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On the continuity and distribution of water substance in atmospheric circulations

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Abstract

The author's studies of distributions of atmospheric water in relation to air circulations are reviewed and summarized here. These studies began in the 1950's during student days at M.I.T. and at the Weather Radar Branch, Air Force Cambridge Research Laboratories. They were extended during 1961–63 at the Travelers Research Center, Hartford, Connecticut, with encouragement and support from Helmut Weickmann, then a principal scientist at the U.S. Army Signal Research and Development Laboratory, Fort Monmouth, New Jersey. The author's work on this subject was substantially completed at the National Severe Storms Laboratory, Norman, Oklahoma.

The studies show the nature of probable connections among distributions of water vapor, cloud, rain, and snow with vertical and horizontal winds, divergence of the wind, compressibility of the atmosphere, and the strength and distribution of various microphysical processes. The findings also aid interpretation of observations and they offer lessons for efforts toward artificial augmentation of precipitation.

1. Introduction

On March 16–17, 1956, a northeaster brought heavy snow and howling winds with temperatures in the teens to eastern New England. The onslaught was recorded on Great Blue Hill, Massachusetts, by various instruments including the 3-cm CPS-9 radar of the Air Force Cambridge Research Laboratories' Weather Radar Branch, directed by David Atlas. Photographs of the plan-position and range-height indicators were subsequently critical components of the thesis I submitted for the Sc.D. at M.I.T., under the guidance of Prof. Henry Houghton (Kessler, 1957). My thesis was a natural adjunct to studies that David Atlas and I made to characterize the distribution of hydrometeors for use by engineers concerned with ablation of airfoil surfaces by hydrometeors (Atlas and Kessler, 1957).

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Mostly an empirical study, my thesis included some theoretical interpretation of observations; it described and somewhat explained the distribution of condensate in the storm. The density of precipitation at various places and heights was deduced by combining measurements of radar reflectivity with surface precipitation rate, moisture content as indicated by surface data and radiosondes, and estimates of vertical and horizontal air currents based mostly on measured winds. Among the tools used was the simplified continuity equation

$$N(V+w)=\text{constant}, \qquad (1)$$

where N is the number density (number/m^3) of precipitation particles uniform at each height, V is their terminal fall velocity and w is the updraft speed. This equation, wherein V and w have opposite signs, represents conservation of particle flux through unit area. If there is no creation of new particles, the equation is valid provided that the cross sectional area occupied by the particles does not change. This is the case if there is no horizontal divergence of air, i.e., if w is constant. The equation was thought to be applicable to a stream of descending precipitation particles originated at some high altitude, perhaps in a cirrus tuft. There could be evaporation from individual particles in sub-saturated air or growth of the particles from the vapor and by accretion of cloud, but without losses or additions to their number.

I and a few others who made use of Eq. (1) during the early- and mid-1950's did not then well realize the restriction on its use represented by invariant w. This error probably arose in extending an application that originated in study of the radar "bright band" phenomenon, associated with the melting zone. Within this thin layer, variations of w in the vertical would be correspondingly small, while increase of V following the melting process is large, but it was forgotten or not realized that substantial vertical variations of w that occur in deeper layers are accompanied by offsetting variations of the horizontal wind. The three dimensional divergence of air is, to a first approximation, zero; if this condition were exactly met, then N would not change at all as a result of $V+w$ becoming small owing to increase of w following descent of precipitation. A tendency toward vertical compression of a packet during descent into a stronger updraft (passage through a zone of vertical convergence) is actually more than compensated by horizontal divergence; the net effect, usually small, reflects merely the expansion of rising air.

Fortuitously, the conditions to which I applied Eq. (1) in my thesis did not preclude its use, and my conclusions pertaining to the storm of 16–17 March 1956 did not require revision. [1] However, the most important consequences of my thesis came through later realization that I had improperly understood Eq. (1). Correction of this deficiency (Kessler and Atlas, 1959) was a cornerstone of my later professional career; understanding and use of correct equations has taught much and given a deeper sense for beauty in precipitation processes.

2. Studies of water substance sans cloud

2.1. The vertical dimension with time variations

When I realized that there was much to be learned from study of more general equations of continuity for water substance, I commenced intense study of such equations and pre-

[1] I don't discuss this further here because many later studies by others are more relevant today.

sented first results at AMS' Seventh Conference on Radar Meteorology (Kessler, 1958, and subsequently formally presented, Kessler, 1959). This work included derivation of model profiles of precipitation, M, that descended in a saturated incompressible atmosphere at constant fall speed V through updrafts defined by a parabolic distribution with zeros at the ground and at height H, and with a maximum at $H/2$. No cloud processes were considered — rather it was assumed that condensate appeared immediately in the form of precipitation, at the rate wG at each altitude, where G is a condensation function.

Continuity equations for precipitation can be derived in the same fashion as the continuity equation for air, as presented in standard texts. In the course of a derivation, terms appear with the product of precipitation and three dimensional divergence. In many applications, the contribution of 3-D divergence is small and can be neglected. In this case the problem has an analytical solution and finite differencing can be avoided. The equations emphasized in the 1958 and 1959 papers were for the incompressible case with constant condensation function and fall velocity:

$$\partial M/\partial t = -(w+V) * \partial M/\partial z + w * G \tag{2}$$

$$w = (4 * w_{max}/H) * (z - z^2/H) \tag{3}$$

$$\mathrm{d}z/\mathrm{d}t = V + w \tag{4}$$

Terms on the right in Eq. (2) represent vertical advection and condensation.

The model density of descending condensate in saturated rising air of constant density increases linearly with updraft w when the magnitude of w is much smaller than that of the fallspeed V, but it increases with a higher power of w as w_{max} approaches the magnitude of V. Vertical profiles of the modelled precipitation density are strongly reminiscent of profiles suggested by radar observations in widespread precipitation. As shown in Fig. 1, the maximum rate of increase of precipitation density following its descent in the atmosphere occurs in the altitude zone of maximum vertical velocity [2], and increases following descent are small where updrafts are weak near the base and top of the column. This seems unsurprising on reflection, but it was exciting to produce these realistic-looking profiles with an analytic model. Inclusion of net three dimensional divergence, a consequence of decline of air density with altitude, only slightly changes the results, except when updrafts closely approach or exceed precipitation fall speeds (see Section 4.1.).

Another interesting finding with this model is that even in a non-advective steady-state situation, the precipitation rate at the ground exceeds the condensation rate in the column above, and increasingly as the fall speed becomes a smaller multiple of the updraft speed. The reason for this lies in the fact that the precipitation arriving in any specified area at the ground beneath an updraft center originates in a model region overhead that is larger than that contained in a cylinder erected on the ground. This purely geometrical result, which reflects horizontal divergence accompanying upward air motion at high altitude and convergence with upward motion at low altitude is discussed further and illustrated by Fig. 2 in Section 2.4.

A further demonstration of kinematic studies concerns variations of the vertical profile of precipitation density with variations of vertical velocity of air, when the fall speed V

[2] More precisely, where the condensation function times the vertical velocity is a maximum. This is usually somewhat below the altitude where the vertical velocity is a maximum.

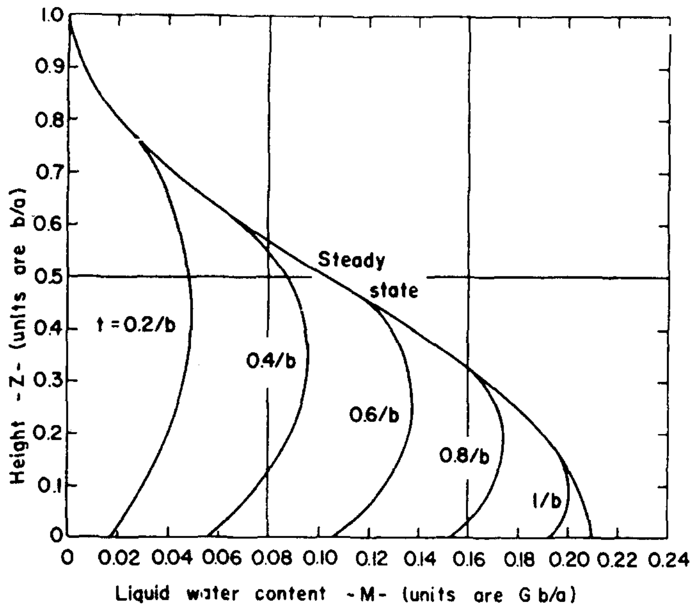

Fig. 1. Time dependent solution of Eq. (2). The height of the steady-state saturated updraft column $H=b/a$ (old notation). The magnitude of the fall velocity V (negative) is four times the maximum updraft, $b^2/4a$, which occurs at $H/2$. The precipitation profile attains a steady state when precipitation starting at height H arrives at the base of the column, a little later than $1/b=-H/V$, which would be the exact time if the fall velocity relative to the ground were V throughout (from Kessler, 1959).

changes discontinuously. Such change of V is an approximation to conditions at the radar bright band, where slow-falling snow melts to fast-falling rain within a short vertical distance. When the air's upward velocity is a substantial fraction of the fall speed of snow, analysis of the continuity equation (Eq. 1 is applicable) shows that the change of precipi-

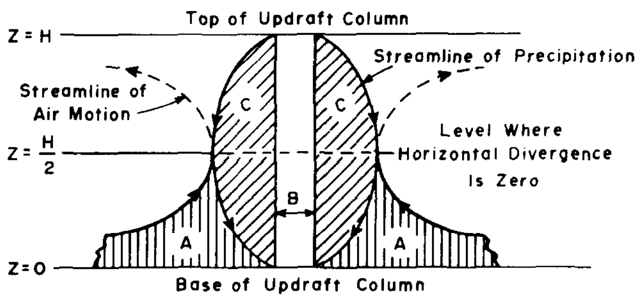

Fig. 2. Horizontal divergence at high altitudes accompanies rising air motion and spreads model precipitation packets horizontally as they descend at constant fall velocity relative to the air in B and C. At low altitudes the packets contract in the horizontally convergent wind field. Precipitation at the ground can therefore be greater than the condensation that occurs vertically overhead. When cloud is collected by precipitation at a particular altitude, but not below, the region contributing moisture can be further increased as indicated by the vertically shaded area A (from Kessler, 1969).

tation density involves a much larger reduction below the bright band than when the air velocity is comparatively small. As demonstrated in Section 2.4., a somewhat more general form of Eq. (1) is a very powerful tool for examining connections between microphysical and kinematic processes.

Finally, consider that the model equations without cloud depict rate of growth of a precipitation packet at any altitude in a column as the product of the vertical velocity and condensation functions at that altitude, and vertical motion of the packet as the sum of the vertical velocity of the air and the velocity of the packet relative to the air. Thinking about this led me to a combination of graphical and analytical methods for solving the time-dependent equations in the case of an incompressible atmosphere. The method of solution is highly instructive in its illustration of first formation of a precipitation maximum aloft in a steady saturated updraft, the maximum then increasing as it descends toward the ground. The steady state in the profile is established, as shown in Fig. 1, when the precipitation from the top of the column arrives at the ground, and the maximum then remains at the ground. This process is reminiscent of many radar observations. [3]

2.2. Two space dimensions with time but without cloud

A necessary next step was to extend the model to (at least) two space dimensions in order to capture elements of the water budget intrinsic to precipitation processes. This involved interesting additional conditions and essential recourse to digital computers.

The computational region was a slice through a mathematically defined roll circulation. On left and right edges, horizontal wind was zero; on the left hand edge there was the same profile of vertical velocity previously calculated and on the right edge was its mirror downdraft. Cloud was still absent, but precipitation, denoted M, was carried away from the updraft column by diverging air at high altitudes and had to evaporate in air that had become dry through descent. This evaporation process was modeled as follows: The amount of condensate formed in ascending saturated air was assumed as before to fall relative to the air at speed V. Negative M-values that appeared in descending air were carried along with the air at the air velocities u and w. Wherever positive M-values overtook negative values, the negative values were increased toward zero and the positive values correspondingly reduced. This represented instantaneous evaporation of precipitation in subsaturated air. M when positive was the precipitation density and M when negative was the saturation deficit.

The subsequent paper (Kessler, 1961) presented the following principal ideas and conclusions. First, the solutions defining distributions of M were shown to be scalable, e.g, the solutions determined for model updrafts of particular height were related to solutions for other heights by simple multiplications of the time and precipitation densities. Specifically, for new updraft height H, new generation (condensation) function G, new vertical air speed w_{max}, and new fall speed $V = w_{max} V / w_{max}$, the new solutions denoted by M are given by $M = MGH/GH$ and the solutions are applicable at new times $T = T(H/H)(w_{max}/w_{max})$.

