

## NOTES AND CORRESPONDENCE

**An Improved Method for Computing Horizontal Diffusion in a Sigma-Coordinate Model and Its Application to Simulations over Mountainous Topography**

GÜNTHER ZÄNGL

*Meteorologisches Institut der Universität München, Munich, Germany*

5 July 2001 and 11 October 2001

## ABSTRACT

A set of modifications is presented to reduce the unphysical impact of horizontal diffusion in numerical models with a terrain-following sigma-coordinate system. At model levels sufficiently far away from the ground, vertical interpolation is used to compute diffusion truly horizontally when the coordinate surfaces are sloping. Close to the ground, where truly horizontal computation of diffusion is not everywhere possible without intersecting the topography, a combination of one-sided truly horizontal diffusion and orography-adjusted diffusion along the sigma surfaces is used for most of the variables. The latter means that the diffusion coefficient is reduced strongly when the grid points involved in the computation of horizontal diffusion are located at greatly different heights. For temperature, one-sided horizontal diffusion is not used because it damps the slope wind circulation in an unphysical way. However, a temperature gradient correction is applied to the terrain-following part of the temperature diffusion. Its purpose is to make diffusion neutral with respect to the current vertical temperature gradient. The modifications have been implemented into the Fifth-Generation Pennsylvania State University–National Center for Atmospheric Research Mesoscale Model (MM5). Idealized simulations of the valley wind circulation in the Inn Valley of the Alps are performed to test the modifications. In its original state, the model turns out to be unable to reproduce this valley wind system because of errors related to diffusion along the coordinate surfaces. However, the model captures all essential features of the observed valley wind with the modified diffusion scheme. Both the temporal evolution and the vertical structure of the valley wind are consistent with observations. This result suggests that the model's ability to simulate flow over mountainous topography is greatly improved by use of the modified scheme.

**1. Introduction**

The treatment of horizontal diffusion in models with a terrain-following sigma-coordinate system has been a long-standing problem. Calculating this diffusion along the sigma surfaces may introduce serious errors over mountainous terrain, particularly for atmospheric properties having a strong vertical gradient. Temperature diffusion along sigma surfaces, for example, tends to cool the air in valleys and to heat that above mountains. A schematic illustration of this behavior is given in Fig. 1a. Likewise, diffusion of the water vapor mixing ratio along sigma surfaces tends to dry the air in valleys and to moisten the atmosphere above mountains. Despite these deficiencies, there are still several mesoscale models in which diffusion is computed along the coordinate surfaces, for example, the Fifth-Generation Mesoscale Model (MM5), developed at The Pennsylvania State

University and the National Center for Atmospheric Research (NCAR; Grell et al. 1995) or the new regional weather forecast model of the German Weather Service (the Lokal Modell; Doms and Schättler 1999).

To prevent spurious temperature or moisture tendencies, diffusion should be computed truly horizontally. At sufficient distance from the ground, this can be accomplished easily with vertical interpolation between the coordinate surfaces. Near the ground, however, truly horizontal computation might be impossible without intersecting the ground. In this case, one may either switch back to diffusion along sigma surfaces (e.g., Ballard and Golding 1991) or use one-sided, truly horizontal diffusion (Li and Atkinson 1999). The former method partly retains the diffusional errors mentioned above, particularly in narrow valleys. On the other hand, one-sided diffusion is usually not applicable at all grid points because, at terrain minima, it might intersect the ground, too. Moreover, when applied to temperature, one-sided horizontal diffusion starting at the slope of a valley tends to damp the slope wind circulation. As an alternative, one may apply the full coordinate transformation to the

---

*Corresponding author address:* Günther Zängl, Abteilung für Theoretische Meteorologie, Theresienstraße 37, München D-80333, Germany.  
E-mail: guenther@meteo.physik.uni-muenchen.de

