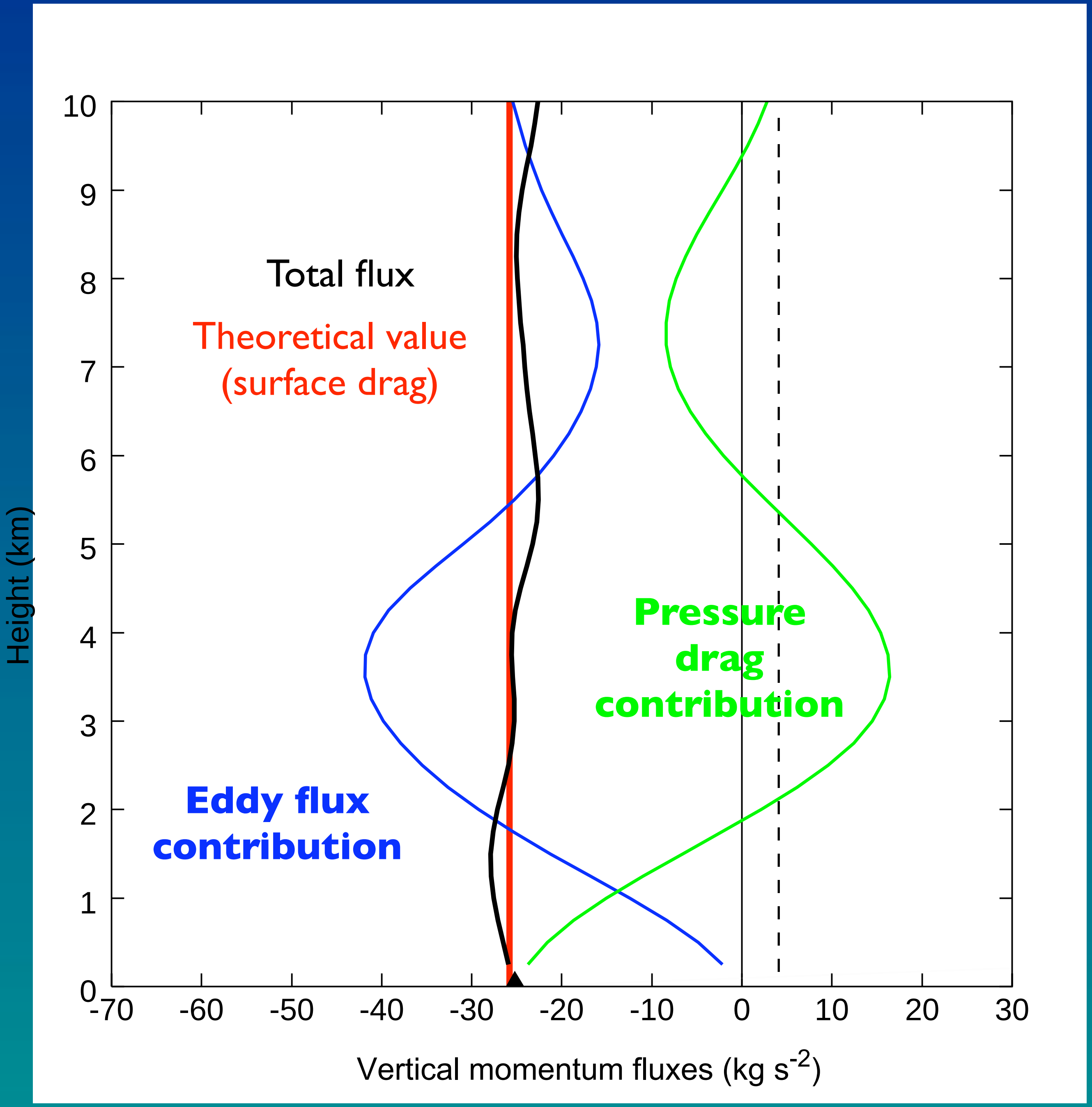
Vertical momentum flux = constant

Slide from Mike Toy

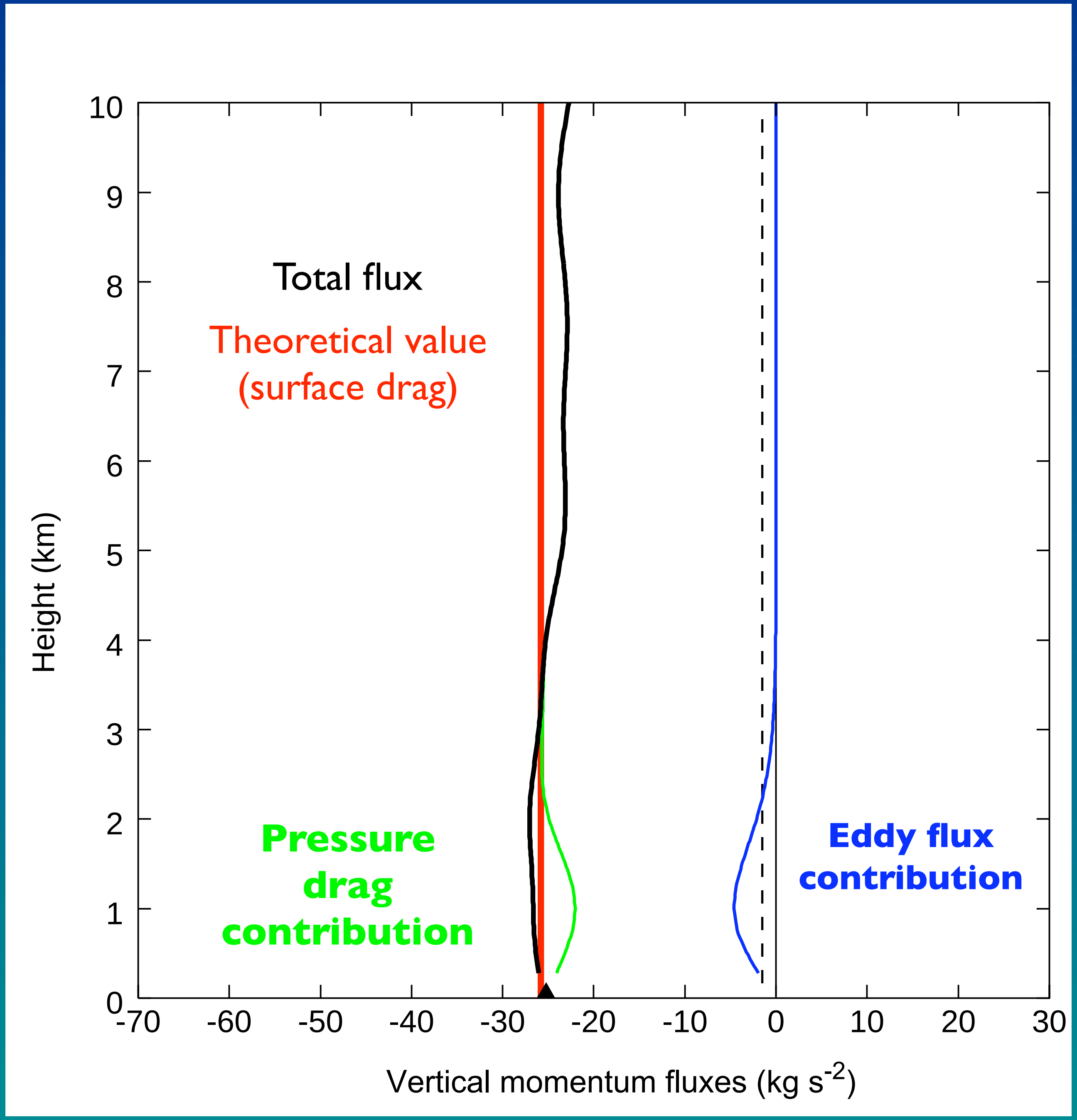
Small-amplitude gravity wave experiment

Profiles of vertical flux of horizontal momentum at t = 1.11 hours

Terrain-following σ (Eulerian) coordinate



θ coordinate (mostly)