[3] I am not strong in mathematics, but from physical reasoning I developed a solution method that involves depiction of the governing partial differential equation as two total differential equations. The equations were old-hat to John Freeman, who immediately provided a lucid and very helpful discussion of their properties in mathematical terms. Many other persons, cited in the Acknowledgements section of my papers, were also very helpful.

Second, notice that it is necessary to change V and w in the same proportion in order to preserve the shape of the derived precipitation field. For a given circulation and condensation function, the sole determinant of the shape of the developing distribution of precipitation is the ratio $w_{max}/|V|$. Accordingly, in a weak circulation, $w_{max}/|V| \ll 1$, in a moderate circulation, $w_{max}/|V| \approx 1$, and in a strong circulation, $w_{max}/|V| \gg 1$. Third, maxima aloft tend to predominate in the derived fields of water substance, even when the fall speeds of precipitation are much larger than the maximum vertical air current. This feature of the models is attributed to continued formation of condensate aloft and to the longer time often required for precipitation to fall to the ground than for the modeled volume of air to overturn substantially. Finally, consideration of the model suggests the importance of certain water budget parameters, as follows:

(1) total water: the mass of all vapor and condensate in the region of interest;
(2) maximum condensable water: the amount of vapor that would condense if all the air in a given region were lifted to the highest elevation in the region;
(3) condensable water: the mass of vapor that would be condensed if an hypothesized wind field prevailed long enough to carry all the air parcels to the highest elevations of their streamlines;
(4) condensed water: the accumulated condensation calculated for a given duration of an hypothesized field of motion;
(5) stored precipitation: the total mass of condensate actually present aloft;
(6) available precipitation: the amount of condensate aloft in excess of any saturation deficit directly beneath it.

Items (1) and (2) were determined analytically from model parameters and the others were calculated through a marching process, with model circulation continued until overturning of the air mass had been completed. A principal finding of the field calculations is that the precipitation efficiency is substantially larger when updrafts are weak than when they are strong. In other words, a larger proportion of model condensed moisture reaches the ground as precipitation when updrafts are small compared to precipitation fall speeds. This is because strong updrafts move condensate to higher altitudes where it is more spread by horizontal divergence to overlie air that was dried by compensating descent. Thus, in weak updrafts the precipitation tends more to descend toward the ground through the same moist current that produced it. Model differences are large: Nearly 90% of condensed moisture ultimately arrives at the ground when the fall speed of precipitation is two to four times the maximum updraft, but less than half the condensed moisture is so deposited when fall speed is one-fourth the maximum updraft. It should be remembered, however, that this finding does not embody probable effects of mixing between a cloud and its surroundings, effects that might be larger when updrafts are weak because of the longer time required for rising air to reach its upper level destinations.

2.3. *A final paper without cloud — generators and stalactites*

Some elements of the papers described above were summarized in a paper published in the *Monthly Weather Review* (Kessler, 1963a), with some further applications of theory. The *MWR* paper includes a detailed derivation of Eq. (1) from more general continuity equations, discussion of the limited conditions under which this equation has validity, and presentation of related equations applicable under more general conditions. Fields of precipitation and saturation deficit were calculated as before, with additional rather similar solutions representing no-evaporation of precipitation once formed. In this paper, the theory was extended to model radar-observed "generators" and "stalactites". Generators are probably cirrus tufts and trails, and stalactites are protuberances (sometimes virga?) of descending precipitation at the interface between widespread precipitation aloft and a lower dry layer that is destabilized by evaporative cooling of its top. Since the fall speed of snow is from about 0.5 to 1 ms^{-1}, the model indicates that the shapes observed can be explained by small convection cells in which vertical air currents range from a small fraction of a meter per second up to these larger amounts.

2.4. *Understanding some effects and interactions involving microphysical processes and air motions without understanding microphysics*

Consider Fig. 2. This illustrates idealized physical conditions that lead to a precipitation rate at the ground exceeding the total condensation rate at the center of a steady-state saturated updraft column. An updraft column is necessarily associated with horizontal convergence of air at low altitudes and horizontal divergence aloft. An ensemble of precipitation particles starting at height H in region B, which may be just the width of a raingage, is first stretched by horizontal divergence as the particles descend along the streamlines that bound region C; then they converge below the level of nondivergence in the figure. The inflowing streamlines at the upper boundary of region A define a region wherein cloud forms and increases in density as it rises with moist air from low altitudes. In the figure, cloud is captured by descending precipitation at the level of non-divergence and returned as precipitation to the ground along the descending streamlines that are the outer boundaries of region C. The level of nondivergence is chosen here to illustrate this transition because some of the effects we seek to illuminate are maximized in this case. In this model case, it is clear that the region that contributes to precipitation at the ground is larger than that vertically overhead. And clearly, the excess of precipitation over condensation at the updraft center must be compensated by losses away from the center.

In order to have some clear and simple illustrations of how microphysical processes can influence effects of the field of motion in a mathematical model, we omit effects of the compressibility of the atmosphere. Compressibility introduces non-linearity to the applicable equations — this greatly complicates their formal analysis without changing the answers very much and without a commensurate contribution to understanding. [4] In an updraft column just 1 km high, effects of air density variations are negligible. We consider the updraft distribution, Eq. (3), as a representation of the vertical velocity depicted in Fig. 2. Other model parameters have the following values: $w_{max} = 0.5$ ms^{-1}; Height of updraft column $H = 1$ km; Condensation function $G = 10^{-3}$ $gm^{-3}m^{-1}$ (i.e., gm^{-4}). And in the following, the air's precipitation content, M, is calculated at the various heights from the solution to the equation

[4] Effects of compressibility are well enough understood. We simply don't wish to have them complicate this portion of the presentation.

Table 1
Models of precipitation development without cloud for two fall speeds in incompressible air

	Height z(km)	Updraft $w(\mathrm{ms}^{-1})$	Ppt.cont. $M(\mathrm{gm}^{-3})$	Ppt.Rate* $R=-M(V+w)$	Cond.Rate* $\int w*G\mathrm{d}z$
$V=-1$	1	0	0	0	0
$(\mathrm{ms}^{-1}\mathrm{L})$	0.5	0.5	0.285	0.143	0.167
	0	0	0.571	0.571	0.333
$V=-0.67$	1	0	0	0	0
$(\mathrm{ms}^{-1}\mathrm{L})$	0.5	0.5	0.709	0.118	0.167
	0	0	1.418	0.946	0.333

*Units of precipitation and condensation rates are $\mathrm{gm}^{-2}\mathrm{s}^{-1}$. To convert to mm/hr, multiply by 3.6. Condensation rates apply to the layer above the height z.

$$M(z)=M(H)+\int_z^H\left[\frac{wG}{w+V}\right]\mathrm{d}z, \tag{5}$$

derived from Eq. (2). We assume constant air density, constant terminal velocity of precipitation particles, no horizontal advection, immediate creation of precipitation from condensate without a cloud phase, and steady state. [5]

Case 1. Illustration of an effect of fall speed of precipitation.

With no cloud phase, condensate attaches immediately to precipitation particles falling at speed V relative to the air. In this case, descent of precipitation is along streamlines bounded by the outer boundary of region C in Fig. 2, and there is no influx of cloud as suggested by the vertically hatched area A in Fig. 2. Table 1 shows some magnitudes of quantities of interest.

Notice first that when the fall speed of precipitation relative to air is slower, the precipitation content M is greater. This is so because the slower falling precipitation grows for a longer time in the updraft, in which water is condensing from the vapor at the same rate in the two cases. Next, notice that the precipitation rate at the level of nondivergence is *less* than the integrated condensation rate above in both cases. This is so because the volume contributing to precipitation at that altitude is less than that contained in a cylinder erected at that altitude; this deficit appears in the diagram as the blank space immediately above the streamline that bounds the upper part of region C in Fig. 2. Finally, notice that the precipitation rate at the ground is *greater* than the integrated condensation vertically aloft and greater for the slower than for the faster fall speed, the latter because the streamlines that enclose region C become more strongly bowed as the terminal fall speed declines and enclose precipitation from a larger area aloft. For cases not shown where the terminal speed becomes much larger than the updraft, the precipitation rate at the ground approaches the condensation rate aloft asymptotically.

[5] The explicit solution to this equation is not presented here because of its considerable length; Eq. (2) is discussed in several of the references and its full solution is given as Eq. (3.7) on p. 5 of the author's AMS monograph (1969). It is fun to use this and related equations with various parameters with a spreadsheet. [Be sure to change the erroneous minus sign in the first line of Eq. (3.7) in the monograph to plus, as indicated by the monograph's errata sheet.]

Now what practical implication might reside in this analysis? Since the parameters are applicable to snow, it seems that, other things being equal, if snow crystals remain unaggregated and relatively slow falling, precipitation at the ground would tend to be more concentrated at the core of a weather disturbance, and the maximum precipitation rate would be larger. Whether such an effect could actually be identified in weather observations is highly problematical, however, given the host of fluctuating parameters characteristic of real weather situations.

Case 2. Illustration of some implications of accretion.

In the 1957 paper by Atlas and Kessler on modelling of widespread precipitation, the following statement appears on the first page: "...the present precipitation models are based on steady-state solutions of the continuity equation for moisture and assume that all the water condensed out of the air goes over to precipitation. In other words, in the hypothetical storms, there is no storage of water in the form of clouds. This is not realistic in the rain below the 0°C level because rain can grow only by the collection of cloud droplets. Thus, there may be two or three times more water in the form of cloud than in the form of rain, especially just below the melting level. Above the level of about -3°C, snow and liquid water cloud rarely coexist..." We here examine more fully some implications of this discussion.

Proceeding from the top down, theory confirms observations that snow and liquid water cloud do not often coexist, this because the equilibrium vapor pressure over ice, being lower than that over water at the same temperature, establishes a vapor pressure gradient between a snow crystal and nearby cloud droplets that is a powerful force toward evaporation of the water motes with deposition on the ice surface. So water droplets tend to evaporate in the presence of ice crystals, and this process continues whether or not there is also accretion. And should the condensation process be so strong that some cloud forms in spite of strong growth of ice crystals directly from the vapor, we expect accretion of water droplets to proceed rather efficiently in snow because of the relative large cross-sectional area of snow aggregates and because of their porous nature and irregular surface. On the other hand, the coexistence of large amounts of cloud with rain mentioned in the 1957 paper suggests that rain is not an efficient collector of cloud. We are not prepared to pursue these interesting aspects further here, but show in Section 4.5. how the situation tends to be controlled strongly by the updraft, which, when faster, forces the cloud content to larger values with associated increased accretion rate.

We now show implications of our model for two contrasting and extreme situations with and without liquid water cloud below a level of melting snow. Our vehicles for doing this are Eq. (5) and Eq. (6), below, which is a generalized form of Eq. (1). Thus, in a saturated atmosphere wherein the fall speed of precipitation changes abruptly, we can model the cloud and precipitation contents appropriately by equating the total flux of moisture on both sides of the velocity discontinuity:

$$(mw)_b + M_b(V+w)_b = (mw)_a + M_a(V+w)_a, \tag{6}$$

where subscripts b and a refer to below and above, m is the content of cloud that moves with the air at velocity w, and M is the precipitation content that falls relative to the air at velocity V. While Eq. (6) can be deduced from comprehensive continuity equations, it is also proper to write it directly as done here as a statement of continuity of water substance

Table 2
Model of precipitation development without cloud below a melting process at 0.5 km

Height z(km)	Updraft $w(\mathrm{ms}^{-1})$	$V+w$	Cld.Cont. $m(\mathrm{gm}^{-3})$	Ppt.Cont. $M(\mathrm{gm}^{-3})$	Ppt.Rate $R=-M(V+w)$	Cond.Rate $\int w*G\mathrm{d}z$
1	0	−1	0	0	0	0
0.5	0.5	−0.5	0	0.285	0.143	0.167
0.5	0.5	−4.5	0	0.032	0.143	0.167
0	0	−5	0	0.068	0.340	0.333

through a thin slab. While V may vary (be discontinuous in the model) as a reflection of the melting process, w must be the same on both sides of the model thin melting layer, since infinite divergence would be implied otherwise.

