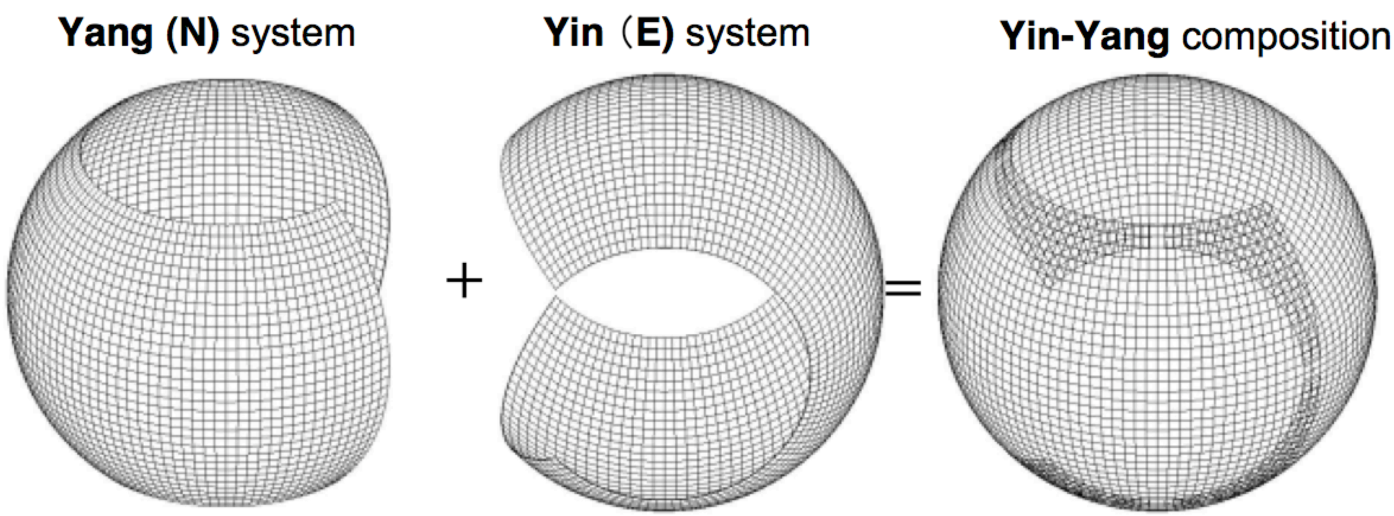
Some Possible Discretizations for the Sphere

Yin-Yang grid

【*Yi Jing*: the Book of Changes】

The universe（both space and time）can be divided into **Yin** and **Yang**, Which is composed with *metals*(金), *water*(水), *wood*(木), *fire*(火) and *soil*(土). For example, the **moon** is due to Yin, and the **sun** belong to Yang. The energy of the atmosphere comes from the sun.



Provided by Dr. Kageyama, ESC, who is the developer of the Yin-Yang grid

(From Peng et al, EGU 2004)

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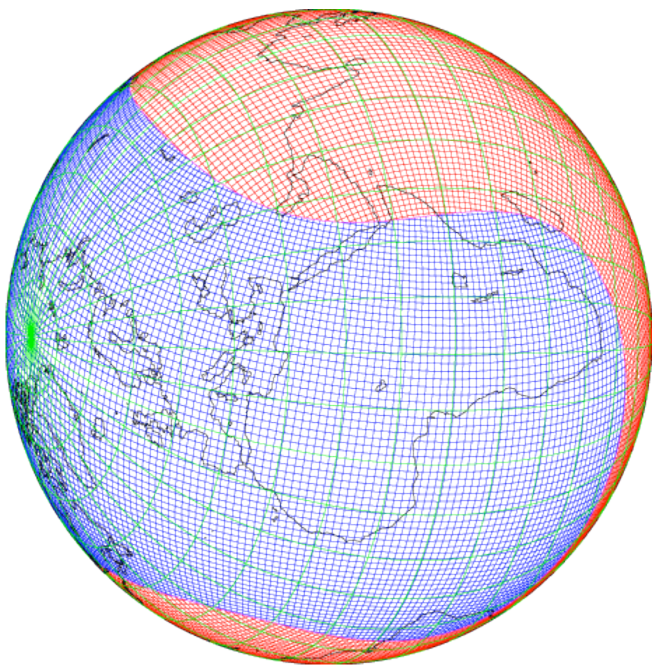
yin-yang grid