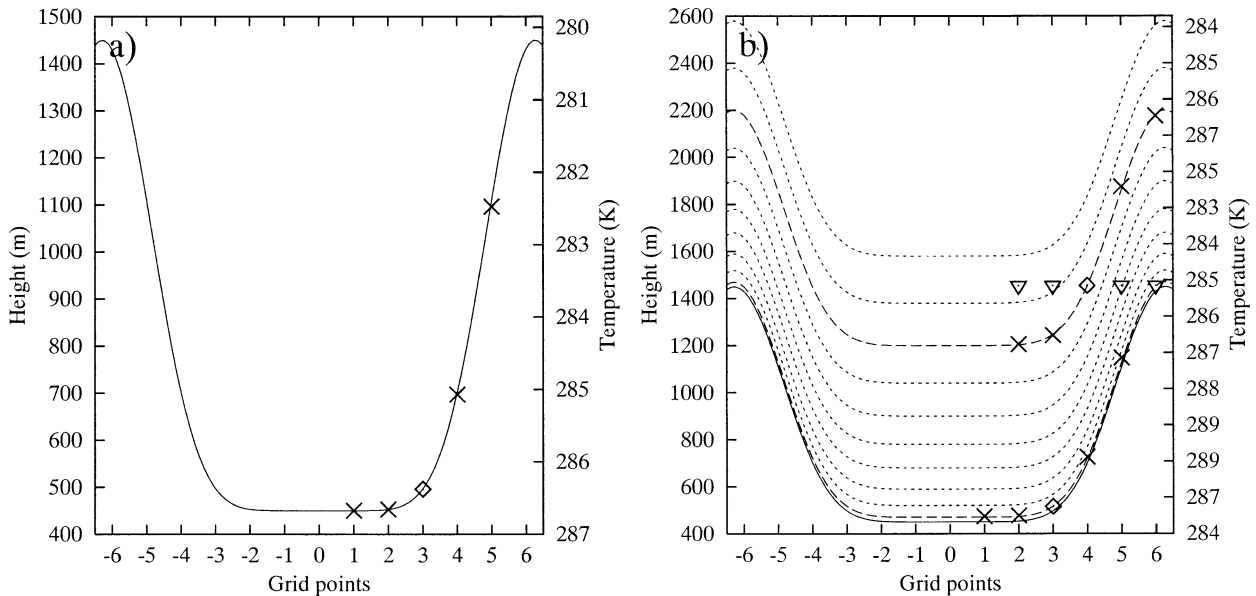


FIG. 1. (a) Schematic illustration of the effect of temperature diffusion along a sigma surface. For the following example, a vertical temperature gradient of  $-6.5 \text{ K km}^{-1}$  is assumed (right ordinate). Fourth-order diffusion (as used in MM5) is discretized as  $D_T = -(K_h/\Delta x^4) [6T_i - 4(T_{i+1} + T_{i-1}) + (T_{i+2} + T_{i-2})]$ , with  $K_h$ ,  $\Delta x$ , and  $i$  being the horizontal diffusion coefficient, the grid distance, and the number of the grid point, respectively. Applying this to gridpoint 3 (marked by the diamond), one obtains  $D_T = -8.7K_h/\Delta x^4$ , i.e., a considerable cooling. For gridpoints 4 (2),  $D_T = -2.1(+0.5)K_h/\Delta x^4$  are found. A long time integration shows that sigma diffusion attempts to establish an isothermal stratification within a valley. (b) To elucidate the possible problems arising from applying the fully transformed diffusion operator, a surface inversion in the valley bottom and an elevated inversion around 2000 m are assumed. When computing the diffusion for gridpoint 3 at the lowermost model level, the contributions from gridpoints 4 and 5 (crosses) are much too warm. A correction of this error by (one sided) vertical derivatives computed at points 4 and 5 is bound to fail because these points are already above the inversion. A similar problem occurs for gridpoint 4 at the eighth model level. Since gridpoints 5 and 6 (crosses) lie within or above the elevated inversion, the correction terms will introduce a large error because they are based on an unrepresentative vertical temperature gradient. A truly horizontal computation, as indicated by the triangles, may be the only way to avoid spurious tendencies irrespective of the thermal stratification.

horizontal diffusion operator, correcting the effect of sloping coordinate surfaces with vertical derivatives. From an analytical point of view, this method is superior to vertical interpolation in that it can be applied everywhere in the same way. However, as illustrated in Fig. 1b, it may be subject to large numerical errors when the height difference between adjacent grid points greatly exceeds the vertical distance between the coordinate surfaces. Moreover, applying the full transformation to fourth-order diffusion (which is often used in mesoscale models) yields a very complicated expression.

This paper describes a new method to minimize diffusion-induced errors in the model levels close to the ground. It is combined with truly horizontal diffusion in the free atmosphere. The new diffusion scheme has been implemented into MM5. Idealized simulations of the valley wind system in the Inn Valley of the European Alps are presented to test the new scheme and to compare it with the existing scheme that computes diffusion along the sigma surfaces. The diffusion scheme is described in section 2. In section 3, the test simulations are presented. Conclusions are drawn in section 4.

## 2. The modified diffusion scheme

Currently, centered fourth-order diffusion along the sigma surfaces (henceforth referred to as sigma diffu-

sion) is implemented in MM5. In analytical form, it reads

$$D_\phi = -K_h \left( \frac{\partial^4 \phi}{\partial x^4} + \frac{\partial^4 \phi}{\partial y^4} \right), \quad (1)$$

$\phi$  being any prognostic variable and  $K_h$  being the diffusion coefficient. Note that the fourth derivative needs a minus sign in order to damp waves. The second-order accurate discretization is

$$\left( \frac{\partial^4 \phi}{\partial x^4} \right)_i \approx \frac{6\phi_i - 4(\phi_{i+1} + \phi_{i-1}) + (\phi_{i+2} + \phi_{i-2})}{\Delta x^4}, \quad (2)$$

$\Delta x$  and  $i$  being the grid increment and the gridpoint index, respectively. To ensure numerical stability,  $K_h$  has a minimum value of  $3 \times 10^{-3} \Delta x^4 / \Delta t$ ,  $\Delta t$  being the time step. It increases as a function of the local deformation of the wind field to parameterize, for example, shear turbulence [see Grell et al. (1995) for further details]. It is pointed out that the minimum diffusion in the model is substantially stronger than atmospheric diffusion under weak wind conditions. Thus, a local reduction of the diffusion coefficient would be acceptable as long as it does not affect numerical stability. Note that vertical diffusion is computed by the boundary layer parameterization in MM5. It is essentially decoupled

from horizontal diffusion and is not considered in this study.

In the modified diffusion scheme, diffusion is computed truly horizontally at all model levels where this is possible without intersecting the lowermost half-sigma surface at any grid point of the current domain. The range of levels fulfilling this condition depends on the steepness of the orography and is computed at the beginning of the integration. In the following, these levels will be referred to as the upper levels. In the example given in Fig. 1b, the eighth level from the bottom is the lower boundary of the upper levels defined in this way. It is mentioned that this distinction is made primarily for reasons of implementation and efficiency. The upper levels are the region in which no other method of computing diffusion is needed.

The vertical interpolation between the coordinate surfaces is done linearly with height (as suggested by the triangles in Fig. 1b) except for the water vapor mixing ratio for which an exponential interpolation is used. At height  $z$ , the mixing ratio  $q$  is set to  $q(h_1) \exp\{(z - h_1)/(h_2 - h_1) \ln[q(h_2)/q(h_1)]\}$ ,  $h_1$  and  $h_2$  being the heights of the adjacent sigma levels at the respective grid point. Linear interpolation of the mixing ratio has been found to yield a systematic upward transport of moisture, which can easily be explained by the fact that the mixing ratio tends to decrease exponentially with height rather than linearly. Sensitivity tests indicated that higher-order interpolation (involving more than the two adjacent sigma levels) does not yield a significant further improvement but is substantially more expensive than linear or exponential interpolation. The diffusion coefficient is kept at its original value in the upper model levels.

In the remaining model levels, diffusion is treated differently for momentum, temperature, and the moisture variables. For momentum, a linear transition from truly horizontal diffusion to sigma diffusion (as currently implemented in MM5) is chosen. Details on this transition are given in the appendix. Since the friction-induced reduction of the wind speed tends to follow the orography, a transition to sigma diffusion is thought to be better justified than a transition to one-sided, truly horizontal diffusion. The latter would imply that near-surface air over a slope is mixed with air farther away from the surface, being less influenced by friction and thus possibly moving significantly faster. Yet, the sensitivity of the model results to the implementation of momentum diffusion is weak so that it does not really matter which of these possibilities is chosen. For many applications, momentum diffusion may even be calculated everywhere along the sigma surfaces without significant impact on the results. The diffusion coefficient is not changed near the ground, because a significant reduction of momentum diffusion yields stability problems.

For the moisture variables (the mixing ratios of water vapor, cloud water, etc.), a combination of one-sided truly horizontal diffusion and orography-adjusted sigma

diffusion (to be described below) is used in the lower levels. The weighting between these types of diffusion and the transition toward the centered truly horizontal diffusion in the upper levels are described in the appendix.