For illustrative purposes we take H, G, and w as in Case 1. But now we compare a situation totally without cloud with a situation where cloud exists in the layers below the melting level. In both we have snow that falls at a speed of 1 ms^{-1} to an altitude of 0.5 km, where it melts to rain falling at 5 ms^{-1}. The first two rows in Tables 2 and 3 duplicate values in the first two rows of Table 1. At 0.5 km, the snow melts to rain falling at 5 ms^{-1}. The value of M after melting is calculating by rearranging Eq. (6). When there is no cloud below the melting level, the terms in m are omitted.

$$M_b=M_a(V+w)_a/(V+w)_b. \tag{6a}$$

We see that M must diminish by a factor of nine as a result of its change of fall speed, in order for the flux to be the same on both sides of the fall speed discontinuity.

Note that with a weaker updraft speed, say 0.1 ms^{-1}, the descent rate of snow *relative to the ground* at $z=0.5$ km would be 0.9 ms^{-1}, and the speed of rain below would be about 4.9 ms^{-1}; the ratio of 5.44 is much nearer the ratio of the terminal velocities of snow and rain, and the change of M through the model melting level is correspondingly reduced. From these comparisons we deduce that a considerable part of the variability shown in radar observations of transitions through the melting zone is probably a consequence of the strength of the airflow with only a somewhat remote connection to cloud physics — does cloud physics accommodate to the airflow?

Below the melting layer, growth of model precipitation continues according to Eq. (5). The precipitation rate at the ground is just a little less than that associated with a fall speed of 5 ms^{-1} throughout the entire height of the updraft column (not tabulated).

Table 3
Model of precipitation development with cloud below a melting process at 0.5 km

Height z(km)	Updraft $w(\mathrm{ms}^{-1})$	$V+w$	Cld.Cont. $m(\mathrm{gm}^{-3})$	Ppt.Cont. $M(\mathrm{gm}^{-3})$	Ppt.Rate $R=-M(V+w)$	Cond.Rate $\int w*G\mathrm{d}z$
1	0	−1	0	0	0	0
0.5	0.5	−0.5	0	0.285	0.143	0.167
0.5	0.5	−4.5	0.5	0.087	0.392	0.167
0	0	−5	0	0.087	0.435	0.333

Now consider the case where condensate is not collected at all by rain but is collected completely and instantaneously by snow. Then with the melting level at the level of non-divergence, the cloud content increases with condensation in rising air to the melting level. The cloud content is then defined by

$$m(z) = \int_0^z G * \mathrm{d}z. \tag{7}$$

In our model updraft column just 1 km high, with $G = 10^{-3}$ gm^{-4}, $m = 0.5$ gm^{-3} at 500 m, and is so entered in Table 3. Now the variant of Eq. (6) to be solved is

$$M_b = [M_a(V+w)_a - (mw)_b] / (V+w)_b. \tag{6b}$$

All of the terms on the right hand side of Eq. (6b) are negative, and the numerator is numerically larger than the numerator in Eq. (6a). Consequently, M below the melting level when there is cloud beneath is larger than it is when the collection process has been efficient beneath. This is as expected! However, it may be surprising to note that even though there is no further increase in precipitation content from our model melting level to the ground (because our model cloud and precipitation have no interaction in that layer), the precipitation rate at the ground in this no-accretion-by-rain case is actually nearly 30% larger than in the case where rain collects cloud efficiently (Table 2)!

A physical explanation for this is indicated by Fig. 2. Without accretion below the level of nondivergence, the region marked A contributes to precipitation at the ground within the cross section marked B, whereas when accretion by rain is an efficient process, condensate formed in region A contributes to rainfall at greater distances from the precipitation core and there is a lesser concentration of precipitation in the core. Thus, we see that micro-physical and kinematical processes are deeply intertwined. At the bottom line, of course, there are deep implications for atmospheric dynamics.

Note also that in this case where there is zero collection of cloud by rain in the rising air beneath, the cloud density just below the model melting level is more than five times the density of precipitation at the same altitude, compatible with the statement in the 1957 paper of Atlas and Kessler. (We have to regard this confirmation as somewhat coincidental. These cases are schematic and heuristic.)

3. Introduction of microphysics to the models

3.1. Background

In 1961, I transferred from the Weather Radar Branch on the summit of Great Blue Hill, Massachusetts, to the Travelers Research Center in Hartford, Connecticut. With reference to the work discussed here, I was at first concerned to complete the paper discussed in Section 2.3. (Kessler, 1963a). However, in late 1961, a correspondence began with Dr. Weickmann who was then at the U.S. Army Electronics Research and Development Laboratory, Ft. Monmouth, New Jersey. Correspondence and discussions led to submission of a proposal, which was accepted, and most of my subsequent development of parameterized cloud physics processes and their introduction to the kinematic models was done under this contract.

Some background is interesting and relevant. According to progress reports of the contract (Kessler et al., 1963), "The purpose of the project is to increase understanding of the roles of cloud conversion, accretion, evaporation, and entrainment processes in shaping the distributions of water vapor, cloud, and precipitation associated with tropical circulations." The word *tropical* was quite important. The U.S. government was directing its attention toward Viet Nam, a tropical country with mountainous terrain and heavy rainfall, and a troop buildup ensued in 1963. There was also fresh memory of impacts on military operations of summer rainfall in Korea. Furthermore, I was trying to keep the focus of the investigation from becoming too broad. Limitation to *tropical rainfall* allowed omission or at least limit to considerations of the ice phase and melting zone, which seemed to me to carry some complexities with potential for bogging down the investigation.

Dr. Weickmann was a principal scientist at USAERDL; I have a letter from him dated 26 April 1963, which he signed as "Acting Chief, Atmospheric Physics Branch". My recollection is that he and one or two other "paperclip scientists" who came to the U.S. with him or at nearly the same time, represented the major component of meteorological talent, if not all of it, in that Branch. [6] There was a problem at the component of USAERDL with which I was acquainted in that the administrative staff seemed not to relate well with the contractors — administrative staff tended to make contractors feel that they were not trusted. And I believe that this situation was regrettably promoted by actual abuses. For example, in my report of a conference at Fort Monmouth on May 10 and 11, 1962, I wrote that at least one third of the contractor or potential contractor presentations were "very poor" and that only a few were reasonable. Dr. Weickmann's evaluations and interactions were especially important in this environment.

3.2. *Conceptual framework*

Early development of this work involved use of one equation to portray precipitation and vapor, and cloud was not portrayed. In this equation, M when positive is precipitation, and when negative it is the saturation deficit. M is increased in rising air or reduced in descending air through use of a generation (condensation) function relatable to a decline of saturation vapor density in the real atmosphere. Clearly, in a model that includes cloud, at least two continuity equations would be required. In one, cloud would be represented by m, and it would share the motion of the air in all respects. It would be increased in rising air and diminished in descending air according to the condensation function used to govern precipitation formation in the earlier models. Thus the assumption about immediacy of evaporation previously applied to precipitation would now be applied (more correctly) to cloud.

[6] Paperclip scientists were well qualified persons who had worked with the Third Reich during World War II, but who were deemed to have done so not out of choice but mainly because they had little choice. Besides Dr. Weickmann at USAERDL, one was H.J. aufm Kampe, who worked closely with Helmut, but may have been in another unit of USAERDL. I was puzzled about his name until Helmut cleared the mystery: aufm is a contraction of auf dem; thus aufm Kampe had forebears in the countryside as do many others.

The second equation would deal with precipitation, M. This would form not from vapor but rather from cloud; thus there would be no condensation function in the M-equation, but an interaction with the equation for m is identified. Furthermore, precipitation once formed can be augmented through an accretion process, and it can be depleted by evaporation in subsaturated air. Before discussing the modelling of these processes, we present the set of three-dimensional equations for precipitation and cloud with microphysics indicated schematically.

$$\frac{\partial m}{\partial t} = -u\frac{\partial m}{\partial x} - v\frac{\partial m}{\partial y} - w\frac{\partial m}{\partial z} + wG + mw\frac{\partial \ln \rho}{\partial z} - AC - CC + EP \tag{8a}$$

$$\frac{\partial M}{\partial t} = -u\frac{\partial M}{\partial x} - v\frac{\partial M}{\partial y} - (V+w)\frac{\partial M}{\partial z} - M\frac{\partial V}{\partial z} + Mw\frac{\partial \ln \rho}{\partial z} + AC + CC - EP \tag{8b}$$

As indicated for Eq. (2), these equations are derived in essentially the same manner as the equation of continuity for air. The symbols have their usual meanings. The terms account for the local rate of change of precipitation and cloud; their horizontal and vertical advection; divergence of precipitation owing to vertical variation of its terminal fall speed; divergence associated with vertical motion in the compressible atmosphere [7]; condensation and evaporation of cloud and changes of the saturation deficit that accompany vertical motion of air; and microphysical processes represented by autoconversion of cloud, collection (accretion) of cloud by precipitation, and evaporation of precipitation. In the form shown the units are density, but the equation is readily cast in mixing ratio units (see p. 4 in Kessler, 1969). The condensation–evaporation function G is related to the lapse of saturation vapor density Q_s through $G = Q_s(\partial \ln\rho/\partial z) - \partial Q_s/\partial z$, and I have usually approximated this as the linear decline, $G = A + Bz$, where $A = 3 * 10^{-3}$ gm^{-4} and $B = -3 * 10^{-7}$ gm^{-5}. These values are roughly applicable to tropical or summer atmospheres. In application, forms of Eq. (8a) and (8b) are used with the w-equation (Eq. 3) to define a model distribution of water substance in updraft column or cross-section.

3.3. Autoconversion process

It should be well understood that none of the work reviewed here represents a new contribution to understanding microphysics at the micro-level. I worked with a strong sense for interactions among processes as discussed here, and in expectation that their study would be facilitated by simple means to portray microphysical processes. The first process to be considered was conversion of cloud to precipitation. How to portray it? I did little more than observe in the literature and with my own eyes that thin water clouds seem to be persistent, and that rain falls from dense clouds. It seemed plausible to assume that up to some threshhold, clouds are stable, and that above that threshhold, a certain fraction of the cloud changes to rain per unit time. This would be modeled by

$$\text{Autoconversion} = k_1 * (m - a), \tag{9}$$

[7] The factor $w\partial \ln \rho/\partial z$ appears through its substitution for the three dimensional divergence of air.

wherein k_1 would be zero up to a threshold a and thereafter adopt an assigned value. The term represented in Eq. (9) is additive in the M-equation and subtractive in the m-equation, and it is applied only when $m > a$. While this treatment teaches nothing about how autoconversion works in nature, it does facilitate evaluation of effects of various thresholds and rates of autoconversion. Different values of these parameters could be used in the model and varying effects observed. The values $k_1 = 10^{-3}\ s^{-1}$ or $10^{-4}\ s^{-1}$ and $a = 0.5$ to $1\ gm^{-3}$, or thereabouts, are often used.

The simplicity of this model was of some concern to Helmut, and we discussed it on the telephone on November 11, 1963. On the following day, I wrote to him in part as follows: "...The fall speed of a cloud particle is estimated by Stokes' Law

$$V = C_1 D^2 \qquad [1]$$

where D is the diameter of the particle.The water content of a cloud whose particles are all of one size is

$$m = \pi \rho N D^3 / 6 = C_2 N D^3, \qquad [2]$$

where N is the number of particles. Following your suggestion, we imagine that the rapidity of cloud conversion is proportional to the product of the fall speed and cross-sections of the drops, i.e, proportional to VND^2. Then

$$\text{cloud conversion rate} = C_3 C_1 N D^4 = C_3 C_1 N (m / C_2 N)^{4/3} = C_3 C_1 N^{-1/3} (m / C_2)^{4/3}. \qquad [3]$$

Now C_1 and C_2 have definite measurable values. N would be assigned according to the condensation or sublimation nucleus regime we wish to model. The big problem is with C_3. Eq. [3] can be rewritten

$$\text{cloud conversion rate} = (K / N^{1/3})(m^{4/3}) \qquad [4]$$

and in this form is not greatly dissimilar from that now being used, viz.,

$$\textit{cloud conversion [rate]} = k_1 (m - a) \qquad [5]$$

In any event, it appears to me that the development leading to Eq. [4] could be much improved to provide a rational basis for selecting K in [4]. I think this would be a big project, but an eminently worthwhile one.In terms of the overall problem, however, I believe that such a refinement to the present model should go hand in hand with a fundamental review of all the assumptions in it, in order that a reasonably balanced set of assumptions are incorporated....In spite of its deficiencies, the present model is valuable for the insights it gives into kinematic relationships and conservation laws governing wind and water distributions and their relations to the strengths of microphysical processes....".

