

Profile relationships: the log-linear range, and extension to strong stability

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SUMMARY

The diabatic mean profile forms in the surface layer are studied, by applying analysis methods having high resolving power to data from O'Neill, U.S.A. (heights up to 6.4 m) and from Kerang and Hay, Australia (heights mostly up to 16 m).

It is found, concordantly from the O'Neill and Australian data, that the log-linear law is valid for z/L values between -0.03 and $+1$, which includes a small range of unstable and a surprisingly wide range of stable conditions. For all quantities studied (wind, potential temperature, and specific humidity), it is concluded that the Monin-Obukhov coefficient α is near 4.5 in unstable and 5.2 in stable conditions, within a standard error of about 10 per cent. The ratios K_H/K_M and K_W/K_M evidently remain constant, equal to unity, over the whole of the log-linear range (and somewhat beyond).

In stable conditions, the log-linear law implies that Ri approaches a critical value α^{-1} , approximately 0.2 , as $z/L \rightarrow \infty$. However, at $z/L \approx 1$ a second régime sets in, in which the profiles are only quasi-determinate, approximating, on the average, a simple logarithmic form (gradients proportional to z^{-1}); this régime covers the range approximately $1 < z/L < (\alpha + 1)$, i.e. $(\alpha + 1)^{-1} < Ri < 1$. A third range of extreme stability, $Ri > 1$, is practically unrepresented in the data examined.

The major part of the unstable range, for $z/L < -0.03$, will be discussed in a later paper.

1. INTRODUCTION

The problem of documenting the diabatic mean profile forms in the atmospheric surface layer (heights up to a few tens of metres) has been outlined in the books by Priestley (1959) and by Lumley and Panofsky (1964). Some of the characteristics are now well established, but, as is still apparent in the recent literature (e.g. discussion by Swinbank and others 1966), there remain important differences of opinion about some aspects, despite the availability of observational data of evidently high quality.

This paper describes in part an endeavour to establish the profile relationships as definitively as possible, from the best available data. To avoid the danger of cross-contamination in evaluating the different parameters which characterize a profile, the methods of analysis are designed to have 'high resolving power,' i.e. to have maximum sensitivity to a required quantity and low sensitivity to errors in other quantities. The analysis also provides to some extent a check on the quality of the observational data.

A guiding principle throughout is to keep the treatment of the data as simple and direct as possible. The quantities examined are, in essence, simple the *inter-height differences* Δ of mean wind speed U , potential temperature θ , or specific humidity q , and various ratios of these differences; these quantities come directly and unequivocally from the experimental data.

Again, from a more fundamental standpoint, it is the vertical *gradients*, or *differences*, which are directly related to the turbulent transfer, while the mean quantities themselves (wind, temperature, etc.) involve an additional parameter which is essentially a constant of integration, this being the roughness length z_0 in the case of the wind profile. Thus, by dealing only with differences, we eliminate these unneeded parameters right from the outset; it is a considerable advantage that the analysis does not depend on evaluation of z_0 (or the corresponding drag coefficient) with its associated uncertainties.

No appeal is made to measured fluxes. This is not meant to underrate the obvious importance of flux measurements, but it is considered that independent profile relationships should also be established as firmly as possible. In the first place, both profile and flux measurements have their own difficulties and pitfalls, and it is advantageous to make possible a large measure of independent checking between the two. In addition, we must recognize basic inconsistencies which may arise under some conditions; for example, with the marked variability of wind strength which typically accompanies clear-day convection, the *direct average* u_* (friction velocity) indicated by the wind profile may well differ from the *root-mean-square* u_* indicated by a drag-plate or eddy-correlation measurement of shearing stress.

The general plan of this work is to start from near-neutral conditions, and then to proceed towards stronger instability on the one hand and stability on the other. The present paper discusses the initial departure from neutrality, as represented by the 'log-linear' profile form. In the unstable (lapse) case, this form is applicable only for small deviations from neutral; the remainder of the unstable régime will be discussed in a later paper. In the stable (inversion) case, however, the log-linear form is found to remain valid up to considerable degrees of stability, and it is convenient to discuss the full range of stable conditions in the present paper.

The complete lapse and inversion profile forms which have been adopted as a result of this investigation have in the meantime been summarized in A.M.* An application of the lapse relationships in evaluating the evaporation from a lake was described earlier (Webb 1960).

2. OBSERVATIONAL DATA

A preliminary assessment of available wind profile data was made, by inspecting the plotted inter-height differences ΔU (for equal height ratios). It was found that most of the published data have obvious imperfections appearing as irregular 'dog-leg' bends; in the following work, appeal is made only to data sources with ΔU 's of apparently high quality in this respect.

The data adopted are those of the Johns Hopkins (J.H.) group at O'Neill, U.S.A., in 1953 (1 hour runs), and those of Swinbank and co-workers (30 min runs) during five expeditions in Australia: at Kerang, Victoria – site 1 in February 1962, and site 2 in October 1963 and September 1964, and at Hay, N.S.W. – site 1 in March 1964, and site 2 in March 1965 (this last providing a few night-time runs only).

The O'Neill measurements are reported in full by Johns Hopkins University (1953) and, for alternate runs only, by Lettau and Davidson (1957). Here we include only runs within the 'general observation periods' defined in the latter publication. The Australian daytime data have been reported by Swinbank and Dyer (1968), and the data for Kerang in February 1962 have also been published by Swinbank (1964). The Australian night-time data of Swinbank have not been published, and are therefore listed for reference in Appendix 1 of this paper.

The Johns Hopkins (J.H.) measurements were at doubled heights from 0.4 to 6.4 m. In the Australian observations, winds were measured at doubled heights from 1 to 16 m, with the addition of 0.5 m in February 1962 only, and 32 m in March 1965; temperature differences, and in many cases humidity differences, were measured between heights 1, 4, and 16 m with the addition of 0.5 and 8 m in February 1962 only, and of 2, 8, and 32 m in March 1965.

To avoid conditions of changing radiation, attention is restricted to day-time observations between 10 and 17 hr (Local Mean Time) and night-time observations between 19 and 06 hr (LMT).

The J.H. temperature data have appreciable random sampling scatter, because temperature T rather than its inter-height difference ΔT was sampled at the different heights in turn (the natural fluctuations of T being much larger than those of ΔT).

* A.M. refers to Webb (1965).

Nevertheless, these data are found to be adequate for their limited applications here, namely, in the evaluation of Richardson number Ri as discussed below, and in the $\Delta U/\Delta\theta$ analysis reproduced in Fig. 9.

A further shortcoming is that the J.H. temperatures were measured over only a 5 min period centred in each 1 hr run. However, it seems probable that they will be reasonably representative, since the observations were in steady conditions; and this prospect is supported by the close association which is found on comparing the J.H. ($\theta_{6.4} - \theta_{0.4}$) values (subscripts indicating height in m) with the U.C.L.A. group's ($\theta_8 - \theta_{0.5}$) values, for which the averaging period was 15 min – the longest available.

At the O'Neill site, the grass, of height 20 to 40 cm, was mowed to about 10 cm, later 6 cm, with an upwind fetch of about 500 m for the prevailing southerly winds. To assess the lack of adjustment to the mown surface for the J.H. wind profiles, we appeal to the theory of Panofsky and Townsend (1964). For fetches as large and heights as small as are relevant here, the results of the Panofsky-Townsend theory are evidently in close agreement with those of the more fundamental numerical treatments of Onishi and Estoque (1968, Figs. 1 and 4) and of Taylor (1969, Fig. 10), though a somewhat greater lack of adjustment is estimated from an alternative theory of Townsend (Blom and Wartena 1969). Most significantly, however, measurements by Bradley (1968) show the transition to be more abrupt near the top of the modified layer, and the adjustment to be more complete in the lower parts which concern us here, than from the Panofsky-Townsend theory, thus suggesting that this theory will yield safe estimates.

With z_0 taken to be 3 cm unmown and 0.6 cm mown, the excess of u_* or $\partial U/\partial z$ (i.e. virtually, ΔU) at the geometric midheight of the uppermost J.H. height interval, relative to that near the ground, is estimated to be 1.4 per cent. The fetch was shorter in some of the J.H. night-time runs, when the surface flow had an easterly component and traversed the nearer lateral boundary; in the worst case, the estimated lack of adjustment becomes 2.9 per cent. These figures are satisfactorily small, particularly since the slight natural decrease of u_* with height (Calder 1939) amounts to around 1 per cent acting in the opposite sense.

The theories mentioned above are based on neutral stratification. In stable conditions, while the fetch effect is expected to be more pronounced, circumstantial evidence suggests that it is still not serious. First, in the J.H. night runs with decreasing fetch there is no increasing trend in the values of α as obtained in the next Section, nor in the ordinate values in Fig. 7 (a) representing the profile shape. Again, more generally, the results from the J.H. data and from the Australian data are found to be essentially concordant, suggesting in retrospect that the J.H. fetch effect is not serious.

As stability parameter we take $Ri_{1.6}$ for the J.H. data or Ri_2 for the Australian data. Ri_2 is evaluated from $(\theta_b - \theta_a)/(U_b - U_a)^2$, with $z = \sqrt{(ab)}$, as discussed in A.M.; the multiplying factor is $(g/\theta) z \ln(b/a)$ in near-neutral conditions (g being the acceleration of gravity and θ the absolute (potential) temperature), and does not vary greatly over the whole diabatic range. The small correction for the buoyancy of water vapour is incorporated by replacing $\Delta\theta$ by $(\Delta\theta + 0.61 \theta \Delta q)$.

In evaluating Ri , the height ratio b/a is generally taken as 4. However, in the case of the J.H. data, for parts of the analysis where Ri is required to be as accurate as possible (Section 3 of this paper), the values of $Ri_{1.6}$ are revised taking $\Delta\theta$ over the full height interval 0.4 to 6.4 m, in order to minimize the errors due to imperfect temperature sampling. From the ratios of revised to original $Ri_{1.6}$, it is estimated that the standard deviation of the revised $Ri_{1.6}$, due to the temperature sampling scatter, is about 7 per cent, in both unstable and stable cases – an acceptably small figure.

3. LOG-LINEAR ANALYSIS

(a) Method of analysis

The 'log-linear' wind profile of Monin and Obukhov (1954), expressing the influence of thermal stratification to a first approximation, is given in gradient form by

$$\frac{\partial U}{\partial z} = \frac{u_*}{kz} \left(1 + \alpha \frac{z}{L} \right) \quad (1)$$

or in integrated form by

$$U = \frac{u_*}{k} \left(\ln \frac{z}{z_0} + \alpha \frac{z - z_0}{L} \right) \quad (2)$$

Here k (≈ 0.41) is von Kármán's constant, α is a numerical constant, and L is Obukhov's scale height defined by

$$L = \frac{-u_*^3}{kgH/c_p\rho\theta},$$

in which H is the vertical heat-flux (upwards counted positive) and ρ and c_p are the air density and specific heat at constant pressure.