Slide from Mike Toy

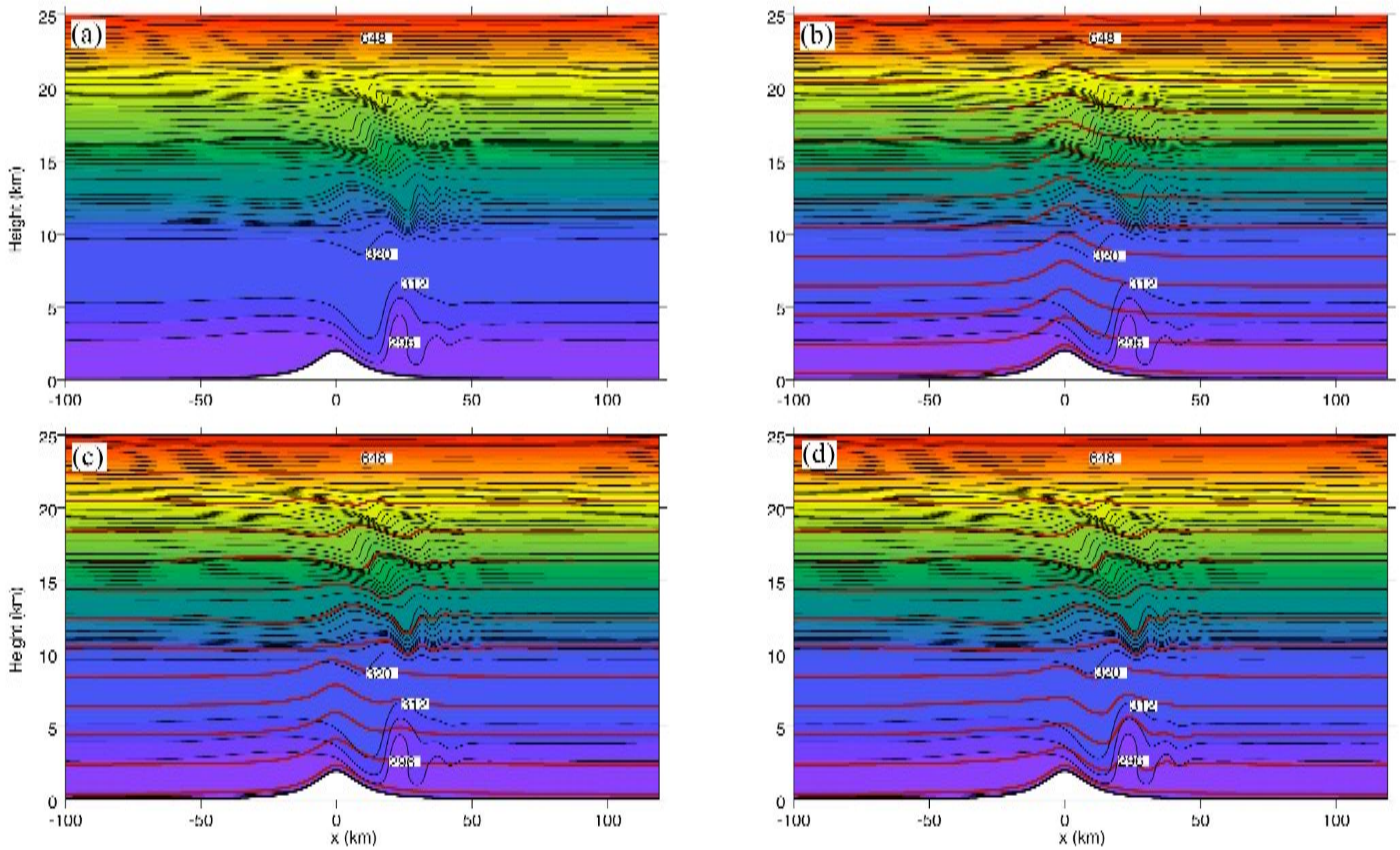


Figure 5.20: Isentropic surfaces (black curves) at time $t = 1$hr10min for the 11 January 1972 Boulder windstorm simulations using the σ coordinate with (a) 500 levels and (b) 125 levels in the lowest 25km, and the hybrid coordinate with 125 levels in the lowest 25km for (c) $\theta_{min} = 20$K and (d) $\theta_{min} = 270$K. The contour interval is 8K and selected isentropes are labeled. The bold red curves in panels (b)-(d) show the locations of every tenth model coordinate surface.

