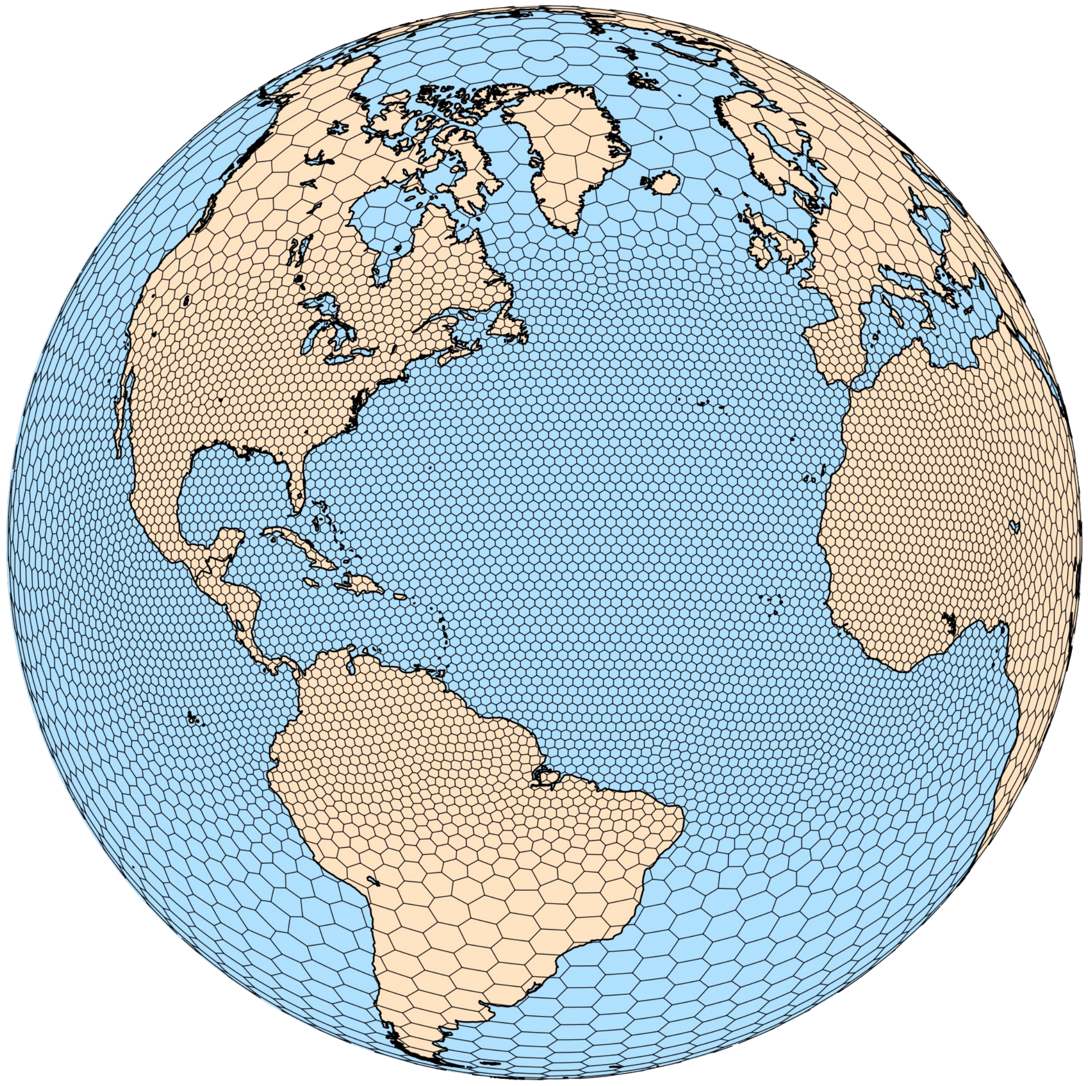
Dynamics and Dynamics Configuration

Time integration

- Time integration
- Timesteps
- Transport
- Filters
- References



MPAS Nonhydrostatic Atmospheric Solver

Equations

- Prognostic equations for coupled variables.
- Generalized height coordinate.
- Horizontally vector-invariant equation set.
- Continuity equation for dry air mass.
- Thermodynamic equation for coupled potential temperature.

Variables: $(U, V, \Omega, \Theta, Q_j) = \tilde{\rho}_d\,(u, v, \omega, \theta, q_j)$ $\qquad \tilde{\rho}_d = \rho_d/\zeta_z$

Vertical coordinate: $z = \zeta + A(\zeta) h_s(x, y, \zeta)$

Prognostic equations:

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\nabla_\zeta\left(\frac{p}{\zeta_z}\right) - \frac{\partial \mathbf{z}_H p}{\partial \zeta}\right] - \eta\,\mathbf{k}\times\mathbf{V}_H - \mathbf{v}_H \nabla_\zeta\cdot\mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K + \mathbf{F}_{V_H}$$

$$\frac{\partial W}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\frac{\partial p}{\partial \zeta} + g\tilde{\rho}_m\right] - \left(\nabla\cdot\mathbf{v}\,W\right)_\zeta + F_W$$

$$\frac{\partial \Theta_m}{\partial t} = -\left(\nabla\cdot\mathbf{V}\,\theta_m\right)_\zeta + F_{\Theta_m}$$

$$\frac{\partial \tilde{\rho}_d}{\partial t} = -\left(\nabla\cdot\mathbf{V}\right)_\zeta$$

$$\frac{\partial Q_j}{\partial t} = -\left(\nabla\cdot\mathbf{V}\,q_j\right)_\zeta + F_{Q_j}$$

Diagnostics and definitions:

$$\frac{\rho_m}{\rho_d} = 1 + q_v + q_c + q_r + \ldots$$

$$p = p_0\left(\frac{R_d \zeta_z \Theta_m}{p_0}\right)^\gamma \qquad \theta_m = \theta\left[1 + (R_v/R_d) q_v\right]$$

Time Integration

3rd Order Runge-Kutta time integration

Advance one time step $\phi^t \to \phi^{t+\Delta t}$

$$\frac{\partial U}{\partial t} = RHS_u$$

$$\frac{\partial W}{\partial t} = RHS_w$$

$$\vdots$$

$$\phi^* = \phi^t + \frac{\Delta t}{3} RHS(\phi^t)$$

$$\phi^{**} = \phi^t + \frac{\Delta t}{2} RHS(\phi^*)$$

$$\phi^{t+\Delta t} = \phi^t + \Delta t \, RHS(\phi^{**})$$

$$\phi_t = ik\phi; \quad \phi^{n+1} = A\phi^n; \quad |A| = 1 - \frac{(k\Delta t)^4}{24}$$ + H.O.T