Our immediate concern is to examine the validity of Eq. (1) for the initial departure from neutral conditions, and at the same time to evaluate the Monin-Obukhov coefficient α .

First, by integrating Eq. (1) between any two heights a and b , and dividing throughout by $\ln(b/a)$, we have

$$\frac{U_b - U_a}{\ln(b/a)} = \frac{u_*}{k} \left(1 + \frac{\alpha}{L} \frac{b - a}{\ln(b/a)} \right) \quad (3)$$

This may be written

$$y = \frac{u_*}{k} \left(1 + \frac{\alpha}{L} x \right), \quad (4a)$$

where, allowing for an assigned zero-plane displacement δ ,

$$\left. \begin{aligned} y &= \frac{U_b - U_a}{\ln \frac{b - \delta}{a - \delta}} \\ x &= \frac{b - a}{\ln \frac{b - \delta}{a - \delta}} \end{aligned} \right\} \quad (4b)$$

We plot the observations for each run in the form y against x , plotting one point for each available pair of heights. Eq. (4a) shows that, if the data do follow the log-linear form, then the points lie on a straight line sloping in the sense determined by the sign of α/L , i.e. negative for unstable conditions, as illustrated in Fig. 1, and positive for stable conditions, as illustrated in Figs. 2 and 3; in truly neutral conditions, the line would be horizontal.

It is apparent from Eq. (4a) that the line cuts the vertical axis ($x = 0$) at u_*/k , and cuts the horizontal axis ($y = 0$) at $-L/\alpha$. From the latter, it follows that if the value of L on the particular occasion were known, then the coefficient α could be estimated from

$$\alpha = -L/X, \quad (5)$$

where X is the intercept on the horizontal axis. However, L is not initially known; instead, the stability parameter available from the profile measurements is Ri . We therefore rearrange Eq. (5) to estimate α from X and Ri , as follows.

According to a further analysis of the data (Section 5), the profiles of $\partial U/\partial z$ and $\partial \theta/\partial z$ remain similar throughout the range of Ri that concerns us at this point. We therefore take, in similarity to Eq. (1),

$$\frac{\partial \theta}{\partial z} = -\frac{T_*}{kz} \left(1 + \alpha \frac{z}{L} \right), \quad (6)$$

where T_* is defined by $H = c_p \rho u_* T_*$; and thence, from Eqs. (1) and (6),

$$\text{Ri} = \frac{z}{L} \left(1 + \alpha \frac{z}{L} \right)^{-1}. \quad (7)$$

This gives, on substituting for α/L from Eq. (5),

$$L = \frac{z}{\text{Ri}_z (1 - z/X)} \quad (8)$$

from which L may be estimated* without prejudice to the value of α . Finally, inserting Eq. (8) into Eq. (5) gives the expression for evaluating α from Ri and X

$$\alpha = \frac{z}{\text{Ri}_z (z - X)}. \quad (9)$$

In plotting the data in terms of Eq. (4b), a suitable value of the zero-plane displacement δ can be assigned, after some initial trial and error, by noting the systematic curvature which appears at the low-abscissa end (small heights) if the adopted value of δ is appreciably in error. For the Johns Hopkins data, it is found appropriate to take δ equal to half the quoted grass height, i.e. $\delta = 5$ cm before 12 August 1953 (on which date the grass was removed), and $\delta = 3$ cm after that date, except that for the last few days, starting 25 August, it is found more suitable to revert to $\delta = 5$ cm. For the Australian data, it is found suitable to take $\delta = 0$ throughout; this is consistent with the experimental policy (Swinbank and Dyer 1968) of measuring all instrument heights from an assumed zero plane at about half the average grass height.

In the analysis to evaluate u_*/k and X , one could, instead of plotting the data as in Figs. 1 to 3, carry out an equivalent regression calculation. However, the visual analysis by plotting is preferred here, as it aids the critical assessment of the method and of the data.

(b) Log-linear results

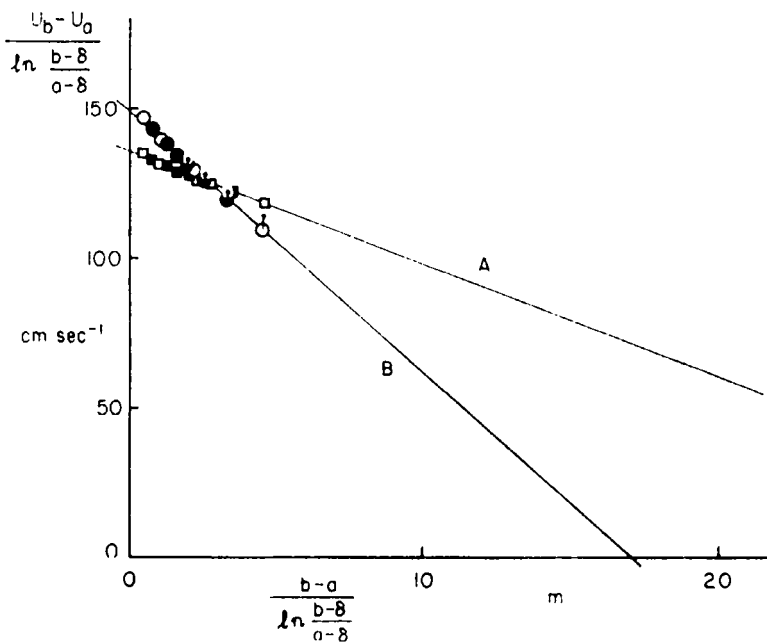
The wind profile analysis is illustrated by examples with small observational scatter in Figs. 1 and 2. The analysis of an inversion run in which the full temperature profile is available is shown in Fig. 3, where it is apparent that the behaviour of θ is of a similar nature to that of U . For every run, the intercept X has been evaluated from the best straight line fitted by eye, α then being evaluated from Eq. (9).

The values of α obtained, together with other details, are given in Table 1 for unstable and Table 2 for stable conditions. Runs in conditions very close to neutral, with $|\text{Ri}_{1.6}|$ or $|\text{Ri}_2|$ values of 0.006 or less, have been excluded. The analysis is mostly limited to U profiles, on account of the random scatter of $\Delta\theta$ and Δq in the J.H. data and the incomplete selection of heights for $\Delta\theta$ and Δq in most of the Australian data. With the J.H. data, analysis effort has been reduced by eliminating the runs with appreciable observational scatter in a preliminary abbreviated analysis, as noted at the foot of Table 1.

In the case of stable conditions, it is found that the log-linear form remains valid up to quite high degrees of stability – a straight-line relationship is consistently obtained, as in Fig. 2, and, also, the resulting values of α are, within the experimental variability, independent of Ri . Eventually, a breakdown occurs as described in Section 4 below.

In unstable conditions, it is found that the log-linear form is valid up to a $z/|L|$ or $|\text{Ri}|$ value of only about 0.03, beyond which there is a smooth departure from this form; thus, the log-linear range considered in this paper is quite small. The form of the departure, established by an analysis to be described in a later paper, is summarized in A.M. Its appearance at $|\text{Ri}|$ about 0.03 is comparable with earlier findings from other data – for the heat flux-gradient relationship by Priestley (1955), for the temperature profile by Webb (1958), and for the wind profile by Taylor (1960).

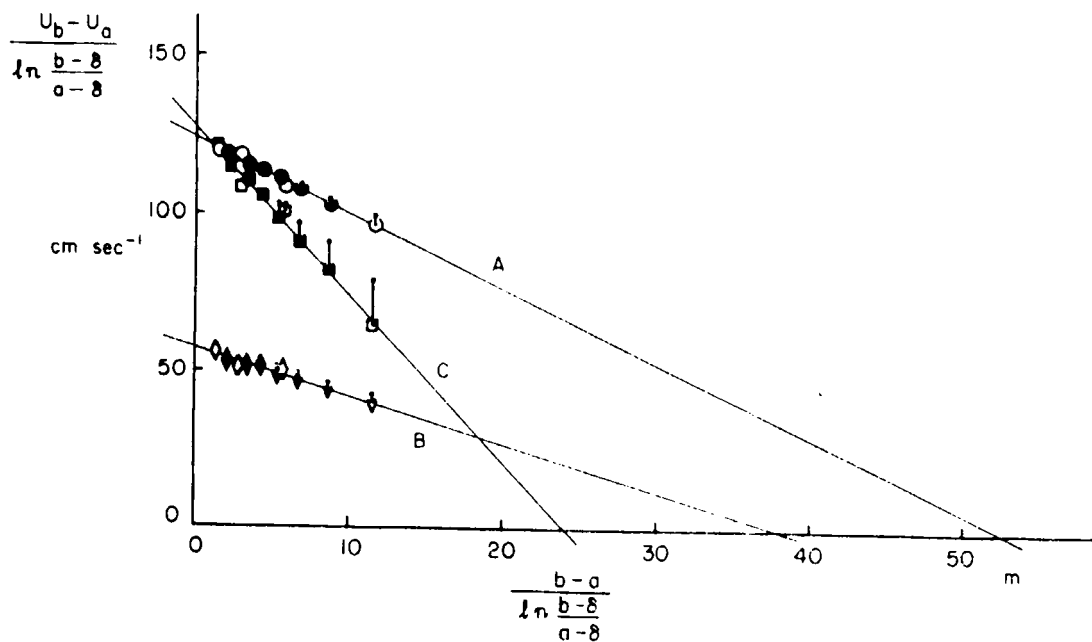
* Though not applicable here, it should be noted that, after the value of α has been determined, the practical evaluation of L from measured Ri would be directly from the inverse of Eq. (7), which is $z/L = \text{Ri}/(1 - \alpha \text{Ri})$.



(a) Johns Hopkins (O'Neill) data

A 7 September 1953 : 16-1700.

B 31 August 1953 : 14-1500.



(b) Australian (Kerang) data

A 22 October 1963 : 1559 (mast 2).

