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Flux-gradient relationships in the constant flux layer

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SUMMARY

An analysis is made of the Monin-Obukhov function Φ_M in the familiar wind profile equation, using data from two recent expeditions to Gurley (New South Wales) and Hay (New South Wales). In one, the friction velocity u_* is determined directly by the eddy correlation method, and in the other, conducted during mid-winter when small heat-fluxes were experienced, by the use of a friction coefficient applied to a low-level wind.

By collating with a similar earlier analysis for heat and water vapour transfer, the variations of Φ_M, Φ_H and Φ_W with stability are presented in tabular form in the z/L range $-0{\cdot}01$ to $-1{\cdot}0$. Within this range the empirical relationships $\Phi_M = (1 - 16\,z/L)^{-\frac{1}{4}}$ and $\Phi_{H,W} = (1 - 16\,z/L)^{-\frac{1}{2}}$, and the implied equality between Ri and z/L, are found to approximate the data to within a few per cent.

1. INTRODUCTION

For some time the relationship between fluxes and gradients in the constant-flux layer under various degrees of instability has been the subject of much enquiry. Following a series of micro-meteorological expeditions (Swinbank and Dyer 1967), the flux-gradient relationships for heat and water vapour, expressed in the form of the Monin-Obukhov universal functions Φ_H and Φ_W as a function of $-z/L$, were satisfactorily specified (Dyer 1967). Φ_H was found to be equal to Φ_W, implying that $K_H = K_W$ over a wide range of $-z/L$.

The corresponding discussion for momentum, in terms of Φ_M, was not presented at that time because of uncertainty regarding the correct values for the friction velocity u_*, which enters the momentum analysis in a more crucial way. This difficulty arose because, unlike the fluxes of sensible heat and water vapour, the eddy-flux of momentum was not measured directly but had to be inferred from a low-level wind by the use of a friction coefficient C_f.

In the absence of a significant body of neutral runs, some uncertainty attaches to such estimates of C_f, since the extrapolation from the non-neutral case frequently involves a knowledge of the wind-profile form itself. Furthermore, the possible variation of C_f with stability becomes an important question. C_f may also vary with wind-speed through a Reynolds Number effect discussed by Deacon (1957).

The purpose of this paper is to present an analysis for Φ_M based on new data, in which these difficulties are very largely overcome. Material from two expeditions is used.

One set of data is taken from the micro-meteorological component of Project Wangara (Clarke, private communication), an experiment conducted at Hay (New South Wales) during July and August, 1967, aimed at investigating the transfer of momentum through the first 1 to 2 kilometres of the atmosphere. Here, the u_* values were again determined by the friction coefficient method but, this being a mid-winter expedition, large heat-fluxes were not encountered. In addition a total of 43 neutral runs were obtained, thus enabling C_f to be well determined, including the evaluation of a slight variation of C_f with wind-speed.

The second experiment was conducted at Gurley (New South Wales) during March 1970. In this case u_* values were obtained from direct measurements of the momentum flux by the eddy-correlation technique.

2. Sites and instrumentation

The site at Hay (New South Wales) used for Project Wangara was similar to other sites used in this area and described previously (Swinbank and Dyer 1967).

The site at Gurley (New South Wales) was a 20,000 acre wheat farm. After the wheat is harvested, it is general practice in this area to plough the land, leaving a uniformly rough surface of soil and stubble. The experiment was timed to take place immediately after the ploughing had been completed. Heavy rain occurred mid-way through the expedition (35 mm on the evening of 16 March) causing a dramatic increase in the water vapour flux on subsequent days. An uninterrupted fetch of 1 mile of ploughed land was available to the north, with much larger fetches to the east and west. To the south there was a fetch of ½ mile within the ploughed paddock, with flat and treeless terrain for a further mile.

The instrumentation for both expeditions followed the pattern of earlier work (Swinbank and Dyer 1967). All runs were of approximately 30 min duration. Mean wind speeds were recorded at heights of 1, 2, 4, 8 and 16 metres. An additional anemometer was placed some distance from the main mast at a nominal height of 50 cm. Temperature differences were measured between 1 and 2 m height (1 to 2 m and 2 to 4 m at Wangara). Standard measurements were made of net radiation and ground heat-flux.

Eddy flux measurements of sensible heat were made at a height of 4 metres, using an improved version of the Fluxatron (Hicks 1971). In addition, for the Gurley experiment, similar measurements were made of the momentum flux, and a few measurements of the water-vapour flux using a new type of water-vapour sensor (Hicks and Goodman 1970).