One-sided diffusion is discretized using the “half” of the centered difference scheme for the fourth derivative:

$$\left(\frac{\partial^4 \phi}{\partial x^4}\right)_i \approx \frac{3\phi_i - 4\phi_{i+1} + \phi_{i+2}}{\Delta x^4}. \quad (3)$$

Although a Taylor expansion indicates that this is not a mathematically consistent approximation of the fourth derivative, this scheme has better numerical properties than the “correct” one-sided approximations. Applying the Taylor-expansion-derived one-sided discretizations of the second and fourth derivatives to an alternating sequence of  $+1$  and  $-1$ , one finds that most of them amplify two-grid waves ( $2\Delta x$ -waves), which makes them inappropriate for discretizing diffusion. This is the case for the first-order-, second-order-, third-order-, and fourth-order-accurate approximations of the second derivative and for the second-order-accurate approximation of the fourth derivative. The only exception among the schemes involving not more than six grid points is the first-order-accurate approximation of the fourth derivative:

$$\left(\frac{\partial^4 \phi}{\partial x^4}\right)_i \approx \frac{\phi_i - 4\phi_{i+1} + 6\phi_{i+2} - 4\phi_{i+3} + \phi_{i+4}}{\Delta x^4}. \quad (4)$$

As compared with (3), which also damps two-grid waves, this scheme has the disadvantage of involving two more grid points. Thus, there will be many cases in which (4) intersects the ground while (3) is still applicable. Another argument for (3) arises from efficiency considerations. This will be discussed at the end of this section.

Orography-adjusted sigma diffusion follows the idea that sigma diffusion may be applied along the axis of a valley without inducing spurious tendencies. However, in the direction across the valley axis, sigma diffusion should be switched off. To decide which directions are along and across the valley axis, respectively, the height difference between the local grid point and the neighboring grid points is considered. Wherever this difference is large, the grid points are assumed not to be in the along-valley direction, and the respective diffusion coefficient is strongly reduced. There are many possible ways to implement such a reduction. Based on extensive tests, we have chosen to reduce the diffusion coefficient in the  $i$  direction (denoting one of the horizontal directions) by a factor of  $5(5 + \Delta h_{i,1}^6 + \Delta h_{i,2}^6)^{-1}$ , where

$$\Delta h_{i,1} = \frac{h_{i+2} + h_{i-2} - 4(h_{i+1} + h_{i-1}) + 6h_i}{100 \text{ m}} \quad \text{and} \quad (5)$$

$$\Delta h_{i,2} = \frac{h_i - 0.25(h_{i+1} + h_{i+2} + h_{i-1} + h_{i-2})}{100 \text{ m}}. \quad (6)$$

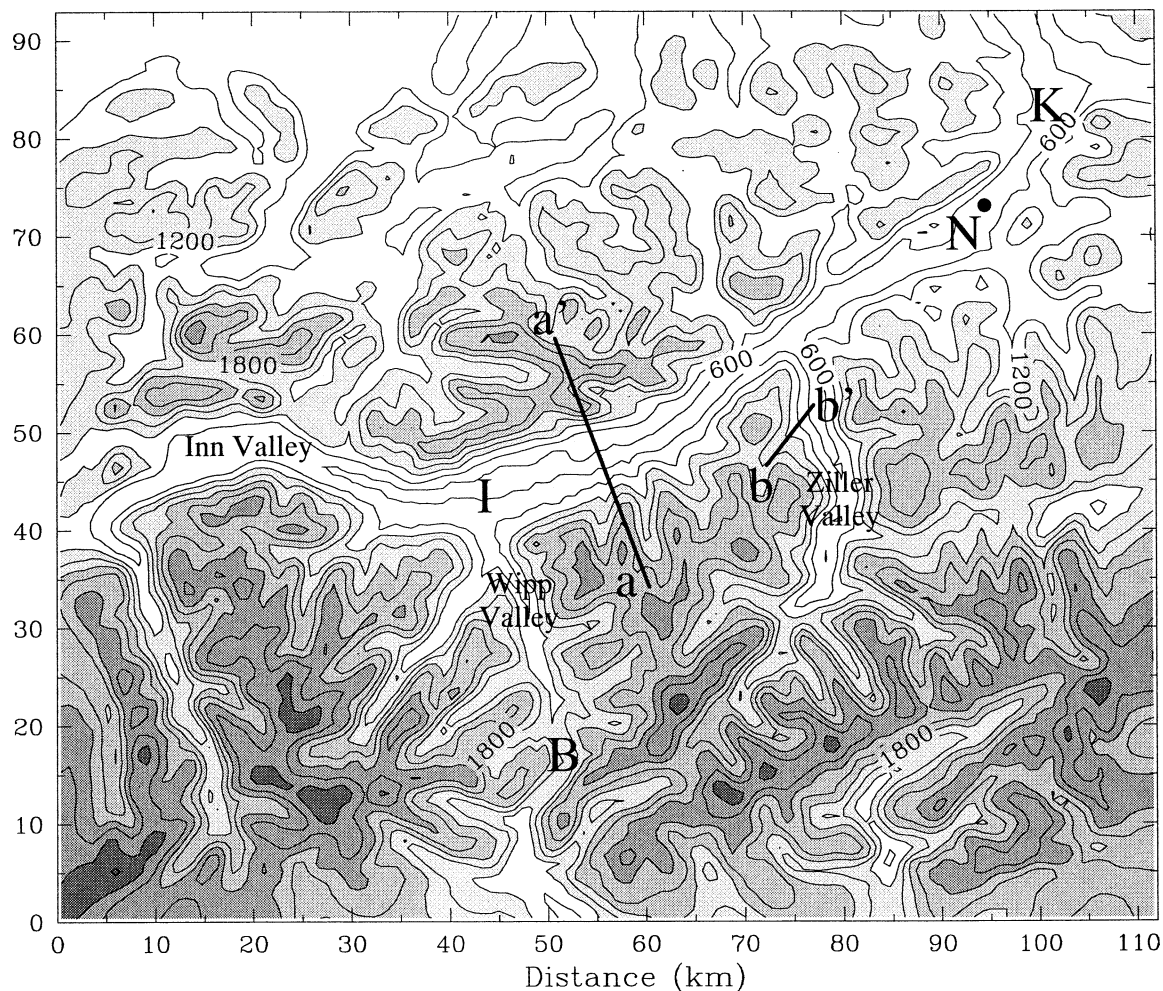


FIG. 2. Orography of the innermost model domain (grid distance 800 m). The contour interval is 300 m; shading steps are every 600 m. The lines  $aa'$  and  $bb'$  denote the positions of the vertical cross sections shown in Figs. 6 and 7, respectively. Locations are Innsbruck (I), Brenner Pass (B), Niederbreitenbach (N), and Kufstein (K). The dot near N marks the grid point from which the vertical profiles shown in Figs. 4 and 5 are taken.