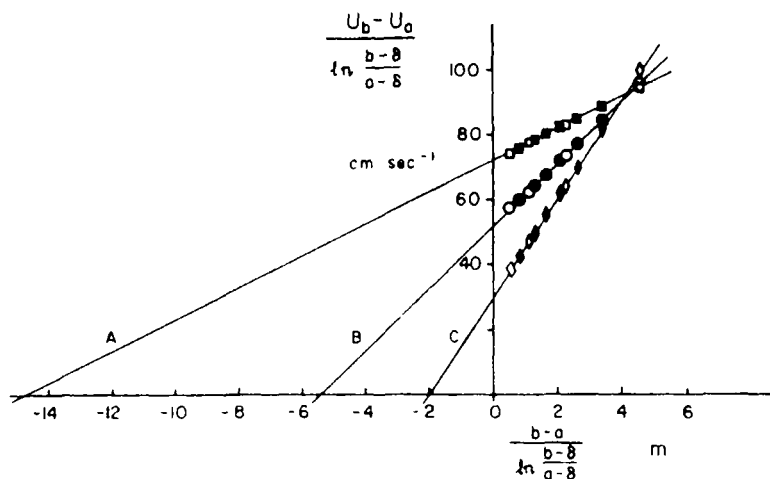
B 18 September 1964 : 1609 (mast 2).

C 15 September 1964 : 1154 (mast 1).

Figure 1. Log-linear analysis of wind profiles (Eq. (4a, b)) - unstable conditions. Open symbols : pairs of adjacent observation heights; black symbols : other height pairs. Straight lines are fitted by eye; intercepts give u_*/k and α (via Eq. (9)). Small points show deviation from log-linear form for $z/|L|$ values above 0.03 (see text).

To make a reasonable number of lapse runs accessible in this paper, the analysis is extended to a little beyond the true log-linear range, and, as illustrated in Fig. 1, each affected ΔU value is formally 'corrected' by a factor calculated from the full profile form given as Eq. (25a, b) in A.M. The range accepted allows up to 5 per cent correction to $(U_{6.4} - U_{3.2})$ for the J.H. data or to $(U_8 - U_4)$ for the Australian data, which corresponds to acceptance of $|Ri_{1.6}|$ or $|Ri_2|$ values up to 0.027.

For the Australian lapse data, the correction to the topmost difference $(U_{16} - U_8)$ is also less than 5 per cent if $|Ri_2|$ is less than 0.013; most of the runs in September 1964 lie within or near this limit, while most of the runs in October 1963 do not (Table 1). The

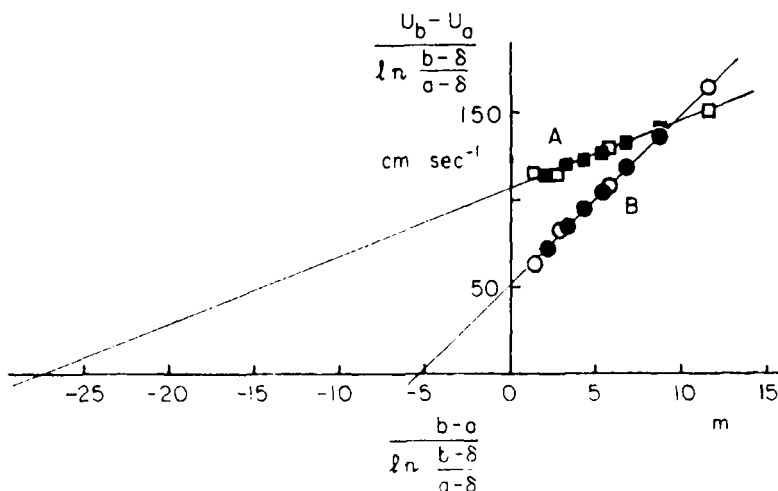


(a) Johns Hopkins (O'Neill) data

A 9 August 1953 : 01-0200.

B 8 August 1953 : 19-2000.

C 19 August 1953 : 00-0100.



(b) Australian (Kerang) data

A 15 September 1964 : 1458 (mast 2).

B 12 October 1963 : 2209 (mast 1).

Figure 2. Log-linear analysis of wind profiles (Eq. (4a, b)) - stable conditions. Open symbols : pairs of adjacent observation heights; black symbols : other height pairs. Straight lines are fitted by eye; intercepts give u_*/k and α (via Eq. (9)).

few runs with $|Ri_2|$ appreciably greater than 0.013 do involve a significant extension beyond the log-linear range, as illustrated by C in Fig. 1 (b); these yield comparable values of α , offering some supplementary support for the form of 'correction' applied.

The method of analysis employed here, while suitable for investigating the log-linear form, is not suitable for determining the departure from this form in the unstable case – the curvature in the 'uncorrected' plotting is distinctly apparent only in runs with very small observational scatter, as in B of Fig. 1 (a) and in C of Fig. 1 (b).

The low and middle cloud characteristics during the runs considered here were as follows. The J.H. day-time runs were almost completely free from low cloud and mostly free from middle cloud; during the Australian day-time runs, low and/or middle cloud were present in all cases. No anomalous effects that could be attributed to broken low or middle cloud were detected, at least up to the small degrees of instability considered here. Both J.H. and Australian night-time runs were free from low cloud and mostly free from middle cloud.

Table 3 gives geometric means of the α values, together with standard deviations and standard errors. On the basis of these results, suitable values to α to adopt at present are

lapse : $\alpha = 4.5$,

inversion : $\alpha = 5.2$.

These are evidently determined to within a standard error of perhaps 10 per cent; thus, while it appears that α is somewhat larger for stable than for unstable conditions, the

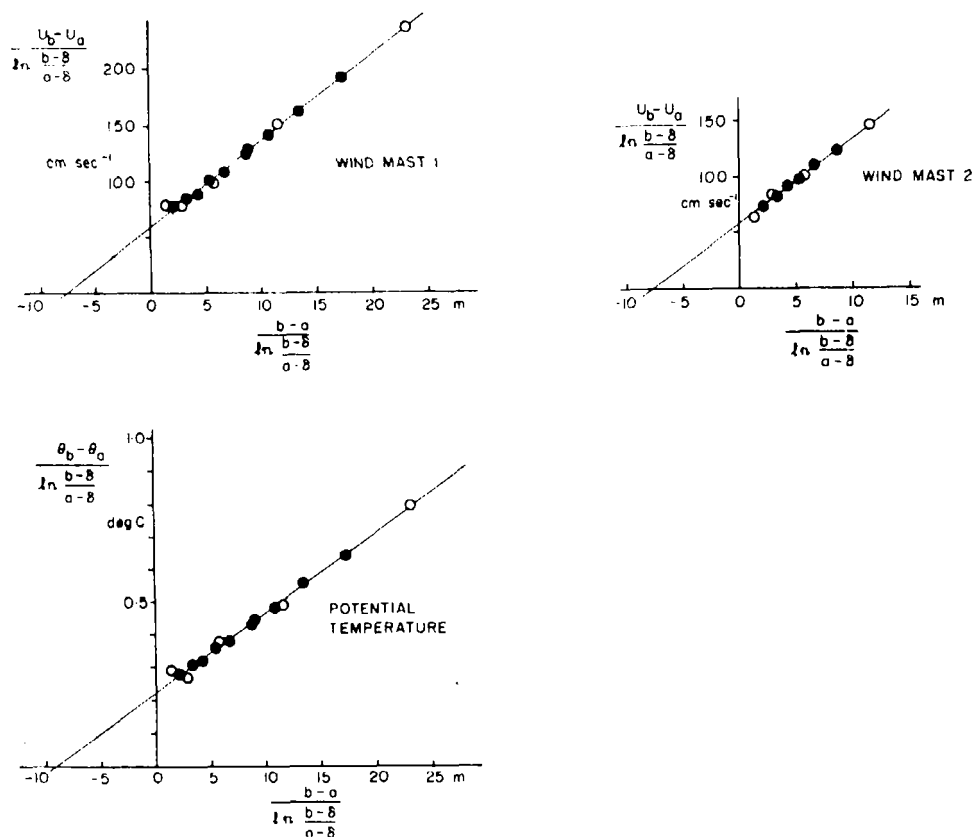


Figure 3. Log-linear analysis of wind and potential temperature profiles (Eq. (4a, b)) – stable conditions. Australian (Hay) data, 12 March 1965 : 0018. Open symbols : pairs of adjacent observation heights; black symbols : other height pairs. Top height is 32 m for wind mast 1 and potential temperature, 16 m for wind mast 2. Straight lines are fitted by eye; intercepts give u_*/k from wind or T_*/k from potential temperature, and α (via Eq. (9)).

TABLE 1. α VALUES FROM INDIVIDUAL RUNS - UNSTABLE CONDITIONS

(a) O'Neill (Johns Hopkins) 1953 data					
Date 1953	Time	$U_{1.6}$ cm s ⁻¹	$-R_{1.6}$	$-L$ m	α for wind
9 August	11-1200	687	0.017	98	(4.5)
	12-1300	806	0.015	114	5.7
	13-1400	806	0.013	130	7.2
	15-1600	734	0.012	140	5.4
13 August	14-1500	742	0.019	86	3.0
	15-1600	697	0.018	92	3.1
	16-1700	628	0.010	161	5.6
24 August	13-1400	860	0.019	90	5.4
	14-1500	908	0.016	111	6.8
	15-1600	868	0.012	140	6.6
	16-1700	850	0.008	197	6.2
25 August	10-1100	867	0.015	108	3.7
31 August	14-1500	832	0.026	66	3.9
	15-1600	815	0.020	83	4.9
	16-1700	815	0.015	107	3.0
7 September	12-1300	836	0.025	66	(3.0)
	16-1700	757	0.011	146	4.0

(b) Australian (Kerang) data									
Wind mast 1						Wind mast 2			
Date 1963	Time of start (30 min runs)	U_2 cm s ⁻¹	$-R_2$	$-L$ m	α for wind	U_2 cm s ⁻¹	$-R_2$	$-L$ m	α for wind
12 October	1148	771	0.026	86	3.7	-	0.026	85	4.1
	1408	709	0.017	122	3.1	-	0.018	118	3.6
	1444	692	0.018	118	2.7	-	0.018	116	(2.6)
13 October	1500	762	0.024	90	4.2	-	0.025	87	(3.5)
	1617	739	0.011	192	1.4	-	0.011	183	[1.4]
14 October	1603	-	0.023	94	3.4	610	0.022	100	(4.0)
22 October	1103	506	0.016	126	[1.8]	507	0.016	128	[2.7]
	1302	709	0.024	89	3.6	717	0.023	95	3.9
	1450	662	0.016	130	4.6	648	0.017	121	[2.8]
	1525	680	0.019	112	3.2	660	0.018	116	4.0
	1559	653	0.010	201	(3.4)	631	0.011	192	3.7
1964									
7 September	1253	863	0.021	104	4.2	881	0.022	98	[3.5]
	1533	717	0.013	157	(3.3)	684	0.016	-	[-]
15 September	1154	737	0.020	112	4.8	713	0.021	100	(3.1)
16 September	1047	1,015	0.010	212	3.9	1,007	0.011	-	[-]
	1123	1,028	0.009	221	(4.3)	1,029	0.012	176	[3.1]
	1200	1,022	0.011	192	5.8	1,005	0.013	162	[4.0]
	1234	978	0.014	156	4.5	958	0.015	142	[2.3]
	1408	949	0.012	167	3.3	960	0.013	163	[3.4]
	1441	898	0.007	285	5.4	904	0.008	-	[-]
18 September	1609	340	0.013	163	(5.9)	342	0.015	144	3.8

Notes for Tables 1 and 2

α is evaluated from the intercept X via Eq. (9). In the α column, the apparent accuracy within which X was estimated is indicated as follows :

4.8 practically no scatter of plotted points (cf. Figs. 1 and 2); X determined to within about 5 per cent.