Slide from Mike Toy

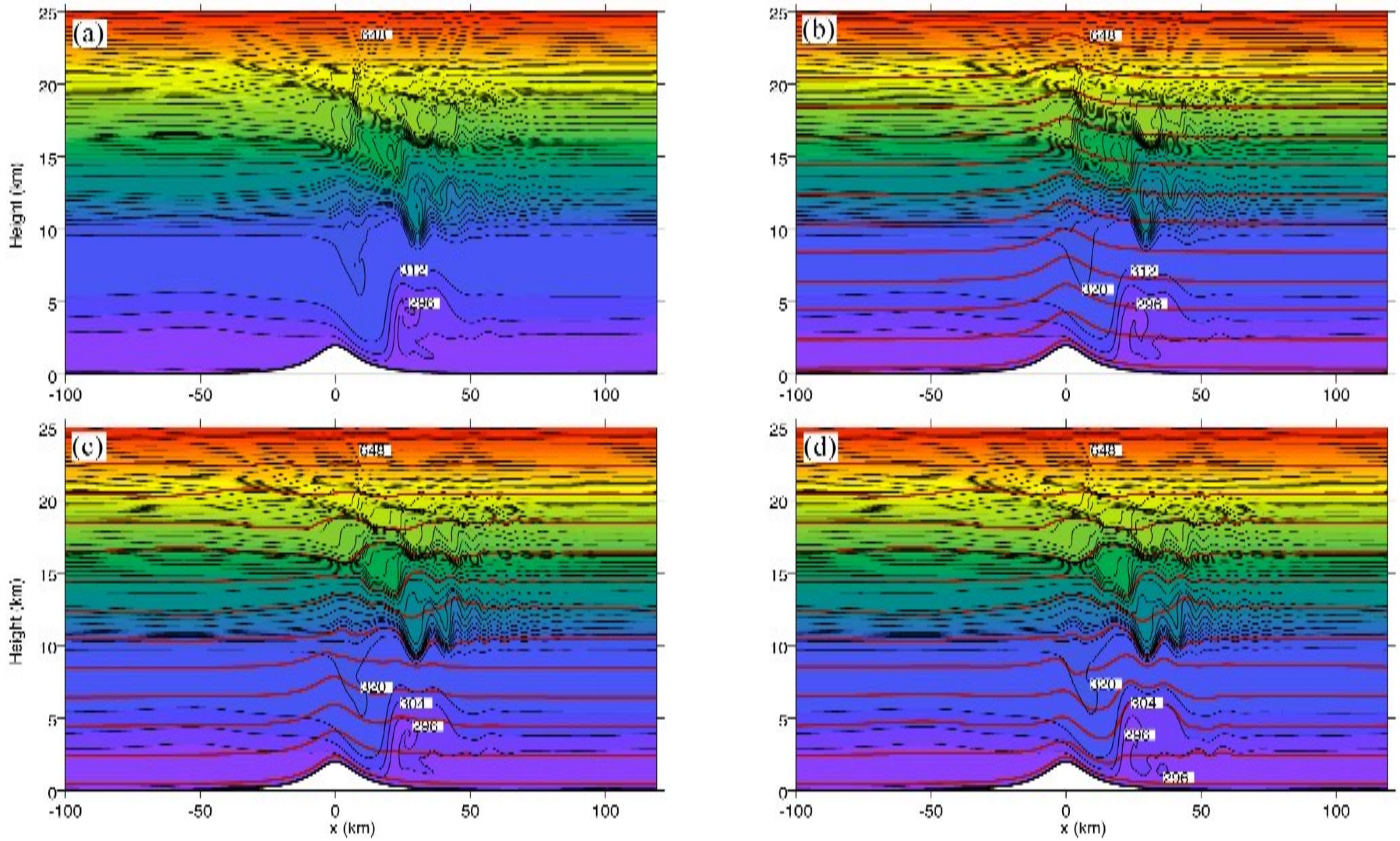


Figure 5.21: Isentropic surfaces (black curves) at time t=2hours for the 11 January 1972 Boulder windstorm simulations using the σ coordinate with (a) 500 levels and (b) 125 levels in the lowest 25km, and the hybrid coordinate with 125 levels in the lowest 25km for (c) θ_{min}=20K and (d) θ_{min}=270K. The contour interval is 8K and selected isentropes are labeled. The bold red curves in panels (b)-(d) show the locations of every tenth model coordinate surface.

Slide from Mike Toy

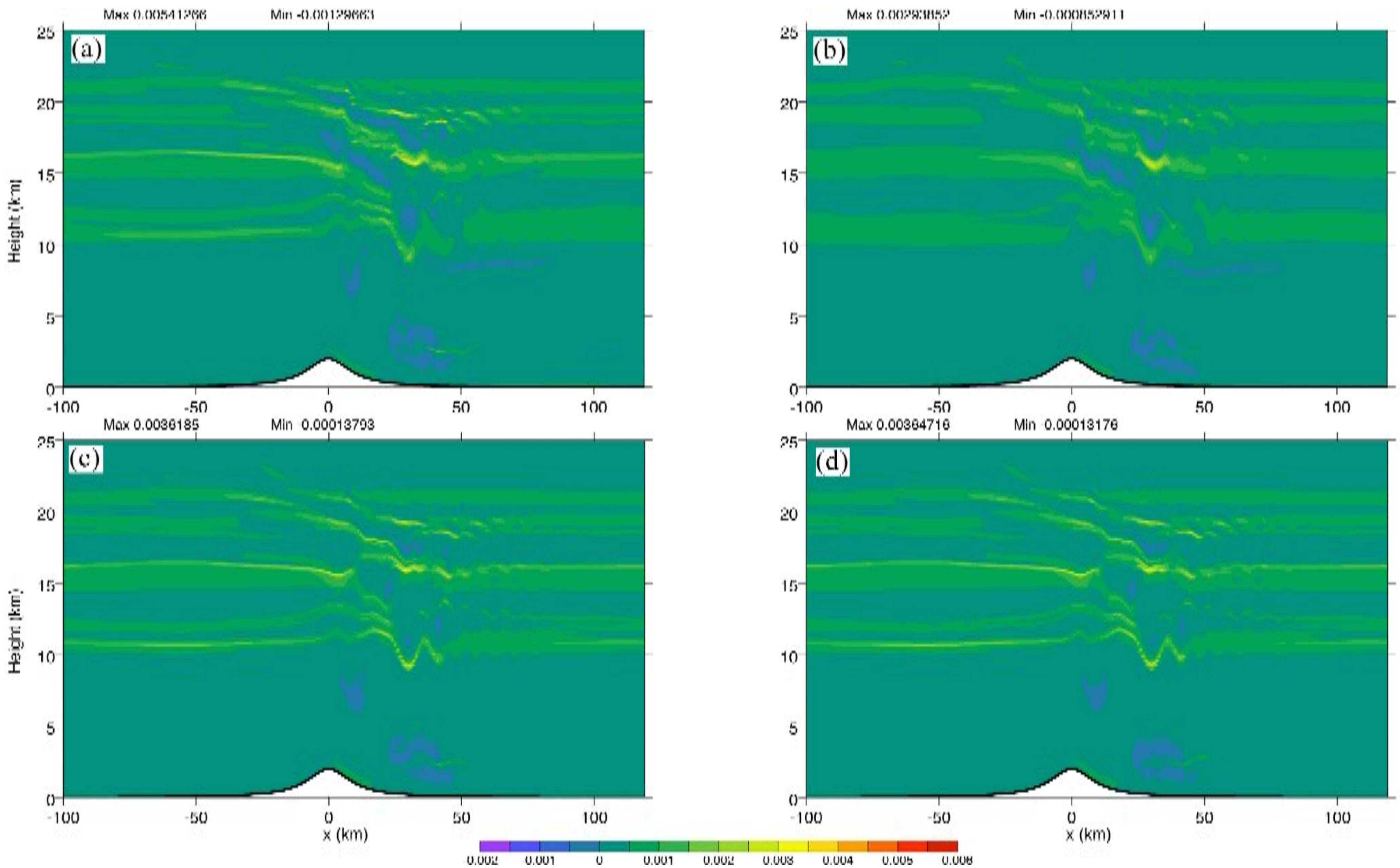


Figure 5.22: Static stability $N^2 = g\theta^{-1}\partial\theta/\partial z$ at time $t = 2$ hours for the 11 January 1972 Boulder windstorm simulations using the σ coordinate with (a) 500 levels and (b) 125 levels in the lowest 25km, and the hybrid coordinate with 125 levels in the lowest 25 km for (c) $\theta_{min} = 20$ K and (d) $\theta_{min} = 270$ K.