Time Integration

2^{nd}-order RK variant – default in MPAS

Advance one time step $\phi^t \rightarrow \phi^{t+\Delta t}$

$$\frac{\partial U}{\partial t} = RHS_u$$

$$\frac{\partial W}{\partial t} = RHS_w$$

-
-
-

$$\phi^* = \phi^t + \frac{\Delta t}{2} RHS(\phi^t)$$

$$\phi^{**} = \phi^t + \frac{\Delta t}{2} RHS(\phi^*)$$

$$\phi^{t+\Delta t} = \phi^t + \Delta t\, RHS(\phi^{**})$$

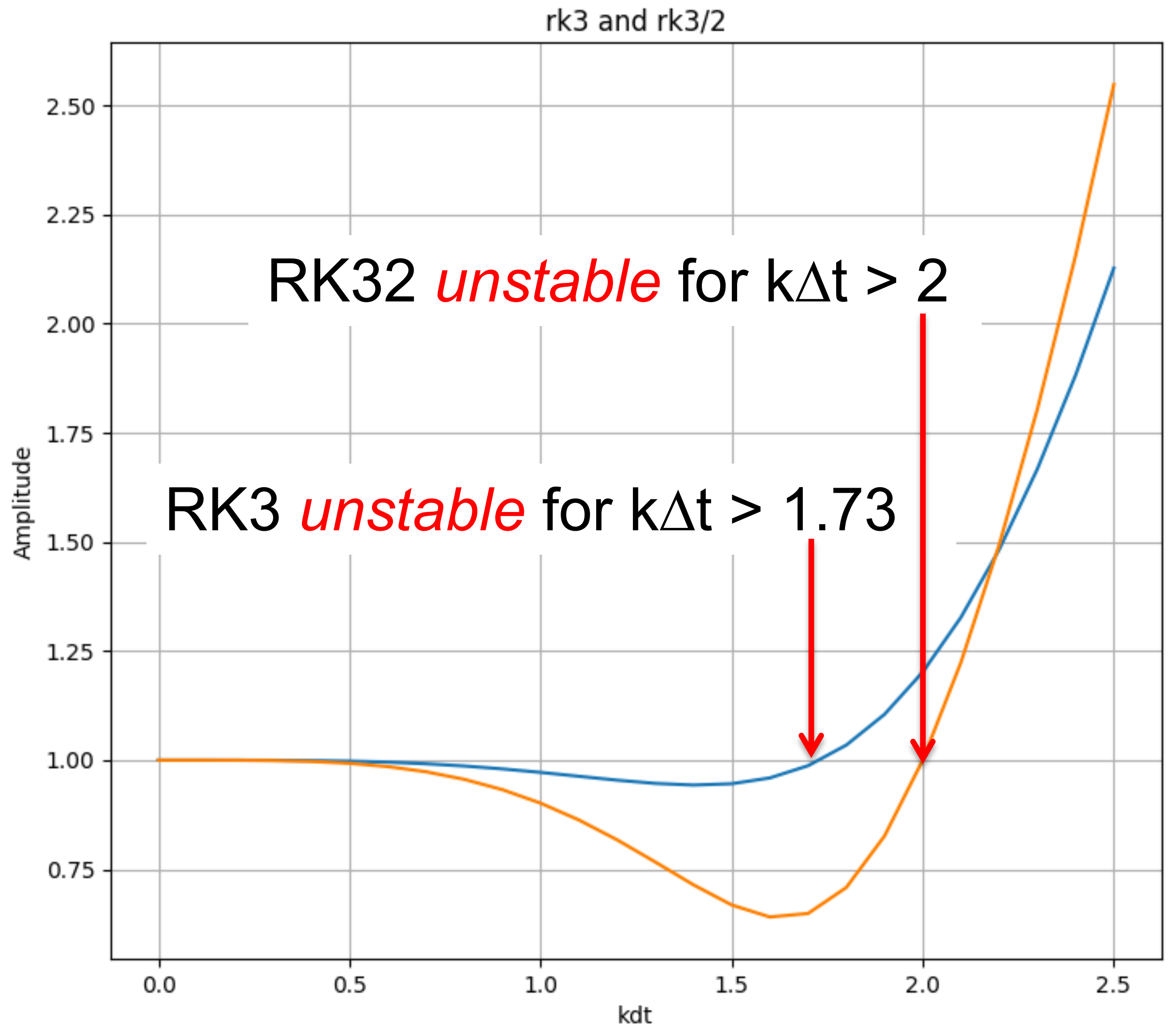
$$\phi_t = ik\phi; \quad \phi^{n+1} = A\phi^n; \quad |A| = 1 - \frac{(k\Delta t)^3}{12}$$ + H.O.T

MPAS
Model for Prediction Across Scales

Time Integration

$$\phi_t = ik\phi; \quad \phi^{n+1} = A\phi^n$$

Exact: $|A| = 1$
RK3 and RK32



In applications we see little difference in MPAS solutions using RK3 compared to those using RK32

Time Integration: Acoustic Modes

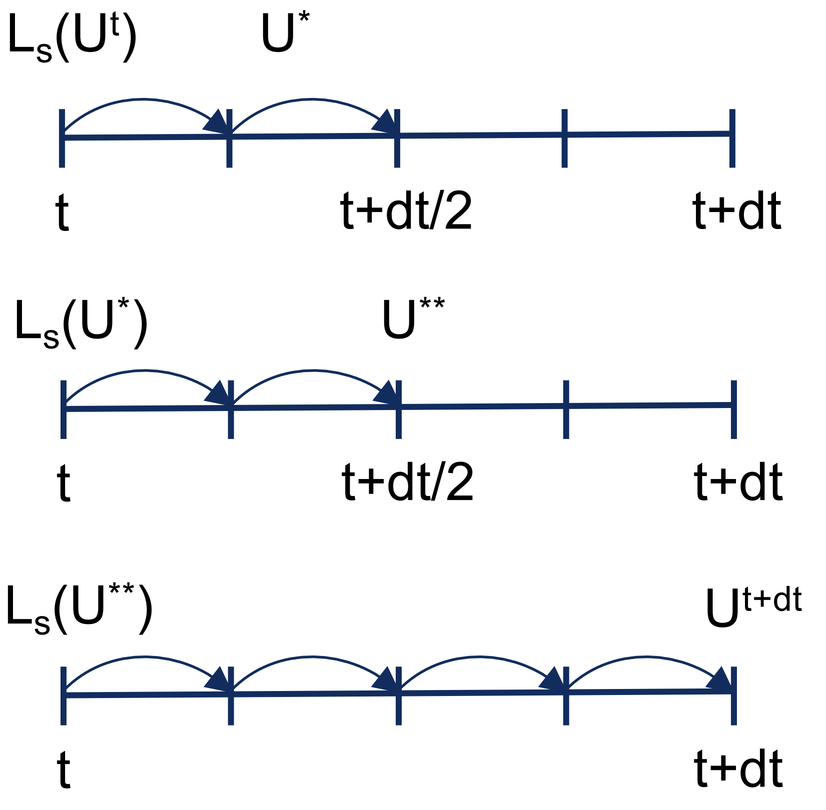
$$U_t = L_{fast}(U) + L_{slow}(U)$$

Split-explicit time integration

fast: acoustic waves and gravity waves.
slow: everything else.

- RK3/2 is 2nd order accurate.
- Stable for centered and upwind advection schemes.
- Stable for Courant number Udt/dx < 2
- Three $L_{slow}(U)$ evaluations per timestep.

3rd order Runge-Kutta, 2nd-order variant, 3 steps
acoustic steps in RK substeps



Time Integration

Default time integration

```
Call physics

Do dynamics_split_steps
        Do rk3_step = 1, 3
                compute large-time-step tendency
                Do acoustic_steps
                        update u
                        update rho, theta and w
                End acoustic_steps
        End rk3_step
End dynamics_split_steps

Do scalar_rk3_step = 1, 3
        scalar RK3 transport
End scalar_rk3_step
```

Dynamics are integrated first (config_split_dynamics_transport = .true.), typically with multiple Runge-Kutta timesteps (dynamics_split_steps > 1)

Scalar transport is integrated separately, after the dynamics

Time Integration

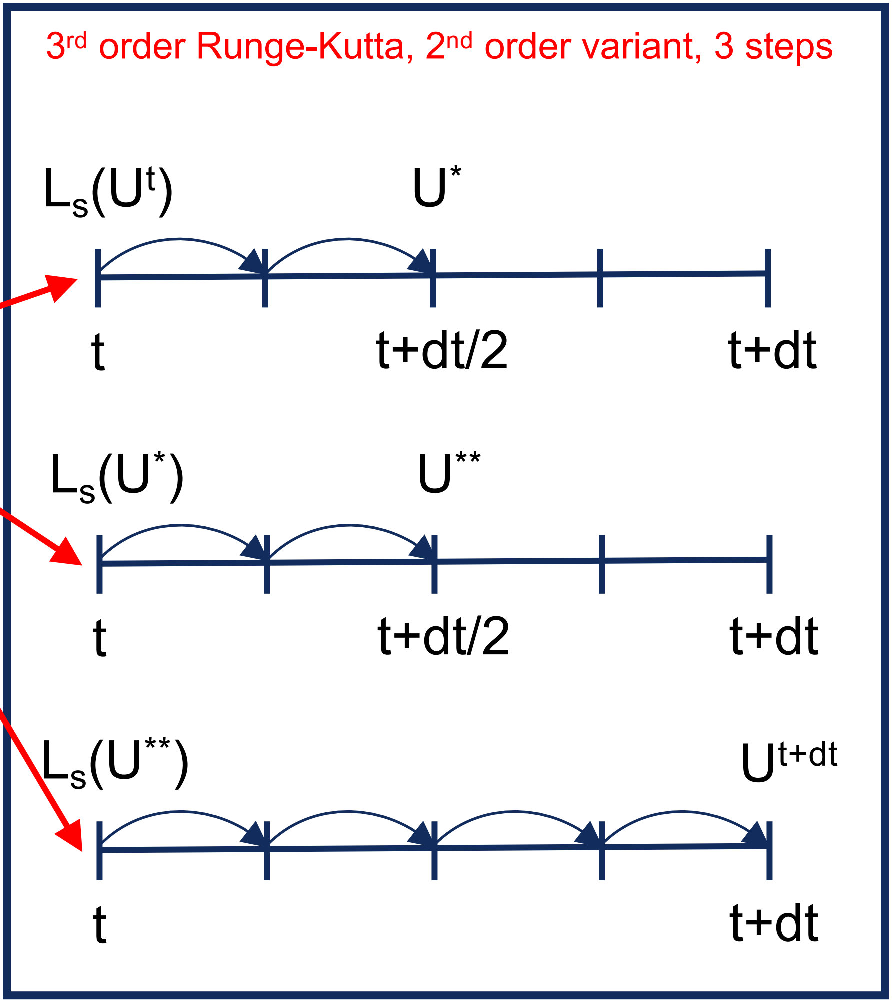
MPAS
Model for Prediction Across Scales

Default time integration

Call physics

```
Do dynamics_split_steps
        Do rk3_step = 1, 3
                compute large-time-step tendency
                Do acoustic_steps
                        update u
                        update rho, theta and w
                End acoustic_steps
        End rk3_step
End dynamics_split_steps

Do scalar_rk3_step = 1, 3
        scalar RK3 transport
End scalar_rk3_step
```