4.8 X determined to within about 15 per cent.

(4.8) X determined to within about 30 per cent.

[4.8] appreciable scatter; only rough estimation of X is possible.

[-] prohibitive scatter and/or insufficient data; no estimation of X is possible.

For the Johns Hopkins data, runs in the lowest classes (square brackets) have been eliminated in a preliminary analysis using adjacent-height intervals only.

possibility that the two are equal cannot be entirely dismissed. These values of α are taken to apply for θ and q as well as for U , as suggested by the few Hay 1965 results for θ and supported more firmly by the evidence to be considered in Section 5.

The standard deviation of individual α values is seen in Table 3 to be around 30 per cent. It is not clear whether this variability is inherent in the atmosphere, or whether, on the other hand, it could be reduced by further refinements of experimental technique.

TABLE 2. α VALUES FROM INDIVIDUAL RUNS - STABLE CONDITIONS

(a) O'Neill (Johns Hopkins) 1953 data					
Date 1953	Time	$U_{1.6}$ cm s ⁻¹	$Ri_{1.6}$	L m	α for wind
8 August	19-2000	280	0.051	23	4.3
	21-2200	231	0.062	13	7.8
	22-2300	261	0.048	23	6.4
9 August	00-0100	298	0.037	34	5.2
	01-0200	366	0.021	68	4.6
	03-0400	328	0.020	71	(4.0)
13 August	22-2300	300	0.053	18	7.7
	23-2400	296	0.056	18	6.7
18 August	19-2000	203	0.082	11	5.5
	20-2100	140	0.130	4.4	4.9
	21-2200	199	0.101	6.3	5.9
	22-2300	251	0.072	14	4.9
	23-2400	310	0.041	31	5.0
19 August	00-0100	205	0.096	9.1	4.6
	01-0200	154	0.107	7.8	4.4
	02-0300	221	0.058	13	8.9
	19-2000	164	0.142	5.8	3.3
8 September	00-0100	435	0.018	77	5.1
	01-0200	370	0.022	60	(6.4)
	02-0300	356	0.023	53	8.7

(b) Australian data											
Location	Date	Time of start (30 min runs)	Wind mast 1				Wind mast 2				
			U_2 cm s ⁻¹	Ri_2	L m	α for wind	U_2 cm s ⁻¹	Ri_2	L m	α for wind	α for pot. temp.
Ker.	17 Feb. 62	2126	173	0.154	1.7	[5.6] ^d					
Ker.	19 Feb. 62	1859	138	0.084	11	(6.6) ^d					
Ker.	12 Oct. 63	2033	309	0.058	26	4.5	-	0.063	21	4.9	
		2108	254	0.081	16	(4.1) ^d	-	0.085	14	(4.6) ^d	
		2209	300	0.054	27	5.0	-	0.060	22	5.5	
Ker.	14 Oct. 63	2011	-	0.032	52	6.3	357	0.036	42	6.8	
		2047	-	0.103	11	[-] ^d	193	0.104	8.8	[5.2] ^d	
Hay	12 Mar. 64	2150	144	0.128	5.6	[5.0] ^d	145	0.129	5.8	[4.9] ^d	
Ker.	15 Sept. 64	1458*	746	0.009	217	3.9	720	0.010	183	6.7	
		1532*	500	0.027	67	3.0	496	0.030	57	4.9	
		1606*	296	0.087	16	[3.5] ^d	316	0.080	17	(3.9) ^d	
Hay	11 Mar. 65	2244	444	0.019	92	(7.0)	432	0.023	74	6.6	(7.0)
	12 Mar. 65	0018	358	0.030	53	6.9	339	0.034	46	6.0	5.6
	12 Mar. 65	2019	183	0.072	13	(7.3) ^d	179	0.081	11	[7.1] ^d	(9.7) ^d

Notes: See notes at foot of Table 1 which are also applicable to Table 2.

Further notes for Table 2: In the α column for the Australian data, the symbol 'd' indicates that a departure of the type illustrated in Fig. 4 is apparent at a height of 16 m or less.

* The day-time temperature inversion on 15 September 1964 is associated with a small downward sensible heat flux accompanying comparatively large upward latent heat flux.

TABLE 3. GEOMETRIC MEAN VALUES OF α
(Values in square brackets in Tables 1 and 2 are excluded)

	Number of values	Geometric mean α	Standard deviation of individual values	Standard error of geometric mean
Unstable conditions				
O'Neill (J.H.) 1953 : U	17	4.6	36 per cent	8 per cent
Kerang 1963 : U	18	3.4	30 per cent	6 per cent
Kerang 1964 : U	12	4.3	24 per cent	6 per cent
Stable conditions				
O'Neil (J. H.) 1953 : U	20	5.5	30 per cent	6 per cent
Kerang 1963 and 1964 : U	14	4.9	27 per cent	7 per cent
Hay 1965 : $\begin{cases} U \\ \theta \end{cases}$	5	6.7	8 per cent	3 per cent
	3	7.2	32 per cent	17 per cent

Earlier analyses in which the log-linear form has been fitted to wind profile observations are those of Liljequist (1957), Taylor (1960), Blackadar, Panofsky, McVehil and Wollaston (1960), McVehil (1964), Zilitinkevich and Chalikov (1968), and Hoeber (1968). All of these deal with stable conditions, with the addition of unstable conditions in the analyses by Taylor and by Zilitinkevich and Chalikov. Except in Liljequist's paper, group values of α are estimated; these are broadly concordant with the present results, though in some cases there is considerable variability between values from different data sources. The α value of 7 arising from McVehil's analysis (stable conditions) is somewhat higher than that adopted here.

4. THE COMPLETE INVERSION PROFILE

(a) Extension to strong stability

In the case of stable stratification, so long as Eqs. (1), (6), and (7) remain applicable, it is apparent that, for increasing z/L , the gradients and Ri approach constant limiting values given by

$$\frac{\partial U}{\partial z} \rightarrow \frac{u_* \alpha}{kL}, \quad \frac{\partial \theta}{\partial z} \rightarrow -\frac{T_* \alpha}{kL}, \quad Ri \rightarrow \frac{1}{\alpha}.$$

(In comparison with the corresponding expressions in near-neutral stratification, these are suggestive of a 'mixing length' scaled to L/α). We note that if α is near 5, then Ri approaches a critical value near 0.2.

Presumably, gradients near these limits could not persist to indefinitely large z/L . Rather, with the severely weakened turbulence, other processes such as gravity waves and katabatic or other meso-scale drifts may be expected to supervene, and the profiles would then depart from the log-linear form.

A departure is indeed found to occur, as illustrated in Fig. 4. In the Australian data, it begins at a value of z/L around 1. The runs in which the feature is evident are indicated by the symbol 'd' in Table 2 (b).

The departure always appears as a relative drop of ΔU , and also of $\Delta \theta$ in the one example available (Fig. 4 (c)), indicating a deviation of $\partial U/\partial z$ and $\partial \theta/\partial z$ to below the log-linear value. The apparent departure ranges from sharp, as in Fig. 4 (a) and (c), to smooth, as in Fig. 4 (b). The runs which exhibit the departure are replotted in Fig. 5, where the abscissa and ordinate are normalized according to the respective intercepts in the plots for the individual runs. (Two cases in which the intercepts are not well

determined are excluded). It is apparent that the departure can, within its wide variability, be reasonably represented by the intermediate behaviour shown by the broken line, which corresponds to a transition at $z/L = 1$ from log-linear to simple logarithmic form. This