Advantages: relatively isotropic, relatively uniform resolution, conformal. Local refinement is possible.

Disadvantages: boundaries (overlaps) need special treatment. Local refinements at boundaries would also need special treatment.

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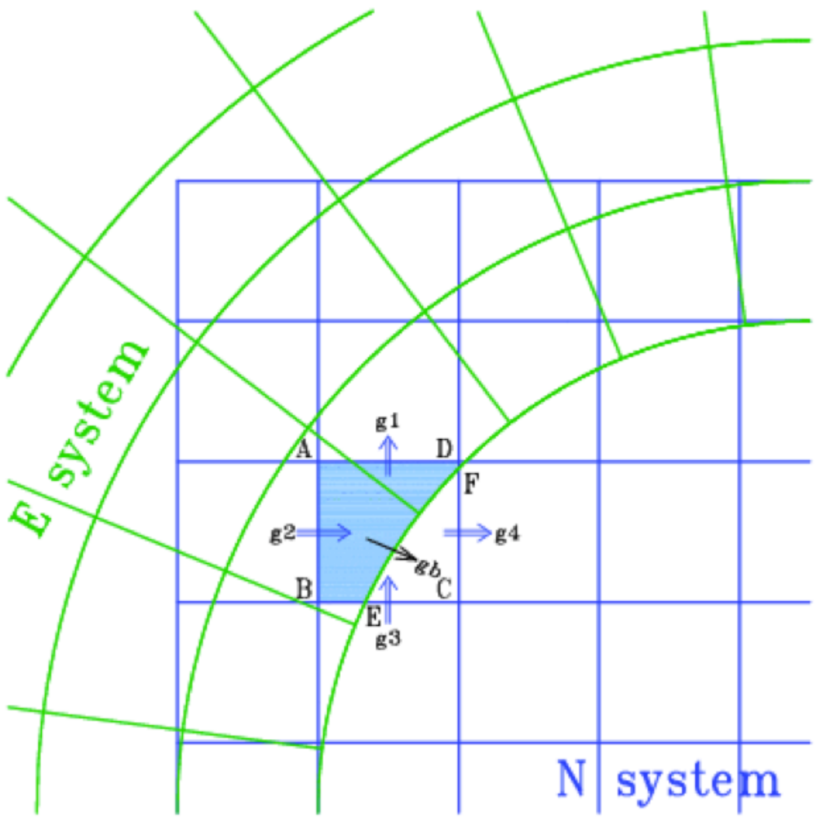
Yin-Yang Grid



The necessary and sufficient condition for global conservation is as

$$\int_{\Gamma_E} g_E d\Gamma = \int_{\Gamma_E} g_N d\Gamma$$

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$d\Gamma$ denotes any part of the boundary of N,E, e.g. EF.

(From Peng et al, EGU 2004)

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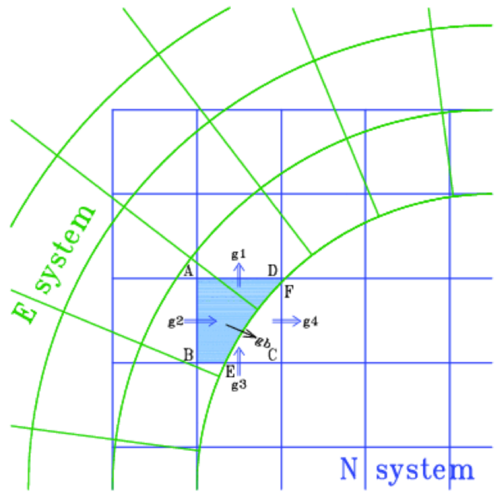