Slide from Mike Toy

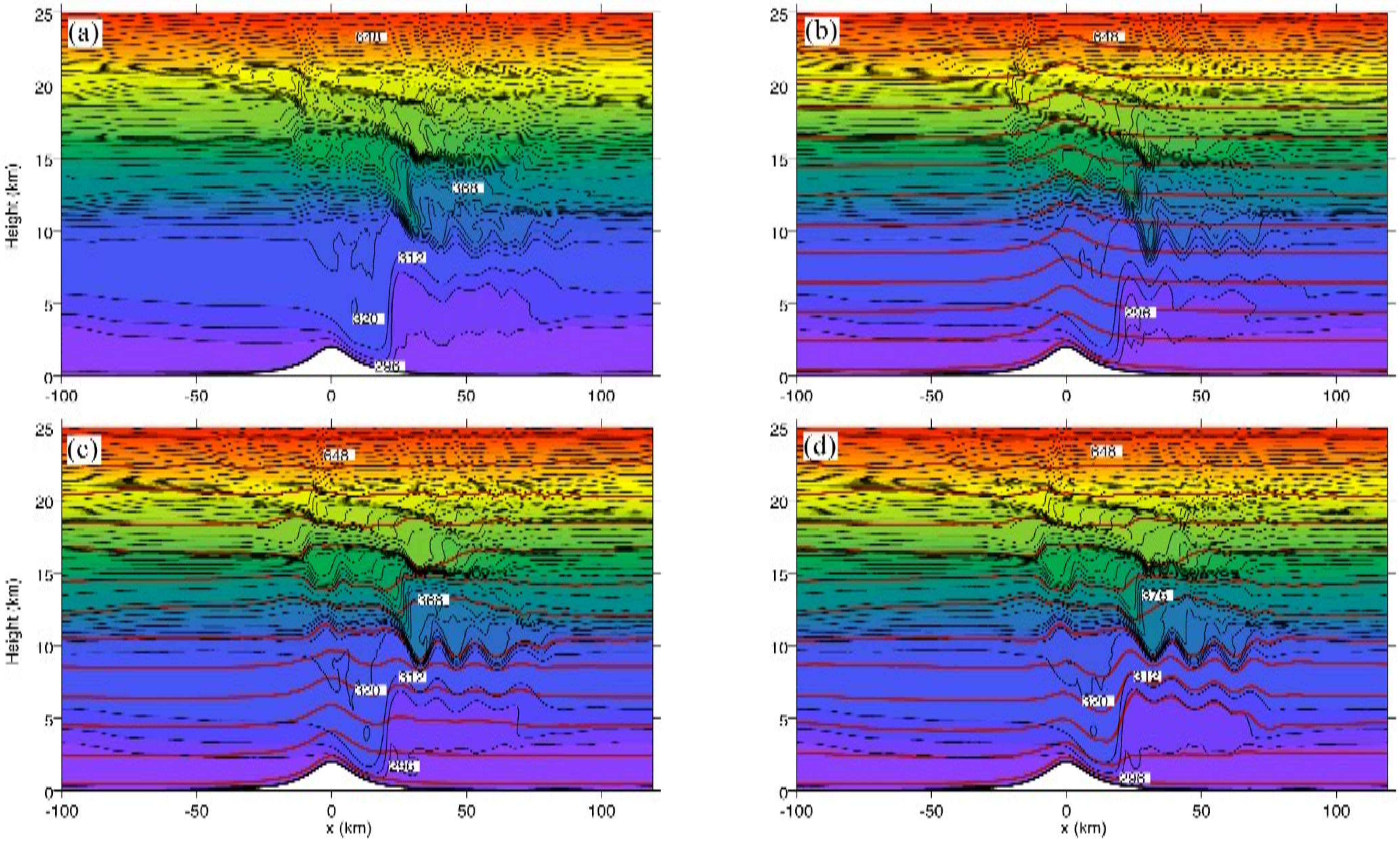
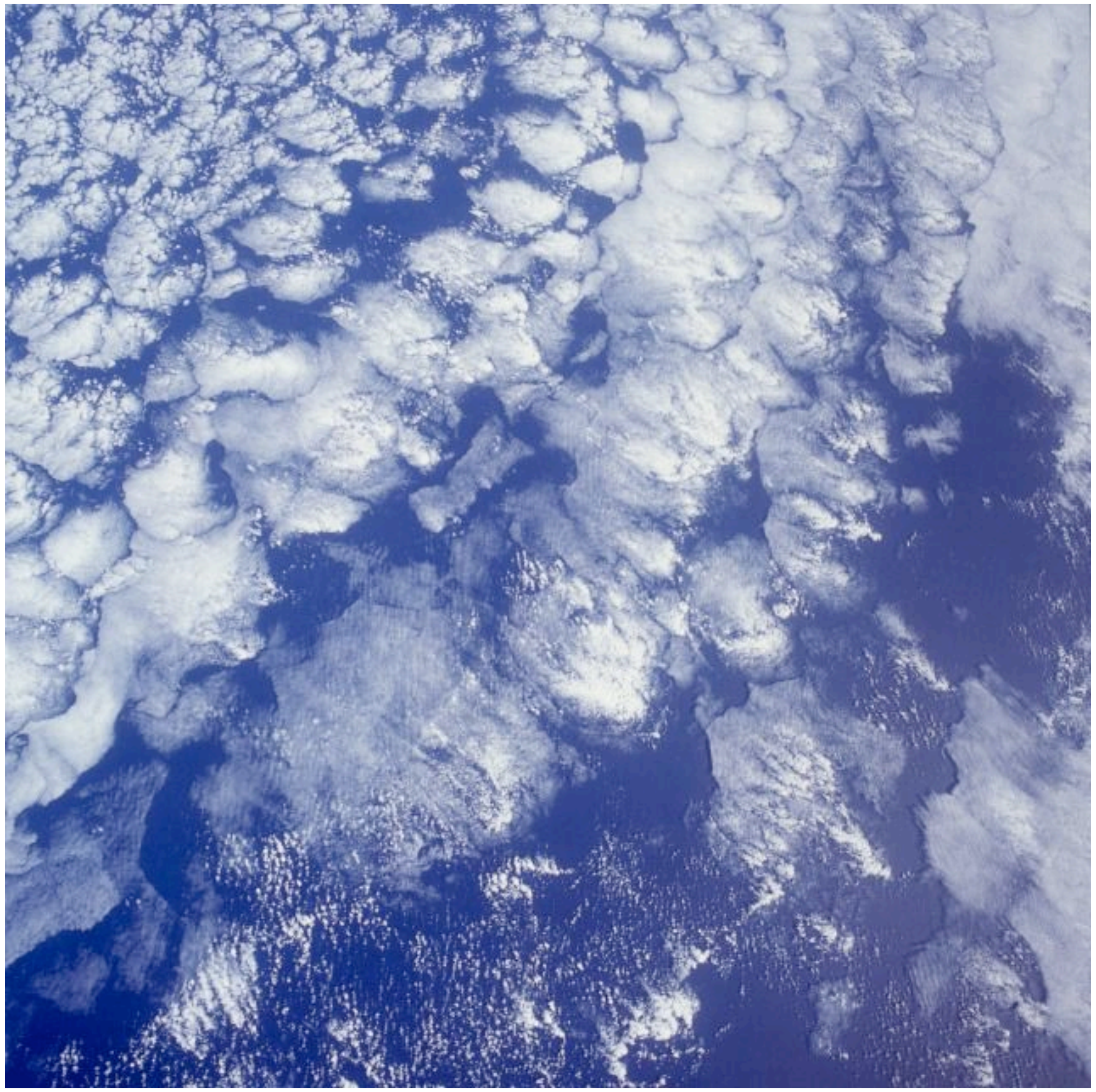