Note that (5) has the same structure as the discretization of the fourth derivative [(2)]. This ensures that sigma diffusion is switched off whenever the shape of the underlying orography is such as to induce a spurious diffusional tendency in the case of a nonzero vertical gradient of the variable under consideration.

This reduction has been found to eliminate the unphysical behavior of sigma diffusion very effectively. Yet, for very complex topography (as, for example, shown in Fig. 2), there may still be a few grid points at which there is essentially no diffusion. Since numerical noise tends to develop at such grid points, a subsidiary second-order sigma diffusion is applied at points at which one-sided truly horizontal diffusion is switched off in all directions and fourth-order sigma diffusion is reduced strongly. The second-order diffusion is reduced in a similar manner as described above when the grid points involved are not at the same height. However, since the discretization of the second deriv-

ative involves only three instead of five grid points, it is easier to apply in narrow valleys.

For temperature, one-sided truly horizontal diffusion is not used because it damps the slope wind circulation in an unphysical way (see Fig. 6 and related discussion). Instead, a transition from centered truly horizontal diffusion to orography-adjusted sigma diffusion is used in the lower model levels. Details on this transition are again given in the appendix. Since temperature proved to be most sensitive to diffusional errors, a temperature gradient correction is applied to the sigma diffusion in addition to orographic adjustment. This correction is based on the fact that, for a given vertical temperature gradient, sigma diffusion induces no errors when the vertical gradient is subtracted from the temperature field before computing the diffusion. Let  $\gamma_v$  be the vertical temperature gradient at the grid point at which diffusion is to be computed. Then, the corrected expression for temperature diffusion along sigma surfaces reads

$$\begin{aligned}
\left(\frac{\partial^4 T}{\partial x^4}\right)_{i,\text{corr}} &= \frac{1}{\Delta x^4} \{6T_i - 4[T_{i+1} - \gamma_i(h_{i+1} - h_i) + T_{i-1} - \gamma_i(h_{i-1} - h_i)] + T_{i+2} - \gamma_i(h_{i+2} - h_i) + T_{i-2} - \gamma_i(h_{i-2} - h_i)\} \\
&= \frac{6(T_i - \gamma_i h_i) - 4(T_{i+1} - \gamma_i h_{i+1} + T_{i-1} - \gamma_i h_{i-1}) + T_{i+2} - \gamma_i h_{i+2} + T_{i-2} - \gamma_i h_{i-2}}{\Delta x^4}.
\end{aligned} \tag{7}$$

It is pointed out that this correction makes sense only in combination with orographic adjustment. The latter ensures that the heights of the grid points involved in the computation of diffusion differ only slightly from each other, so that the vertical temperature gradient  $\gamma_i$  can be regarded as representative. Otherwise, the same problem as with the full coordinate transformation would occur (see the introduction and Fig. 1b). Here,  $\gamma_i$  is not allowed to be superadiabatic or to exceed  $+30 \text{ K km}^{-1}$  to preclude unphysical positive feedback effects. The correction is applied to the standard fourth-order diffusion and to the subsidiary second-order diffusion. Sensitivity tests showed that it improves the model's capability of reproducing the nocturnal ground inversion.

Last, a few aspects of computational efficiency are considered. Not surprising, the vertical interpolation necessary for truly horizontal diffusion brings about a substantial increase of computing costs. This increase is particularly true for MM5 because no invertible relation between the index of a model level and the corresponding sigma value is available. Provided that sufficient memory is available, a significant part of these additional costs can be saved by computing the coefficients needed for vertical interpolation only once at the beginning of the simulation and storing them. For linear interpolation, eight three-dimensional fields are required for each model domain since the fourth-order diffusion operator involves eight remote grid points. In these fields, the index of one of the adjacent model levels plus the weighting coefficient for the vertical interpolation can be stored. However, different interpolation coefficients are required for each grid staggering level. For the Arakawa-B grid used in MM5, this makes a total of 24 three-dimensional fields for horizontal wind, vertical wind, and the remaining variables. Since the errors induced by computing momentum diffusion along the sigma surfaces are comparatively small, it may be an acceptable compromise to restrict truly horizontal diffusion to temperature and the moisture variables. Note that the scheme used for one-sided diffusion [(3)] does not require any further interpolation coefficients. However, using (4) instead of (3) would double the additional memory requirement.

### 3. Application of the modified diffusion scheme

Idealized numerical simulations of the valley wind system of the Inn Valley have been performed to test the modified diffusion scheme. Valley wind simulations

are particularly appropriate for such an assessment because the possible spurious impact of diffusion is strongest when advection is weak. The improvements achieved with the modifications described above become clearest under such conditions. The Inn Valley has been chosen because a large number of detailed measurements are available for this valley (e.g., Pamperin and Stilke 1985; Vergeiner and Dreiseitl 1987). Since these measurements show many characteristic features recurring every fine weather day, the simulations could be based on a highly idealized setup, with only orography being taken from the data.

#### a. Setup of the simulations

The simulations presented in this study have been performed with MM5, version 3.3 (Grell et al. 1995). Five interactively nested model domains are used, the finest horizontal spacing being 800 m. The orography is interpolated from high-resolution ( $30''$ ) data available from the U.S. Geological Survey. The model orography of the innermost domain is displayed in Fig. 2, showing the lower Inn Valley together with its tributaries. In the vertical, 39 sigma levels are used, corresponding to 38 half-sigma levels at which most of the variables are computed. The lowermost half-sigma level is located at about 18 m above ground. It will be referred to as surface level in the remainder of this paper. The vertical spacing is about 50 m close to the ground and decreases up to 800 m near the upper boundary, which is chosen to be at 100 hPa.

To keep the setup as simple as possible, the absence of any horizontal pressure and temperature gradients is assumed. This assumption corresponds to vanishing large-scale winds. For consistency, the meridional gradient of solar radiation is disregarded. A constant latitude of  $47.5^\circ\text{N}$  (close to Niederbreitenbach; see Fig. 2) is used for the computation of radiation. The simulation starts at a fictitious date of 15 October, at 0000 UTC (the date being relevant for radiation only) and is carried out for 30 h. Temperature and moisture are specified according to what is typical for mid-October in the presence of continental air. At 1000 hPa, a temperature of  $10^\circ\text{C}$  is assumed. The vertical temperature gradient is set to  $-3.5 \text{ K km}^{-1}$  below 850 hPa and to  $-6.5 \text{ K km}^{-1}$  between 850 hPa and the tropopause, which is taken to be at 225 hPa. Farther above this level, an isothermal stratosphere is prescribed. Relative humidity is set to 50% below 600 hPa and then gradually decreases to