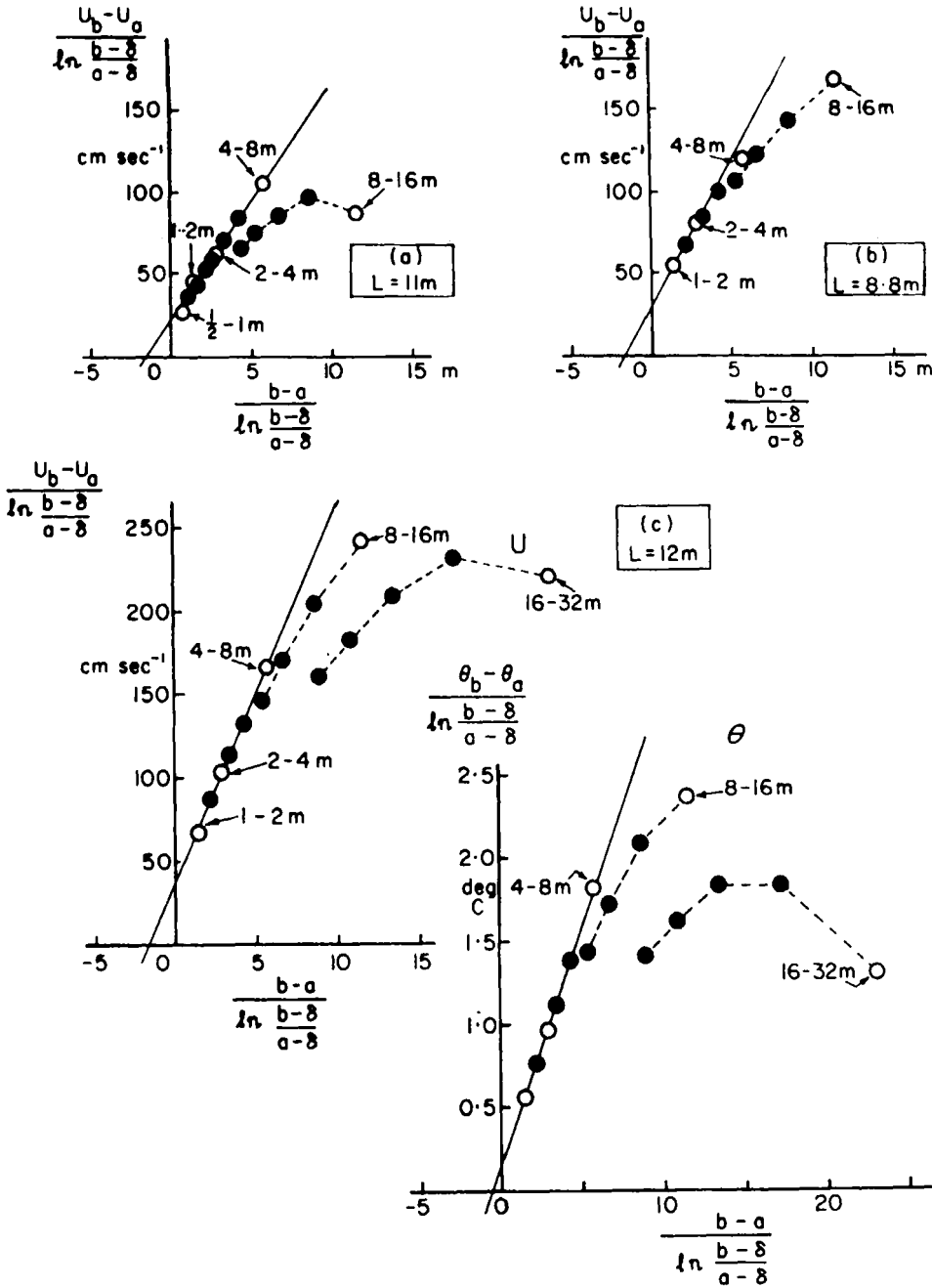


Figure 4. Same form of plotting as in Figs. 2 and 3; examples in stronger stability, showing departure from log-linear form which begins at height about L . Points with 16 m or 32 m as the upper of the pair of heights are shown connected by pecked lines. Australian data

- (a) Kerang 19 February 1962 : 1859.
- (b) Kerang 14 October 1963 : 2047 (mast 2).
- (c) Hay 12 March 1965 : 2019 (U is average of masts 1 and 2).

is expressed by taking an extension of Eq. (1) in the form

$$\frac{\partial U}{\partial z} = \frac{u_*}{kz} (1 + \alpha) \text{ for } z/L \geq 1. \quad (10)$$

With similarity of U and θ profiles (see Section 5), the corresponding extension of Eq. (6) for the θ profile is

$$\frac{\partial \theta}{\partial z} = -\frac{T_*}{kz} (1 + \alpha) \text{ for } z/L \geq 1. \quad (11)$$

Again, from Eqs. (10) and (11) the corresponding extension of Eq. (7) for Ri is

$$Ri = (z/L) (1 + \alpha)^{-1} \text{ for } z/L \geq 1. \quad (12)$$

A departure of the above kind is not generally to be expected in the Johns Hopkins data, as the z/L values reached are not sufficiently large – only in the two most stable runs does L drop to somewhat below the topmost observation height. In point of fact, no departure is found in any of the runs.

The available Australian runs with Ri_2 near to or greater than the critical value 0.2 are reproduced in Fig. 6. In Fig. 6 (a) and (b), with Ri_2 near 0.2, the behaviour of U is essentially similar to that beyond the departure point as illustrated by the open symbols in Fig. 4, with a further upward trend apparent in Fig. 6 (b). The plotted θ data, while too fragmentary to indicate any definite form, do warn that there is probably no systematic relationship between U and θ in individual runs.

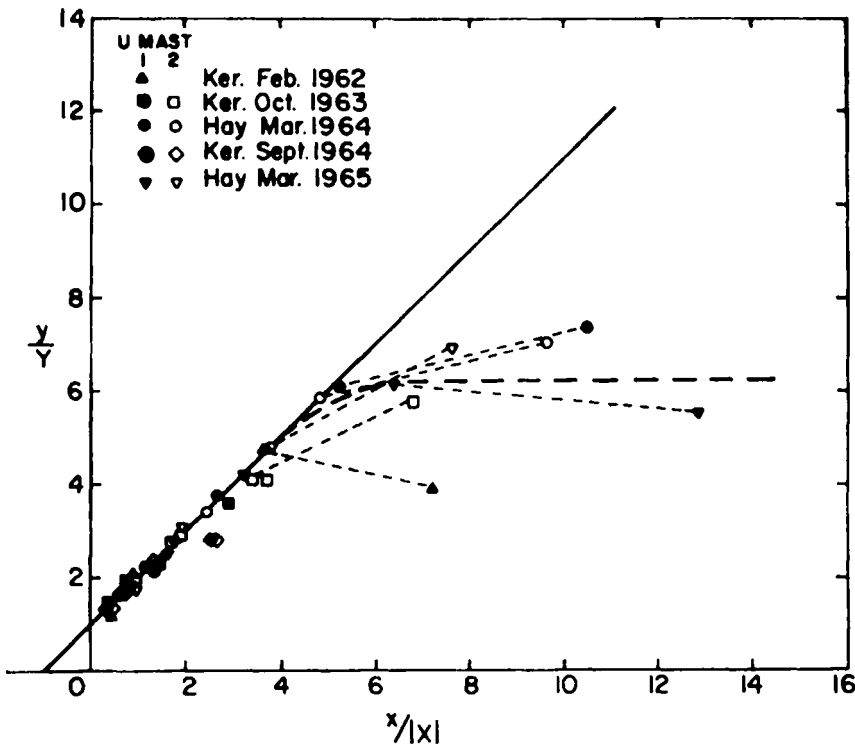


Figure 5. Australian data, wind profiles in stable conditions : runs in which the departure from log-linear form (cf. Fig. 4) occurs. Co-ordinates are quantities x and y defined by Eq. (4b), normalized with respect to the intercepts X and Y in the plots for the individual runs. Points represent adjacent height pairs only. Sloping straight line represents the log-linear form, Eq. (1); broken line represents departure given by Eq. (10); pecked lines where space allows connect points from the same run.

Fig. 6 (c) shows the only available U profile in very strong stability ($Ri_2 = 1.4$). At the lowest heights, there is a suggestion of a compressed version of the type of profile appearing in Fig. 6 (b). The major feature is the final upward trend, which has a linear form, such as might be expected in laminar flow, or, at any rate, with constant average transfer coefficient, if the shearing stress is constant with height. Of course, in such strong stability, it is debatable whether the shearing stress might be independent of height, and, in any case, a great deal of variability of profile form from one occasion to another is to be expected.

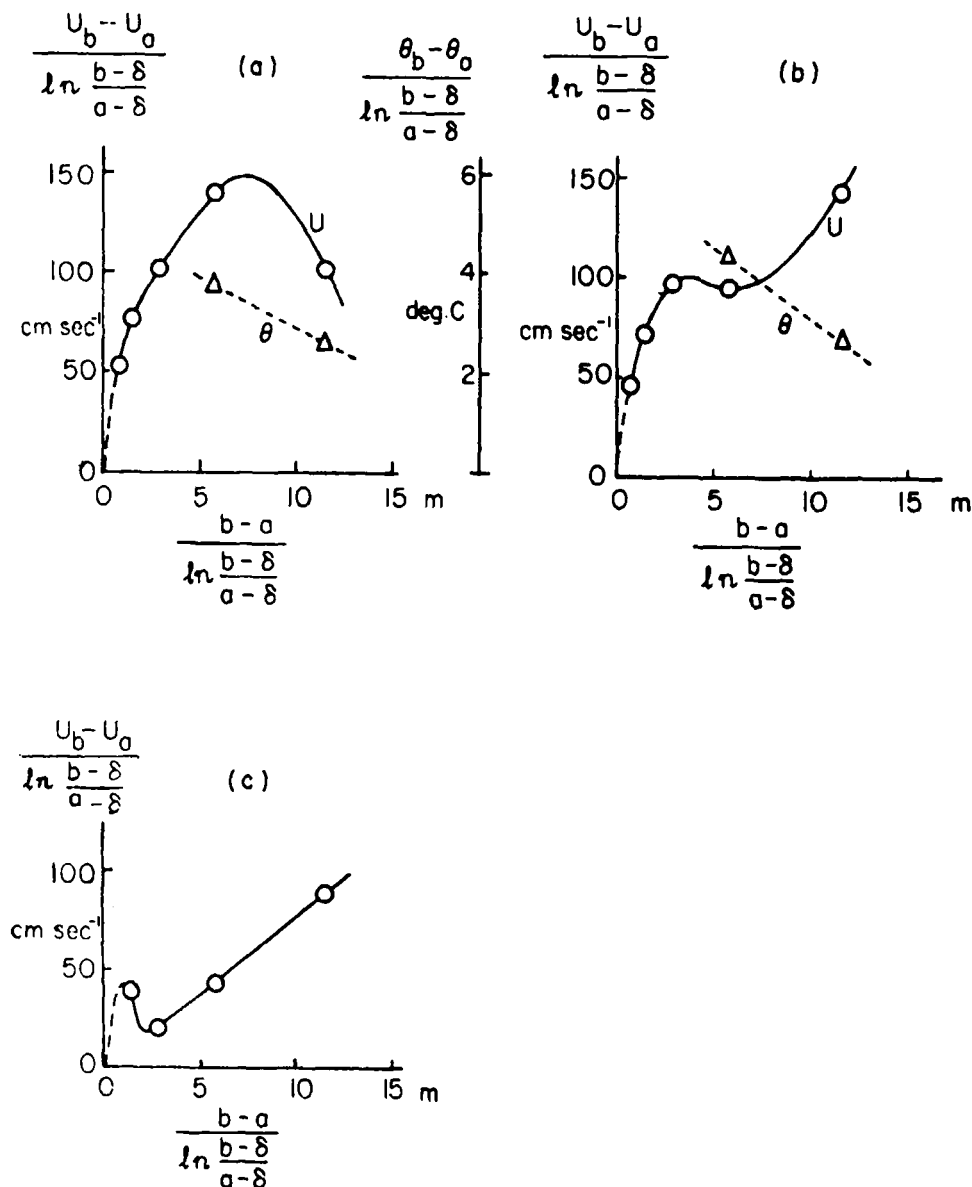


Figure 6. Same form of plotting as in Figs. 2, 3, and 4 : runs in very strong stability, with Ri_2 about 0.2 or greater. Points represent adjacent height pairs only. Australian (Kerang) data

- (a) 17 February 1962 : 2159. $Ri_2 = 0.20$.
- (b) 17 February 1962 : 2230. $Ri_2 = 0.19$.
- (c) 14 October 1963 : 2144 (mast 2). $Ri_2 = 1.40$

(b) Overall consideration of the inversion profile

As an overall test of the proposed inversion wind profile, Fig. 7 shows shape indicator ratios $(U_{6.4} - U_{1.6})/(U_{1.6} - U_{0.4})$ plotted against $Ri_{1.6}$ for the Johns Hopkins data and $(U_{16} - U_4)/(U_4 - U_1)$ plotted against Ri_2 for the Australian data. The approximate agreement of the plotted points with the curves representing Eqs. (1) and (10) with $\alpha \approx 5$, and with $\delta \approx 4$ cm for the J.H. data, is apparent.