Figure 5.23: Isentropic surfaces (black curves) at time $t = 3$hours for the 11 January 1972 Boulder windstorm simulations using the σ coordinate with (a) 500 levels and (b) 125 levels in the lowest 25km, and the hybrid coordinate with 125 levels in the lowest 25km for (c) $\theta_{min} = 20$K and (d) $\theta_{min} = 270$K. The contour interval is 8K and selected isentropes are labeled. The bold red curves in panels (b)-(d) show the locations of every tenth model coordinate surface..

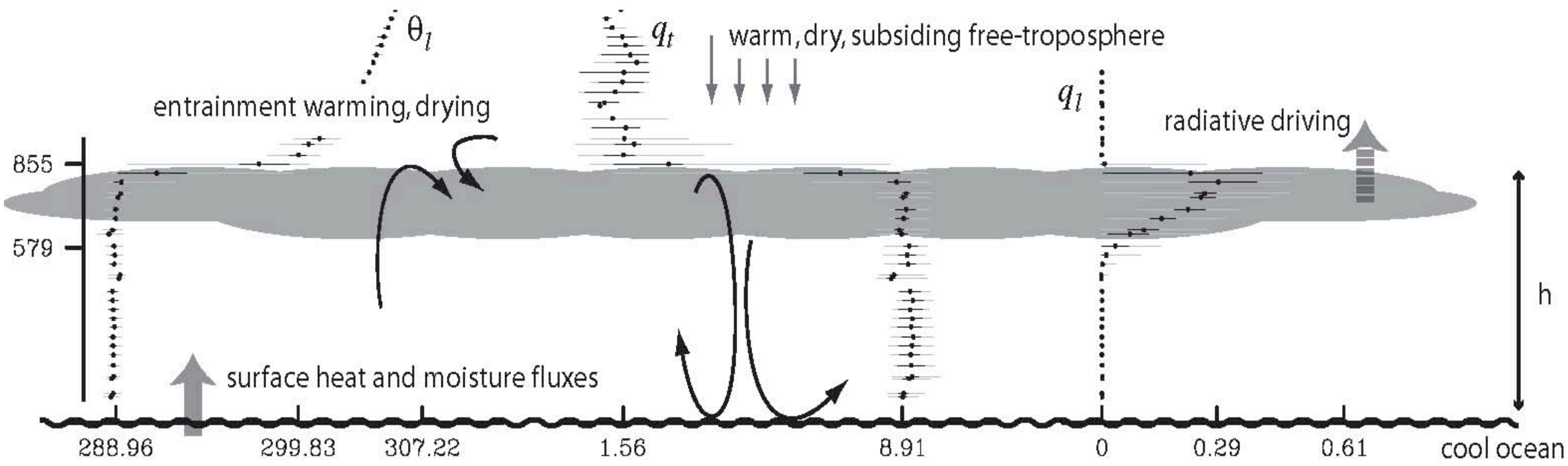
Slide from Mike Toy

The embedded PBL:
A 20-year-old "wild idea"

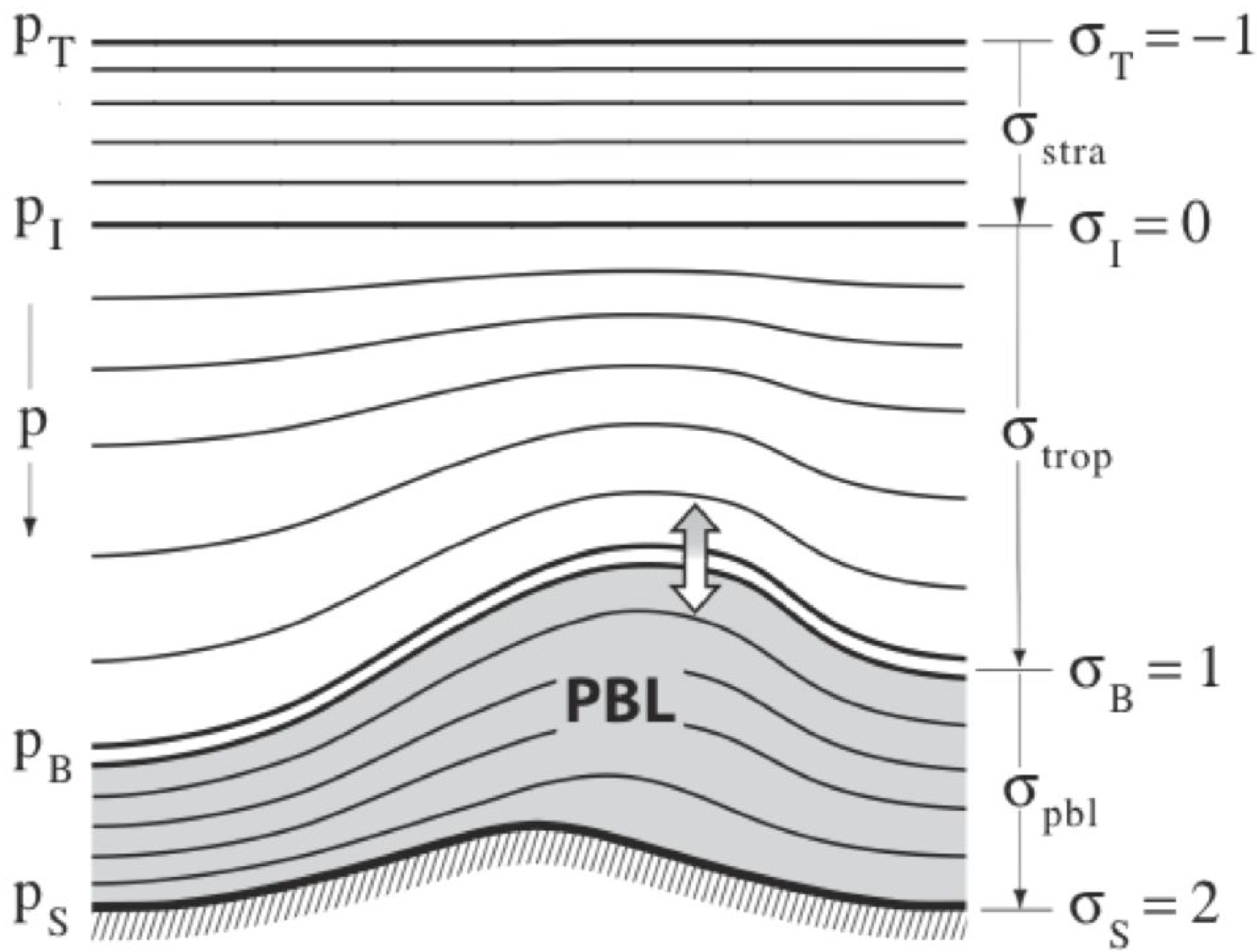
- The PBL is the layer that is directly and strongly influenced by the lower boundary, primarily through turbulent processes.
- The PBL's depth varies dramatically in both time and space.
- The top of the PBL is usually very well defined.
- Air crosses the PBL top as a result of turbulent and convective processes that must be parameterized.