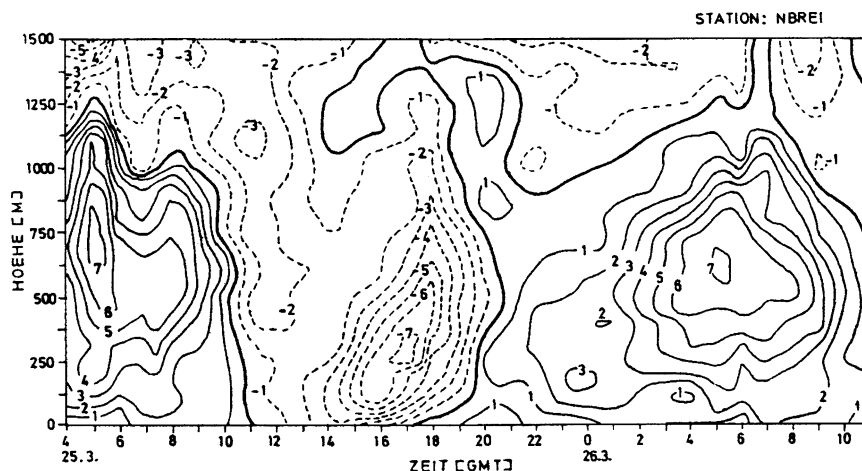


FIG. 3. Time–height (Zeit–Höhe) section of the valley wind measured at Niederbreitenbach on 25–26 Mar 1982, adapted from Pamperin and Stilke (1985). Negative wind speeds (dashed lines) denote upvalley winds; positive wind speeds (solid lines) denote downvalley wind. Contour interval is  $1 \text{ m s}^{-1}$ .

20% at the tropopause. Above the tropopause, a value of 5% is assumed.

Because of the idealized model setup, relatively simple physics parameterizations are sufficient. Moisture is included to get realistic surface energy fluxes, but sophisticated precipitation physics are not needed since the initial conditions preclude any formation of rain. Moreover, no convection scheme is required. The planetary boundary layer (PBL) is parameterized using the Blackadar scheme (Zhang and Anthes 1982). More sophisticated schemes based on turbulent kinetic energy have also been tested without finding significant differences. The radiation scheme includes the effects of moisture and clouds (Grell et al. 1995). It has been modified by the author to account for sloping orography and shading (Garnier and Ohmura 1968). At the upper model boundary, a rigid-lid condition is sufficient because the basic state chosen for the simulations does not support stationary vertically propagating gravity waves.

Four simulations based on the same initial conditions are presented. Two of them use existing model options. First, actual temperature  $T$  is used for computing the diffusion along the sigma levels. Since sigma diffusion tries to equalize temperature differences on a model surface, this option tends to cool valleys and to heat mountains (see also Fig. 1a). In the second simulation, the so-called perturbation temperature  $T'$  is used for computing the diffusion;  $T'$  is the difference between the actual temperature and a reference temperature profile that usually has a vertical gradient around  $-6 \text{ K km}^{-1}$  in the lower troposphere. Thus, the second option (which is recommended by NCAR) tries to establish a vertical temperature gradient of  $-6 \text{ K km}^{-1}$  within valleys. As a consequence, valleys tend to be warmer than with the first option. In the following, these two versions of sigma diffusion will be referred to as  $T$  diffusion and  $T'$  diffusion, respectively. In the third simulation, the

modified scheme described in section 2 is used. Last, a sensitivity experiment is performed in which temperature diffusion is treated in the same way as moisture diffusion. It highlights the importance of treating temperature diffusion in a special way.

#### b. The valley wind system in the Inn Valley

Because the lower Inn Valley has little slope, its valley wind system primarily arises from the volume effect (Steinacker 1984). Because of the presence of mountains, the air volume to be heated or cooled by radiation and turbulent transports is lower within the valley than over the plain. Thus, the diurnal temperature range is enhanced in the valley, leading to relatively low surface pressure in the afternoon and to relatively high pressure in the second half of the night. As a consequence, upvalley (downvalley) wind is observed in the afternoon (night). As is typical for large valleys, the valley wind circulation is delayed by several hours with respect to the diurnal cycle of solar radiation. An interesting feature of the Inn Valley's wind system is a nocturnal low-level jet in the valley exit region (to the north of Kufstein; see Fig. 2). Its wind maximum typically occurs 200–300 m above ground and can be as large as  $15 \text{ m s}^{-1}$  (Pamperin and Stilke 1985).

A time–height section of the valley wind speed typical for the interior of the valley is shown in Fig. 3 (Pamperin and Stilke 1985). Measurements were taken on 25–26 March 1982 at Niederbreitenbach (see Fig. 2 for location). Maximum wind speeds are around  $7 \text{ m s}^{-1}$  for both the upvalley and the downvalley wind. In contrast to what is found near the valley exit, the downvalley wind maximum is located at a substantially greater height than the upvalley wind maximum. In correspondence, the surface wind speed is larger in the afternoon