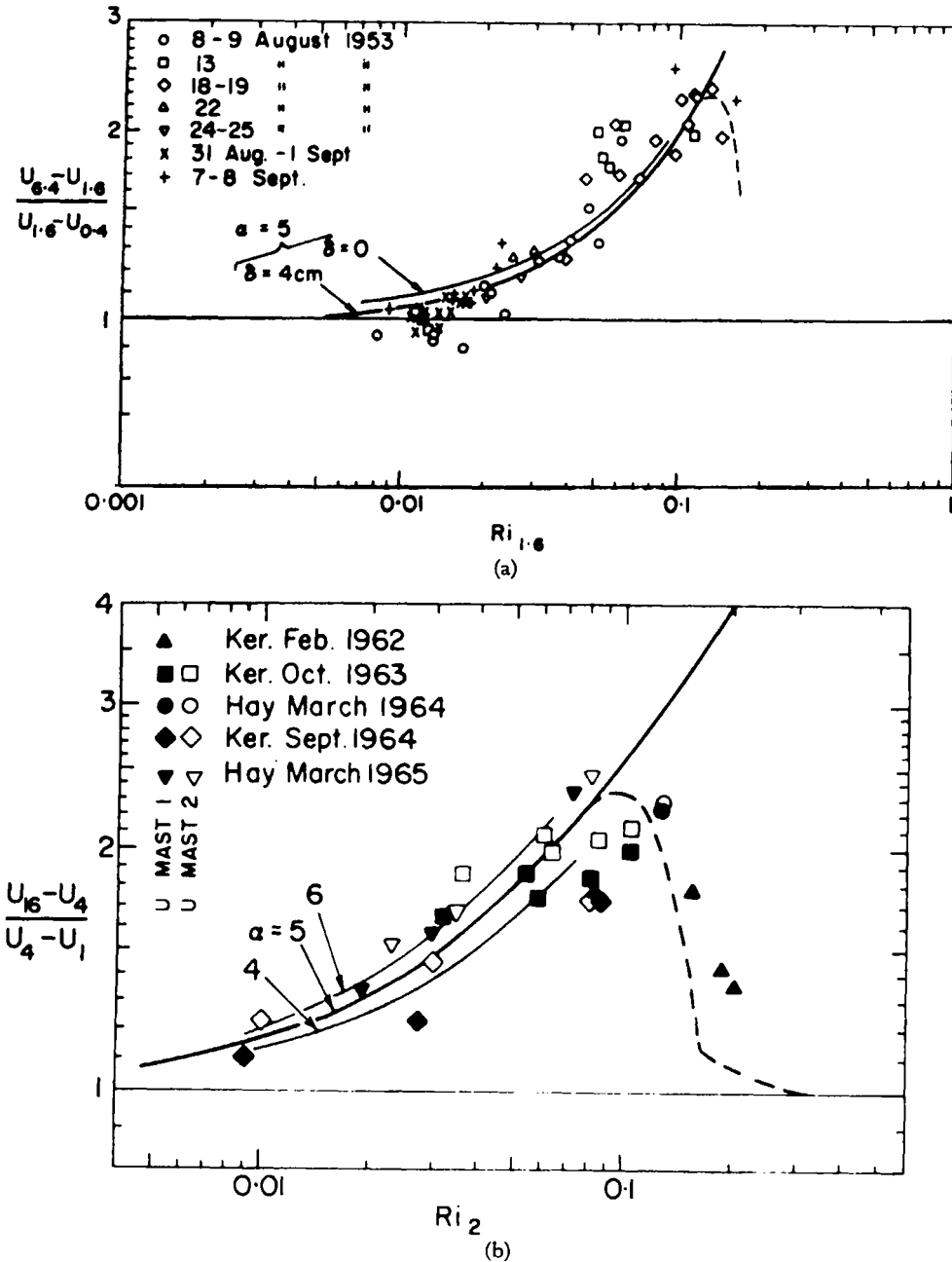


Figure 7. Stable conditions. Wind profile shape indicator ratio, (a) Johns Hopkins 1953 data $(U_{6.4} - U_{1.6})/(U_{1.6} - U_{0.4})$ plotted against $Ri_{1.6}$, and (b) Australian data, $(U_{16} - U_4)/(U_4 - U_1)$ plotted against Ri_2 . Full curves represent log-linear form, Eq. (1), with indicated values of α , and with zero-plane displacement δ as indicated in (a) and zero in (b); broken curves represent departure given by Eq. (10).

In Fig. 8, the behaviour of Ri is illustrated by a plot of Ri_3 against Ri_2 for the Australian data. The evident features are the approach towards a limiting value $1/\alpha$ as in Eq. (7), and then a deviation to continue increasing as in Eq. (12).

It appears in Fig. 8 that the z proportionality of Eq. (12), corresponding to the z^{-1} proportionality of Eqs. (10) and (11), is probably applicable up to $Ri =$ nearly 1, i.e. (see Eq. (12)) up to $z/L \approx 6$. For Ri greater than 1, while a great deal of variability is to be expected in this very strong stability, the one run available in Fig. 8 shows Ri roughly constant (actually decreasing somewhat) with height, which would conform with approximately linear profiles of U and θ on this particular occasion.

5. COMPARISON OF U , θ , AND q : THE RATIOS K_H/K_M AND K_W/K_M

General agreement has not yet been reached concerning the behaviour of K_H/K_M , the ratio of the transfer coefficients for heat and momentum. Findings for near-neutral and stable conditions have been reported by Swinbank (1955), Kondo (1962), McVehil (1964), Gurvich (1965), Mordukhovich and Tsvang (1966), and Zilitinkevich and Chalikov (1968).

With given values of the fluxes, assumed to be constant with height, the dependence of K_H/K_M on z/L , or on Ri , is proportional to that of $(\partial U/\partial z)/(\partial \theta/\partial z)$, which, in turn, is closely represented by $\Delta U/\Delta \theta$. Here we examine the height dependence of $\Delta U/\Delta \theta$, taking the differences Δ between adjacent observation heights and taking z as the geometric mean height for each interval. Similarly, examination of $\Delta U/\Delta q$ indicates the behaviour of K_W/K_M (K_W being the transfer coefficient for water vapour). The relationship between the profiles of U and θ , as obtained below, has been anticipated earlier in the paper to derive the θ profile from a knowledge of the U profile.

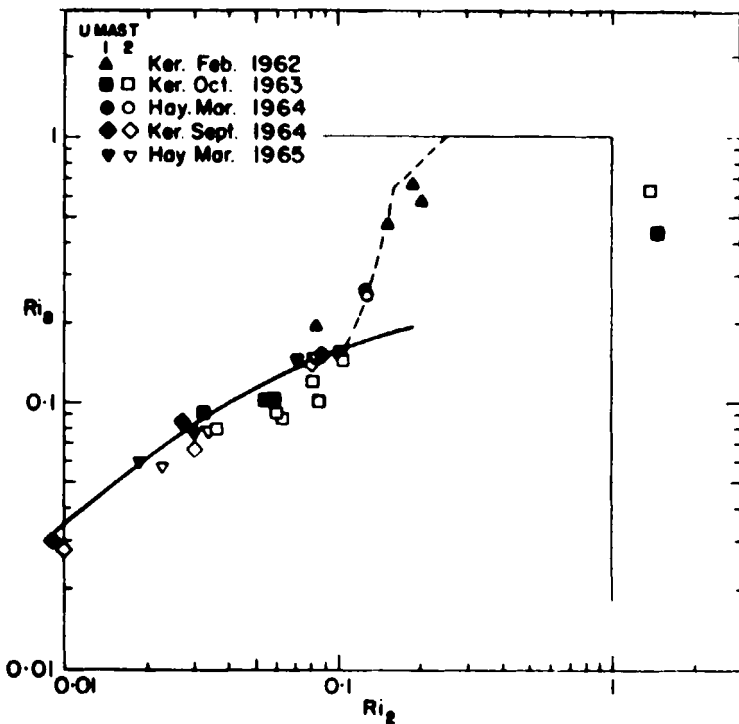


Figure 8. Stable conditions, Australian data: Ri_3 plotted against Ri_2 . Full curve represents log-linear form, Eq. (7), with $\alpha = 5.2$; broken curve represents departure given by Eq. (12).

Runs with very small gradients are excluded. In the Australian data, wind mast 1 is used when there is a choice, and for Δq the crystal hygrometer value is used where possible. Unfortunately, the only suitable Δq values are those from the Australian lapse data – in the J.H. data the experimental scatter of Δq is prohibitive, while in most of the Australian inversion data Δq is either not available or too small for accuracy.

For each of the J.H. runs, the four values of $\Delta U/\Delta \theta$ are taken relative to a reference value $(\Delta U/\Delta \theta)_{0.03}$ at the height where $|Ri| = 0.03$, the latter having been obtained from a smooth curve drawn through the four values plotted against $\log z^*$. Geometric means of the grouped J.H. ratios $(\Delta U/\Delta \theta)/(\Delta U/\Delta \theta)_{0.03}$ are plotted against Ri in Fig. 9 (a) for unstable conditions up to a little beyond the log-linear range, and in Fig. 9 (b) for stable conditions. (Three runs with an apparent temperature anomaly, 11–14 h on 25 August 1953, have been excluded). To incorporate runs in the most strongly stable conditions, the analysis is extended with the reference at $Ri = 0.1$, contributing the point at the right-hand end of Fig. 9 (b); the reference value on either basis, 0.03 or 0.1, is denoted by $(\Delta U/\Delta \theta)_{ref}$.

We conclude from Fig. 9 that there is no significant variation of $\Delta U/\Delta \theta$, nor, therefore, of K_H/K_M , over the ranges of Ri covered. Certainly there is no evidence of any rise of K_H/K_M in unstable conditions as $|Ri|$ increases up to 0.07, nor of any fall of K_H/K_M in stable conditions as Ri increases up to 0.15. It is gratifying to note the good definition of this analysis in spite of the poor temperature sampling, the latter being largely responsible for the experimental variability indicated in Fig. 9.