The complete data for Gurley is given in the Appendix. The Wangara data is to be published separately.

3. Φ_M-analysis

Values Φ_M are calculated for each successive height interval from the familiar expression,

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz} \Phi_M (z/L) \qquad (1)$$

making the assumption that $\partial u/\partial z$ at the geometric mean height is given by the finite difference form $(\Delta u/\Delta z)/(\sqrt{2} \ln 2)$. For the two-fold height intervals used, this form is exact for a neutral logarithmic profile, and with increasing instability an error in $\partial u/\partial z$ of less than 0·5 per cent might be expected. The ½ to 1 metre data are not used in this analysis, since the precise height of the ½ metre anemometer relative to the main mast is not known. A value of 0·41 is used for k.

The Monin-Obukhov length L is calculated from the relation

$$L = \frac{-\rho C_p \theta u_*^3}{gk(H + 0{\cdot}070 E)} \qquad (2)$$

when the latent heat-flux E is inferred from an energy balance.

The Φ_M data thus computed are grouped into convenient ranges of z/L and plotted in Fig. 1 as geometric mean values. For the Φ_M analysis, 52 runs are available from the Gurley data, and 44 from Wangara.

4. The accuracy of H and u_* measurements

Before discussing the Φ_M results in detail, it is pertinent to consider the accuracy with which H and u_* are determined by the eddy-correlation method.

In the Fluxatron technique, long time-constant filters are imposed in each channel to remove the mean values as well as long-period eddies which do not contribute to the net flux-transport process. Originally the time-constant of these filters was set, somewhat arbitrarily, at 40 s (Dyer, Hicks and King 1967). Recently, to ensure a satisfactory response in light winds, a value of 160 s was selected. It should be noted that 97·5 per cent of the flux carried by eddies of 160 s period is thereby recorded. On the basis of spectral measurements by Panofsky and Mares (1968), the low-frequency loss ranges from 1·4 per cent at 4 m s^{-1} wind speed to 3·4 per cent at 1 m s^{-1}.

At the high frequency end, the limit is set by the response time of the propeller anemometers measuring u and w, which have a nominal distance constant of about 0·6 m. It is estimated, again from Panofsky and Mares (1968), that this would cause a loss of 4 per cent at all wind-speeds.

On the basis of frequency response, therefore, we might expect the covariances $\overline{w'u'}$ and $\overline{w'T'}$ to be underestimated by 4 to 7 per cent, according to wind speed.

For those runs where an independent measurement of the water vapour flux is available, an overall energy balance $(H + E)/(R - G)$, of 1·00 (standard error 0·04) was obtained.

5. Discussion

The heavy line of Fig. 1 represents a line of best fit, drawn by eye, to the experimental points. The data beyond $|z/L| = 1$ have been ignored for two reasons. There has been some evidence (Webb 1964) that a new régime of turbulence begins at or a little below $|z/L| = 1$. There is therefore no reason to expect that the same smooth curve will necessarily extrapolate into the region beyond : also the scatter of the data in this region is too great to establish an independent curve with confidence.

The dotted line of Fig. 1 is deduced from the shape function analysis of earlier data (Swinbank and Dyer 1967) assuming that, at $|z/L| = 0{\cdot}001$, $\Phi_H - \Phi_M = 1{\cdot}0$. This analysis has the feature that it relies solely on profile observations, and thus the accuracy of values of H and u_* does not affect it.

It is encouraging that the two quite different approaches agree to within a few per cent. Similar agreement was found previously for the corresponding Φ_H and Φ_W analysis (Dyer 1967). The earlier determination of Φ_H is included in Fig. 1.

The experimental findings for Φ_M are summarized numerically in Table 1, together with the earlier results for Φ_H and Φ_W. The latter were recalculated using $k = 0{\cdot}41$ rather than 0·40 as used previously.