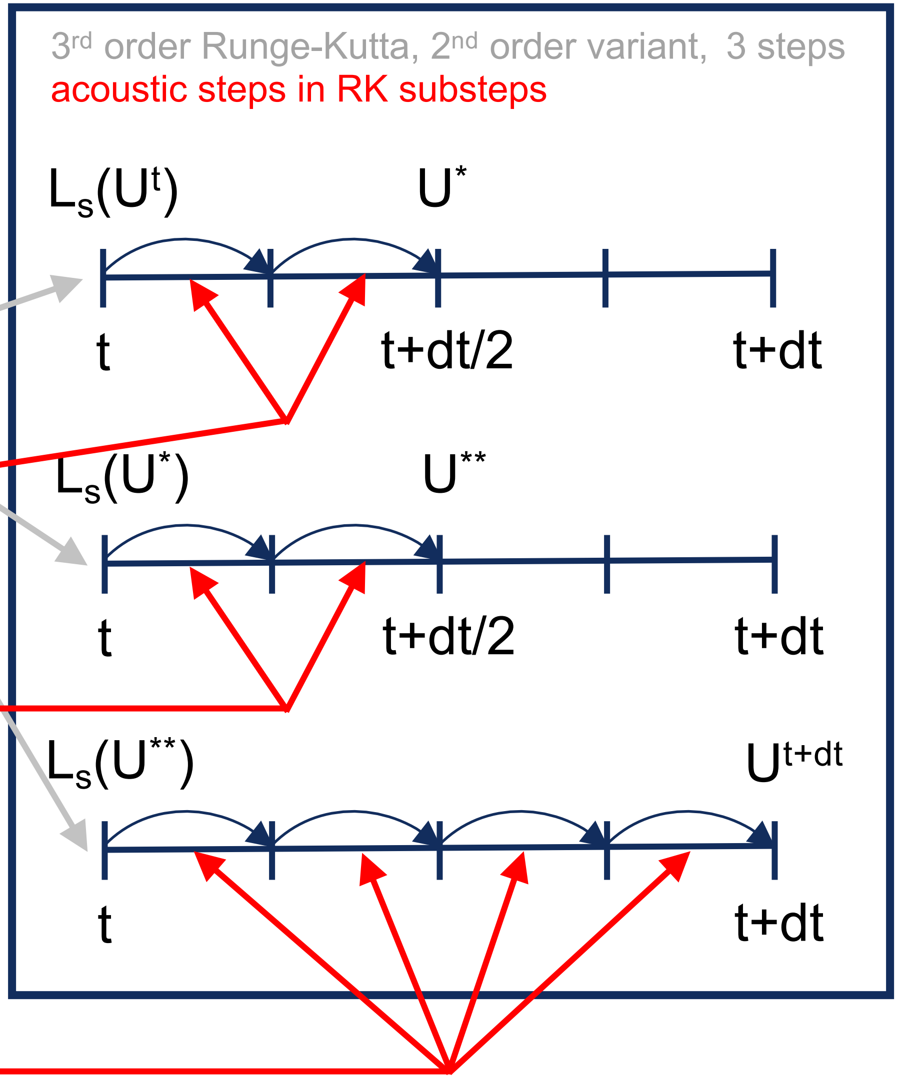
Time Integration

Default time integration

Call physics

Do dynamics_split_steps
    Do rk3_step = 1, 3
        compute large-time-step tendency
        Do acoustic_steps
            update u
            update rho, theta and w
        End acoustic_steps
    End rk3_step
End dynamics_split_steps

Do scalar_rk3_step = 1, 3
    scalar RK3 transport
End scalar_rk3_step



Time Integration

Default time integration

Call physics

Do dynamics_split_steps
    Do rk3_step = 1, 3
        compute large-time-step tendency
        Do acoustic_steps
            update u
            update rho, theta and w
        End acoustic_steps
    End rk3_step
End dynamics_split_steps

Do scalar_rk3_step = 1, 3
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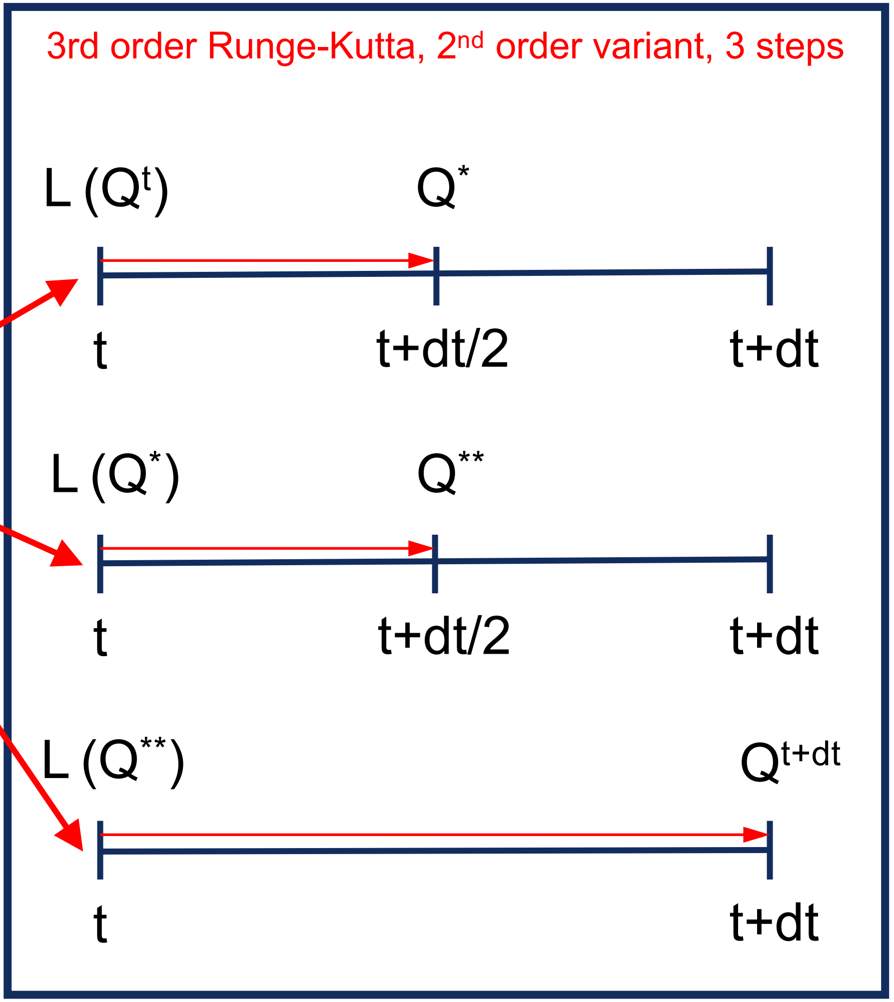


MPAS
Model for Prediction Across Scales

Time Integration

Default time integration

```
Call physics

Do dynamics_split_steps
        Do rk3_step = 1, 3
                compute large-time-step tendency
                Do acoustic_steps
                        update u
                        update rho, theta and w
                End acoustic_steps
        End rk3_step
End dynamics_split_steps

Do scalar_rk3_step = 1, 3
        scalar RK3 transport
End scalar_rk3_step
```

```
&nhyd_model
    config_dt = 90
    config_start_time = ”2010-10-23_00:00:00"
    config_run_duration = ”5_00:00:00”
    config_split_dynamics_transport = true
    config_dynamics_split_steps = 3
    config_number_of_sub_steps = 4
```

Default time integration

In the file “namelist.atmosphere”

Time Integration

MPAS
Model for Prediction Across Scales

Default time integration

Call physics

Do dynamics_split_steps
    Do rk3_step = 1, 3
        compute large-time-step tendency
        Do acoustic_steps
            update u
            update rho, theta and w
        End acoustic_steps
    End rk3_step
End dynamics_split_steps

Do scalar_rk3_step = 1, 3
    scalar RK3 transport
End scalar_rk3_step

```
&nhyd_model
    config_dt = 90
    config_start_time = "2010-10-23_00:00:00"
    config_run_duration = "5_00:00:00"
    config_split_dynamics_transport = true
    config_dynamics_split_steps = 3
    config_number_of_sub_steps = 4
```