Potent processes at the PBL top



From Bjorn Stevens



Konor et al. 2009

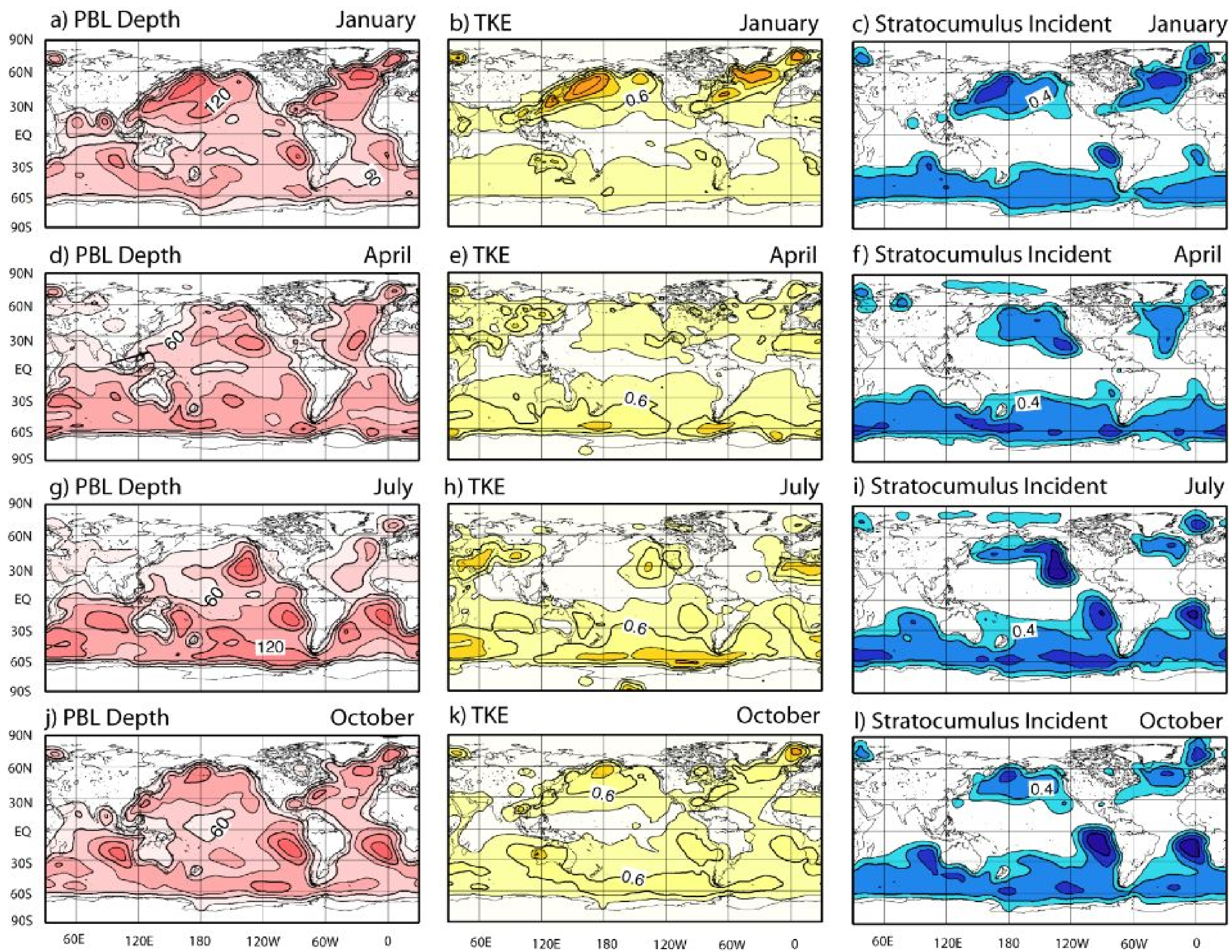