The necessary and sufficient condition for global conservation is as

$$\int_{\Gamma_E} g_E \, d\Gamma = \int_{\Gamma_E} g_N \, d\Gamma$$

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The sufficient condition is

$$g_E \, d\Gamma = g_N \, d\Gamma$$

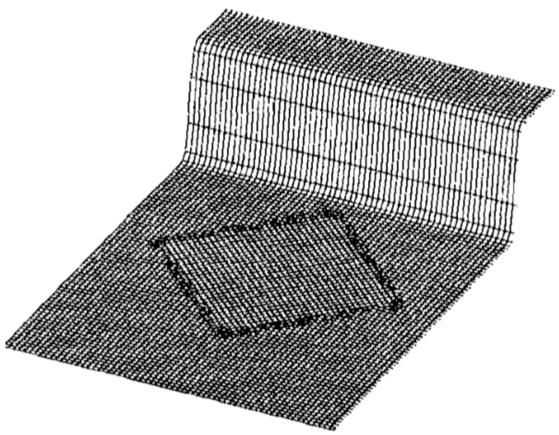
$d\Gamma$ denotes any part of the boundary of N,E, e.g. EF.

Linear integration (i.e. piecewise-constant fluxes) is stable, conservative, but inaccurate for higher-order transport schemes.

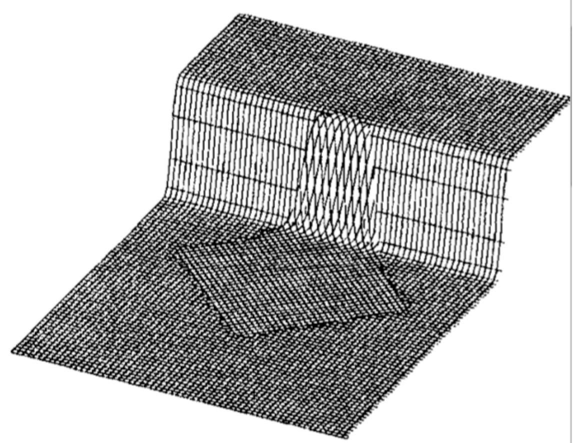
Higher-order flux integration can be designed to be conservative and accurate, but have not proven stable.

Conservative Interpolation for Overlapping Grids

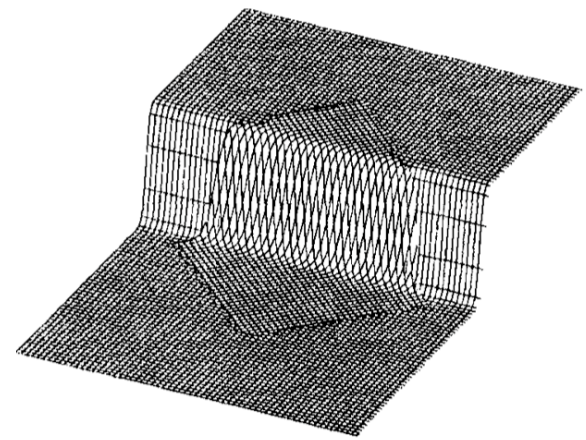
Chessire and Henshaw (SIAM J. Sci. Comput. 1994)