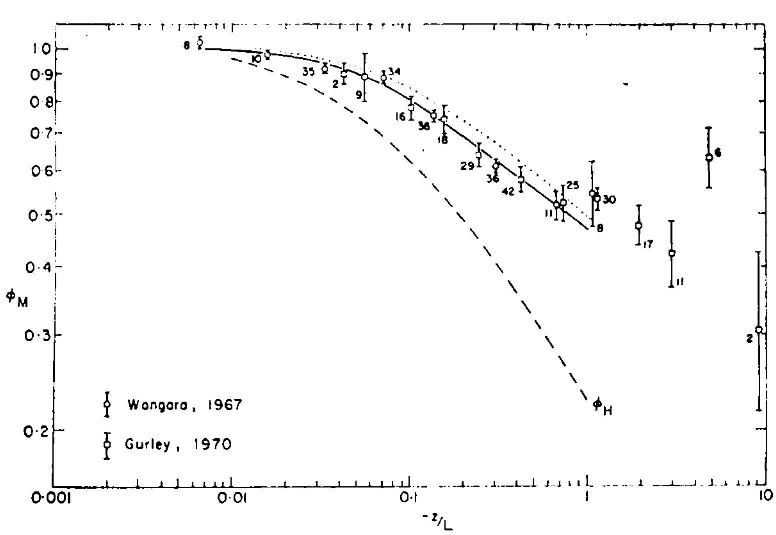

Figure 1. Geometric mean values of Φ_M as a function of $-z/L$ for the two experiments. The dotted line represents Φ_M determined from shape function analysis (Swinbank and Dyer 1967) and the dashed line is the earlier determination of Φ_H (Dyer 1967).

The data of Table 1 lead to a comparison of Ri with z/L, since, from the definitions of the various quantities

$$\mathrm{Ri} = z/L \cdot \frac{\Phi_H}{\Phi_M{}^2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It is seen that Ri and z/L are approximately equal, the largest difference being about 6 per cent at $z/L = -0{\cdot}05$. This approximate equality has already been foreshadowed in comments by Pandolfo (1966), and Businger (1966). The basic implications are not immediately obvious, but warrant further careful consideration.

TABLE 1. Experimentally determined values of Φ_M, Φ_H and Φ_W as a function of z/L

$-z/L$	Φ_M	Φ_H, Φ_W	$\Phi_H/\Phi_M{}^2$	ψ_M	ψ_H, ψ_W
0·01	0·99	0·95	0·97	0·009	0·047
0·02	0·97	0·89	0·95	0·024	0·098
0·05	0·90	0·76	0·94	0·082	0·252
0·1	0·81	0·62	0·95	0·185	0·465
0·2	0·69	0·48	1·02	0·354	0·771
0·5	0·55	0·32	1·05	0·707	1·324
1·0	0·47	0·23	1·03	1·032	1·819

Included also in Table 1 are the numerical values for the integrated forms ψ_M, ψ_H and ψ_W, first suggested by Panofsky (1963), where

$$\psi = \int_0^{z/L} \frac{(1 - \Phi)}{z/L} \, d\,(z/L)$$

The ψ-functions are useful in practical applications where finite differences are measured.

6. Comparison with other workers

It is generally accepted that dimensionless scaling according to z/L is valid, as demonstrated recently by Swinbank (1968) and as implied in the present analysis. This appears to provide a satisfactory physical basis of interpretation up to a limit in the region of $|z/L| = 1$.

A number of flux-gradient relationships have appeared in the literature. Some are entirely empirical, whilst others have some physical content. Frequently, an assumption is made regarding the transfer coefficients K_H and K_M. It is difficult to decide which have the greater merit in physical understanding and it is partly for this reason that a purely numerical expression of the experimental findings has been used in this paper.

A direct comparison between the results of this work and previous experimental or theoretical treatments is beset with a number of difficulties. Swinbank (1964, 1968) has presented two analyses of earlier data but, as his u_* values were not determined directly, such a comparison would be inconclusive. The KEYPS profile (Lumley and Panofsky 1964) contains K_H/K_M in an unspecified form, leading to a similar uncertainty.

Businger (1966) has suggested the empirical forms $\Phi_M = (1 - 16\,z/L)^{-\frac{1}{4}}$ and Φ_H, $\Phi_W = (1 - 16\,z/L)^{-\frac{1}{2}}$. A comparison of these expressions with the numerical values of Table 1 reveals departures of a few per cent at various parts of the z/L range considered. In some contexts these differences may be relatively trivial and the above formulae are then a convenient expression of the present result. Most workers will be aware of the danger of assuming that these formulae have the correct asymptotic form at very high instabilities, i.e. beyond the range of the present measurements. The applicability of the z/L framework has already been questioned in this regard.

A recent profile analysis (Webb 1970) argues that K_H/K_M is equal to unity from neutral to Ri = $-$ 0·07, but this conclusion is based on his Fig. 9; the present results, noting that $K_H/K_M = \Phi_M/\Phi_H$, would fit his diagram equally well. In our case we find that $K_H/K_M = 1{\cdot}12$ at $z/L = -0{\cdot}03$, with a steady increase up to a value of 2·07 at $z/L = -1$.