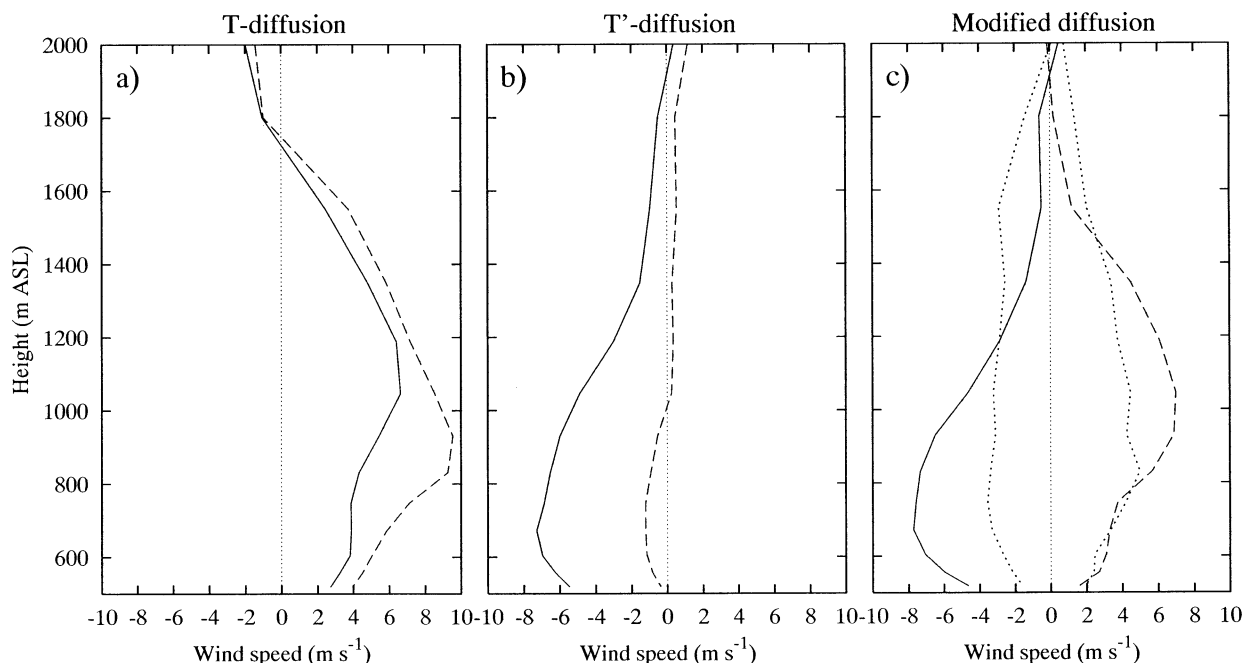


FIG. 4. Simulated wind profiles at Niederbreitenbach (see Fig. 2 for the location of the grid point) for  $t = 16$  (solid lines) and 30 h (dashed lines). As in Fig. 3, positive (negative) values denote downvalley (upvalley) wind. Simulations are performed with (a)  $T$  diffusion, (b)  $T'$  diffusion and (c) modified diffusion. In (c), dotted lines refer to the sensitivity experiment with one-sided horizontal temperature diffusion.

than during the night when stable stratification reduces turbulent mixing.

### c. Results of the simulations

Simulated vertical wind profiles at Niederbreitenbach are displayed in Fig. 4 for  $t = 16$  (solid lines) and 30 h (dashed lines). Figures 4a, 4b, and 4c refer to  $T$  diffusion,  $T'$  diffusion, and modified diffusion, respectively. In Fig. 4c, the results for the sensitivity experiment without special treatment of temperature diffusion are added with dotted lines. As in Fig. 3, negative (positive) values denote upvalley (downvalley) wind.

It is obvious that the model fails to reproduce the observed valley wind circulation with the original sigma-diffusion scheme. With  $T$  diffusion, downvalley wind is simulated throughout the day. It is only slightly weaker in the afternoon than during the night. On the other hand, permanent upvalley wind is obtained with  $T'$  diffusion. It is weak at the end of the night and of realistic strength in the afternoon. In brief, the variable used for computing the temperature diffusion determines the direction of the valley wind.

With the modified diffusion scheme, the simulated valley wind circulation is remarkably close to reality. The wind maxima of both the upvalley wind and the downvalley wind are within  $1 \text{ m s}^{-1}$  of the observed values. Moreover, the day–night asymmetry of the surface wind speed and of the height of the wind maximum is captured well by the simulation. At the surface, the simulated wind speed is somewhat larger than observed

because it refers to 18 m above ground. It is noted that the low-level jet in the exit region of the Inn Valley is also reproduced well by the simulation (not shown). It is probably related to a transition from subcritical to supercritical flow. A separate study will be devoted to this phenomenon. The sensitivity experiment with one-sided horizontal temperature diffusion (dotted lines) shows too weak winds. Yet, the direction of the valley wind remains correct.

To explain the differences between sigma diffusion and the modified diffusion scheme, the vertical temperature profiles at Niederbreitenbach are considered (Fig. 5). With the modified scheme (solid lines), a realistic diurnal temperature range is obtained. In particular, enhanced nocturnal cooling is evident up to a height of 1000 m above ground. Such a nocturnal temperature profile is typical for the Inn Valley (e.g., Verginer and Dreiseitl 1987). It is a consequence of the slope wind circulation that is associated with weak upward motion over the interior of the valley, leading to adiabatic cooling. Over the plain, nocturnal cooling is usually confined to a shallower layer above the ground. This difference in the depth of nocturnal cooling is the primary reason for the development of nocturnal downvalley winds since a cooler air column above the valley is related to higher surface pressure.

With sigma diffusion, however, a realistic simulation of the diurnal temperature range is impossible. In both cases, the day–night difference of the surface temperature is several times too weak. In comparing the profiles for  $T$  diffusion (dashed lines) and  $T'$  diffusion

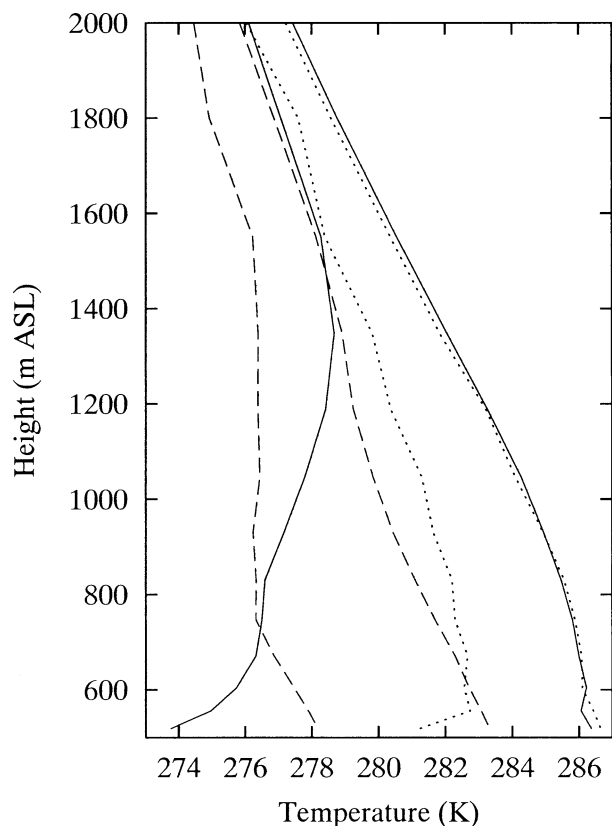


FIG. 5. Simulated temperature profiles at Niederbreitenbach (see Fig. 2 for location) for modified diffusion (solid lines),  $T$  diffusion (dashed lines), and  $T'$  diffusion (dotted lines). In each case, the colder (warmer) profile refers to  $t = 30$  h ( $t = 16$  h).

(dotted lines), it becomes clear why the direction of the valley wind is opposite for these two options. With  $T$  diffusion, the whole valley atmosphere is several degrees colder than with  $T'$  diffusion. This results from the fact mentioned above that sigma diffusion attempts to establish a certain vertical temperature gradient within a valley.