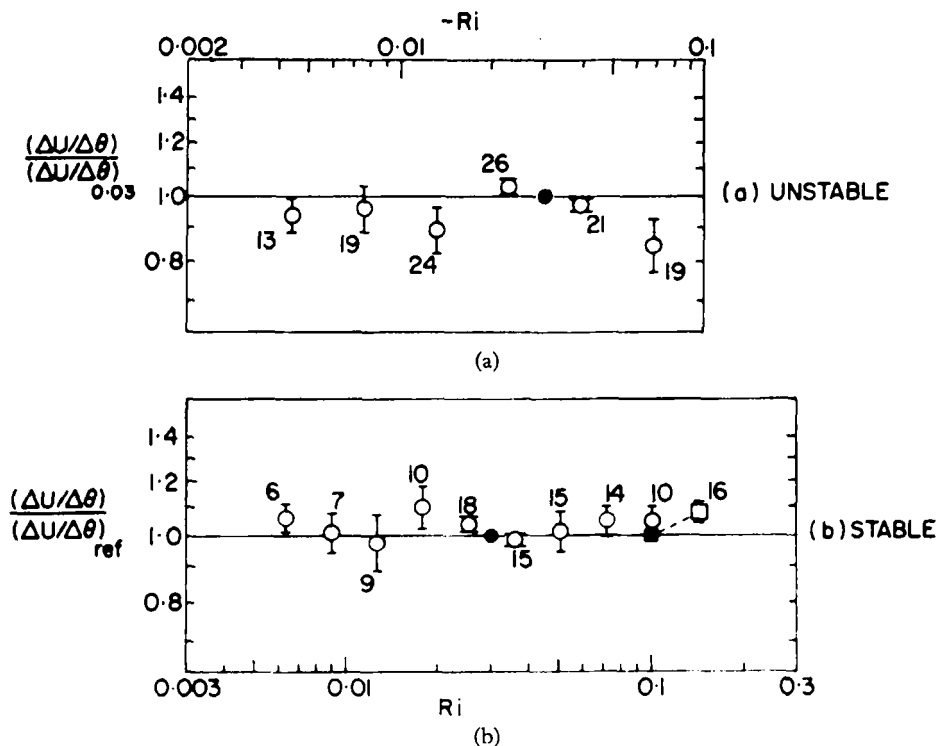


Figure 9. Johns Hopkins 1953 data, (a) unstable and (b) stable conditions. $(\Delta U/\Delta \theta)/(\Delta U/\Delta \theta)_{ref}$ plotted against Ri at the geometric mean height (estimated from $Ri_{1.6}$) as an indicator of the behaviour of K_H/K_M . Δ denotes difference between adjacent observation heights (height ratio 2). Reference value 'ref' is at $|Ri| = 0.03$ (circles) or, in (b) only, at $Ri = 0.1$ (squares). All points are relative to respective reference point shown in black. Number of values and standard error of average are shown at each point.

* Though this interpolation involves very little subjectivity, any possibility of personal bias was avoided by having it carried through by a disinterested assistant; in any case, the relative trend of the four ratios is, of course, not affected by the value of $\Delta U/\Delta \theta)_{0.03}$.

For the Australian data, the runs are taken individually rather than grouped, as there are fewer runs and generally fewer levels of temperature measurement. The reference value in each run is now taken simply as the value of $(\Delta U/\Delta\theta)$ or $(\Delta U/\Delta q)$ for the lowest height interval available. The analysis is shown in Fig. 10.

In the unstable case, Fig. 10 (a), it appears that $\Delta U/|\Delta\theta|$ increases significantly with $z/|L|$, while $\Delta U/|\Delta q|$ does not; the geometric mean of $(\Delta U/\Delta\theta)_{4:16}/(\Delta U/\Delta\theta)_{1:4}$ (where subscripts indicate height intervals) is 1.30, with standard error 5 per cent, while that of $(\Delta U/\Delta q)_{4:16}/(\Delta U/\Delta q)_{1:4}$ is 1.01, with s.e. 4 per cent. This apparent dissimilarity between θ and q seems anomalous, in view of the evidence of Crawford (1965), Swinbank and Dyer (1967), and Dyer (1967) that θ and q have similar form over a wide range of instability.

The suggested interpretation of these results is that the true profiles of θ and q are indeed similar, and that the apparent behaviour of $\Delta U/\Delta q$ is realistic, while that of $\Delta U/\Delta\theta$ is not, for some unknown reason. Unfortunately, it is not possible to test this proposition for different sites, as the only available Australian data at small instability are from Kerang site 2.

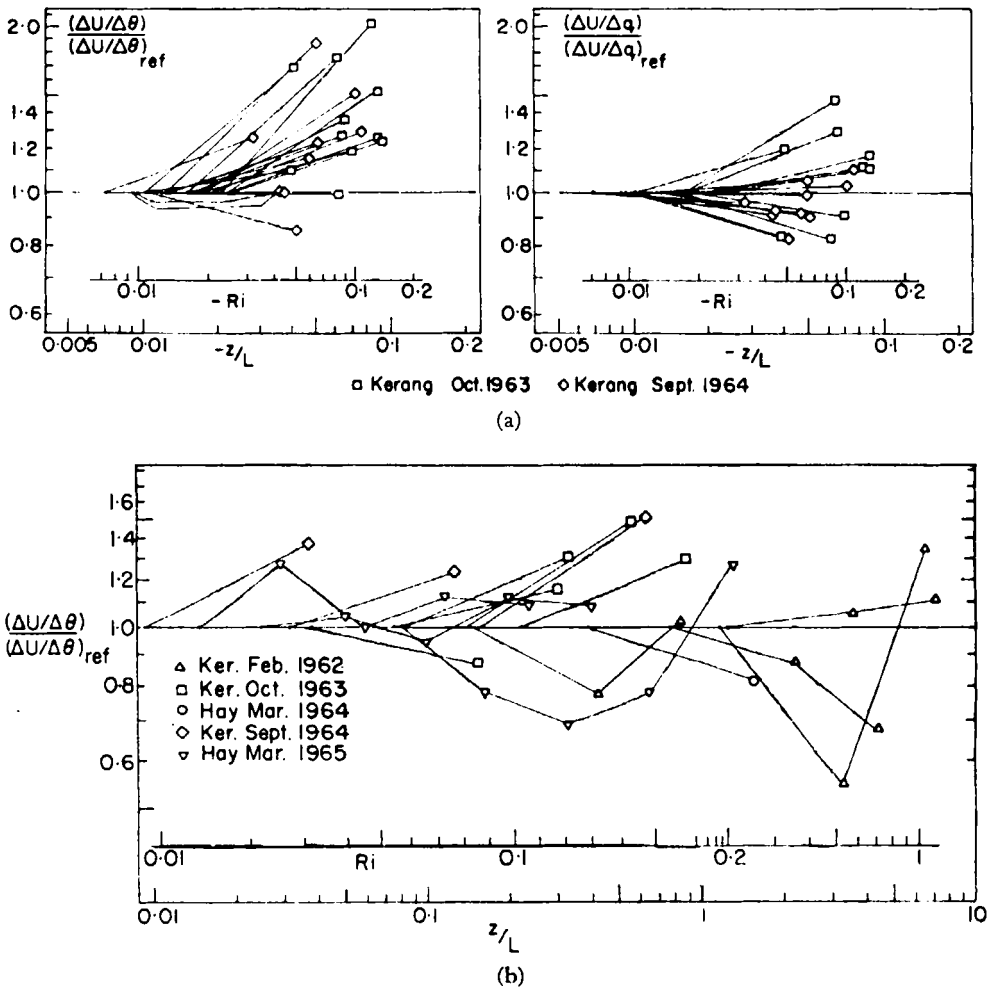


Figure 10. Australian data, (a) unstable conditions (measurements available for heights, 1, 4, and 16 m only), and (b) stable conditions. $(\Delta U/\Delta\theta)/(\Delta U/\Delta\theta)_{ref}$ as an indicator of K_H/K_M , and, in (a) only, $(\Delta U/\Delta q)/(\Delta U/\Delta q)_{ref}$ as an indicator of K_W/K_M , plotted against z/L and Ri at the geometric mean height (estimated from Ri_2). Δ denotes difference between adjacent observation heights. For each run, the reference value is that from the lowest pair of heights available; lines interconnect points from the same run.

Thus, we adopt the conclusion, consistent with that from the J.H. data, that in unstable conditions the profiles of U , θ , and q remain similar, i.e. K_H/K_M and K_W/K_M remain constant, for $z/|L|$ ranging up to about 0.07 ($|Ri|$ ranging up to about 0.1). It should be mentioned that this conclusion is not incompatible with the finding of Swinbank and Dyer (1967, Fig. 1) that, over a wide range of instability, U is not similar to θ and q . For we are dealing here with smaller degrees of instability: if $z/|L|$ at their top height (16 m) is less than 0.07, then their abscissa $|Ri_2|$ is less than about 0.009, in which range they do not indicate any significant difference between the profiles.

For inversion conditions, the conclusion from the Australian data, Fig. 10 (b), is similar to that from the J.H. data, over a more extensive range. It is evident that $\Delta U/\Delta \theta$, and therefore K_H/K_M , have no significant variation – certainly no decrease – as z/L increases up to 1.5, and perhaps up to 6, i.e. as Ri increases up to about 0.2, and perhaps up to about 1.

It will be noted that the conclusions from Figs. 9 and 10 are essentially independent of the exact profile forms used in estimating the z/L or Ri values from $Ri_{1.6}$ or Ri_2 . Any profile anomaly would do no more than cause small relative shifts along the horizontal axis in Figs. 9 and 10.

In strong stability, the constancy of the downward eddy heat flux with height comes into question, as some increase with height may be expected in association with radiation divergence (Funk 1960, 1962). There would then be a corresponding relative increase of $\Delta \theta$ with height, which would appear in Figs. 9 (b) and 10 (b) as a decrease of the ordinate with increasing z/L . Thus, the fact that no such systematic decrease is evident suggests that, at least in these observations, the change of H with height is not large, for Ri values up to near 0.2 and perhaps up to about 1.

Evaluation of the radiation divergence $\text{div } R$ itself is not feasible, as the observations did not include surface temperature; however, Funk's (1960) values of $\text{div } R$ calculated from other observations can be taken as an indication. It is found that, provided $Ri_{1.6}$ or Ri_2 is reasonably below 0.2, the increase of $-H$ would generally be no more than about 2 per cent per doubling of height.* This would have only a small effect in Figs. 9 (b) and 10 (b), which supports the conclusion already suggested. The effect would of course be larger for runs with Ri around 0.2 or greater, as the magnitudes of H would probably be small; while in the extreme of stability, with Ri greater than 1 (U_1 generally less than 1 m s^{-1}), radiation divergence undoubtedly has an important influence on the temperature profile.