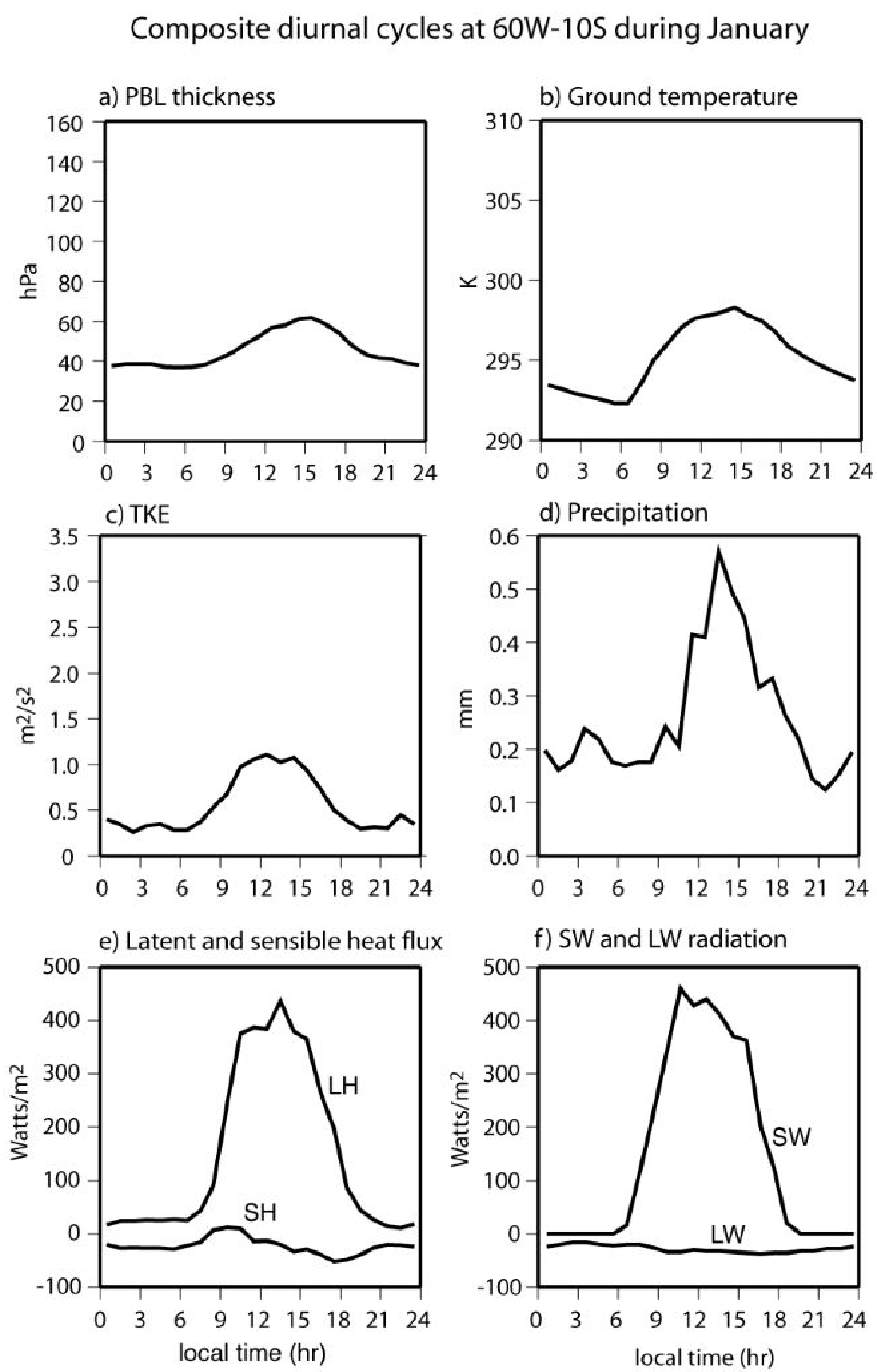
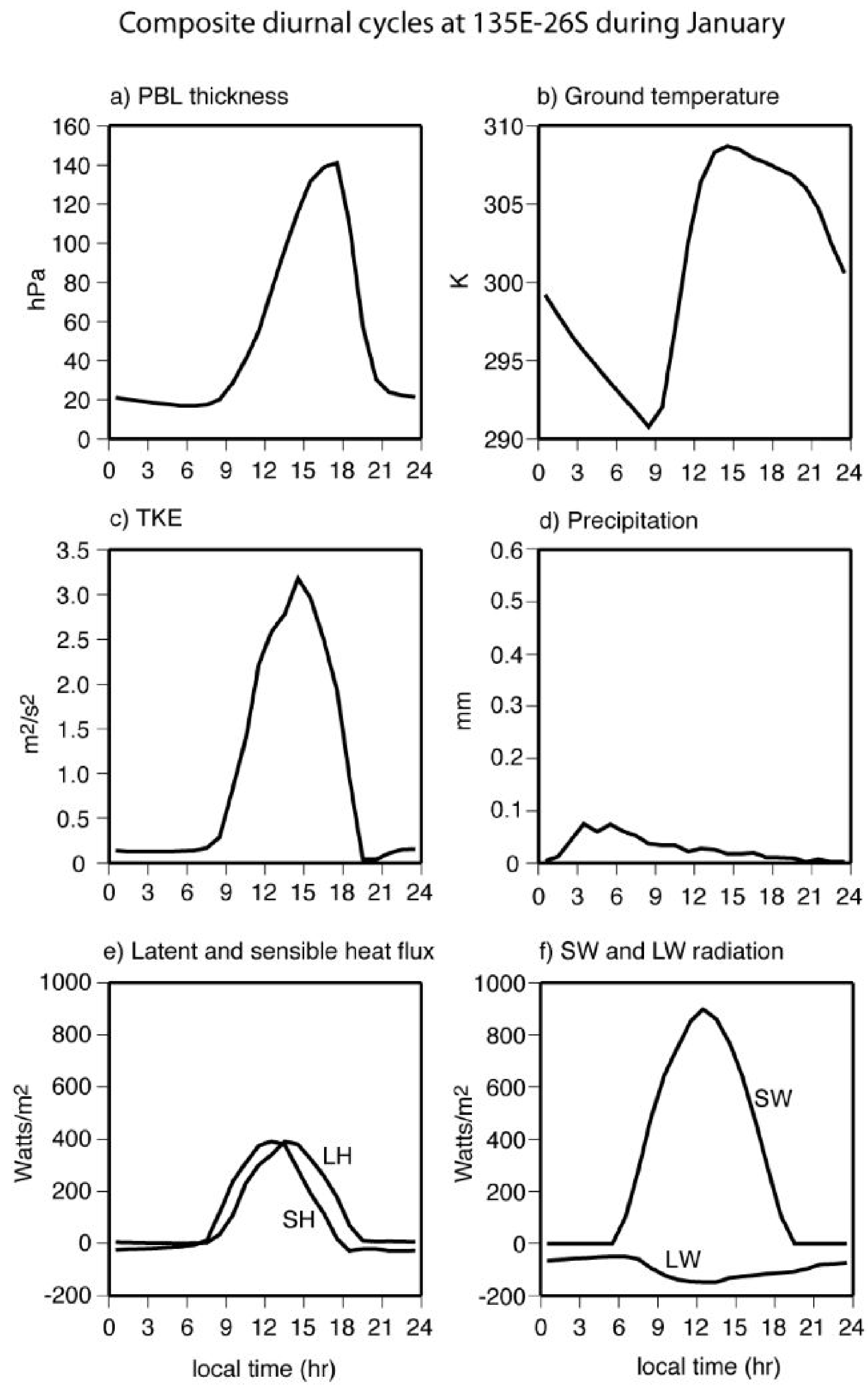