Default time integration

$$\Delta t \text{ (dynamics)} = \frac{\text{config\_dt}}{\text{config\_dynamics\_split\_steps}}$$

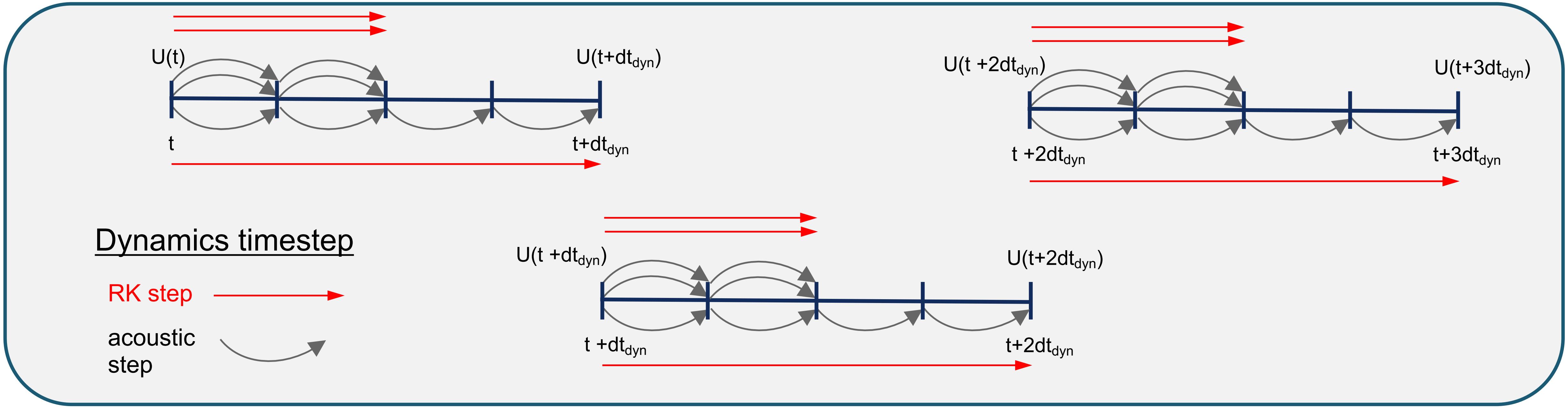
$$\Delta t \text{ (acoustic)} = \frac{\Delta t \text{ (dynamics)}}{\text{config\_number\_of\_sub\_steps}}$$

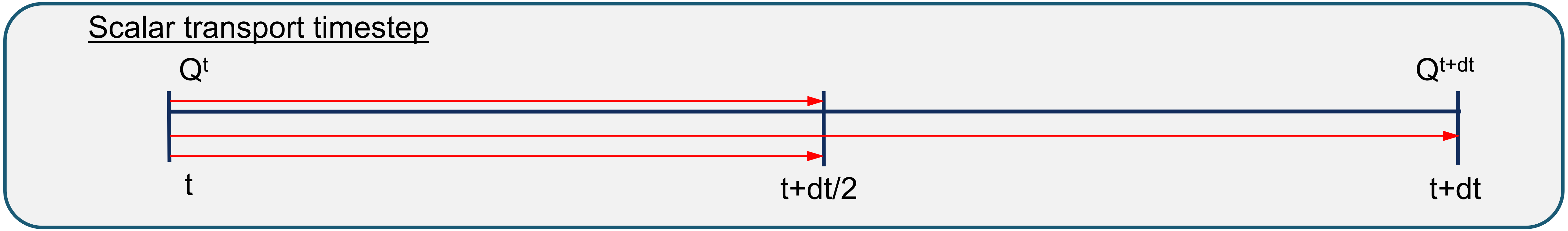
$$\Delta t \text{ (scalar transport)} = \text{config\_dt}$$

MPAS
Model for Prediction Across Scales

Time Integration – default configuration

config_dynamics_split_steps = 3, config_number_of_sub_steps = 4,
config_time_integration_order = 2





MPAS
Model for Prediction Across Scales

Time Integration

Testing the Timestep Configuration

```
&nhyd_model
    config_dt = 90.
```

Timestep in seconds

Similar to WRF, the model timestep (in seconds) initially should be set to be 6 times the finest nominal mesh spacing in km. For example – 15 km fine-mesh spacing would use a 90 second timestep.

We have found that a larger timestep (e.g. 8x) is often stable.

If MPAS integrations become unstable (producing NaNs) after just a few timesteps, the issue may be the acoustic modes.

1) Reduce the main timestep (*config_dt*) and see if the simulations are stable.
2) If stable with a reduced timestep, try the original timestep with a reduced acoustic timestep: *config_number_of_sub_steps* > 2 (even integer)
3) The acoustic and dry dynamics timestep can also be reduced by increasing *config_dynamics_split_steps* > 3 (can be odd or even)
4) If none of these work, then the problem is likely not the dynamics. Check the initial conditions.

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Vertical coordinate: $z = \zeta + A(\zeta)h_s(x, y, \zeta)$

Prognostic equations:

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\nabla_\zeta\left(\frac{p}{\zeta_z}\right) - \frac{\partial \mathbf{z}_H p}{\partial \zeta}\right] - \eta\,\mathbf{k}\times\mathbf{V}_H - \mathbf{v}_H \nabla_\zeta\cdot\mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K + \mathbf{F}_{V_H}$$

$$\frac{\partial W}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\frac{\partial p}{\partial \zeta} + g\tilde{\rho}_m\right] - (\nabla\cdot\mathbf{v}\,W)_\zeta + F_W$$

$$\frac{\partial \Theta_m}{\partial t} = -(\nabla\cdot\mathbf{V}\,\theta_m)_\zeta + F_{\Theta_m}$$

$$\frac{\partial \tilde{\rho}_d}{\partial t} = -(\nabla\cdot\mathbf{V})_\zeta$$

$$\frac{\partial Q_j}{\partial t} = -(\nabla\cdot\mathbf{V}\,q_j)_\zeta + F_{Q_j}$$

Dry-air flux divergence

Flux divergence

Diagnostics and definitions:

$$\frac{\rho_m}{\rho_d} = 1 + q_v + q_c + q_r + \ldots$$

$$p = p_0\left(\frac{R_d \zeta_z \Theta_m}{p_0}\right)^\gamma \qquad \theta_m = \theta[1 + (R_v/R_d)q_v]$$

Operators on the Voronoi Mesh
Flux divergence and transport

Transport equation, conservative form:

$$\frac{\partial(\rho\psi)}{\partial t} = -\nabla \cdot \mathbf{V}(\rho\psi)$$

Finite-Volume formulation, Integrate over cell:

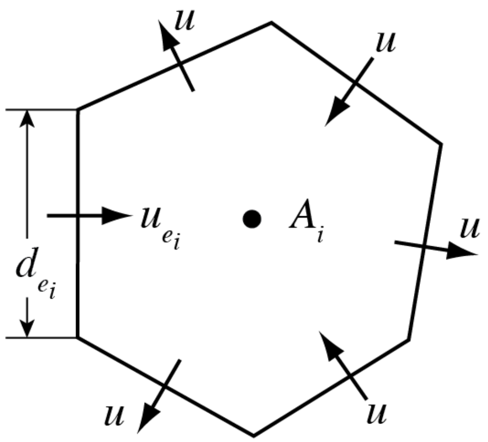
$$\int_D \left[\frac{\partial}{\partial t}(\rho\psi) = -\nabla \cdot \mathbf{V}(\rho\psi)\right] dV$$

Apply divergence theorem:

$$\frac{\partial(\overline{\rho\psi})}{\partial t} = -\frac{1}{V}\int_\Sigma (\rho\psi)\, \mathbf{V}\cdot\mathbf{n}\, d\sigma$$

Discretize in time and space:

$$(\overline{\rho\psi})_i^{t+\Delta t} = (\overline{\rho\psi})_i^t - \Delta t\, \frac{1}{A_i}\sum_{n_{e_i}} d_{e_i}\overline{(\rho\mathbf{V}\cdot\mathbf{n}_{e_i})\psi}$$



Velocity divergence operator is 2nd-order accurate for edge-centered velocities.

Operators on the Voronoi Mesh
Flux divergence and transport

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Discretize in time and space:

$$(\overline{\rho\psi})_i^{t+\Delta t} = (\overline{\rho\psi})_i^t - \Delta t\, \frac{1}{A_i} \sum_{n_{e_i}} d_{e_i} \overline{(\rho\mathbf{V}\cdot\mathbf{n}_{e_i})\psi}$$

In MPAS, the mass flux is a prognostic variables at the cell edge.