I did not pursue this further, largely because my training and abilities did not well equip me to do so, and partly because other avenues of investigation seemed more interesting to me. Subsequently, of course, much progress in the study of rain formation was made by others who treat details of combination processes involving discretized model distributions of cloud water.

3.4. Accretion process

Accretion occurs as slow-falling cloud droplets are overtaken and touched by larger precipitation particles. To model this process, I turned to the inverse-exponential distribution of precipitation first proposed by Marshall and Palmer (1948), $N=N_0 \exp(-\lambda D)$, where N is the number density of particles in unit size range of the distribution and D is diameter. Except for droplets of smallest sizes, this model is a fair approximation to reality, even though it defines a number density for drops larger than occur in nature. Its mathematical treatment leads readily to tractable explicit representations.

The rate at which cloud is accumulated by a single precipitation particle of diameter D_i falling at velocity V_i and collecting cloud particles with capture efficiency E is

$$\delta M_i / \delta t = -\pi D_i^2 E_i V_i m / 4, \tag{10}$$

the minus sign appearing because V_i is negative. To determine the accretion rate for a precipitation packet, we substitute for V using

$$V = -130 D^{0.5}\ ms^{-1}, \tag{11}$$

after Spilhaus (1948), then multiply the result by the M–P distribution and integrate over all diameters. The result is

$$\mathrm{d}M/\mathrm{d}t_{\text{accretion}} = 6.96 * 10^{-4} E N_0^{1/8} m M^{7/8}\ (\mathrm{gm}^{-3}\mathrm{s}^{-1}). \tag{12}$$

This is the form I have usually used. A derivation that includes effects of the variation of air density with altitude, which allows particles at higher altitudes to fall faster, leads to the addition of $\exp(kz/2)$ as a multiplier of the right hand side, where $k=(1/z)[\ln(\rho_0/\rho_z)]$ with ρ = air density; $k \approx 10^{-4}\ \mathrm{m}^{-1}$. Eq. (12) appears additively in the M-equation and is subtracted in the m-equation; it is applied with $M>0$ only when $m>0$.

The appearance of the one-eighth power on N_0 in Eq. (12) suggests that the deficiency of the M–P distribution with respect to its representation of the small-drop end of the precipitation spectrum is unimportant in this application. And although the M–P distribution identifies drop sizes larger than those that occur in natural rain, their number is so small as to be without important consequence here.

3.5. *Evaporation of rain*

A method similar to that used to derive the accretion equation is used for evaporation. Instead of the process represented by Eq. (10), we use an expression offered by Kinzer and Gunn, and given in table 117 of the Smithsonian Meteorological Tables (1958). The derivation detailed on pp. 29, 30 of the author's monograph (1969) leads to

$$\mathrm{d}M/\mathrm{d}t_{\text{evaporation}} = 1.93 * 10^{-6} N_0^{7/20} m M^{13/20}\ (\mathrm{gm}^{-3}\mathrm{s}^{-1}) \tag{13}$$

This equation is applied with $M>0$ only when $m<0$. A constant value of N_0 in Eq. (13) does some violence to the physics of the evaporation process, since this process actually decreases the relative number of small droplets, and in some other respects also, Eq. (13) is a rather rough approximation to evaporation.

3.6. *Fall velocity of rain*

In the early models, V is constant over all space and time except for discontinuous jumps such as discussed in Section 2.4. A more realistic appoach describes variation of V with M.

We use the terminal velocity of the particle that divides the M–P distribution into two parts of equal water content:

$$V_{\mathrm{median}} = -38.3 * N_0^{-1/8} M^{1/8}, \tag{14}$$

wherein the term $\exp(kz/2)$ may be entered as a multiplier to account for effects on V of the variation of air density with altitude as discussed in connection with Eq. (12), above. Eq. (14) is derived on pp. 27, 28 of the author's monograph (1969).

4. Results of studies of comprehensive models

The approximately two years of work at the Traveler's Research Center supported by Dr. Weickmann's unit at USAERDL is documented in reports of the project and in several presentations and other papers (Kessler et al., 1962–1964) and ultimately with further studies (Kessler et al., 1972–1987). [8] Here we present a summary of findings. It is important to bear in mind that with the exception of Section 4.12., all of the model data of this section and above apply to situations in which the air circulation is prescribed — there is no feedback from the microphysics to the dynamics. Nevertheless, as shown in Section 4.12. and elsewhere, the kinematic models have implications for dynamic processes. [9]

4.1. High-speed updrafts

When the maximum updraft on a parabolic profile exceeds the fall speed of precipitation present at that place, there usually exist two points on the profile where updraft and fall speed are equal. If we omit the small variation of V with M, the steady state precipitation content at heights of balance (zero advection) is then approximately defined as

$$M \approx -G/\mathrm{d}\ln\rho/\mathrm{d}z \approx 10^4 G. \tag{15}$$

Eq. (15) defines the precipitation density as that at which its rate of creation by condensation (about equal to the rate at which cloud is locally accreted) is exactly offset by the three dimensional expansion of rising air. Notice that in the high speed updraft case described by Eq. (15), M is independent of updraft w, but in physical terms, it should be realized that rates of both condensation and three dimensional divergence depend on w, so that what has happened in Eq. (15) is that w appearing in both numerator and denominator have canceled each other out. Eq. (15) defines, in the lower summer atmosphere, a very large density, about 30 gm^{-3}, that would occur after sufficient time where vertical advection is zero. Since each gram of condensate typically represents a negative buoyancy equivalent to about $\frac{1}{4}C$,

[8] The references include all of the formal presentations made during the period of the contract on the work of the contract.

[9] I wish also to acknowledge important encouragement for my major publication (Kessler, 1969). Apart from support within the National Severe Storms Laboratory and its Hqs. organization, the Environmental Research Laboratories, three persons, particularly, made a great difference. These were Helmut Weickmann, Helmut Landsberg, and Murray Mitchell, then editor of the AMS Monograph series. All three are now deceased, and I regret that I can not thank them personally again for their support that motivated me to make a large work effort and to submit my work collected to that time for publication as a monograph.

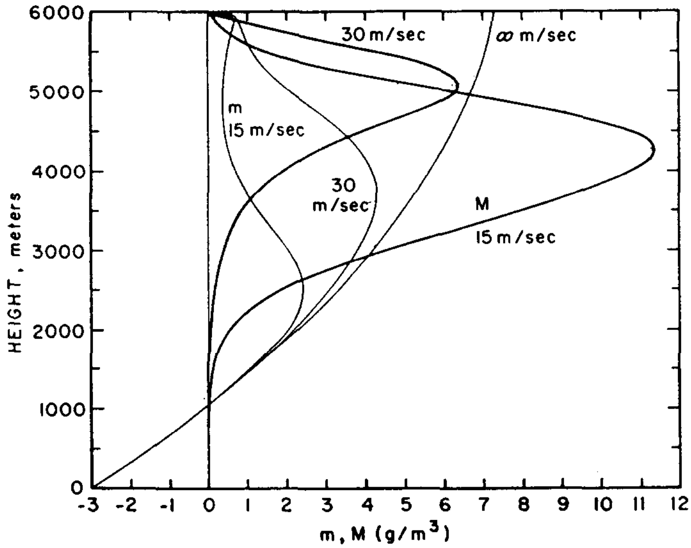

Fig. 3. Model steady-state profiles of coexisting cloud (light lines) and precipitation in summer updraft columns, for two speeds of strong updrafts. The precipitation maxima occur above those of cloud and both maxima are at higher altitudes when updrafts are stronger (From Kessler, 1969).

this means practically that the burden of such a large amount of condensed water would substantially exceed the buoyancy of sensible warmth in the rising current, and the updraft must stop before such large values of precipitation are attained. In this we have a partial explanation for the showery nature of much precipitation.

On the other hand, since G decreases with height to practically zero while $\mathrm{d}\ln\rho/\mathrm{d}z$ remains about the same with height, much smaller values of equilibrium density are defined in the upper atmosphere. When the condensation level is at a low altitude, and the updraft moderate, i.e., around 3–10 ms^{-1}, precipitation tends to occur below the updraft maximum and a shower regime is practically inevitable. However, when the updraft is very strong and the lifted condensation level is sufficiently elevated, there may not be a balance level below the updraft maximum because there is no precipitation there — the precipitation formed from cloud aloft must necessarily descend outside of the updraft core. But note especially that the model content of condensed water is largest in enduring updrafts that have about the same speed as precipitation fallspeeds, and is much reduced when updrafts are very strong with water maxima aloft. This model, illustrated for two strong updraft speeds in Fig. 3, has a counterpart in severe thunderstorms.

These important relationships between updraft speed and its burden of condensate, also treated in Section 4.12., are detailed in the author's monograph and in two subsequent papers (Kessler, 1975a, 1982a) And there is an analogy of great practical significance in the field of mining, as presented by Carte (1968) at the same conference where the hail studies summarized in the following subsection were first presented (Kessler et al., 1968). Quoting from the introduction to Carte's paper: "Updraft ventilation shafts almost 2 km in depth and more than 5 m in diameter are used by some South African gold mines. Nearly saturated air enters at the base and is carried upwards at speeds from a few to 20 ms^{-1}. Condensation occurs during ascent. The condensed water may reach the top of the shaft as cloud droplets or as raindrops, in concentrations up to several gm^{-3}... ...water in upcast shafts can have a profound influence on the airflow resistance. If the airspeed is slightly less than a critical value (approximately 10 ms^{-1}), water drops accumulate in mid-shaft. The accumulation continues until the airflow is impeded to such an extent that the fan drawing the air upwards becomes overloaded and stalls. The airflow then ceases and water cascades to the bottom. After this the fan restarts and the cycle is repeated. (Great losses in power and ventilation efficiency are therefore suffered if the airspeed is kept near the critical value.)..."

4.2. A model for the growth of hail and an error in a model for hail prevention

Here we consider the growth of a single precipitation particle by accretion of constant cloud. The accretion Eq. (10), and the fall speed Eq. (11) can be combined with the restraint that the updraft w is the linear function $w = W + Cz$, to give a differential equation whose solution is

$$z = \frac{V_0 + (W + C_{z_0} - K/C}{C}[\exp(Ct) - 1] + \frac{Kt}{C} + z_0, \tag{16}$$

where $K = (130)^2 Em/4\rho_w$; ρ_w is the density of liquid water or ice, i.e., $\rho_w \approx 10^6\ \mathrm{gm}^{-3.}$ All quantities except z and t are constant. From this equation, we deduce the following:

(1) When $C > 0$, i.e., updraft increasing with height, and with the initial motion upward, i.e., $(V_0 + w_0) > 0$, the upward speed increases indefinitely with time if $(V_0 + w_0) > K/C$. In this case, the rate of increase of w following the rising motion is greater that the rate of increase of terminal fallspeed that accompanies accretion of cloud. If $(V_0 + w_0) < K/C$, the initial upward motion becomes downward motion, increasing in magnitude with time. These situations cover conditions below an updraft maximum.

(2) When the updraft decreases with height, i.e., at altitudes above an updraft maximum, the particle motion becomes or remains downward, and $V + w$ approaches the magnitude K/C asymptotically. In other words, the descent rate $V + w$ tends toward that for which the increase of terminal fallspeed following accretion during descent is exactly compensated by the increased updrafts encountered during descent. Since the growth and motion of a particle tends to bring it to any height z with a fallspeed near $V_z = K/C - w_z$, irrespective of the particle's starting size and altitude, a vertical sorting process must operate when particles are initially distributed in size and altitude. A layer in which the updraft decreases with increasing height is a filter allowing passage of only those large particles whose fallspeeds exceed the maximum updraft.

Particles that remain for some time above an updraft maximum while their terminal fall speeds increase by accretion to the value required to fall through that maximum, are spread by horizontal divergence. This can be an important effect [10], and may account in part for relatively wide spacing of hailstones sometimes observed with hailfalls of large sizes.

Fig. 4 shows time to descend to the ground and final diameters of particles that grow by accretion, as determined by numerical integration of an equation applicable to a parabolic

[10] Important dilution in this case with enduring strong updrafts also occurs as a result of three dimensional divergence of rising air, not represented in Eq. (14).