Somewhat surprising is the evidence that K_H/K_M remains constant up to fairly high degrees of stability, instead of decreasing on account of buoyancy forces. Presumably this constancy is a result of the high correlation which exists between the turbulent fluctuations of temperature and those of wind speed, as in examples reproduced in Fig. 13 of Priestley (1959) and in Fig. 1 of A.M.; because of this correlation, thermal buoyancy would affect both heat and momentum transfers almost equally. Undoubtedly, the high correlation itself arises simply because the turbulent fluctuations are produced by vertical movements through the mean gradients, in much the same way for both temperature and horizontal speed.

Similarly, as illustrated in Fig. 11, temperature and humidity fluctuations are also observed to be highly correlated in stable conditions (as well as in unstable conditions, as illustrated by Swinbank and Dyer 1967). From this we infer that in stable conditions $K_W = K_H$ and the profiles of θ and q are similar in form.

* Here, it is tentatively assumed that the convergence of H decreases inversely with height, i.e. the increment of H is the same for each doubling of height, since this is roughly the behaviour of $\text{div } R$ in examples calculated by Deacon (1950) and by Funk (1961). For most of the runs quoted here in Table 2, in which $Ri_{1.6}$ or Ri_2 is less than 0.2, H as estimated from the profiles (see Discussion) is in the range -2 to -3.5 mW cm^{-2} . The calculated $\text{div } R$ at a height of 1 m (Funk 1960) ranges from 0.03 to $0.1 \text{ mW cm}^{-2} \text{ m}^{-1}$ in cases with essentially clear sky and wind speeds greater than 1 m s^{-1} . With the convergence of H taken to be somewhat smaller than $\text{div } R$ (Funk 1960), its value as estimated at the geometric mean height between 1 and 2 m indicates that H changes by up to about 2 per cent between these two heights.

Thus, the humidity profile adopted is given by

$$\frac{\partial q}{\partial z} = -\frac{q_*}{kz} \left(1 + \alpha \frac{z}{L}\right) \quad (13)$$

in unstable conditions for $z/|L|$ up to about 0.03 and in stable conditions for z/L up to about 1; and

$$\frac{\partial q}{\partial z} = -\frac{q_*}{kz} (1 + \alpha) \quad (14)$$

(subject to large variability) in stable conditions for z/L greater than about 1. Here q_* is the specific humidity scale defined by equating the vertical flux of water vapour to $\rho u_* q_*$.

As an independent test of the profile forms in the stable case, we can appeal to the evaporation measurements of Crawford (1965) which he plots in dimensionless form as E^* in his Fig. 2. It is found that E^* calculated from the adopted profile forms does in fact agree closely with Crawford's well-defined results for Ri values up to 0.1, and falls within the very large experimental scatter (though not at the mean) for $Ri > 0.1$. Again, the relationships adopted here are consistent with the conclusion of Rider (1954) that $K_W = K_M$ in both unstable and stable conditions, and with that of Harbeck (1967) and Högström (1967) that $K_W = K_M$ in near-neutral conditions.

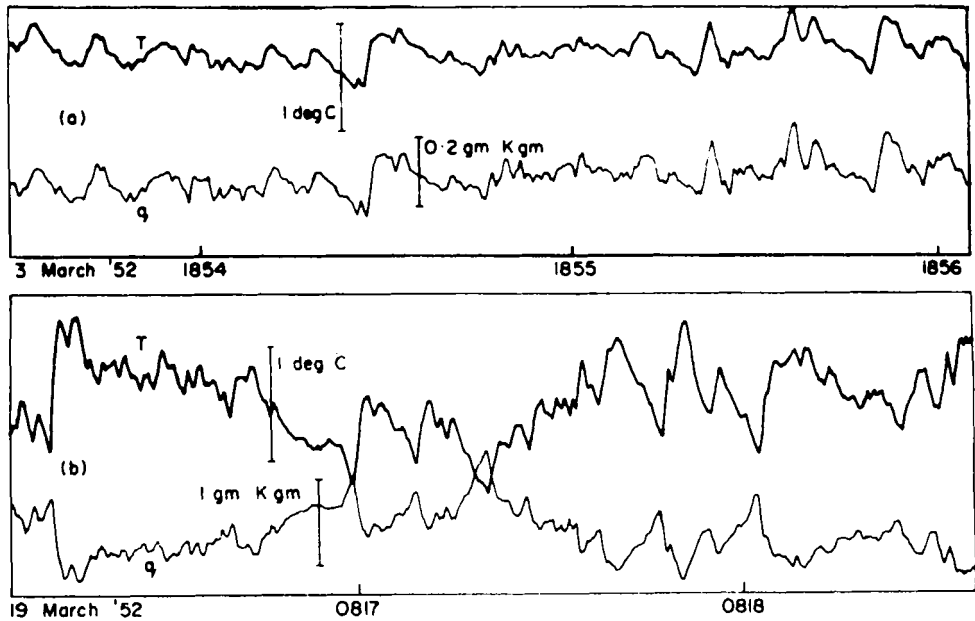


Figure 11. Illustrations of high correlation between fluctuations of temperature T and specific humidity q in stable conditions. Observations at Edithvale, Vic., Australia, by McIlroy (1955) during a series of measurements reported by Swinbank (1955).

- (a) Water vapour flux downwards. 3 March 1952, run 1851-56, height 1.9 m; $Ri_{1.9} = +0.038$. Correlation coefficient $R_{Tq} = +0.91$.
- (b) Water vapour flux upwards. 19 March 1952, run 0815-20, height 1.5 m; $Ri_{1.5} = +0.039$. Correlation coefficient $R_{Tq} = -0.97$.

6. DISCUSSION

In Figs. 1 to 3, the inherent difficulty of evaluating α accurately is evident in the long extrapolation needed to identify the intercept X on the horizontal axis. On the other hand, it is apparent that u_*/k is accurately determined by the intercept on the vertical axis, as only a short extrapolation is needed.

It is interesting to note that, by this method, u_* for individual runs may be evaluated from high-quality wind profile data alone, in the absence of any accurate measurement of temperature gradient or stability parameter. The data must, of course, be within the log-linear range, i.e. in the small range of unstable conditions with $z/|L|$ up to about 0.03, or in the large range of stable conditions with z/L up to about 1.

If T_*/k is also similarly evaluated from the temperature profile, then the heat flux follows immediately from $H = c_p \rho u_* T_*$, and the Obukhov length from $L = -(u_*/k)^2 (g/\theta)^{-1} (T_*/k)^{-1}$. Alternatively, after u_* is determined, the heat flux may be evaluated from $H = -c_p \rho u_*^2 \Delta\theta/\Delta U$, where Δ represents the difference over a height interval which need not be entirely within the log-linear range: $z/|L|$ may safely be up to 0.07 in unstable conditions, and tentatively may, with increasing risk of variability, be up to 1.5 or possibly higher in stable conditions. Similarly, the flux of water vapour, etc. may be evaluated from the measured profile of specific humidity etc. A crude indication of L and thence H can also be obtained from the wind profile alone, by reading off both intercepts and using Eq. (5) with the adopted value of α , provided the data are within the log-linear range.

In the graphical method described in this paper, or in the equivalent regression calculation or a related one such as that of McVehil (1964), both u_*/k and α are evaluated. In contrast are methods which appeal to measured u_* to preassign the value of u_*/k in a regression to evaluate α . In these the evaluation of α is subject to far more serious errors, which arise from quite small errors in u_*/k , as indicated by Taylor (1960) and by Swinbank (1964). This acute sensitivity to u_*/k is also readily apparent graphically – in Figs. 1 to 3, a small forced shift of the intercept u_*/k along the vertical axis would swing the line to produce a comparatively large shift of the intercept on the horizontal axis, hence a large error in α .

Referring now to stable conditions, we note from Eq. (1) that in the log-linear range K_M has the form

$$\begin{aligned} K_M &= kzu_* (1 + \alpha z/L)^{-1} \\ &= kzu_* (1 - \alpha \text{Ri}). \end{aligned}$$

Thus, K_M approaches 0 as Ri approaches the critical value α^{-1} , approximately 0.2, suggesting that this value represents the limit for the régime of fully turbulent flow. However, before this limit is quite reached, a second régime, with quasi-determinate profiles of U and θ , sets in at $z/L \approx 1$, i.e. $\text{Ri} \approx (1 + \alpha)^{-1}$, and extends over perhaps a six-fold height range up to Ri near 1.

Finally, for Ri greater than 1 there is the régime of extreme stability. Here, the one U profile available indicates a linear form; but presumably, in general, the profile forms will be highly variable, with appreciable influence of radiation divergence on the temperature profile, and with probable occurrence of a laminated thermal structure as observed in the sea (Roden 1968, Woods 1968a, b, Cooper and Stommel 1968) and wind drift counterflows like that illustrated by U.S. Weather Bureau (1955, frontispiece).

Turbulence observations have in fact indicated a demarcation between turbulent and smoother flow at a value of Ri around 0.2 or somewhat greater, but do not appear to have indicated any further transition at $\text{Ri} \approx 1$. These are reported by Deacon (1953), Portman, Elder, Ryznar and Noble (1962), Lyons, Panofsky and Wollaston (1964), and Kaimal and Izumi (1965). Okamoto and Webb (paper in preparation) have found that temperature fluctuations at height 2 m are of a turbulent nature when Ri is less than about 0.2, but are variable in behaviour – intermittently quiet with occasional isolated smooth pulses, or wave-like, or turbulent – when Ri is greater than 0.2.

The simplicity of the log-linear profile, extending over a wide range of stable conditions, suggests that there must be a simple governing mechanism, though as yet this has not been elucidated. It is interesting to note that the critical Ri value about 0.2, which is indicated by the present analysis, is just below the critical value $1/4$ which arises from stability theory (see review by Yih 1965).

ACKNOWLEDGMENTS

It is fitting to acknowledge the high quality of the experimental data on which this investigation depends. The writer is grateful to Mr. W. C. Swinbank for making available his unpublished night-time data, which, at his suggestion, are reproduced here for reference (Appendix 1). Appreciation is expressed to Messrs. G. F. Rutter, J. Stevenson, J. Caligari, and Mrs. E. Osborn, for their assistance at various stages of the analysis.

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