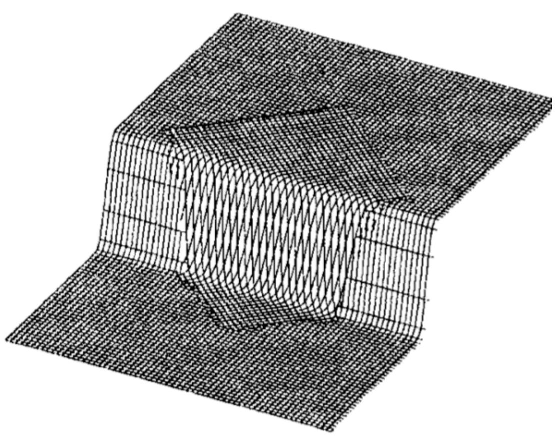
$t = 0$



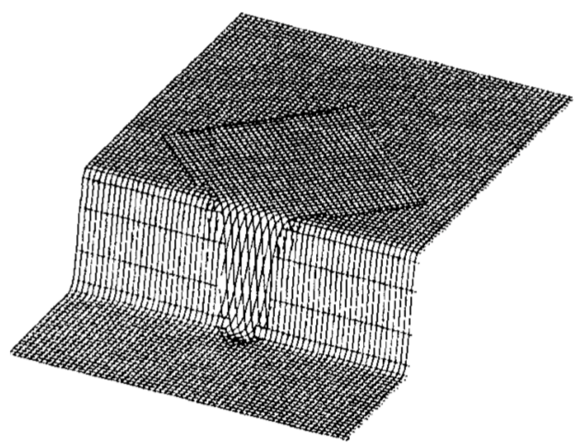
$t = 20$



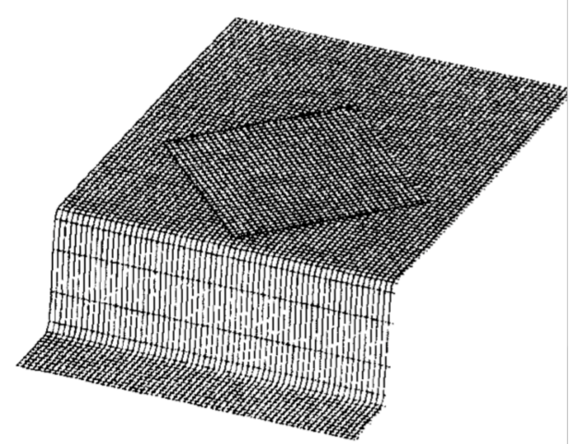
$t = 40$



$t = 60$



$t = 80$



$t = 100$

Conservative Interpolation for Overlapping Grids
Chessire and Henshaw (SIAM J. Sci. Comput. 1994)

(8)
$$\mathbf{F}_{1,N_1+1} = \sum_{j=M_1}^{M_2} \gamma_j \mathbf{F}_{2,j} \qquad 0 \le M_1 \le M_2 \le N_2,$$

(9)
$$\mathbf{F}_{2,0} = \sum_{j=L_1}^{L_2} \beta_j \mathbf{F}_{1,j} \qquad 0 \le L_1 \le L_2 \le N_1.$$

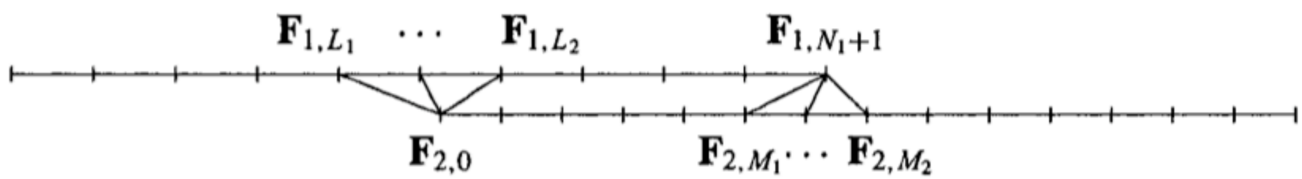


FIG. 4. *Interpolation of endpoints.*

Introduce the discrete approximation to the integral $I(t)$,

$$S(t) = \sum_{k=1}^{2} \sum_{i=0}^{N_k} \alpha_{k,i} \mathbf{u}_{k,i} h_{k,i}.$$