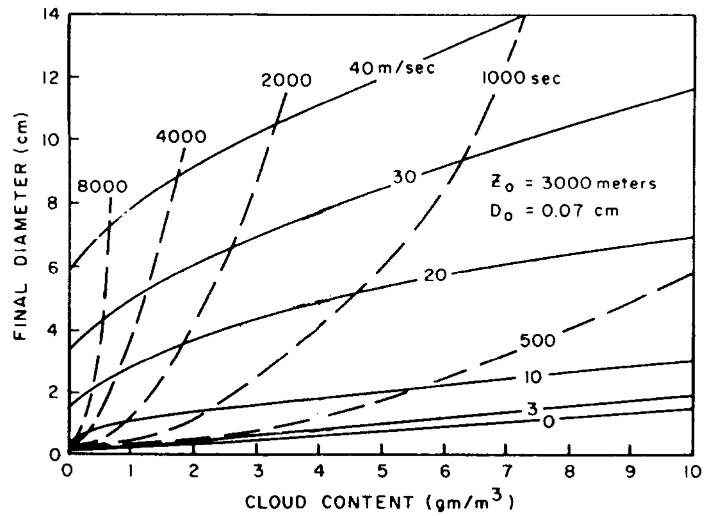

Fig. 4. Final size of an accreting particle and time of descent to $z=0$ from starting height $z_0=3000$ m, in relation to cloud density and maximum speed of a parabolic updraft (from Kessler, 1969).

vertical profile of vertical velocity in a column 10 km high. Particles with initial diameter 0.7 mm are introduced at a height of 3000 m and grow by accretion of constant cloud of density shown on the abscissa, in updrafts whose maximum values are shown on the solid lines sloping upward from the left.

During the 1960's, there was a substantial program in the Caucasus region of the former Soviet Union to reduce damage from hail. This program used rocket-borne seeding agents with objective to increase the number of accreting particles and thereby to reduce their average size. Highly controversial, a theory was presented to underpin the experiments (Sulakvelidze et al., 1967). In particular, it was noted that it is particularly above the maximum of a strong updraft, in the so-called accumulation zone, where descent of suspended particles is regulated by their growth, because higher terminal fall speeds are matched by stronger updrafts encountered as growing particles descend to lower altitudes.

The relatively long residence time of particles above the updraft maximum is properly related to large size there and to preponderance of a large particle fraction in cases where precipitation is prevented by strong updrafts from descending to the ground. However, although the size of individual particles can be closely related to their residence time and to cloud content and maximum updraft speed, the aggregate particle density in fast updrafts is strongly regulated by three dimensional divergence of rising air. Sulakvelidze's conclusion about the condensed water mass differs particularly from ours in that his calculated content increases with the square of strong updraft speeds, while our theory predicts approximate independence between these quantities when the updraft is strong. And as discussed in subsection a above, our theory predicts *smaller water contents when the updraft is much faster than precipitation fall speed than when updrafts and fall speeds are nearly equal.* It appears that Sulakvelidze did not develop his conclusion from systematic consideration of continuity equations.

An interesting analysis of the accumulation zone has been given by Nadibaidze (1971), who examined the vertical profile from a Langrangian perspective by following the growth and concentration of individual particles. In agreement with the theory presented here, Nadibaidze related accumulation of a large particle fraction at great altitudes to counteracting processes of growth and dilution, rather than directly to an updraft barrier to precipitation descent.

4.3. Cloud density without precipitation in relation to vertical displacement of air

The density of cloud does not always tend to increase in a parcel of rising air, even in the absence of mixing with cloud-free ambient air. Cloud formed and carried upward with rising moist air eventually diminishes in density owing to continued expansion of the rising parcel and diminished condensation rate at cold temperatures. The altitude where cloud density is a maximum in a rising current is a height where the rate of cloud formation is fully compensated by the rate of expansion of the air; it is higher as the lifted condensation level is higher. The maximum possible cloud amount near 8 gm^{-3} is presented in saturated summer air risen from near sea-level without losses. In the absence of mixing and autoconversion of cloud, the profile of cloud in a rising column is not a function of updraft speed, but only of initial vapor content and height at which the air began its rise. The implication that *cloud density may sometimes increase with descending motion* may help to explain the relative persistence of some cirrus clouds. A mathematical treatment of this subject is given on pp. 14, 15 of the author's monograph (1969).

4.4. Time of onset of precipitation in relation to autoconversion and accretion

The onset of precipitation is advanced in time by making autoconversion and/or accretion large, but the influence of variations of either diminishes as the other becomes relatively large. Of course, onset is delayed indefinitely if the coefficient k_1 in Eq. (9) is set to zero or if the autoconversion threshold a is set at a level that exceeds the maximum amount of cloud created in rising air currents. A more detailed discussion is given on pp. 31, 32 of the monograph.

4.5. Sorting of precipitation particles by gravity

A wide range of sizes of precipitation particles that may be present in a packet of precipitation aloft are sorted through their differential fall speeds during descent toward the ground. Fig. 5 illustrates a semi-quantitative criterion for gravity-sorting effectiveness. The ratio of the vertical depth h of a precipitation packet at the time precipitation begins at the ground to the depth S at the start of its descent is the stretching factor SF. When SF is very large, the distribution of raindrops at one time at the ground becomes very narrow.

The stretching factor as defined is

$$SF = h/S = (H/S)(1 - V_{min}/V_{max}) + 1. \tag{17}$$

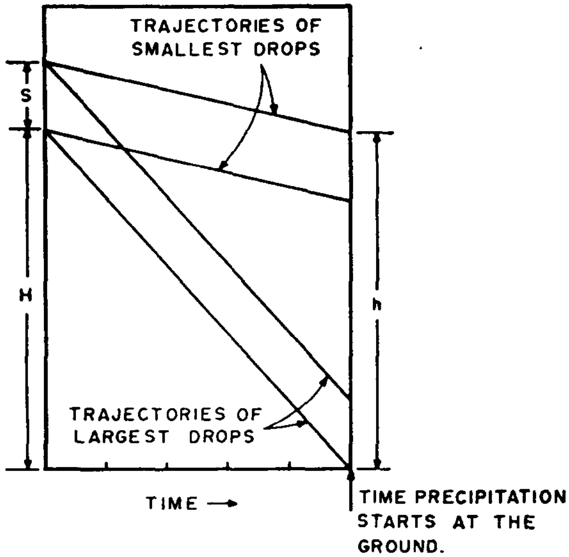

Fig. 5. Kinematic geometry of the stretching factor h/S (from Kessler, 1969).

When V_{min} is near zero, i.e., when the smallest drops are very small, Eq. (15) reduces to $SF=(H/S)+1$. When the difference between V_{max} and V_{min} is relatively small, as in snow, H/S must be relatively large for gravity to produce appreciable sorting of particle sizes.

4.6. Steady-state cloud amount in precipitation

The model steady-state cloud content is only weakly dependent on updraft speed and shape of the drop-size distribution. Thus, the accretion equation, Eq. (12), shows that rain is formed from cloud approximately in proportion to the product *mM*. In such a case, if the updraft and precipitation content M are doubled, then the doubled rate of precipitation formation is maintained with no increase of cloud at all. A more detailed analysis indicates that the steady state cloud content is proportional, very roughly, to $(w/N_0)^{2/9}$ and to $H^{-7/9}$. The latter dependence can be understood in terms of the increased precipitation content in deeper updrafts and consequent increased effectiveness of cloud accretion. The inverse dependence on N_0 reflects the greater power of accretion when precipitation is comprised of a greater proportion of large drops.

The equilibrium cloud content is strongly inversely dependent on the efficiency with which precipitation collects cloud. If collection efficiency is low, the cloud content is pushed by the updraft to larger values at which the accretion process transfers cloud to precipitation at the same rate as cloud is created.

4.7. Cloud and precipitation excess at onset of precipitation

The time-dependent vertical profiles of precipitation and cloud in the model are characterized by initial excesses whose magnitudes increase with the difference between the equilibrium cloud content and the magnitude of the conversion threshold a in Eq. (9). The excesses also tend to be larger when the autoconversion rate k_1 is smaller. An onset pulse or gush of model precipitation at Earth's surface represents the rapid deposition of condensation accumulated as cloud over a relatively long period; this gush can reflect variations of k_1 and a with altitude. Thus, the gush is particularly large when the cloud conversion process begins at high altitude and the resulting precipitation is augmented by accretion as it descends through dense cloud at lower altitudes. If a starting gush of precipitation is a natural result of processes that are not strongly electrical, it becomes more difficult to define those attributed to collapse of the electrical field during lightning flashes, as described by Moore et al. (1966). See also Kessler (1966).

4.8. Precipitation rate and amount in relation to updraft speed and duration

The amount of condensation is proportional to the product of updraft speed and duration in saturated air but precipitation at the ground is not so simply defined because other processes occur during the time required for resultant cloud to coalesce into precipitation particles and then to fall out. Results are quite different between cases when updraft is strong with vertical displacement completed quickly, and updraft slow and enduring. When the updraft is very strong, cloud may be carried to very high altitudes, spread there by divergence, and a relatively large proportion of condensate returned to vapor by evaporation in unsaturated air that lies between cloud and ground. In addition, strong updrafts activate more condensation nuclei than weak updrafts do, so condensate is spread over a larger number of more nearly uniform cloud drops when updrafts are strong, and coalescence and accretion processes, which depend largely on differential fall speeds, are weakened in strong updrafts. When precipitation does finally form there, it has a dense cloud to feed on, and is likely to fall in torrents. At the other extreme of weak updrafts, there is plenty of time for precipitation to form from cloud and to fall out even while condensation continues. In this case, onset is apt to be gradual and precipitation rates at the ground would be comparable to condensation rates above.

Returning to our kinematic model, we define the displacement parameter $D = w_{max}T/H$, where w_{max} is entered rather than w because we continue to conceive of air initially saturated throughout and an updraft column with zeros at its base and top and a maximum between. With rapid displacement, the total precipitation beneath an updraft column is proportional to $\boldsymbol{D}$ when $\boldsymbol{D}$ is $\ll 1$ (provided that the density of cloud formed in the rising column exceeds the autoconversion threshold) and is insensitive to $\boldsymbol{D}$ when $\boldsymbol{D} \geq 1$.

Figs. 6 and 7 show precipitation at the ground for displacement parameters 1/6 and 1, respectively, in a model parabolic updraft and with model microphysical processes discussed in Section 3. The updraft has the various indicated speeds until vertical displacement is complete, and remains at zero as the precipitation process is completed. In Fig. 6, very little cloud is exported to the environment even when updrafts are strong, and the total precipitation at the ground is about the same in all cases. Notice that the precipitation curves have nearly the same shape and amplitude for updrafts speeds of 2, 5 and ∞ ms^{-1}. The reason for this similarity is that in all of these cases, condensation and cloud formation are virtually complete before precipitation development is far advanced. The earlier onset time with faster updrafts reflects merely the earlier completion of the cloud-forming process. The

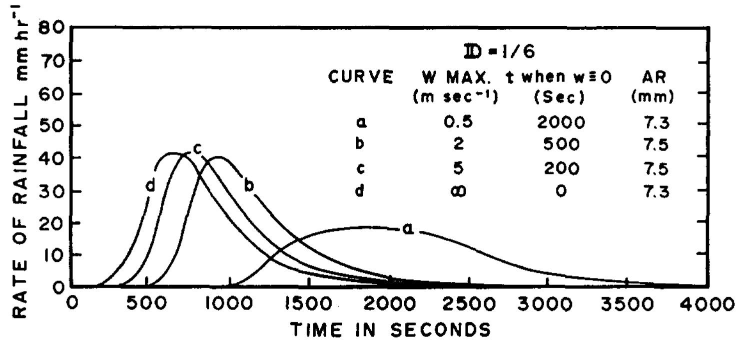

Fig. 6. Surface rainfall rate vs. time for various rates of small vertical displacement of air in a saturated summer atmosphere. AR means accumulated rainfall (from Kessler, 1969).