FIG. 4. Monthly-mean PBL-depth (hPa, with 30 hPa contour interval, left column), TKE (m^2s^{-2}, with 0.2 m^2s^{-2} contour intervals, middle column) and stratocumulus incidence (with 0.1 contour interval, right column) for January (uppermost row), April (second from top), July (third from top) and October (lowermost row).

Konor et al. 2009





Konor et al. 2009

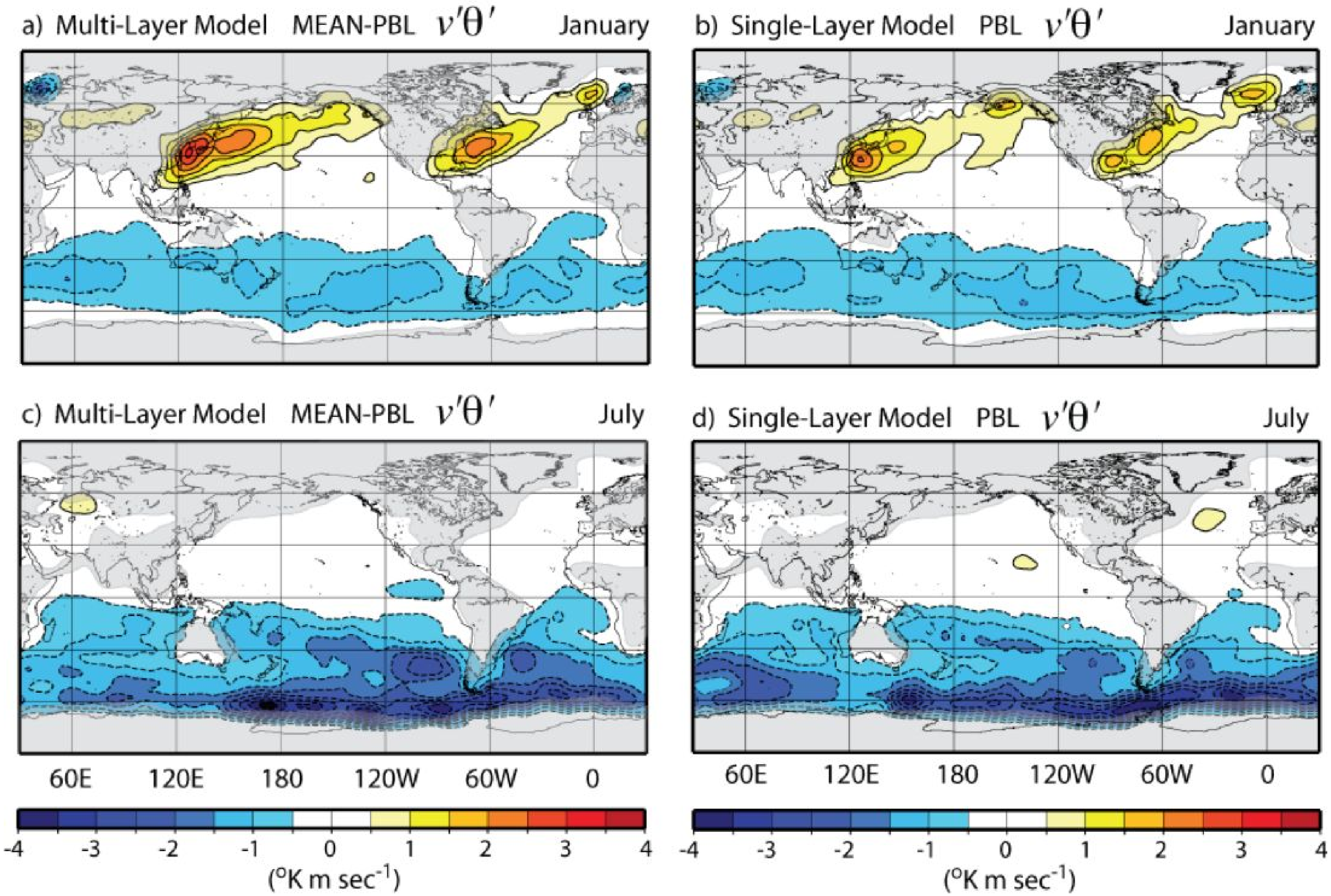


FIG. 13. Monthly-mean longitudinal potential temperature flux by eddies (K m sec^{-1}). Upper and lower panels are for January and July, respectively. Left and right columns show results from the multi-layer CONTROL and single-layer simulation, respectively.

Konor et al. 2009

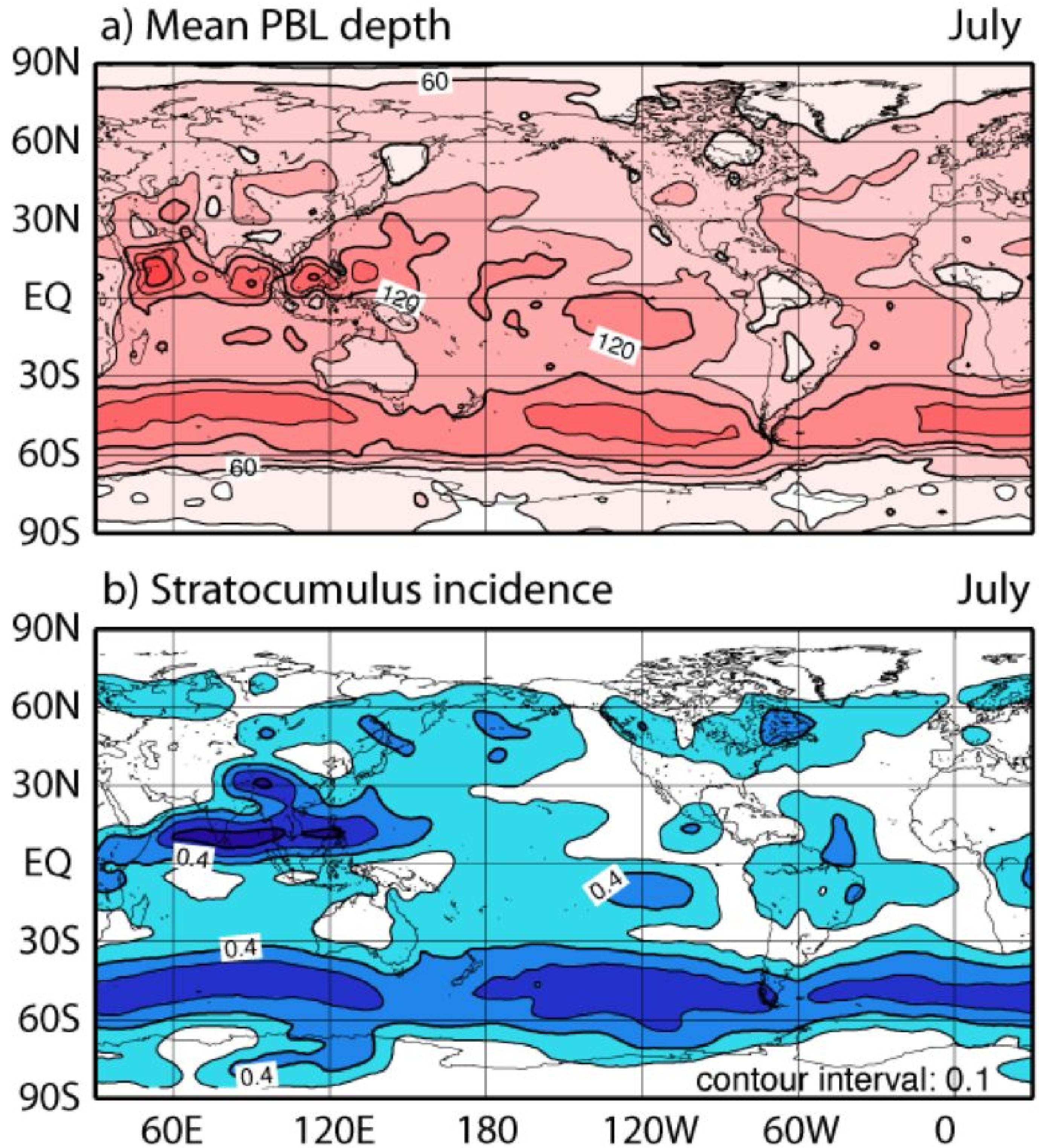


FIG. 16. Monthly-mean PBL depth (hPa, above) and stratocumulus incidence for July from *fixed-sigma* simulation. See Fig. 4 for contour intervals.

Konor et al. 2009

Conclusions

- Hybrid sigma-theta coordinates behave well in hydrostatic models.
- At higher resolution, hybrid z-theta is probably a better choice.
- The hybrid coordinate may need smoothing via an ALE approach.
- Special approaches are needed to deal with the PBL. An embedded PBL can give good results.