Using one-sided, truly horizontal temperature diffusion has no direct impact on the vertical temperature gradient in a valley. However, this type of diffusion damps the development of a slope wind circulation because it mixes the radiatively heated (or cooled) air above the slope with the cooler (warmer) air in the free valley atmosphere. As a consequence, the diurnal temperature range in the Inn Valley has a lower amplitude than with the standard modified diffusion scheme (not shown). To demonstrate the effect of one-sided diffusion on slope winds, two vertical cross sections are shown in Fig. 6. They are taken at 2000 UTC and run along a steeply sloping tributary of the Ziller Valley that is not fully resolved in the model orography (see Fig. 2). A pronounced downslope flow is obtained with the standard modified diffusion (Fig. 6a), whereas only weak indications of a slope wind are present with one-sided horizontal diffusion (Fig. 6b). Although one-sided horizontal diffusion probably is appropriate for moisture (as suggested by more realistic simulations not presented here), it is not recommended for temperature.

A peculiar effect of sigma diffusion on the free atmosphere is noted. Figure 7 displays a vertical cross section across the Inn Valley to the east of Innsbruck (see Fig. 2) at  $t = 16$  h. With modified diffusion (Fig. 7a), the isentropes above crest height are almost per-

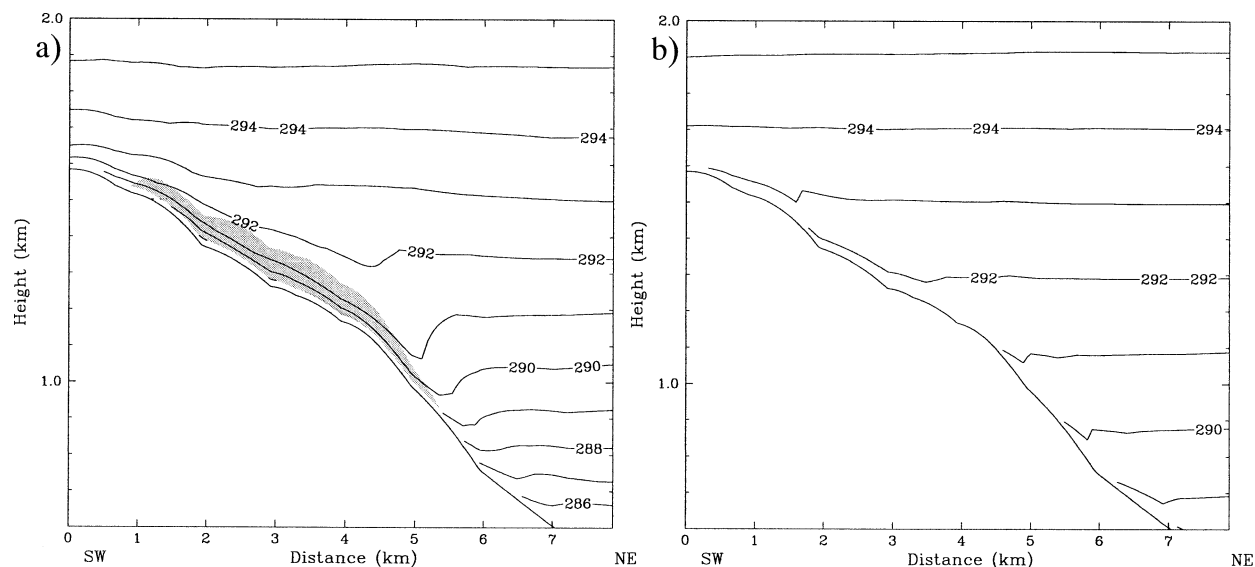


FIG. 6. Vertical cross section of potential temperature along the line  $aa'$  given in Fig. 2:  $t = 20$  h. Contour interval is 1 K; shading denotes downslope wind speeds above  $2 \text{ m s}^{-1}$ . (a) Simulation with modified diffusion; (b) sensitivity experiment with one-sided horizontal temperature diffusion.

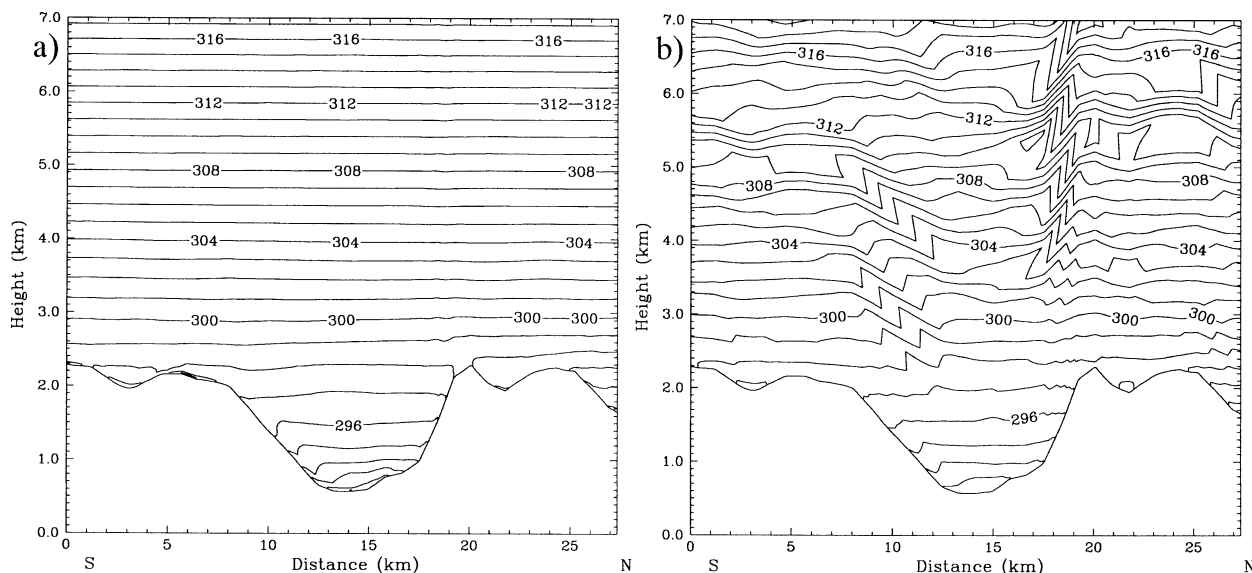


FIG. 7. Vertical cross section of potential temperature along the line  $bb'$  given in Fig. 2:  $t = 16$  h. Contour interval is 1 K. Simulations with (a) modified diffusion and (b)  $T'$  diffusion.

fectly horizontal, as might be expected in the absence of large-scale winds. However, with  $T'$  diffusion, a strange, noisy pattern appears that is reminiscent of breaking gravity waves (Fig. 7b). A similar pattern is obtained with  $T$  diffusion (not shown). It can be explained by the fact that sigma diffusion induces horizontal temperature gradients over mountainous topography. As a result, pressure gradients occur and gravity waves are excited. Since sigma diffusion provides a stationary forcing and no large-scale winds are present, the gravity waves encounter a critical level and break. Although the valley wind profile obtained with  $T'$  diffusion in the afternoon is reasonable (Fig. 4b), the simulation as a whole is by no means realistic.