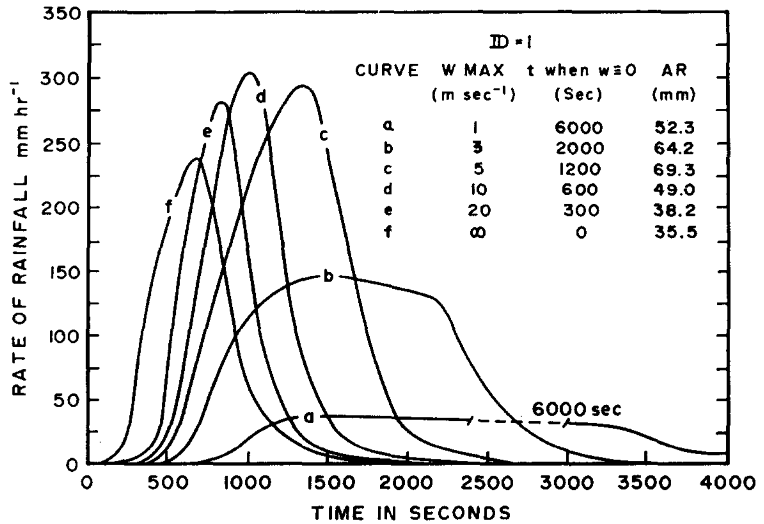

Fig. 7. Same as Fig. 6 for a substantial vertical displacement of air.

similarity of the curves is indicative of limitations in our ability to deduce the strength of vertical currents from features of precipitation at the ground.

Fig. 7 depicts cases where the displacement of a typical air air parcel in the updraft column is a large fraction of the total depth of the column. Precipitation rate and total accumulation are maxima when updrafts and precipitation fall speeds are about the same. While the updraft persists in such cases, much precipitation is suspended in the middle of the updraft column where it grows to large values by collecting cloud that would otherwise be carried to high levels and somewhat lost. An additional boost to the precipitation rate in intermediate updrafts, however, is given by strong horizontal convergence in the lower atmosphere, as explained in connection with Fig. 2.

4.9. *Water budget in a roll circulation with microphysics*

The calculations discussed in Section 2.2. were repeated with two initial moisture conditions, two updraft speeds, and some variation in the parameters of microphysical inter-

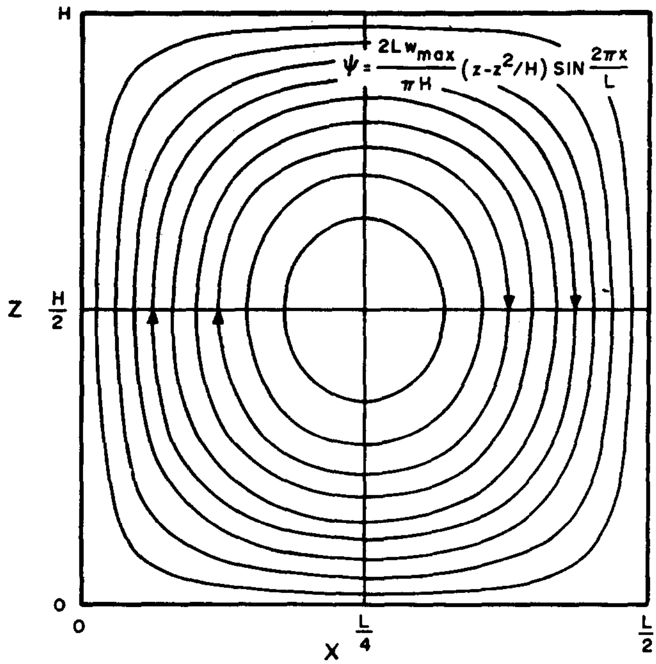

Fig. 8. Model roll circulation used in the calculations illustrated in Fig. 9.

actions. The roll circulation is as depicted in Fig. 8, and results of a set of calculations with unsaturated initial conditions and maximum updraft 10 ms^{-1} are shown in Fig. 9 (from Kessler, 1969, p. 56). The model circulations are continued without change until overturning is substantially complete at $T_1 = 1.35H/w_{max}$, and are then set to zero as precipitation continues to fall out. Budget calculations emphasize reduced efficiency in strong circulations owing to greater evaporation of precipitation in subsaturated air in the environment of the updraft region. It is notable though not surprising that evaporation is greater in cases where the initial condition is subsaturation, and the amount of rain at the ground is further reduced because the amount of condensate in drier atmospheres is smaller. With initial saturation, total rainfall amount divided by the amount condensed (the "efficiency") is ≈82% in the weak updraft ($w_{max} = 2.5$ ms^{-1}). Efficiency is reduced to 77% when N_0, the drop size distribution parameter in precipitation, is increased by a factor of 100, which causes precipitation evaporation rate to increase by the factor five, fall speed of precipitation to decrease to about half its usual values, and accretion rate to approximately double. Efficiency was ≈72% with normal N_0 and maximum updraft increased to 10 ms^{-1}. In the initially undersaturated condition depicted in Fig. 9, the efficiency is just 53%.

The amounts of rainfall at the ground in the model calculations range over nearly a factor of three — there is not only reduced efficiency in stronger and drier circulations, but less moisture is condensed in the drier circulations. It is difficult to escape a conclusion that efforts to augment natural precipitation through artificial adjustment of microphysical processes will require large efforts to produce small results, and that reduction of the moisture

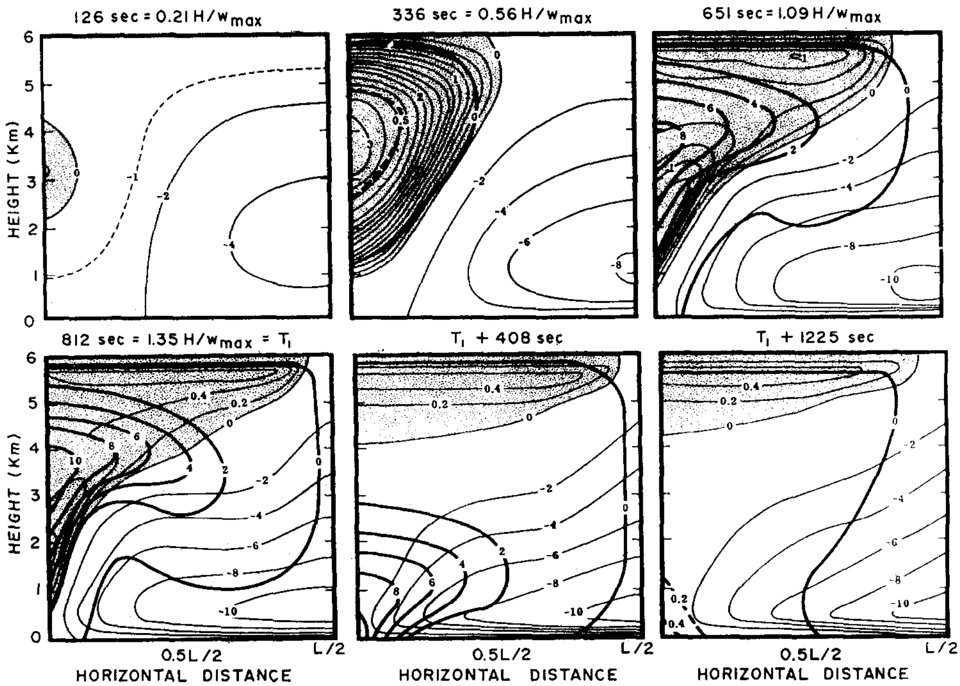

Fig. 9. Distributions of cloud, precipitation, and saturated deficit in a model circulation initially unsaturated. The air circulation is steady with maximum updraft (and downdraft) 10 ms^{-1} until substantial overturning has occurred at 812 s; the circulation is then set to zero while precipitation falls out (from Kessler, 1969).

content of air through success at one place will reduce the likelihood and amount of rain at places nearby. [11]

4.10. Condensate content in sloping updrafts

The situation of interest is illustrated in Fig. 10. An air parcel moves with a horizontal component $w*\tan\Theta$ as a precipitation particle falls relative to the air at velocity V (negative) and traverses the distance $Z = D/\tan\Theta$ in time $T = -D/V\tan\Theta$. The time T and updraft w with microphysical parameters such as the autoconversion threshold determine at what height precipitation develops and its density at exit. If equal-size precipitation particles originate as shown on the upwind side of a sloping column that endures longer than the residence time of the particles, then all particles therein will have come from the upwind side and the largest water content will be found along the downwind side. The particles exit the column after having fallen relative to it the distance $Z = D*\text{cotan}\Theta$. This situation is discussed further in a short published note (Kessler, 1975b).

[11] On the other hand, one may hope to stimulate a process that would not otherwise occur, as by release of latent heat of fusion through seeding of undercooled liquid cloud.

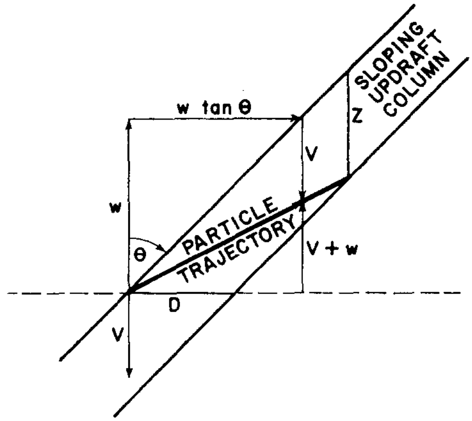

Fig. 10. Kinematic geometry of a sloping updraft column (from Kessler, 1975a, b).

4.11. Estimation of precipitation rate and accumulation by radar

Radar-rainfall relationships, as presented by various authors, embody incomplete adjustments for a multitude of effects because they represent empirical fitting of average values of rainfall measured at Earth's surface with measurements of radar reflectivity aloft. An apparently commonplace underestimation of precipitation beneath downdrafts as reported, for example, by Brandes and Wilson (1987) may be explicable in terms of kinematic theory. It is true that in showers, the precipitation tends to form in updrafts, but it tends to fall out in downdrafts (see Section 5.3.). At Earth's surface, the vertical air velocity is necessarily zero, but horizontal divergence of air descending near the ground spreads descending precipitation over an area larger than it occupies aloft, where the radar measurements are made. Evaporation of precipitation is a slow process in a moist downdraft, and the contrary tendency of precipitation density to increase owing to increasing density of the descending air is also usually small. Hence, we expect radar reflectivity measurements made aloft to be possibly adequate for measuring *rates* but to underestimate *local accumulations*, because the area covered by precipitation at the ground would be underestimated in the radar measurements. A more detailed treatment of this problem is given in Kessler (1987).

4.12. On quasi-steady thunderstorms and the period of showers

The behavior of the model distributions of total water load in updrafts has been roughly described in two differential equations that also include inferences on buoyancy:

$$\mathrm{d}w/\mathrm{d}t = B - K_1 w - K_2 L, \tag{18a}$$

and

$$\mathrm{d}L/\mathrm{d}t = C_1 w - C_2 L, \tag{18b}$$

where B is a forcing function that may be identified with thermal buoyancy, L is the condensed water load, the term $K_1 w$ represents drag, $K_2 L$ represents the contribution of condensate to negative buoyancy, $C_1 w$ represents the condensation process, and $C_2 L$ represents the greater tendency toward fallout of larger amounts of precipitation. Although the equations apply to air parcels, the total derivatives may be replaced by partials that apply to average conditions in the updraft column (see pp. 72, 73 in Kessler, 1969). The equations may then be combined to give a second order equation. When the water load is independent of updraft speed ($\mathrm{d}L/\mathrm{d}t = 0$) as in the case of sustained high-speed updrafts with elevated lifted condensation level, the solutions are characterized by a non-oscillatory adjustment of w toward equilibrium among forces of buoyancy, water load and drag:

$$\bar{w} \rightarrow (B - K_2 L) / K_1, \tag{19}$$

and, when $(K_1 - C_2)^2 < 4K_2 C_1$ with dependence of L on w, there is a solution with damped oscillations with period

$$T = 2\pi [K_2 C_1 - (K_1 - C_2)^2/4]^{-1/2}. \tag{20}$$

C_1, an approximation to the condensation function, has an average value of about $1.5 * 10^{-3} \mathrm{g m}^{-4}$ in a summer atmosphere, and K_2, the coefficient of equivalent buoyancy, is about $10^{-2}\ \mathrm{m^4 g^{-1} s^{-2}}$. A brief discussion on page 73 of Kessler (1969) relates K_1 to work of Simpson and Wiggert (1969) and assigns a value to K_1 from about 10^{-4} and 10^{-3} $\mathrm{s^{-1}}$ in updrafts from 1 to 10 $\mathrm{ms^{-1}}$. C_2 in weak updrafts is the precipitation fallspeed divided by the height of the updraft column and in strong updrafts it is the three dimensional divergence term in the continuity equation, $-w \mathrm{d} \ln \rho / \mathrm{d}z$, in either case about $10^{-3} \mathrm{s}^{-1}$ for an updraft column 10 km high. Then T, the period of condensation oscillations, is around 2000 s, in reasonable agreement with observations and also with 20–30 min periods calculated by Srivastava (1967) from a more complete dynamical model. An implication of Eq. (20) that is probably more significant than its numerical value is its prediction that the period of showers should be longer in colder atmospheres (smaller condensation function).