7. Concluding remarks

We have enumerated, with some confidence, the flux-gradient relationships for momentum, heat and water vapour transfer in the z/L range $-$ 0·01 to $-$ 1·0. These results should find ready application in many problems in micrometeorological research, since atmospheric instability rarely exceeds $|z/L| = 1$ in the first few metres of the atmosphere. Certainly the results should lead to more reliably determined fluxes than the use of near-neutral approximations.

The extension to greater instabilities must be approached with some caution. This is particularly relevant in the context of numerical forecasting or general circulation studies, but lies outside the scope of the present paper.

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Appendix

Micrometeorological data obtained at Gurley, New South Wales, during March 1965. Heat energy units are mW cm^{-2} and Reynolds stress is in dynes cm^{-2}. Temperatures are in °C and wind speeds in m s^{-1}. Potential temperature differences, $\Delta\theta$, were measured between 1 and 2 m height. Cloud covers are reported in eighths

Time	R	G	H	E	τ	$-\Delta\theta$ (1-2 m)	u_M	u_1	u_2	u_4	u_8	u_{16}	Cloud
13 March 1970													
0930	38	8·0	17·3	–	0·430	–	2·32	2·59	2·80	2·97	3·16	3·36	2 Ci
1000	50	10·0	18·3	–	0·514	0·75	2·26	2·48	2·67	2·83	3·00	3·17	2 Ci
1030	58	11·0	24·3	–	0·345	0·46	2·35	2·57	2·77	2·91	3·04	3·26	3 Ci
1100	58	10·5	23·6	–	0·399	0·63	2·16	2·40	2·55	2·70	2·81	2·98	3 Ci
1230	56	10·0	30·9	–	0·622	0·74	2·30	2·52	2·71	2·81	2·90	3·01	4 Ci
14 March 1970													
0930	45	4·5	17·9	–	1·123	–	4·68	5·34	5·84	6·23	6·61	6·90	2 Cu 2 Ac
1000	45	4·5	22·3	–	1·138	0·50	4·40	5·02	5·49	5·88	6·18	6·48	6 Ac As
1030	51	8·5	20·0	–	1·223	0·55	4·73	5·46	5·96	6·38	6·72	7·06	6 Ac As
1100	27	4·5	25·4	–	1·138	0·51	3·89	4·37	4·75	5·07	5·35	5·64	8 Ac As
1130	29	5·5	17·7	–	1·000	0·50	3·55	4·02	4·34	4·59	4·83	5·08	8 Ac As
1200	17	4·0	11·1	–	0·715	0·29	2·75	3·06	3·31	3·52	3·68	3·88	3 Sc 8 As
1230	16	4·0	10·1	–	0·340	0·23	2·98	3·30	3·59	3·82	4·00	4·16	1 Sc 8 As
1300	17	3·5	8·9	–	0·537	0·30	2·87	3·17	3·46	3·71	3·89	4·08	8 Ac As
1330	17	3·5	7·6	–	0·468	0·29	2·80	3·09	3·35	3·60	3·78	3·99	8 As
1400	15	3·5	10·8	–	0·584	0·38	3·23	3·65	3·99	4·26	4·46	4·64	8 As
1430	12	3·0	6·0	–	0·545	0·36	2·80	3·18	3·47	3·71	3·89	4·05	8 Ac As
1700	1	0·0	5·2	–	0·453	0·15	1·90	2·16	2·38	2·56	2·66	2·76	8 Ac As
1730	1	− 1·0	4·4	–	0·449	0·12	2·04	2·33	2·59	2·81	3·01	3·25	7 Ac As
15 March 1970													
1000	16	0·5	3·5	–	0·738	0·03	3·01	3·46	3·85	4·29	4·60	4·88	8 Ac As
1030	20	0·5	3·6	–	0·807	0·05	3·14	3·60	4·02	4·41	4·75	5·08	8 Ac As
1100	35	2·5	5·7	–	0·953	0·17	3·19	3·54	3·91	4·24	4·56	4·84	8 Ac As
1130	29	2·5	7·1	–	0·869	0·39	4·02	4·55	5·05	5·50	5·87	6·16	8 Ac As
1200	25	0·0	4·7	–	0·761	0·29	3·50	4·11	4·59	4·94	5·26	5·55	8 Ac As
1230	30	1·5	9·0	–	1·161	0·42	4·39	5·02	5·54	5·97	6·37	6·71	8 Ac As
1300	19	1·0	4·7	–	0·769	0·34	4·30	4·88	5·47	5·90	6·33	6·70	8 Ac As
1330	21	2·0	7·7	–	1·192	0·39	3·85	4·35	4·82	5·20	5·53	5·83	8 Ac As
1400	37	5·0	8·1	–	1·208	0·47	4·42	5·08	5·62	6·07	6·50	6·84	7 Sc