#### 4. Summary

In this study, a set of modifications has been presented to reduce the unphysical impact of horizontal diffusion in numerical models with a terrain-following sigma-coordinate system. Using vertical interpolation, diffusion is computed truly horizontally at all model levels for which this is possible without intersecting the topography. Close to the ground, diffusion is treated differently for momentum, moisture, and temperature. A simple transition to diffusion along the coordinate surfaces (sigma diffusion) is chosen for momentum. For the moisture variables, one-sided truly horizontal diffusion is used in combination with orography-adjusted sigma diffusion. The latter means that the diffusion coefficient is strongly reduced when the grid points involved in the computation of diffusion are located at greatly different heights. For temperature, one-sided horizontal diffusion is not used because it affects the slope wind circulation in an unphysical way. However, a temperature gradient

correction is applied to the terrain-following part of diffusion so as to ensure that diffusion behaves neutrally with respect to the local vertical temperature gradient. The modifications have been implemented into the mesoscale model known as MM5.

Idealized simulations of the valley wind circulation in the Inn Valley of the Alps have been performed to validate the modified diffusion scheme. It has been found that the model is not able to reproduce this valley wind circulation with the original sigma-diffusion scheme. However, the model captures all essential features of the valley wind with the modified diffusion scheme. Both the temporal evolution and the vertical structure of the valley wind are consistent with observations. This result indicates that unphysical impacts arising from numerical diffusion have been eliminated successfully and that the relevant physical processes are represented properly in the model.

*Acknowledgments.* I am grateful to Joe Egger and the anonymous reviewers for their helpful comments, which led to a significant improvement of the presentation.

#### APPENDIX

##### Transition between Upper-Level and Low-Level Diffusion

In the lower model levels, where centered truly horizontal diffusion is not applicable at all grid points, a transition toward other methods of computing diffusion is needed. In the following,  $k$  denotes the index of the model level, counted from top to bottom as is the convention in MM5. Moreover,  $k_d(k_l)$  denotes the lowermost model level at which centered truly horizontal diffusion can be com-

puted at all grid points of the model domain (at the local grid point). Here  $k_d$  constitutes the lower boundary of the upper levels as defined in section 2;  $k_l$  usually depends on the direction ( $x$  or  $y$ ) and is larger than  $k_d$  at most of the grid points because the steepest slope occurring in a model domain determines  $k_d$ .

For momentum and temperature, centered truly horizontal diffusion is computed wherever possible. In addition, sigma diffusion is computed for  $k$  greater than  $k_d$ , with orographic adjustment being restricted to temperature. Then, at each grid point  $(i, j, k)$ , the following weighting is applied:

$$D_\phi(i, j, k) = \lambda(i, j, k)D_{\phi,\sigma}(i, j, k) + [1 - \lambda(i, j, k)]D_{\phi,h}(i, j, k),$$

$$\lambda(i, j, k) = \begin{cases} \frac{h(i, j, k_d) - h(i, j, k)}{h(i, j, k_d) - h[i, j, k_l(i, j) + 1]}, & k \leq k_l(i, j) \\ 1, & k > k_l(i, j), \end{cases} \quad (\text{A1})$$

with  $D_{\phi,\sigma}$  and  $D_{\phi,h}$  denoting sigma diffusion and truly horizontal diffusion, respectively. The weighting is done for each horizontal direction separately since  $k_l(i, j)$  usually depends on the direction.

For moisture, let  $k_{l+}$  and  $k_{l-}$  denote the lowermost model level at which one-sided truly horizontal diffusion extending to higher and lower indices, respectively, can be computed. As before,  $k_{l+}$  and  $k_{l-}$  depend on the direction, and the weighting is taken to be

$$D_\phi(i, j, k) = \frac{\lambda_+(i, j, k) + \lambda_-(i, j, k)}{2} D_{\phi,\sigma}(i, j, k) + [1 - \lambda_+(i, j, k)]D_{\phi,h+}(i, j, k) + [1 - \lambda_-(i, j, k)]D_{\phi,h-}(i, j, k),$$

$$\lambda_\pm(i, j, k) = \begin{cases} \frac{1}{2} \frac{h(i, j, k_d) - h(i, j, k)}{h(i, j, k_d) - h(i, j, k_m)}, & k \leq k_{l\pm}(i, j) \\ 1, & k > k_{l\pm}(i, j), \end{cases} \quad (\text{A2})$$

with  $k_m$  being the number of vertical levels. Note that the truly horizontal part of diffusion has more weight than for the variables not using one-sided diffusion. Because of the discretization chosen for one-sided diffusion [(3)], (A2) implies that centered truly horizontal diffusion is used wherever possible.

#### REFERENCES

- Ballard, S. P., and B. W. Golding, 1991: Basic model formulation. Short range forecasting research. Mesoscale Documentation Paper 4. Met Office, Bracknell, UK, 42 pp.
- Doms, G., and U. Schättler, 1999: The nonhydrostatic limited-area model LM (Lokal-Modell) of DWD. Part I: Scientific documentation. Deutscher Wetterdienst, Offenbach, Germany, 169 pp.
- Garnier, B. J., and A. Ohmura, 1968: A method of calculating the direct shortwave radiation income of slopes. *J. Appl. Meteor.*, **7**, 796–800.
- Grell, G. A., J. Dudhia, and D. R. Stauffer, 1995: A description of the Fifth-Generation Penn State/NCAR Mesoscale Model (MM5). Tech. Note NCAR/TN-398+STR, National Center for Atmospheric Research, Boulder, CO, 122 pp.
- Li, J.-G., and B. W. Atkinson, 1999: Transition regimes in valley airflows. *Bound.-Layer Meteor.*, **91**, 385–411.
- Pamperin, H., and G. Stille, 1985: Nächtliche Grenzschicht und LLJ im Alpenvorland nahe dem Inntalausgang (Nocturnal boundary layer and low-level jet in the Alpine foreland close to the exit of the Inn Valley). *Meteor. Rundsch.*, **38**, 145–156.
- Steinacker, R., 1984: Area–height distribution of a valley and its relation to the valley wind. *Contrib. Atmos. Phys.*, **57**, 64–71.
- Vergeiner, I., and E. Dreiseitl, 1987: Valley winds and slope winds—Observations and elementary thoughts. *Meteor. Atmos. Phys.*, **36**, 264–286.
- Zhang, D.-L., and R. A. Anthes, 1982: A high-resolution model of the planetary boundary layer—Sensitivity tests and comparisons with SESAME-79 data. *J. Appl. Meteor.*, **21**, 1594–1609.