A physical explanation for the oscillation lies in a developing load of condensed water that leads to reduction or nullification of the buoyancy of excess sensible temperature and hence of the acceleration of the upward current while the vertical air velocity is upward. Then further development of condensation products in the rising current may lead to downward acceleration and later to reversal of the sign of the velocity, while fallout of precipitation may allow updrafts to resume.

If the water load does not increase with updraft speed and does not attain the magnitude that offsets buoyancy, then the steady state solution is approached asymptotically. This can conform to the high-speed updraft case with elevated lifted condensation level discussed previously, and may be thought to correspond to nature's quasi-steady severe thunderstorms.

5. Parameterization of dynamical processes

5.1. Conceptual background

A model atmosphere can be assigned a distribution of temperature and moisture and the buoyancy of a parcel or column of rising air in that environment can be tracked. As cloud and precipitation develop, thermal buoyancy is counteracted by the weight of condensate, and the net buoyancy of individual air parcels can become quite irregular in its distribution, even discontinuous. A problem exists in the management of air velocities and accelerations in regions of irregularity or discontinuity. Rigorous treatments today demand mathematical representation of dynamic pressure, but a simplification is suggested by the realization that an effect of dynamic pressure is to smooth the distribution. Parcel dynamics are checked by the forces needed to push quiescent adjacent air out of the way of rising or descending elements.

This conception is behind the following idea — Let the velocities in an air column be directed by the average buoyancy in the whole column. This idea would be applied while realizing that it is a gross simplification, but with anticipation of interest in its implications and wonder about whether such a model might contribute ideas appropriate to further study. How to investigate dynamical interactions without sophisticated dynamics?! I accordingly proceeded with development that added this feature to the continuity equations and microphysical parameterizations described above. Results were first presented at the International Meteorological Conference at Tel Aviv, Israel, Nov. 30–Dec. 4, 1970, a conference also attended by Helmut Weickmann, then director of NOAA's Atmospheric Physics and Chemistry Laboratory.

I had not seen Helmut for some time, and there was little opportunity at this busy conference. One evening, however, Helmut invited me to accompany him on a little walk to an old nearby Catholic church and monastery, which had connections with Germany. We had a very interesting tour, and we left in the company of two monks in traditional garb. As we walked down cobbled streets, I listened to their conversation in German, which I followed only imperfectly, but well enough to understand one of monks to compliment Helmut on his command of German, and to ask Helmut where he learned it! I looked at Helmut in astonishment, and we both burst out laughing! So Helmut had really become an American!, and this was an item of banter between us when we met during future gatherings of meteorologists and NOAA administrators. [12]

5.2. *Equations of the model for parameterized dynamics*

Our model of columnar convection augments the velocity Eq. (2) and the continuity Eq. (8a, b) (without the terms for horizontal advection) with equations that embrace interactions among the velocity parameter w_{max}; thermal buoyancy a; the load of condensation products, $L=M+m$ (for $m>0$), which contribute negative buoyancy; the lapse rate and moisture distribution in an entraining environment; and the heat transferred during condensation and evaporation. Vertical averaging of condensed water load and effective buoyancy greatly simplifies the calculations while raising questions of rigor and applicability, but facilitates coverage of a great parameter space and elicitation of working relationships that are provocative, somewhat testable and suggestive.

[12] Another event during our stay in Israel was my utter collapse at a reception at the residence of the mayor of Tel Aviv. This was embarrassing to me but interesting to most of the many meteorologists and others present. It may have been due to overindulgence in strong Israeli coffee and unfamiliar food during meetings with scientists working on contracts and grants from NOAA and also touring. After nearly three days of education in hospital, I was pronounced cured and was able to make my presentation and to continue on my trip. Thanks to the several people including Prof. J. Neumann, now deceased, and his wife who were especially supportive during this trial.

Here is a summary sketch of detailed derivations and discussion presented in Kessler and Bumgarner (1971) and Kessler (1974). To Eq. (8b) is added a term $-k_5M$, which defines a rate of mixing of columnar air whose content of precipitation is M, with environmental air whose precipitation content is zero. The term $-k_5(m-m_0)$ added to 8(a) similarly defines mixing of columnar m with ambient m_0. Then there is a buoyancy equation to guide the development of vertical velocity:

$$\frac{\partial w_{\max}}{\partial t}=1.5\left[B-\left|\frac{B}{H}\right|^{1/2}w_{\max}\right]-k_5w_{\max} \tag{21}$$

where total buoyancy B is given by

$$B=\boldsymbol{a}-k_8L \tag{22}$$

and $L=m+M$, with negative values of m taken as zero; $k_8=0.01$ $m^4g^{-1}s^{-2}\approx g/\rho$ is a factor to convert L to an equivalent acceleration. Although ρ varies by a factor of about 2.5 through the 10 km depth of calculations described below, the significance of this variation is minimized by our use of average values of water load. The values of m and M are calculated at each time step. Eq. (21) is related to a maximum updraft attained by an air parcel at $H/2$ after acceleration from $z=0$ under the influence of total buoyancy B. [13]

An equation for thermal buoyancy $\boldsymbol{a}$ follows from consideration of its development as air is displaced from equilibrium levels in an atmosphere not neutrally stratified. For example, in the case of continuing rising motion in a column extending upward from the surface, the entire column tends to become composed of parcels originating near the surface. We relate a parameter S_m to the acceleration identified with the average temperature difference that develops between an environmental sounding and the temperature profile associated with ascent of moist (saturated) surface air. Thus a specific average acceleration from thermal causes, S_m, is approached as rising motion of saturated air continues. Extension of this line of thought to include upward and downward motion of both saturated and unsaturated air parcels, with effects of mixing to include evaporation of both cloud and precipitation in entrained ambient air, leads to

$$\frac{\partial a}{\partial t}=\frac{w_{\max}}{H}[F_1S_m-(1-F_1)S_d-a\ sgn\ w_{\max}]-k_5\boldsymbol{a}-k_6(\varepsilon_m+\varepsilon_M) \tag{23}$$

where F_1 is the fraction of grid points where cloud exists, weighted according to the condensation rate wG at each point. When S_m and S_d are equal, the implied tropospheric lapse rate is midway between moist and dry adiabatic values. The examples presented here are calculated with $S_m=S_d=0.2$ ms^{-2}. An approximation to the dry adiabatic lapse rate in a summer atmosphere in the United States has $S_m=0.6$ ms^{-2} and $S_d=0$.

[13] An explicit parameterization of dynamic pressure is also interesting though not introduced to this model. In Kessler (1985), the dynamic pressure is related to the gradient of pressure required to push aside the air in front of a buoyant parcel and to restore it in the wake. This approach leads to the following term to be appended to a buoyancy equation: $4w^2(L_x^2/L_z^3)$ (sgn w), where L_x and L_z are horizontal and vertical dimensions. Since the present model treats a column rather than a bubble, introduction of this term to the present model might be seen as an extraordinary stretch! Note, however, that the term obviously promotes convection in columns narrow and tall, but columns too narrow are eroded by eddy diffusion (see footnote 17).

The last two terms in brackets in Eq. (23) account for evaporative cooling. The average rate of evaporation of precipitation, is calculated from grid-point values of the term *EP* in Eq. (8a, b). Precipitation evaporates only in unsaturated air. The first term within the last brackets represents the rate of cloud evaporation, calculated where $m > 0$ and averaged over the depth H. This calculation is based on the diminution of cloud as defined by the mixing term that is added to Eq. (8b). Thus, at a particular height we have $\mathrm{d}m/\mathrm{d}t = -k_5(m - m_0)$, where $-k_5 m$ represents the dilution of cloud by mixing with saturated cloud-free air, and $k_5 m_0$ represents the further depletion that occurs from instant cloud evaportation in unsaturated ambient air (where m_0 is negative). The average cloud evaporation, as contrasted to dilution, thus is given by

$$\varepsilon_m = \frac{k_5}{n} [\sum_{m>0} m_0] \tag{24}$$

where n is the number of grid points in the vertical column and the summation of ambient saturation deficits is derived from all altitudes where the convective column is cloudy. The coefficient $k_6 \approx 0.1$ ($\mathrm{ms}^{-2} \mathrm{g}_{\mathrm{H2O}} \mathrm{m}^{-3}$) converts the rate of evaporation to the equivalent rate of change of the acceleration attributable to evaporative cooling. [14]

I am grateful to Rex Inman, deceased, formerly a group leader at the National Severe Storms Laboratory and later Chair of the Meteorology Dept. at the University of Oklahoma, for informing me of a close connection between Eqs. (21) and (23) in this paper and velocity and buoyancy equations used by Priestly (1953). This connection is discussed with some other important details of the calculations in NSSL Tech. Memorandum No. 53 (Kessler and Bumgarner, 1971). I am grateful also to Robert Davies-Jones of NSSL, who checked much of my arithmetic in this and other areas and often improved both the mathematics and prose, and to Frank Ludlam, deceased, who had a considerable and encouraging interest in this work. [15]

5.3. Results

(1) Numerical solutions — The *Tellus* paper (Kessler, 1974) presents substantial intermediate discussion of model properties through use of simplified versions of the equations. Here we proceed directly to results of numerical integration of the complete model. We deal exclusively with updrafts initiated by a starting perturbation buoyancy, i.e., an initial assigned value of $\boldsymbol{a}$. There are five types of solutions given by the complete model: (1) In stable dry cases, simple dampled oscillations; (2) with weak absolute instability, disturbances restored monotonically and asymptotically to zero when the mixing rate is sufficiently large; (3) in moist stable or conditionally unstable cases, cloud and precipitation development leading to a model shower, which is a kind of condensation oscillation with period in remarkable agreement with the prediction of Eq. (20), followed by damped simple

[14] $K_6 = Lg/C_p T\rho$, where L is the latent heat of evaporation, C_p is the specific heat at constant pressure, and is the air density.

[15] After it was rather summarily rejected by the *Quarterly Journal of the Royal Meteorological Society*, Frank said that I should send it to *Tellus*. When I suggested that it might be further improved by attention to remarks offered by a reviewer, he asked if I did not more wish to see it published than further improved.

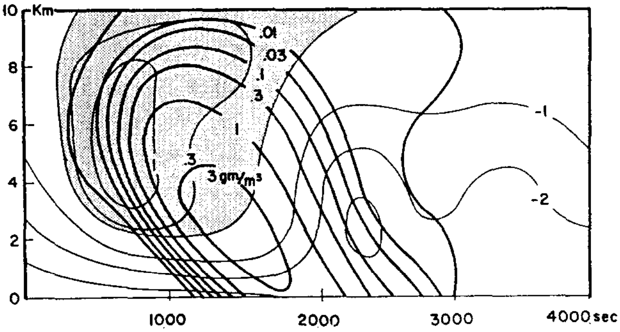

Fig. 11. Time–height diagram showing development of cloud (shaded), precipitation (heavy lines) and saturation deficit (all in gm^{-3}) in a model updraft column with parameterized dynamics. The associated updraft that develops after an initial thermal perturbation of about 0.7°C, attains a speed of about 6 ms^{-1} before it declines with descent of precipitation and then undergoes a damped oscillation (from Kessler, 1974).

oscillations; (4) an updraft that becomes steady, and is accompanied by steady rain at the ground; (5) an updraft that becomes steady and strong (w_{max} from 15 to 40 m/s) without precipitation in the lower model atmosphere, but, in a moist atmosphere with substantial cloud and precipitation aloft and precipitation at the ground implied outside the area beneath the strongest updraft.

The types of solution tend to drift from simple damped oscillations toward high speed updrafts as the environmental lapse rate and the perturbation buoyancy increase and as the mixing rate decreases, i.e., as the implied horizontal dimension of the updraft area increases. [16] With a given conditionally unstable lapse rate, the shower mode is less favored and a steady mode more favored when the moisture content of the environment is large, but steady high speed updrafts are most favored by less-than-saturated moisture conditions, i.e., by a somewhat elevated condensation level, or by a large threshold for cloud conversion. The transitions between some solution types appear to be discontinuous for changes of input parameters across thresholds, and this extreme sensitivity to initial conditions and forking behavior that characterizes some portions of the parameter space suggests that the solutions might properly considered to be ''chaotic'' (Lorenz, 1963, et.seq.). However, the solutions do not possess other properties of chaos, such as non-predictable though determinate behavior and random relationships.