Scalar mixing ratios are defined at cell centers. Their definition at the cell edges defines the *transport scheme.*

More generally, a transport scheme defines the temporally and spatially integrated scalar mass flux through the edge over timestep Δt.

Operators on the Voronoi Mesh
Flux divergence and transport

$$\phi^t \rightarrow \phi^{t+\Delta t}$$

Runge-Kutta time integration

$$\phi^* = \phi^t + \frac{\Delta t}{3} RHS(\phi^t)$$

$$\phi^{**} = \phi^t + \frac{\Delta t}{2} RHS(\phi^*)$$

$$\phi^{t+\Delta t} = \phi^t + \Delta t \, RHS(\phi^{**})$$

$$(\overline{\rho\psi})_i^{t+\Delta t} = (\overline{\rho\psi})_i^t - \Delta t \frac{1}{A_i} \sum_{n_{e_i}} d_{e_i} \overline{(\rho \mathbf{V} \cdot \mathbf{n}_{e_i}) \psi}$$

Transport – Unstructured MPAS Mesh

$$\frac{\partial \mathbf{V}_H}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\nabla_\zeta\left(\frac{p}{\zeta_z}\right) - \frac{\partial \mathbf{z}_H p}{\partial \zeta}\right] - \eta\,\mathbf{k}\times\mathbf{V}_H$$

$$- \mathbf{v}_H \nabla_\zeta \cdot \mathbf{V} - \frac{\partial \Omega \mathbf{v}_H}{\partial \zeta} - \rho_d \nabla_\zeta K + \mathbf{F}_{V_H}$$

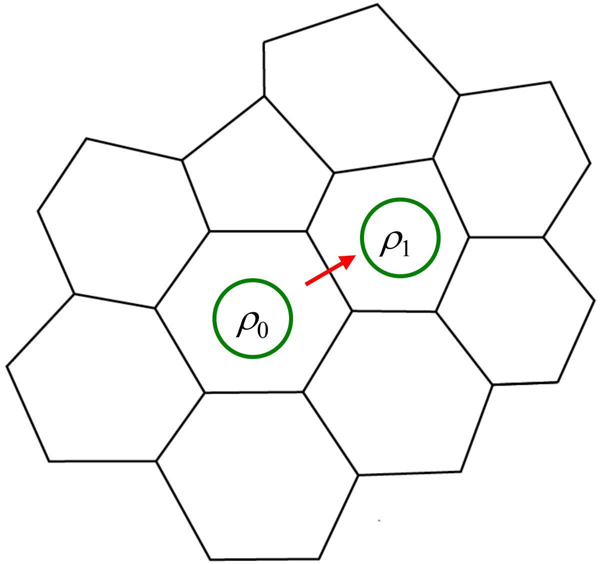
$$\frac{\partial W}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\frac{\partial p}{\partial \zeta} + g\tilde{\rho}_m\right] - \left(\nabla\cdot\mathbf{v}\,W\right)_\zeta + F_W$$

$$\frac{\partial \Theta_m}{\partial t} = -\left(\nabla\cdot\mathbf{V}\,\theta_m\right)_\zeta + F_{\Theta_m}$$

$$\frac{\partial \tilde{\rho}_d}{\partial t} = -\left(\nabla\cdot\mathbf{V}\right)_\zeta$$

$$\frac{\partial Q_j}{\partial t} = -\left(\nabla\cdot\mathbf{V}\,q_j\right)_\zeta + F_{Q_j}$$

$$\mathbf{V} = \rho\mathbf{v}; \quad \mathbf{v} = (u,\, w)$$



For the horizontal dry-air mass flux, the value of the density ρ_d at a cell face is set equal to the average of the densities from the two cells sharing the face:

$$\rho_{\text{edge}} = (\rho_0+\rho_1)/2, \quad \mathbf{V}_{\text{edge}} = u_e\,(\rho_0+\rho_1)/2$$

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$$\frac{\partial W}{\partial t} = -\frac{\rho_d}{\rho_m}\left[\frac{\partial p}{\partial \zeta} + g\tilde{\rho}_m\right] - (\nabla\cdot\mathbf{v}\,W)_\zeta + F_W$$

$$\frac{\partial \Theta_m}{\partial t} = -(\nabla\cdot\mathbf{V}\,\theta_m)_\zeta + F_{\Theta_m}$$

$$\frac{\partial \tilde{\rho}_d}{\partial t} = -(\nabla\cdot\mathbf{V})_\zeta$$

$$\frac{\partial Q_j}{\partial t} = -(\nabla\cdot\mathbf{V}\,q_j)_\zeta + F_{Q_j}$$

Flux divergence

Diagnostics and definitions:

$$\frac{\rho_m}{\rho_d} = 1 + q_v + q_c + q_r + \ldots$$

$$p = p_0\left(\frac{R_d\zeta_z\Theta_m}{p_0}\right)^\gamma \qquad \theta_m = \theta[1 + (R_v/R_d)q_v]$$

Transport – Unstructured MPAS Mesh

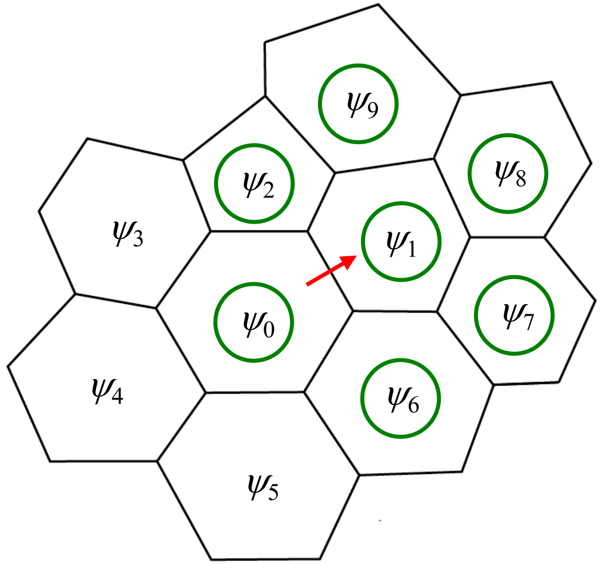
3rd and 4th-order fluxes (e.g. WRF):

$$F(u,\psi)_{i+1/2} = u_{i+1/2}\left[\frac{1}{2}(\psi_{i+1}+\psi_i) - \frac{1}{12}\left(\delta_x^2\psi_{i+1}+\delta_x^2\psi_i\right) + sign(u)\,\frac{\beta}{12}\left(\delta_x^2\psi_{i+1}-\delta_x^2\psi_i\right)\right]$$

where $\delta_x^2\psi_i = \psi_{i-1} - 2\psi_i + \psi_{i+1}$ *(Hundsdorfer et al, 1995; Van Leer, 1985)*

Recognizing $\delta_x^2\psi = \Delta x^2 \frac{\partial^2\psi}{\partial x^2} + O(\Delta x^4)$ we recast the 3rd and 4th order flux as

$$F(u,\psi)_{i+1/2} = u_{i+1/2}\left[\frac{1}{2}(\psi_{i+1}+\psi_i) - \Delta x_e^2\frac{1}{12}\left\{\underline{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1}} + \left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\}\right.$$
$$\left. + sign(u)\,\Delta x_e^2\frac{\beta}{12}\left\{\underline{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1}} - \left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\}\right]$$



where *x* is the direction normal to the cell edge and *i* and *i+1* are cell centers. *<u>We use the least-squares-fit polynomial to compute the second derivatives.</u>*

Transport – Unstructured MPAS Mesh

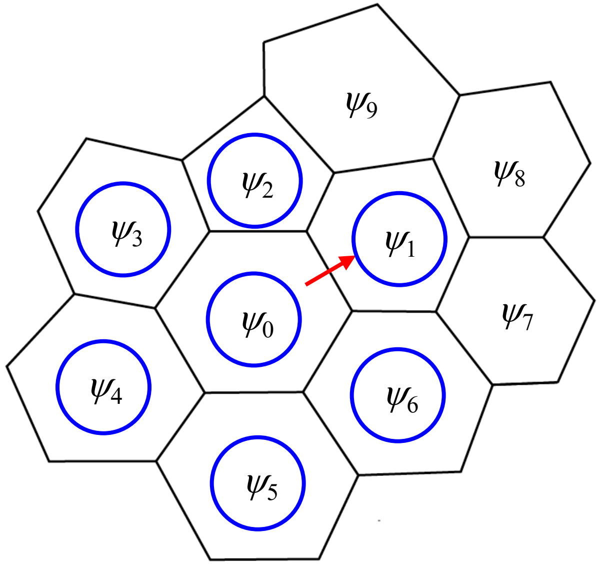
3rd and 4th-order fluxes (e.g. WRF):

$$F(u,\psi)_{i+1/2} = u_{i+1/2}\left[\frac{1}{2}(\psi_{i+1}+\psi_i) - \frac{1}{12}\left(\delta_x^2\psi_{i+1}+\delta_x^2\psi_i\right) + sign(u)\,\frac{\beta}{12}\left(\delta_x^2\psi_{i+1}-\delta_x^2\psi_i\right)\right]$$

where $\delta_x^2\psi_i = \psi_{i-1} - 2\psi_i + \psi_{i+1}$ *(Hundsdorfer et al, 1995; Van Leer, 1985)*