APPENDIX—*continued*

MICROMETEOROLOGICAL DATA OBTAINED AT GURLEY, NEW SOUTH WALES, DURING MARCH 1965. HEAT ENERGY UNITS ARE mW cm^{-2} AND REYNOLDS STRESS IS IN dynes cm^{-2}. TEMPERATURES ARE IN °C AND WIND SPEEDS IN m s^{-1}. POTENTIAL TEMPERATURE DIFFERENCES, $\Delta\theta$, WERE MEASURED BETWEEN 1 AND 2 m HEIGHT. CLOUD COVERS ARE REPORTED IN EIGHTHS

Time	R	G	H	E	τ	$-\Delta\theta$ (1-2 m)	u_M	u_1	u_2	u_4	u_8	u_{16}	Cloud
16 March 1970													
1230	66	8·0	20·2	-	0·730	0·55	2·82	3·13	3·36	3·58	3·75	3·99	4 Cu
1430	12	5·0	14·7	-	0·337	0·38	3·00	3·32	3·59	3·86	4·06	4·30	6 Cu
1530	12	6·0	6·0	-	0·322	0·15	2·42	2·82	3·08	3·34	3·49	3·61	8 Cu
17 March 1970													
1000	40	− 1·5	− 2·0	-	0·430	-	2·72	3·07	3·33	3·54	3·71	3·89	-
1030	45	− 2·5	7·7	-	0·537	-	2·99	3·36	3·65	3·88	4·04	4·20	-
1100	48	− 1·0	7·4	-	0·430	-	2·40	2·75	2·97	3·17	3·33	3·49	-
18 March 1970													
1000	40	− 1·5	6·4	-	0·360	0·16	2·60	2·85	3·06	3·25	3·37	3·50	1 Ac
1030	45	− 2·5	7·4	-	0·391	0·30	2·50	2·73	2·91	3·07	3·22	3·32	-
1100	48	− 1·0	10·3	-	0·175	0·35	2·47	2·64	2·81	2·97	3·15	3·24	-
1130	51	− 2·0	10·9	-	0·414	0·37	2·50	2·71	2·88	3·04	3·24	3·29	-
1430	48	8·0	16·1	-	0·214	0·39	1·63	1·77	1·89	1·97	2·05	2·10	2 Cu
1530	37	5·0	17·5	-	0·483	0·43	1·64	1·79	1·87	1·94	2·11	2·22	3 Cu
1600	36	3·5	8·3	-	0·445	0·02	1·33	1·46	1·57	1·65	1·75	1·91	3 Cu
1630	26	3·0	7·2	-	0·545	0·06	2·15	2·35	2·52	2·65	2·78	2·86	3 Cu
1700	12	1·0	6·1	-	0·730	0·07	2·23	2·49	2·70	2·87	3·02	3·15	1 Cu
19 March 1970													
1000	48	10·0	17·3	-	0·815	0·52	4·89	5·51	6·00	6·42	6·87	7·23	4 Cs
1030	54	10·0	13·7	25·5	0·892	0·50	4·76	5·30	5·74	6·10	6·48	6·77	5 Cs
1100	58	11·0	14·2	28·7	0·699	0·46	4·43	4·95	5·36	5·69	6·04	6·29	3 Cs
1200	60	13·0	16·4	27·2	0·445	0·53	4·08	4·54	4·89	5·16	5·40	5·57	6 Ci Cs
1300	54	12·5	15·2	26·2	0·322	0·42	3·14	3·44	3·67	3·90	4·08	4·29	2 Cu 4 Ci Cs
1400	52	12·0	13·6	31·8	0·437	0·49	2·84	3·08	3·30	3·50	3·68	3·85	6 Cu
1430	48	9·0	10·3	27·9	0·275	0·52	2·48	2·68	2·88	3·06	3·25	3·43	6 Cu
1500	44	9·0	11·3	20·7	0·476	0·60	2·96	3·31	3·55	3·76	3·96	4·09	4 Cu 3 Ci
1530	28	7·0	11·1	-	0·483	0·55	3·12	3·54	3·80	4·03	4·22	4·36	3 Cu 3 Ci
1600	14	5·0	9·4	-	0·476	0·38	2·62	2·97	3·21	3·43	3·59	3·71	3 Cu 8 Ci Cs