Several solution types are portrayed in Figs. 11 and 12. Among the parameters used in the calculations with an unsaturated starting column, only the starting perturbation buoyancy is different in the two cases: 0.025 and 0.05 ms^{-2}, respectively, equivalent to average starting temperature excesses of approximately 0.7 and 1.4C. In *Tellus* there is an extended mathematical description of parameter behavior. In physical terms, the development shown in Fig. 11 is regulated as follows: An upward air current develops immediately after the start in response to the input perturbation thermal buoyancy. The thermal buoyancy immediately starts to decline, partly as a result of mixing of columnar and ambient air, but more

[16] The parameter k_5 is roughly related to an implied size of the updraft column. The equation proposed by Richardson (1926) and subsequently used by Priestly (1953) becomes, in mgs units, $k_5 = 0.117D^{-2/3}\ s^{-1}$, where D is diameter in meters.

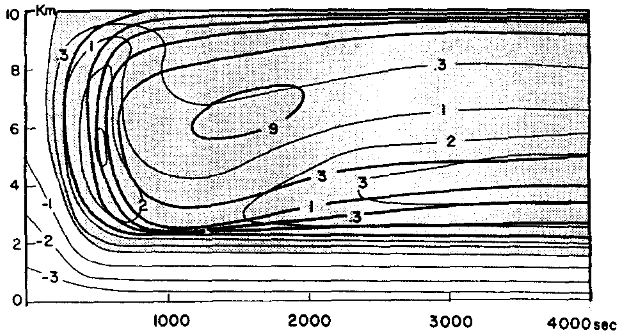

Fig. 12. Same as Fig. 11, except that the starting thermal perturbation is twice as strong. Here the updraft becomes "high speed" and steady, and precipitation can not descend through the updraft column that created it.

because the environmental lapse rate is stable for vertical motions of dry air. After saturation is attained, however, condensation occurs with release of latent heat, and there is a contribution to positive buoyancy. Therefore, the thermal buoyancy declines less rapidly and even begins to increase after 300 s. However, the accumulation of condensation products also continues, offsets the contribution of latent heat, and after 300 s produces decreasing net buoyancy and contributes with the mixing process to diminishing updraft speed. As the updraft weakens, descent of precipitation toward the ground is hastened and as precipitation falls into the subcloud layer, its partial evaporation there produces a substantial contribution to negative thermal buoyancy. The downdraft starting at about 1600 s is attributable to dominant water load effects in the presence of a residual small positive net thermal buoyancy. As precipitation falls rapidly from the descending air column and the cloud evaporates, there is the case of descending dry motion in a stable environment, and a restoring upward buoyancy increases. The subsequent record is that of a simple damped oscillation.

The early developments illustrated in Fig. 12 are similar to Fig. 11. Because of the stronger starting thermal perturbation, however, the vertical velocity is larger when condensation begins. This produces a more rapid recovery of thermal buoyancy, whose rate of increase exceeds the contrary tendency of the increasing water load. Therefore, the updraft continues to increase, precipitation is held aloft, and the high-speed steady updraft case develops.

Fig. 13 shows the range of solution types obtained for various values of environmental moisture and starting thermal perturbation, for lapse rate midway between the moist and dry adiabatic rates. The ambient moisture profile in all cases is defined by $m = C_0 - (2C_0/H)z + (C_0/H^2)z^2$; the value of C_0 is shown on the abscissa of Fig. 13. A different portrayal of the forms of convection defined by this model system is presented by Fig. 14.

(2) Inferences

(i) Nocturnality of storms and their size — Effects of the mixing term and thermal buoyancy in this system bring forcibly to attention the concept of critical size and magnitude of disturbances required to initiate convection. If too small and too weak, a disturbance is damped via the mixing process, even if the environment is statically unstable. This suggests that the nocturnality of thunderstorms in many parts of the world may be, in part, a result of a nighttime boundary layer that is warm enough to be unstable in relation to the middle

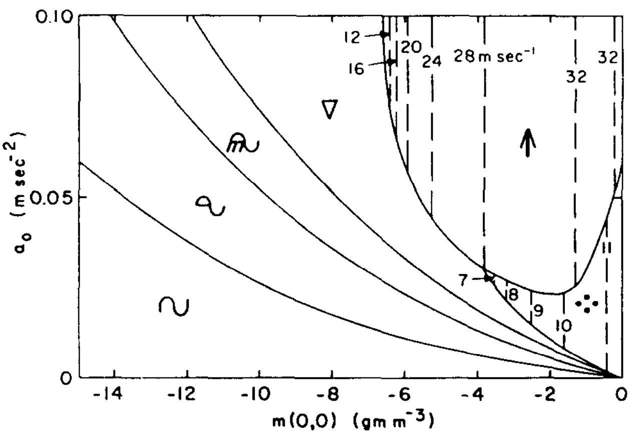

Fig. 13. Types of solutions of the complete numerical model in relation to environmental moisture and starting thermal perturbation. Symbols have obvious meanings. Isopleths in the right side of the figure indicate the steady values of maximum vertical velocity eventually attained. All illustrated solutions apply to a lapse rate midway between moist and dry adiabats and mixing rate $k_5 = 3 * 10^{-4}$ s^{-1} (from Kessler, 1974).

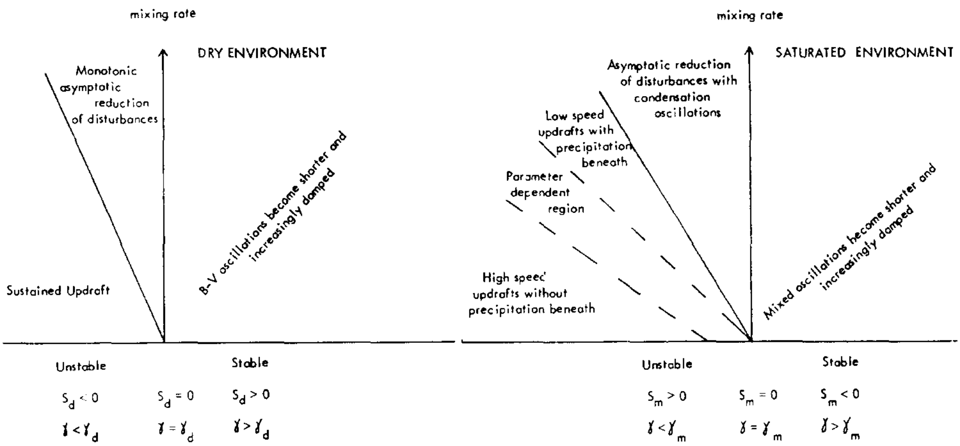

Fig. 14. Modes of model air motion, lapse rate, and mixing rate in dry and saturated model environments. Boundaries of the parameter-dependent region depend on the amplitude of model disturbances and microphysical processes. Complicated conditionally unstable cases are discussed in Kessler (1974). Juxtaposed indications of γ and S_d or S_m indicate equivalences in standard and model notations (from Kessler, 1974).

troposphere, but statically stable with respect to itself, i.e., on the small scale. Then the shears that accompany any vertical velocity are less apt at night to lead to breakdown via the process described by Richardson (1920). Besides time of occurrence, there are implications in this for the motion and development of existing storms, and for the size of storms characteristic of particular air masses. These concepts are more fully developed in the *Tellus* paper.

(ii) A caution toward artificial augmentation of precipitation — Fig. 11 shows a model situation in which onset of precipitation and evaporation as it descends in the sub-cloud layer promotes cessation of the vertical current that produced it. The model with parame-

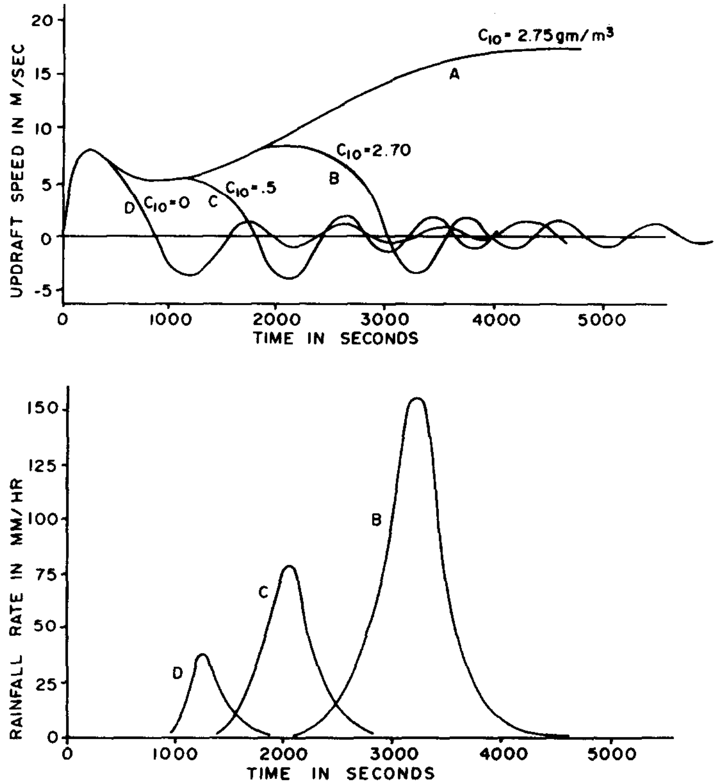

Fig. 15. Illustrating reduction of model precipitation at the ground caused by a hastening of cloud conversion aloft. With parameters tuned to show this result, earlier evaporation of precipitation into the subcloud layer causes reduction of a thermally buoyant updraft before the updraft is better established. Early precipitation formation also represents an early enhanced contribution to negative buoyancy in the lower part of a cloud by virtue of the weight of condensate (from Kessler, 1972).

terized dynamics indicates that cloud seeding may promote this effect. For example, Fig. 15 illustrates precipitation and updraft development in cases where all parameters are the same except for the cloud conversion threshold [a in Eq. (9) but shown as C_{10} in the figure]. Along the curves marked B, the conversion threshold is 2.70 gm^{-3} and precipitation formation is greatly delayed. When precipitation finally forms, it accretes a dense cloud with resultant heavy precipitation at the ground below the model cloud. Curves C and D illustrate further lowering of the conversion threshold with resultant earlier formation of precipitation. The descent of precipitation with evaporation in the subcloud layer reduces buoyancy and hence reduces the upward acceleration and velocity of the air that is feeding the condensation process, and the amount of model precipitation is consequently reduced.

The curve marked A further illustrates the great complexity of the system being considered. In this case, the conversion threshold is so large that the convection mode is changed from a shower to the high-speed steady updraft case, wherein all of the precipitation that arrives at the ground is outside of the steady updraft column. A more complete discussion of this and of several other aspects of weather modification is given in Kessler (1972).

One should not assume from this that cloud seeding should not be considered as a strategy for increasing precipitation. The situation illustrated in Fig. 15 was tuned to produce a particular outcome and different outcomes can be shown with the same model tuned in a different way. The lesson is that there can be no reasonable expectation of success at augmentation of shower precipitation through approaches that fail to address the natural system comprehensively. And complex as the model solutions seem to be, nature is even more complex!

6. Concluding remarks

The author's papers on the continuity and distribution of water substance in atmospheric circulations are summarized here. Generally, detailed derivations are not given here, but the present paper strives to acquaint the reader with basic principles and lines of thought that were utilized in developing the model studies, and with principal conclusions.

At this writing, twenty-five years after publication of my monograph, references to the parameterization of microphysical processes continue to appear in current literature and are still finding use. Although much more sophisticated methods for treating microphysical processes exist, it often continues to be the case that the atmosphere can not be measured well enough to justify use of advanced methods. With notable exceptions, only small variations in precipitation and air velocity outcomes are produced by large variations of microphysical processes in the models treated here.

I am pleased to read occasional references to my simple parameterizations, but isn't the elucidation of the kind of kinematical relationships discussed here at least as much an essence of meteorology as the parameterization of microphysics? It seems that many striking kinematical relationships among condensation, precipitation fall speed, updraft speed, changes thereof and precipitation at the ground are hardly ever discussed in the literature or even mentioned. I think that this subject should be more taught at universities, because it helps understanding of connections between wind and rain that play at the heart of life on Earth, and I think that Helmut would agree with this.

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