Recognizing $\delta_x^2\psi = \Delta x^2 \frac{\partial^2\psi}{\partial x^2} + O(\Delta x^4)$ we recast the 3rd and 4th order flux as

$$F(u,\psi)_{i+1/2} = u_{i+1/2}\left[\frac{1}{2}(\psi_{i+1}+\psi_i) - \Delta x_e^2\frac{1}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1} + \underline{\left(\frac{\partial^2\psi}{\partial x^2}\right)_i}\right\}\right.$$
$$\left. + sign(u)\,\Delta x_e^2\frac{\beta}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1} - \underline{\left(\frac{\partial^2\psi}{\partial x^2}\right)_i}\right\}\right]$$

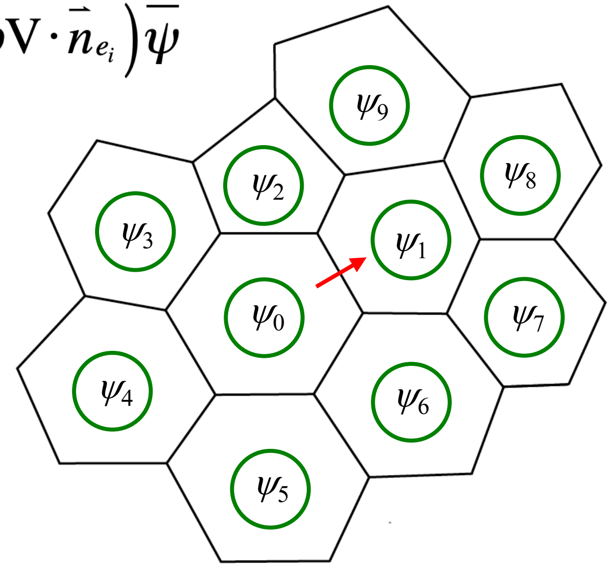


where *x* is the direction normal to the cell edge and *i* and *i+1* are cell centers. *<u>We use the least-squares-fit polynomial to compute the second derivatives.</u>*

Flux divergence, transport, and Runge-Kutta time integration

Scalar transport equation for cell i: $$\frac{\partial(\rho\psi)_i}{\partial t} = L(\mathrm{V},\rho,\psi) = -\frac{1}{A_i}\sum_{n_{e_i}} d_{e_i}\left(\rho \mathrm{V}\cdot\vec{n}_{e_i}\right)\overline{\psi}$$

1. Scalar edge-flux value ψ is the weighted sum of cell values from cells that share edge and all their neighbors.
2. Each edge-flux is used to update the two cells that share the edge.
3. Three edge-flux evaluations and cell updates are needed to complete the Runge-Kutta timestep.
4. Weights are pre-computed and stored for use during the integration.

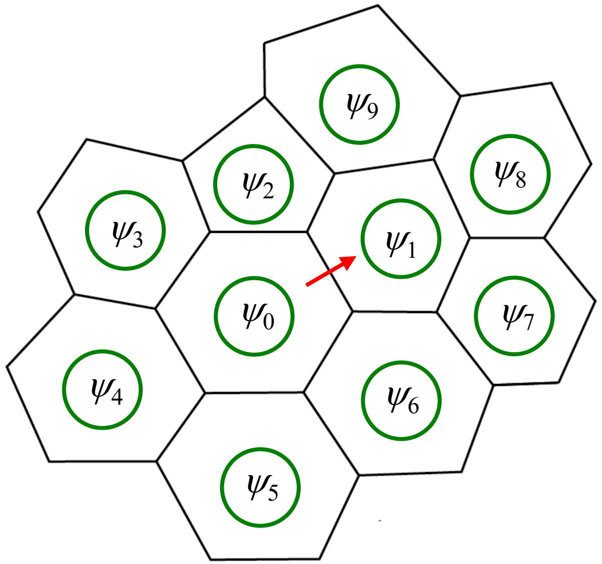


$$(\rho\psi)^{*} = (\rho\psi)^{t} + \frac{\Delta t}{3} L(\mathbf{V},\rho,\psi^{t})$$

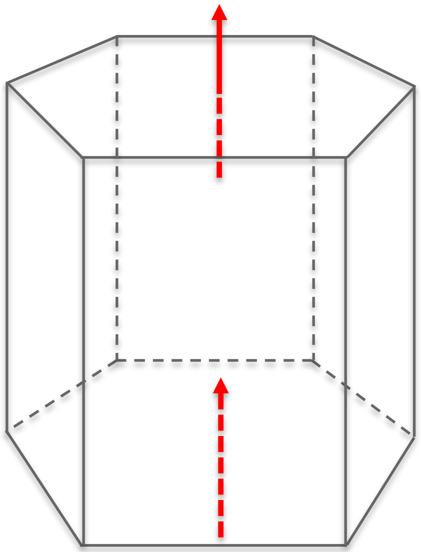
$$(\rho\psi)^{**} = (\rho\psi)^{t} + \frac{\Delta t}{2} L(\mathbf{V},\rho,\psi^{*})$$

$$(\rho\psi)^{t+\Delta t} = (\rho\psi)^{t} + \Delta t\, L(\mathbf{V},\rho,\psi^{**})$$

Flux divergence and transport. Conservation



Horizontal (scalar) mass fluxes



Vertical (scalar) mass fluxes

The mass (or scalar mass) flux on a cell edge (face) is used to update both cells sharing that edge (face), thus mass (and scalar mass) is conserved exactly.

Scalar transport: Positive-definite and monotonic renormalization

Scalar update, last RK3 step: $$(\rho\phi)_i^{t+\Delta t} = (\rho\phi)_i^t - \frac{1}{V_i}\sum_{n_{e_i}} \underbrace{A_{e_i}\overline{(\rho\mathbf{V}\cdot\mathbf{n_{e_i}})\phi}}_{\text{fluxes } f_i} \qquad (1)$$

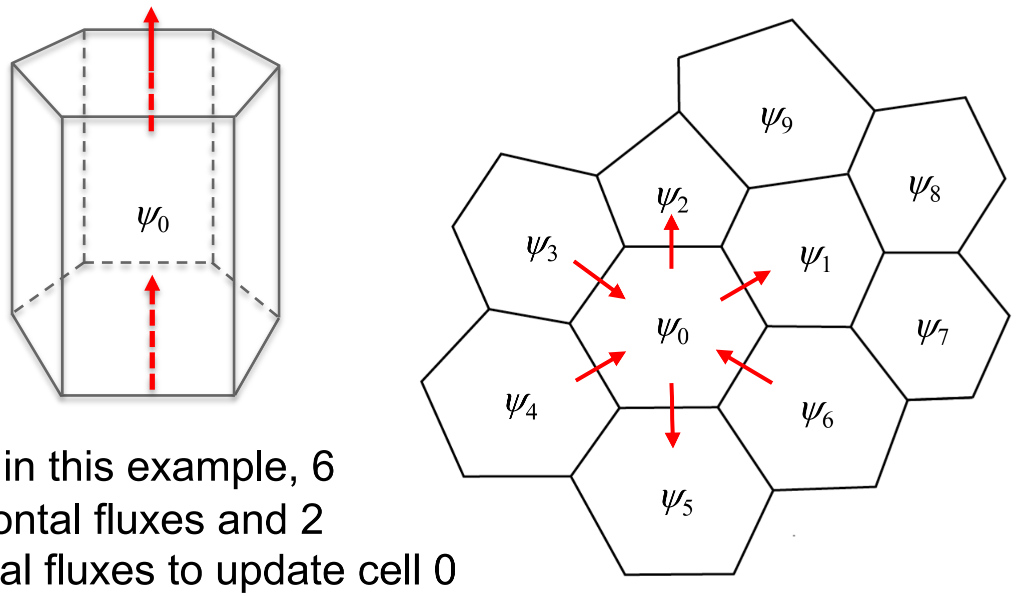
Renormalization

(1) Decompose flux: $f_i = f_i^{upwind} + f_i^c$

(2) Renormalize high-order correction fluxes f_i^c such that solution is positive definite or monotonic:

$f_i^c = R(f_i^c)$

(3) Update scalar equation (1) using $f_i = f_i^{upwind} + R(f_i^c)$



n = 8 in this example, 6 horizontal fluxes and 2 vertical fluxes to update cell 0

Conservative Transport with RK3 Time Integration: *Examples*

$$F(u,\psi)_{i+1/2} = u_{i+1/2}\left[\frac{1}{2}\left(\psi_{i+1}+\psi_i\right) - \Delta x_e^2 \frac{1}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1} + \left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\} + sign(u)\,\Delta x_e^2 \frac{\beta}{12}\left\{\left(\frac{\partial^2\psi}{\partial x^2}\right)_{i+1} - \left(\frac{\partial^2\psi}{\partial x^2}\right)_i\right\}\right]$$

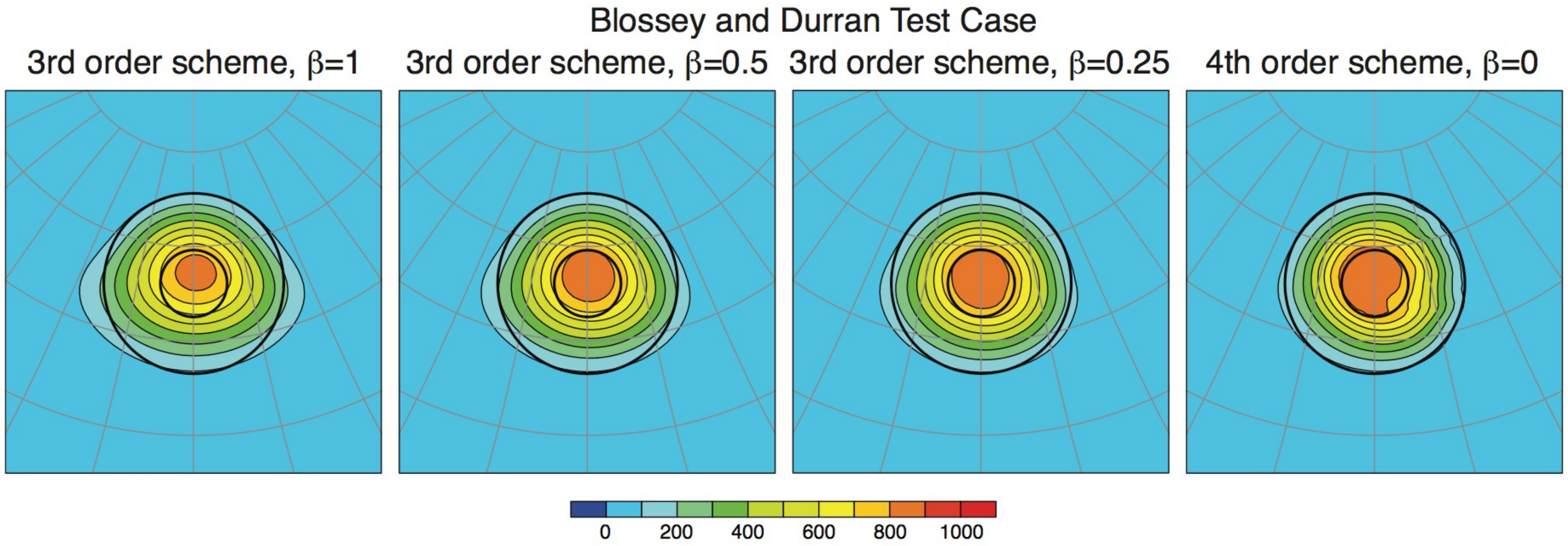


FIG. 7. Deformational flow test case results at time T using (11) with different values of the filter parameter β. The simulations were performed on the 40962-cell grid.

Configuring the dynamics

Transport

namelist.atmosphere (default settings)

```
&nhyd_model
        ⋮
    config_horiz_mixing = '2d_smagorinsky'
    config_les_model = 'none'
    config_les_surface = 'none'
    config_surface_heat_flux = 0.0
    config_surface_moisture_flux = 0.0
    config_surface_drag_coefficient = 0.0
    config_visc4_2dsmag = 0.05
    config_mix_scalars = false
    config_scalar_advection = true
    config_monotonic = true
    config_coef_3rd_order = 0.25
    config_epssm = 0.1
    config_smdiv = 0.1
/
```

Scalar advection is on

Monotonic constraint is applied

Upwind coefficient (0 <-> 1),
0 increases damping.
= 0, 4th order scheme,
> 0, 3rd order scheme.

Operators on the Voronoi Mesh
Resolved and turbulent transport

$$\frac{\partial(\rho\phi)}{\partial t} = \underline{-\nabla \cdot \mathbf{V}\phi}$$

Transport by the resolved flow

$$= \underline{\nabla \cdot (\rho K \nabla \phi)}$$

Turbulent transport, e.g. Smagorinsky

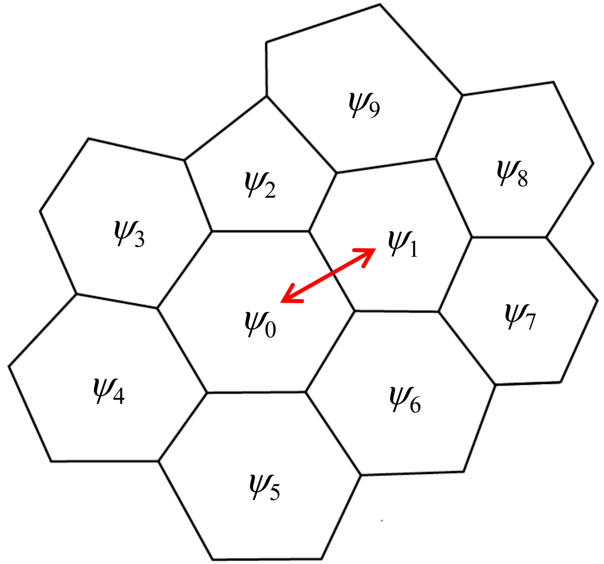
K is an eddy viscosity (m^2/s)

$$= \underline{-\nabla \cdot (\rho\, \nu_4 \nabla(\nabla \cdot \nabla\phi))}$$

4th-order filter cast as a turbulent transport

ν_4 is a hyperviscosity (m^4/s)

Operators on the Voronoi Mesh
Resolved and turbulent transport



continuous operators		discrete operators
$\nabla \cdot \mathbf{V}\phi$	$\longrightarrow$	$\frac{1}{A_i}\sum_{n_{e_i}} d_{e_i}\overline{(\mathbf{V}\cdot\mathbf{n_{e_i}})\phi}$
$\nabla \cdot (\rho K \nabla\phi)$	$\longrightarrow$	$\frac{1}{A_i}\sum_{n_{e_i}} d_{e_i}\overline{\rho K}(\mathbf{n}_{e_i}\cdot\nabla\phi)$
$\nabla \cdot (\rho\,\nu_4 \nabla(\nabla\cdot\nabla\phi))$	$\longrightarrow$	$\frac{1}{A_i}\sum_{n_{e_i}} d_{e_i}\overline{\rho\nu_4}\left(\mathbf{n}_{e_i}\cdot\nabla\left[\frac{1}{A_j}\sum_{n_{e_j}} d_{e_j}(\mathbf{n}_{e_j}\cdot\nabla\phi)\right]\right)$

Operators on the Voronoi Mesh
Filters for horizontal momentum

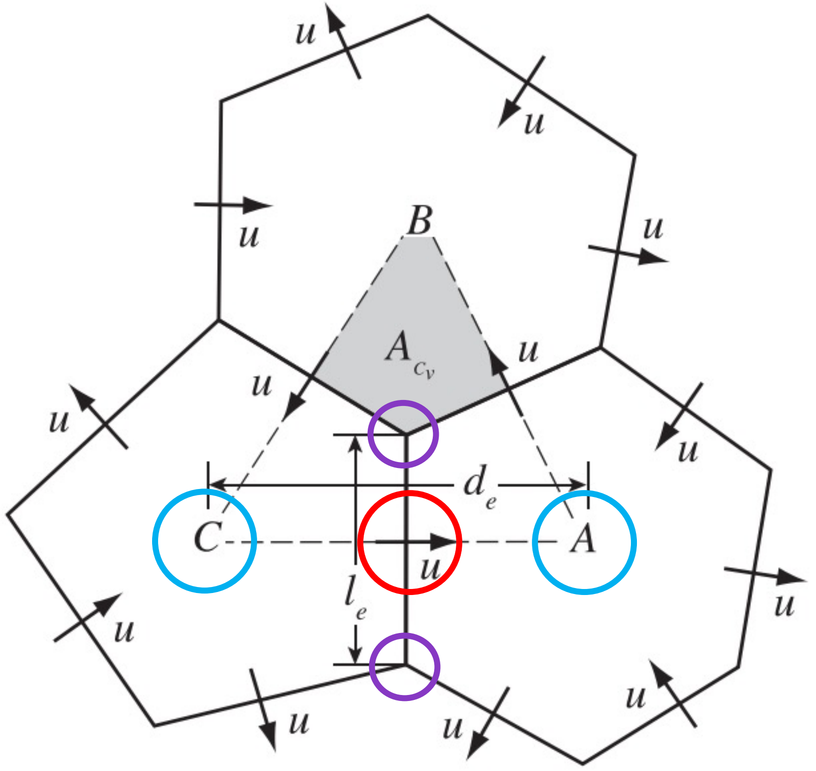
2nd order filter

$$\frac{\partial u_i}{\partial t} = \cdots + K_u \nabla^2 u_i$$

$$\nabla^2 u_i = \frac{\partial}{\partial x_i}\nabla_\zeta \cdot \mathbf{v} - \frac{\partial \eta_r}{\partial x_j}$$

$$\nabla^2 u_i = \frac{\partial}{\partial x_i}\nabla_\zeta \cdot \mathbf{v} - \frac{\partial}{\partial x_j}[\mathbf{k}\cdot(\nabla \times \mathbf{v})]$$

discrete form in MPAS.
η_r is the relative vertical vorticity



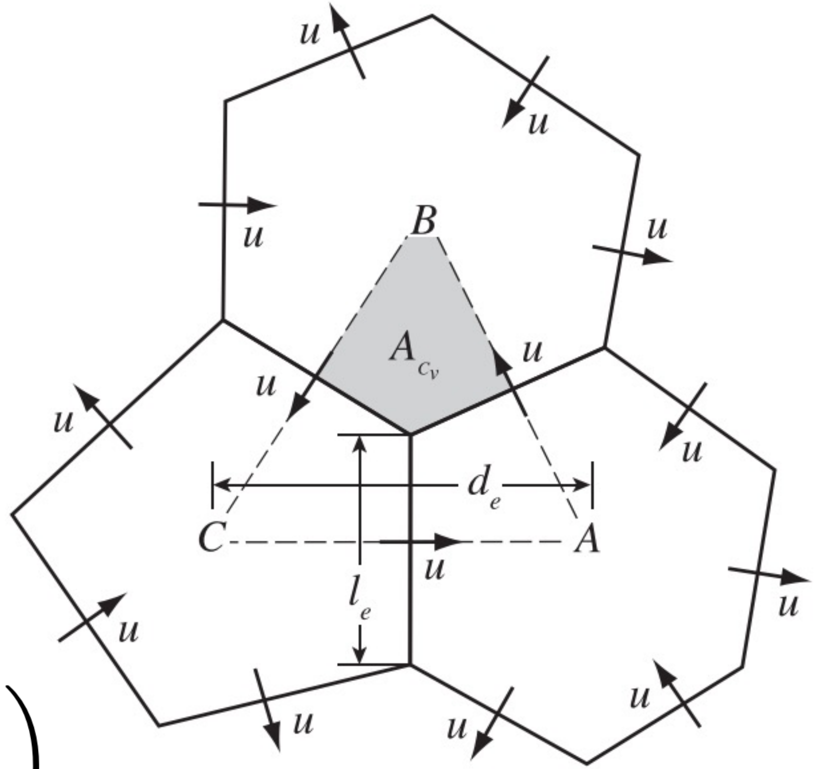
Operators on the Voronoi Mesh
Filters for horizontal momentum

4th order filter

$$\frac{\partial u_i}{\partial t} = \cdots - \nu^u_4 \nabla^4 u_i$$

$$\nabla^4 u_i = \nabla^2 \left(\nabla^2 u_i \right)$$

$$\nabla^4 u_i = \frac{\partial}{\partial x_i} \nabla_\zeta \cdot \nabla^2 u_i - \frac{\partial}{\partial x_j} \left[\mathbf{k} \cdot (\nabla \times \nabla^2 u_i) \right]$$



In MPAS

$$\frac{\delta u_i}{\delta t} = -\nu_4 \left(\gamma_u \frac{\partial}{\partial x_i} \nabla_\zeta \cdot \nabla^2 u_i - \frac{\partial}{\partial x_j} \left[\mathbf{k} \cdot (\nabla \times \nabla^2 u_i) \right] \right)$$

Configuring the dynamics

Dissipation

namelist.atmosphere

```
&nhyd_model
      ⋮
    config_horiz_mixing = '2d_smagorinsky'
    config_les_model = 'none'
    config_les_surface = 'none'
    config_surface_heat_flux = 0.0
    config_surface_moisture_flux = 0.0
    config_surface_drag_coefficient = 0.0
    config_visc4_2dsmag = 0.05
    config_mix_scalars = false
    config_scalar_advection = true
    config_monotonic = true
    config_coef_3rd_order = 0.25
    config_epssm = 0.1
    config_smdiv = 0.1
/
```

Alternately "2d_fixed"

4th order background filter coef, used with 2d_smagorinsky

υ_4 $(m^4/s) = \Delta x^3$ x config_visc4_2dsmag

The dissipation options are not applied to the scalar integration in the default configuration. Here, MPAS-A relies on the monotonic limiter to filter the scalars.

MPAS
Model for Prediction Across Scales

Configuring the dynamics

Dissipation

namelist.atmosphere

```
&nhyd_model
      ⋮
    config_horiz_mixing = '2d_smagorinsky'
    config_les_model = 'none'
    config_les_surface = 'none'
    config_surface_heat_flux = 0.0
    config_surface_moisture_flux = 0.0
    config_surface_drag_coefficient = 0.0
    config_visc4_2dsmag = 0.05
    config_mix_scalars = false
    config_scalar_advection = true
    config_monotonic = true
    config_coef_3rd_order = 0.25
    config_epssm = 0.1
    config_smdiv = 0.1
    config_del4u_div_factor = 10.
/
```

For the horizontal momentum:
$\upsilon_{4,D}$ *(m^4/s) =* υ_4 *x config_del4u_div_factor*

Hidden in the MPAS namelist.atmosphere
config_del4u_div_factor = 10. (default)

Configuring the dynamics
Dissipation

namelist.atmosphere

```
&nhyd_model

  ⋮

config_h_mom_eddy_visc2 = 0
config_h_mom_eddy_visc4 = 0
config_v_mom_eddy_visc2 = 0
config_h_theta_eddy_visc2 = 0
config_h_theta_eddy_visc4 = 0
config_v_theta_eddy_visc2 = 0
config_horiz_mixing = "2d_fixed"
```

fixed viscosity ($m^2 s^{-1}$)

fixed hyper-viscosity ($m^4 s^{-1}$)

Hidden in the MPAS V8 namelist.atmosphere

2d_fixed option is used primarily in idealized cases.

Further information can be found in the MPAS web pages:
https://www2.mmm.ucar.edu/projects/mpas/site/index.html

Draft MPAS technical note:
https://www2.mmm.ucar.edu/projects/mpas/mpas_website_linked_files/MPAS-A_tech_note.pdf

MPAS-Atmosphere Users Guide
https://www2.mmm.ucar.edu/projects/mpas/mpas_atmosphere_users_guide_8.4.0.pdf

Time Integration
References

General description, dynamics
Skamarock, W. C., J. B. Klemp, M. G. Duda, L. Fowler, S.-H. Park, and T. D. Ringler, 2012: A Multi-scale Nonhydrostatic Atmospheric Model Using Centroidal Voronoi Tesselations and C-Grid Staggering. Mon. Wea. Rev., 140, 30903105. doi:10.1175/MWR-D-11-00215.1

Runge-Kutta scheme:
Wicker, L. J., and W. C. Skamarock, 2002: Time Splitting Methods for Elastic Models Using Forward Time Schemes. *Mon. Wea. Rev.,* **130**, 2088-2097.
https://doi.org/10.1175/1520-0493(2002)130<2088:TSMFEM>2.0.CO;2

Detailed presentation on the acoustic time splitting:
Klemp. J. B., W. C. Skamarock, and J. Dudhia, 2007: Conservative Split-Explicit Time Integration Methods for the Compressible Nonhydrostatic Equations. *Mon. Wea. Rev.*, **135**, 2897-2913, doi:10.1175/MWR3440.1
(specifically section 2 and Appendix section (a) which deal with height-coordinate models, i.e. MPAS)

Transport
Skamarock, W. C. and A. Gassmann, 2011: Conservative Transport Schemes for Spherical Geodesic Grids: High-Order Flux Operators for ODE-Based Time Integration. Mon. Wea. Rev., 139, 2562-2575, doi:10.1175/MWR-D-10-05056.1